Report_Reproducibility_Orujlu

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1 Reproducibility Report

- 1.0.1 Student: Omar Faig Orujlu(03750822)
- 1.0.2 Paper: Associative Memory in Iterated Overparameterized Sigmoid Autoencoders

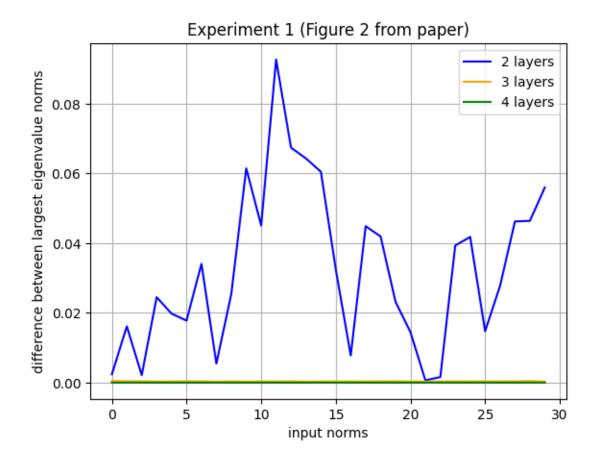
2 Experiment 1 - Single Training Example

In this experiment the same results as in Figure 2 in the paper is reproduced. As stated in the paper the largest (Operator) norm of initial Jacobian for 2 layer sigmoid network is concentrated around 1/2. Also , the initial and final Jacobians are very similar (almost equal) as the number of layers are increased. In the below diagram the results for 3 and 4 layers are the same as in the paper and for 2 layer case the theoretical results are reproduced .

```
[1]: import copy
     import numpy as np
     import torch.nn.functional as F
     import torch
     import torch.nn as nn
     import torchvision
     import matplotlib.pyplot as plt
     from torch import linalg as LA
     from torchsummary import summary
     from torch.nn.utils import weight_norm
     import utils
     from matplotlib.ticker import PercentFormatter
     diff_list_2=[]
     diff_list_3=[]
     diff_list_4=[]
     epochs = 0
     outputs = []
     losses = []
     for k in range(2,5):
             if k == 2:
                 for i in range(1,31):
                     input = utils.sample_input_(i,dim=32)
```

```
model = utils.
→Autoencoder_2_layers_(input_dim=32,hidden_dim=1000)
               optimizer = torch.optim.SGD(params=model.parameters(),lr=1)
               initial state dict = copy.deepcopy(model.state dict())
               loss function = nn.MSELoss()
               while True:
                   epochs+=1
                   reconstructed = model(input)
                   loss = loss_function(reconstructed, input)
                   optimizer.zero_grad()
                   loss.backward()
                   losses.append(loss.detach().numpy())
                   outputs.append((epochs, input, reconstructed))
                   optimizer.step()
                   if loss<1e-7:
                       trained_state_dict = model.state_dict()
                       _,diff_2 = utils.
dcalculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,input)
                       diff_list_2.append(diff_2)
                       break
          k = +1
      if k == 3:
               for i in range(1,31):
                   input = utils.sample_input_(i,dim=32)
                   model = utils.
→Autoencoder 3 layers(input dim=32, hidden dim=1000)
                   optimizer = torch.optim.SGD(params=model.parameters(),lr=1)
                   initial_state_dict = copy.deepcopy(model.state_dict())
                   loss function = nn.MSELoss()
                   while True:
                       epochs+=1
                       reconstructed = model(input)
                       loss = loss_function(reconstructed, input)
                       optimizer.zero_grad()
                       loss.backward()
                       losses.append(loss.detach().numpy())
                       outputs.append((epochs, input, reconstructed))
                       optimizer.step()
                       if loss<1e-7:
                           trained_state_dict = model.state_dict()
                           diff_list_3.append(utils.
Graduate_jacobian_3_layers(initial_state_dict,trained_state_dict,input))
                           break
               k = +1
      if k == 4:
               for i in range(1,31):
```

```
input = utils.sample_input_(i,dim=32)
                    model = utils.
 →Autoencoder_4_layers(input_dim=32,hidden_dim=1000)
                    optimizer = torch.optim.SGD(params=model.parameters(),lr=1)
                    initial_state_dict = copy.deepcopy(model.state_dict())
                    loss function = nn.MSELoss()
                    while True:
                        epochs+=1
                        reconstructed = model(input)
                        loss = loss_function(reconstructed, input)
                        optimizer.zero_grad()
                        loss.backward()
                        losses.append(loss.detach().numpy())
                        outputs.append((epochs, input, reconstructed))
                        optimizer.step()
                        if loss<1e-7:</pre>
                            trained_state_dict = model.state_dict()
                            diff_list_4.append(utils.
 acalculate_jacobian_4_layers(initial_state_dict,trained_state_dict,input))
                            break
                k=+1
plt.plot(diff_list_2,color="blue",label="2 layers")
plt.plot(diff_list_3,color="orange",label="3 layers")
plt.plot(diff_list_4,color="green",label="4 layers")
plt.xlabel('input norms')
plt.ylabel('difference between largest eigenvalue norms')
plt.title('Experiment 1 (Figure 2 from paper)')
\#plt.xlim(-1,32)
plt.grid()
plt.legend()
plt.show()
```



3 Experiment 2 - Multiple Training Example

3.0.1 Linear Region

This experiment is based on Section 5.3 where the eigenvalue distribution in linear region is demonstrated. Here, a 2 layer sigmoid autoencoder is trained with 2,5, and 8 points and these points have input radius of 1. According to lemma 4 of the paper there should be n-1 eigenvalues with norm around 1. This can be seen in the generated(reproduced) diagram, where 10%,40% and 70% of all eigenvalues are near 1. Since there is eigenvalues with norm 1, it can also be concluded that network operates in linear region

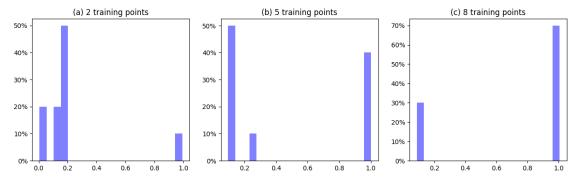
```
[8]: x_2_points = utils.generate_x_training_points(1,10,2)
x_5_points = utils.generate_x_training_points(1,10,5)
x_8_points = utils.generate_x_training_points(1,10,8)
for i in range(0,3):
    if i ==0:
        model = utils.Autoencoder_2_layers_(input_dim=10,hidden_dim=1000)
        optimizer = torch.optim.SGD(params=model.parameters(),lr=3)
        initial_state_dict = copy.deepcopy(model.state_dict())
        loss_function = nn.MSELoss()
```

```
epochs = 0
      outputs = []
      losses = []
      while True:
          epochs+=1
          reconstructed = model(x_2_points)
          loss = loss_function(reconstructed, x_2_points)
        # if epochs%1000==0:
           # print(epochs , ": Current loss:", loss)
          optimizer.zero grad()
          loss.backward()
          losses.append(loss.detach().numpy())
          optimizer.step()
          if loss<1e-7:
              trained_state_dict = model.state_dict()
              eigen_values_p1, = utils.
acalculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_2_points[0])
              eigen_values_p2,_ = utils.
-calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_2_points[1])
                              = torch.cat((eigen_values_p2,eigen_values_p1))
              eigen_values
              eigen_values_normed_2p = torch.abs(eigen_values)
              break
  i+=1
  if i ==1:
      model = utils.Autoencoder_2_layers_(input_dim=10,hidden_dim=1000)
      optimizer = torch.optim.SGD(params=model.parameters(),lr=3)
      initial state dict = copy.deepcopy(model.state dict())
      loss function = nn.MSELoss()
      epochs = 0
      outputs = []
      losses = []
      while True:
          epochs+=1
          reconstructed = model(x 5 points)
          loss = loss_function(reconstructed, x_5_points)
          optimizer.zero_grad()
          loss.backward()
          losses.append(loss.detach().numpy())
          optimizer.step()
          if loss<1e-7:
              trained_state_dict = model.state_dict()
              eigen_values_p5_1,_ = utils.
acalculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_5_points[0])
              eigen_values_p5_2, = utils.
calculate jacobian 2 layers (initial state dict, trained state_dict, x_5_points[1])
```

```
eigen_values_p5_3, = utils.
-calculate jacobian 2 layers (initial state dict, trained state_dict, x_5_points[2])
              eigen_values_p5_4,_ = utils.
-calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_5_points[$])
              eigen_values_p5_5,_ = utils.
→calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_5_points[4])
              eigen_values_p5
                                 = torch.
-cat((eigen_values_p5_1,eigen_values_p5_2,eigen_values_p5_3,eigen_values_p5_4,eigen_values_p
              eigen_values_normed_5p = torch.abs(eigen_values_p5)
              break
  i+=1
  if i ==2:
      model = utils.Autoencoder_2_layers_(input_dim=10,hidden_dim=1000)
      optimizer = torch.optim.SGD(params=model.parameters(),lr=3)
      initial_state_dict = copy.deepcopy(model.state_dict())
      loss_function = nn.MSELoss()
      epochs = 0
      outputs = []
      losses = []
      while True:
          epochs+=1
          reconstructed = model(x_8_points)
          loss = loss_function(reconstructed, x_8_points)
          optimizer.zero_grad()
          loss.backward()
          losses.append(loss.detach().numpy())
          outputs.append((epochs, x_8_points, reconstructed))
           optimizer.step()
           if loss<1e-7:
              trained_state_dict = model.state_dict()
              eigen_values_p8_1,_ = utils.
→calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[0])
              eigen values p8 2, = utils.
→calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[1])
              eigen_values_p8_3,_ = utils.
→calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[2])
              eigen_values_p8_4,_ = utils.
-calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[$])
              eigen_values_p8_5, = utils.
calculate_jacobian_2 layers_(initial_state_dict,trained_state_dict,x_8_points[4])
              eigen_values_p8_6,_ = utils.
-calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[$])
              eigen_values_p8_7,_ = utils.
acalculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[$])
              eigen values p8 8, = utils.
acalculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_8_points[7])
```

```
eigen_values_p8
                                   = torch.
 -cat((eigen_values_p8_1,eigen_values_p8_2,eigen_values_p8_3,eigen_values_p8_4,eigen_values_p
                eigen_values_normed_8p = torch.abs(eigen_values_p8)
                break
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(15, 4))
axes[0].hist(eigen_values_normed_2p,weights=np.
 ones(len(eigen_values_normed_2p)) / len(eigen_values_normed_2p), bins=20,

color='blue', alpha=0.5)
axes[1].hist(eigen_values_normed_5p,weights=np.
 ones(len(eigen_values_normed_5p)) / len(eigen_values_normed_5p), bins=20, ∪
 ⇔color='blue', alpha=0.5)
axes[2].hist(eigen_values_normed_8p,weights=np.
 ones(len(eigen_values_normed_8p)) / len(eigen_values_normed_8p), bins=20,
 ⇔color='blue', alpha=0.5)
#plt.hist(eigens, weights=np.ones(len(eigens)) / len(eigens),bins=70)
#plt.hist(eig, weights=np.ones(len(eig)) / len(eig),bins=70)
axes[0].yaxis.set_major_formatter(PercentFormatter(1))
axes[1].yaxis.set_major_formatter(PercentFormatter(1))
axes[2].yaxis.set_major_formatter(PercentFormatter(1))
# Set titles for each subplot
axes[0].set_title('(a) 2 training points')
axes[1].set_title('(b) 5 training points')
axes[2].set_title('(c) 8 training points')
# Set common y-axis label
fig.text(0.3, -0.05, 'Eigenvalue distribution of 2 - layer sigmoid network_
 strained with input dimnesion 10', va='center', rotation='horizontal')
plt.show()
```



Eigenvalue distribution of 2 - layer sigmoid network trained with input dimnesion 10

3.1 Beyond Linear Region

Here the results from Figure 3 is reproduced. However, in the paper authors have experimented with 5,20, and 40 training points with hidden layer size from 1000 to 1000000. In the figure below, the results for 2,5, and 10 points for hidden layers size 1000 and 10000 are depicted. That's why results could not be reproduced exactly, however, similar behavior can be observed. Note: I tried to run the experiment with 20 training points but it took 30+ hours(for input norm 1.0).

```
[5]: hidden_layers=[1000,10000]
     results 1={}
     results_2={}
     results_3={}
     for t in range(0,3):
         if t==0:
             epochs = 0
             outputs = []
             losses = []
             for k in range(0,len(hidden_layers)):
                 largest_eigen_list=[]
                 for i in range (3,53):
                         x_2-points = utils.
      generate x training points(i,32,2)#increase i too hundred and generate
      \rightarrowpoints at i/2 norms
                         model = utils.
      -Autoencoder_2_layers_(input_dim=32, hidden_dim=hidden_layers[k])
                         optimizer = torch.optim.SGD(params=model.parameters(),lr=5)
                         initial_state_dict = copy.deepcopy(model.state_dict())
                         loss_function = nn.MSELoss()
                         epochs = 0
                         outputs = []
                         losses = []
                         while True:
                             epochs+=1
                             reconstructed = model(x 2 points)
                             loss = loss_function(reconstructed, x_2_points)
                             optimizer.zero_grad()
                             loss.backward()
                             losses.append(loss.detach().numpy())
                             optimizer.step()
                             if loss<1e-7:#should be 1e-7 according to paper
                                  trained_state_dict = model.state_dict()
                                  eigen_values_p3,_ = utils.
      -calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_2_points[2])
                                  larg eigen=torch.max(torch.abs(eigen values p3))
                                  largest_eigen_list.append(larg_eigen)
                                  break
```

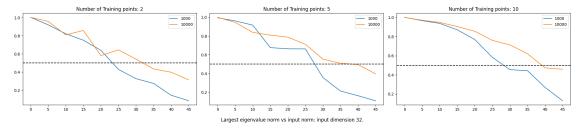
```
results_1[k]=largest_eigen_list
  if t==1:
      epochs = 0
      outputs = []
      losses = []
      for k in range(0,len(hidden_layers)):
           largest_eigen_list=[]
           for i in range (3,53):
                   x_5_points = utils.
⇒generate_x_training_points(i,32,5)#increase i too hundred and generate_
\rightarrow points at i/2 norms
                   model = utils.
-Autoencoder_2_layers_(input_dim=32, hidden_dim=hidden_layers[k])
                   optimizer = torch.optim.SGD(params=model.parameters(),lr=5)
                   initial_state_dict = copy.deepcopy(model.state_dict())
                   loss_function = nn.MSELoss()
                   epochs = 0
                   outputs = []
                   losses = []
                   while True:
                       epochs+=1
                       reconstructed = model(x_5_points)
                       loss = loss_function(reconstructed, x_5_points)
                       optimizer.zero_grad()
                       loss.backward()
                       losses.append(loss.detach().numpy())
                       optimizer.step()
                       if loss<1e-7: #should be 1e-7 according to paper
                           trained_state_dict = model.state_dict()
                           eigen_values_p3,_ = utils.
→calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_5_points[2])
                           larg_eigen=torch.max(torch.abs(eigen_values_p3))
                           largest_eigen_list.append(larg_eigen)
                           break
           results_2[k]=largest_eigen_list
  if t==2:
      epochs = 0
      outputs = []
      losses = []
      for k in range(0,len(hidden_layers)):
           largest_eigen_list=[]
           for i in range (3,53):
                   x_10_points = utils.
⇒generate_x_training_points(i,32,10) #increase i too hundred and generate_
\rightarrowpoints at i/2 norms
                   model = utils.
Autoencoder_2_layers_(input_dim=32,hidden_dim=hidden_layers[k])
```

```
optimizer = torch.optim.SGD(params=model.parameters(),lr=5)
                    initial_state_dict = copy.deepcopy(model.state_dict())
                    loss_function = nn.MSELoss()
                    epochs = 0
                    outputs = []
                    losses = []
                    while True:
                        epochs+=1
                        reconstructed = model(x 10 points)
                        loss = loss_function(reconstructed, x_10_points)
                        optimizer.zero grad()
                        loss.backward()
                        losses.append(loss.detach().numpy())
                        optimizer.step()
                        if loss<1e-7:#should be 1e-7 according to paper
                            trained_state_dict = model.state_dict()
                            eigen_values_p3, = utils.
 acalculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_10_points[2])
                            larg eigen=torch.max(torch.abs(eigen values p3))
                            largest_eigen_list.append(larg_eigen)
                            break
            results_3[k]=largest_eigen_list
x_values = list(range(0, len(results_1[1]), 5))
fig, axs = plt.subplots(1, 3, figsize=(20, 4))
# Plotting the line graphs for each dictionary
axs[0].plot(x_values, results_1[0][::5], label='1000')
axs[0].plot(x_values, results_1[1][::5], label='10000')
axs[0].set_title('Number of Training points: 2')
axs[0].set_xticks(x_values)
axs[0].legend()
axs[1].plot(x values, results 2[0][::5], label='1000')
axs[1].plot(x_values, results_2[1][::5], label='10000')
axs[1].set_title('Number of Training points: 5')
axs[1].set_xticks(x_values)
axs[1].legend()
axs[2].plot(x_values, results_3[0][::5], label='1000')
axs[2].plot(x_values, results_3[1][::5], label='10000')
axs[2].set_title('Number of Training points: 10')
axs[2].set_xticks(x_values)
axs[2].legend()
# Adding the horizontal line to each diagram
axs[0].axhline(0.5, linestyle='--', color='black')
```

```
axs[1].axhline(0.5, linestyle='--', color='black')
axs[2].axhline(0.5, linestyle='--', color='black')

fig.text(0.5, -0.05, 'Largest eigenvalue norm vs input norm: input dimension 32.
    ', ha='center', fontsize=12)

plt.tight_layout()
plt.show()
```



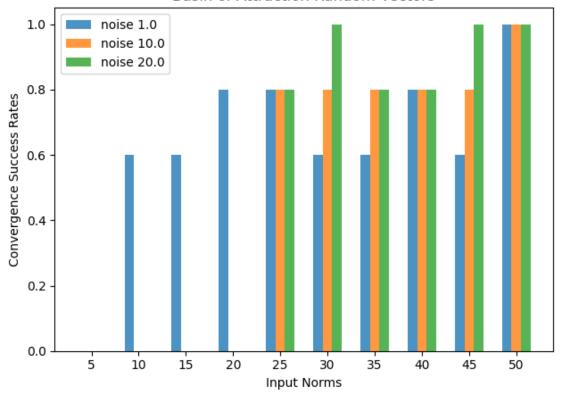
4 Experiment 4 - Basin of Attraction

Here the results from section 5.4 are reproduced. 5 training points with dimension 32 are used for training the model.. As stated in the paper, attractor formulation in small norms fails, this can also be observed in the following diagrams.

```
[6]: epochs = 0
     outputs = []
     losses = []
     input_norms = list(range(5, 51, 5))
     std_list = [1, 10, 20]
     conver = {}
     for t in std_list:
         for i in input_norms:
             input = utils.generate_x_training_points(i, 32, 5)
             model = utils.Autoencoder_2_layers_(input_dim=32, hidden_dim=10000)
             optimizer = torch.optim.SGD(params=model.parameters(), lr=3)
             loss_function = nn.MSELoss()
             while True:
                 epochs += 1
                 reconstructed = model(input)
                 loss = loss_function(reconstructed, input)
                 optimizer.zero_grad()
                 loss.backward()
                 losses.append(loss.detach().numpy())
                 #outputs.append((epochs, input, reconstructed))
                 optimizer.step()
                 if loss < 1e-7:
```

```
noisy = input + np.sqrt(t) * torch.randn_like(input)
                #print("original input",input)
                #print("original noisy output", noisy)
                for k in range(0, 51):
                    noisy = model(noisy)
                    #noisy = noisy result
                    # print("noisy output after iteration no ",k, "noisys",noisy)
                    loss = loss function(noisy, input)
                    #print("loss_ of convergence", loss_, "num of iteration", ___
 \hookrightarrow k, "norm", i, "noise", t)
                    if k==50 and not (loss_<1e-2):
                        convergence_rate = 0
                         conver.setdefault(t, []).append(convergence_rate)
                         #print("converged", 0 , "after iteration no ", k, "norm_
 →of vector", i, "convergence rate", convergence_rate, "noise", t)
                        break
                    if loss_ < 1e-2 :</pre>
                        convergence = 0
                         convergence_rate=0
                         #c#onverged_samples = torch.sum(torch.all(torch.
 \hookrightarrow abs(input - noisy) < 0.1, dim=1)).item()
                         #convergence rate = converged samples / input.shape[0]
                        if loss_function(noisy[0], input[0]) < 0.01:
 ⇔convergence += 1
                        if loss_function(noisy[1], input[1]) < 0.01:
 ⇔convergence += 1
                        if loss_function(noisy[2], input[2]) < 0.01:
 ⇔convergence += 1
                        if loss_function(noisy[3], input[3]) < 0.01:
 ⇔convergence += 1
                        if loss_function(noisy[4], input[4]) < 0.01:
 ⇔convergence += 1
                        convergence_rate = convergence / len(input)
                        conver.setdefault(t, []).append(convergence_rate)
                         \#print("converged", convergence, "after iteration no ", 
 •k, "norm of vector", i, "convergence rate", convergence rate, "noise", t)
                        break
                break
x_{labels} = list(range(5, 51, 5))
data1 = conver[1]
data2 = conver[10]
data3 = conver[20]
# Plotting
```

Basin of Attraction Random Vectors

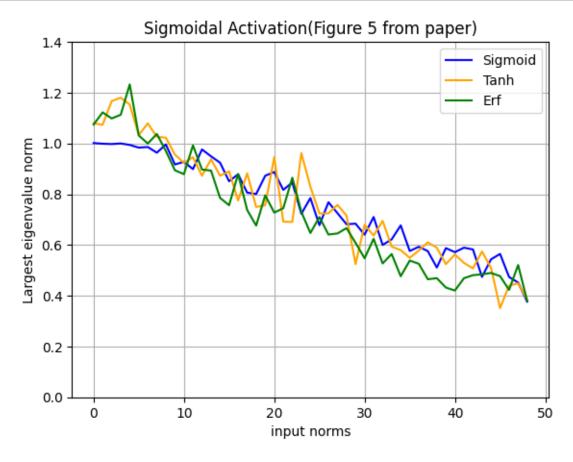


5 Experiment 5 - Sigmoidal Activation

Here the results from Section 5.5 are reproduced. A two layer sigmoid, tanh and erf autoencoder are trained. However, here sigmoid network has 10 training points and others 20. Results are very similar to the ones in figure 5. in the paper and as it is stated in paper, erf and tanh have eigenalues bigger than 1.0 at small input norms.

```
[7]: largest eigen list sigmoid=[]
     largest_eigen_list_tanh=[]
     largest_eigen_list_erf=[]
     epochs = 0
     outputs = []
     losses = []
     for i in range (2,51):
                 x_5_points = utils.generate_x_training_points(i,32,20)
                 model = utils.
      Autoencoder_2_layers_erf(input_dim=32,hidden_dim=10000)
                 optimizer = torch.optim.SGD(params=model.parameters(),lr=5)
                 initial state dict = copy.deepcopy(model.state dict())
                 loss function = nn.MSELoss()
                 epochs = 0
                 outputs = []
                 losses = ∏
                 while True:
                     epochs+=1
                     reconstructed = model(x_5_points)
                     loss = loss_function(reconstructed, x_5_points)
                     optimizer.zero_grad()
                     loss.backward()
                     losses.append(loss.detach().numpy())
                     optimizer.step()
                     if loss<1e-7:
                         trained state dict = model.state dict()
                         eigen_values_p3,_ = utils.
      -calculate_jacobian_2_layers_erf(initial_state_dict,trained_state_dict,x_5_points[2])
                         larg_eigen=torch.max(torch.abs(eigen_values_p3))
                         largest_eigen_list_erf.append(larg_eigen)
                         break
     for i in range (2,51):
                 x_5_points = utils.generate_x_training_points(i,32,10)
                 model = utils.Autoencoder_2_layers_(input_dim=32,hidden_dim=10000)
                 optimizer = torch.optim.SGD(params=model.parameters(),lr=5)
                 initial_state_dict = copy.deepcopy(model.state_dict())
                 loss function = nn.MSELoss()
                 epochs = 0
```

```
outputs = []
            losses = []
            while True:
                epochs+=1
                reconstructed = model(x_5_points)
                loss = loss_function(reconstructed, x_5_points)
                optimizer.zero_grad()
                loss.backward()
                losses.append(loss.detach().numpy())
                optimizer.step()
                if loss<1e-7:
                    trained_state_dict = model.state_dict()
                    eigen_values_p3, = utils.
 -calculate_jacobian_2_layers_(initial_state_dict,trained_state_dict,x_5_points[2])
                    larg_eigen=torch.max(torch.abs(eigen_values_p3))
                    largest_eigen_list_sigmoid.append(larg_eigen)
                    break
for i in range (2,51):
            x_5_points = utils.generate_x_training_points(i,32,20)
            model = utils.
 Autoencoder_2_layers_tanh(input_dim=32,hidden_dim=10000)
            optimizer = torch.optim.SGD(params=model.parameters(),lr=5)
            initial_state_dict = copy.deepcopy(model.state_dict())
            loss_function = nn.MSELoss()
            epochs = 0
            outputs = []
            losses = ∏
            while True:
                epochs+=1
                reconstructed = model(x_5_points)
                loss = loss_function(reconstructed, x_5_points)
                optimizer.zero_grad()
                loss.backward()
                losses.append(loss.detach().numpy())
                optimizer.step()
                if loss<1e-7:
                    trained_state_dict = model.state_dict()
                    eigen_values_p3,_ = utils.
 acalculate_jacobian_2_layers_tanh(initial_state_dict,trained_state_dict,x_5_points[2])
                    larg_eigen=torch.max(torch.abs(eigen_values_p3))
                    largest_eigen_list_tanh.append(larg_eigen)
                    break
plt.plot(largest_eigen_list_sigmoid,color="blue",label="Sigmoid")
plt.plot(largest_eigen_list_tanh,color="orange",label="Tanh")
plt.plot(largest_eigen_list_erf,color="green",label="Erf")
```



6 Possible Extension

In the paper , authors argue that same/similar results can be obtained if tanh/erf functions are used for above experiments. I want to repeat experiments 1-3 with tanh and erf activation functions.

[]: