

Solving the Nurse Scheduling Problem using Differential Evolution

1-202000605

عمر فوزي محمد صالح

2-202000529

عبد الرحمن محمد عبد الوهاب محمد عفيفي

3-202000441

شهاب الدين مختار السعيد زياده

4-202000539

عبد الرحمن معوض عبد العزيز الجوهري

5-202000090

احمد موسي محمد

6-202000086

احمد مصطفى توفيق محمد

7-202000264

حسام أبو الحسن عمر بدر



Introduction and Overview

Project Idea

The objective of this model is to maximize the percentage of time that nurses have the same specialty required by the surgeries and also maximize nurse satisfaction of shift plan.

Furthermore, an assignment model developed using mathematical programming uses a chosen shift plan as an input and aim to assign each specific nurse to the shifts described by that plan.

The act of assigning each nurse to a specific shift for each day of a scheduling horizon, while ensuring to fulfill the demand of the operating rooms (ORs) is very time consuming.

We discuss how the differential evolution generates multiple alternative shift plans, defined as the number of nurses required to work each shift.

In this paper we propose an approach that utilizes differential evolution for solving complex nurse scheduling problems.

Current methods used in practice often develop solutions that are suboptimal, resulting in low nurse utilization, low nurse satisfaction, patient delays, or overtime pay for nurses.

Nurse scheduling is a type of manpower allocation problem that tries to satisfy hospital managers' objectives and nurses' preferences as much as possible by generating fair shift schedules. This paper presents a nurse scheduling problem based on a real case study, and proposes two metaheuristics - a differential evolution algorithm (DE) and a greedy randomised adaptive search procedure (GRASP) - to solve it. To investigate the efficiency of the proposed algorithms, two problems are solved. Furthermore, some comparison metrics are applied to examine the reliability of the proposed algorithms. The computational results in this paper show that the proposed DE outperforms the GRASP.

Similar applications in the market

1) Eschedule

Medical centers turn to Eschedule as a nurse scheduling software because of its time sensitivity.

Eschedule lets you automate scheduling, company wide-training, and is suitable for most web-enabled devices.

You can even receive notifications by email or SMS

The screenshot displays the Eschedule web application interface. At the top, the logo "eschedule" is visible with the tagline "your time is our essence". A navigation bar includes links for Home, Shift Board, Calendar, eSchedules, Employees, Absences, Reports, and Settings. A user notification in the top right corner reads: "Howdy Dan! You have 52 new messages".

The main interface is divided into several sections:

- Left Sidebar:** Contains a calendar for March 2014, a dropdown for "eSchedule: Location: 001 - Company | Sc", a "Position:" dropdown set to "Show All", and a "Show Time Off" checkbox. Below these are icons for Pending, Approved, Warnings, Time Off, and Holiday.
- Center Panel:** Displays a grid of employee schedules for the week of March 2nd to 8th, 2014. The grid is organized by employee rows and date columns. Employees listed include Bob Steele, Dan Brown, Gary Gerr, Harold Cho, and Jeff Smith. Each cell in the grid shows the employee's name, position, and shift details (e.g., "10a-6p001 - Company Associate").
- Right Panel:** Contains a "Today" button and a "Mar. 2 — 8, 2014" date range selector, along with "Day" and "Week" view toggles.

2)Shiftboard

Schedule Flex is a nurse scheduling software that assists you in sharing information company-wide, updates workers when there are open shifts, and has access via mobile devices.

Created by Shift board, Schedule Flex is designed with healthcare in mind. Even flexible scheduling is available.

shiftboard. Find People

Shifts

DATE RANGE: VIEW:

October 31, 2020 - November 6, 2020

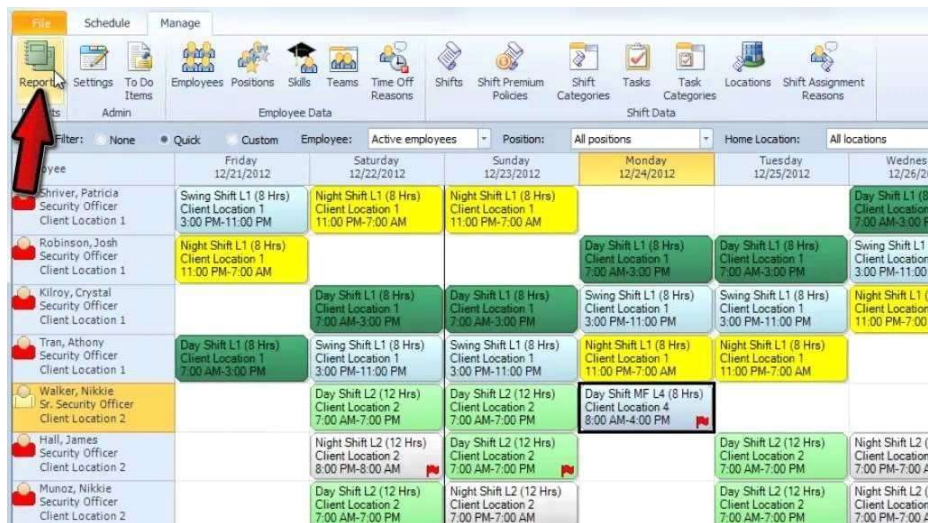
Sat 31	Sun 01	Mon 02	Tue 03	Wed 04	Thu 05
RN 7am - 1pm ELLY THORPE	RN 7am - 1pm MICHAEL HARRIS	CNA 3pm - 5pm PAUL ALBION	RN 7am - 1pm TIM DAVIS	CNA 3pm - 5pm ERIN WILSON	RN 7am - 1pm STEPH CHANG
RN 7am - 1pm CAROL KURTIN	RN 7am - 1pm CAROLYN THOMAS	CNA 3pm - 5pm LYNN ELLIS	RN 7am - 1pm SARAH DUFFY	CNA 3pm - 5pm ALEXIA LEE	RN 7am - 1pm CEDRIC WIS
RN 7am - 1pm DANIEL TURNER	RN 7am - 1pm DANIEL TURNER	CMT 5pm - 9pm	RN 7am - 1pm CECELIE WIS	CMT 5pm - 9pm	RN 7am - 1pm DEBORAH HILL
			CMT 5pm - 9pm		CMT 5pm - 9pm

3)Snap Schedule

Snap Schedule is an online nurse scheduling software designed to make scheduling shifts easy.

Snap Schedule saves you from double-booking shifts and increases productivity.

Working with pen and paper makes it hard to keep on track of schedules. The staff has a clear view of their working week.



The screenshot displays the Snap Schedule software interface. At the top, there are three main tabs: 'File', 'Schedule', and 'Manage'. Below these, a navigation bar contains various icons for different functions. A red arrow points to the 'Reports' icon. The main area shows a weekly schedule grid for several employees. The grid columns represent days of the week (Friday 12/21/2012 to Wednesday 12/26/2012). The rows represent individual employees. Each cell in the grid shows the shift assignment, including the shift type (e.g., Swing Shift L1, Night Shift L1, Day Shift L1), duration (e.g., 8 Hrs), and client location (e.g., Client Location 1). The shifts are color-coded: yellow for Night Shift, green for Day Shift, and blue for Swing Shift. A red flag icon is visible in the cell for Walker, Nikkie on Monday 12/24/2012, indicating a conflict or issue with the schedule.


Employee	Friday 12/21/2012	Saturday 12/22/2012	Sunday 12/23/2012	Monday 12/24/2012	Tuesday 12/25/2012	Wednesday 12/26/2012
Shriver, Patricia Security Officer Client Location 1	Swing Shift L1 (8 Hrs) Client Location 1 3:00 PM-11:00 PM	Night Shift L1 (8 Hrs) Client Location 1 11:00 PM-7:00 AM	Night Shift L1 (8 Hrs) Client Location 1 11:00 PM-7:00 AM			Day Shift L1 (8 Hrs) Client Location 1 7:00 AM-3:00 PM
Robinson, Josh Security Officer Client Location 1	Night Shift L1 (8 Hrs) Client Location 1 11:00 PM-7:00 AM			Day Shift L1 (8 Hrs) Client Location 1 7:00 AM-3:00 PM	Day Shift L1 (8 Hrs) Client Location 1 7:00 AM-3:00 PM	Swing Shift L1 (8 Hrs) Client Location 1 3:00 PM-11:00 PM
Kilroy, Crystal Security Officer Client Location 1		Day Shift L1 (8 Hrs) Client Location 1 7:00 AM-3:00 PM	Day Shift L1 (8 Hrs) Client Location 1 7:00 AM-3:00 PM	Swing Shift L1 (8 Hrs) Client Location 1 3:00 PM-11:00 PM	Swing Shift L1 (8 Hrs) Client Location 1 3:00 PM-11:00 PM	Night Shift L1 (8 Hrs) Client Location 1 11:00 PM-7:00 AM
Tran, Anthony Security Officer Client Location 1	Day Shift L1 (8 Hrs) Client Location 1 7:00 AM-3:00 PM	Swing Shift L1 (8 Hrs) Client Location 1 3:00 PM-11:00 PM	Swing Shift L1 (8 Hrs) Client Location 1 3:00 PM-11:00 PM	Night Shift L1 (8 Hrs) Client Location 1 11:00 PM-7:00 AM	Night Shift L1 (8 Hrs) Client Location 1 11:00 PM-7:00 AM	
Walker, Nikkie Sr. Security Officer Client Location 2		Day Shift L2 (12 Hrs) Client Location 2 7:00 AM-7:00 PM	Day Shift L2 (12 Hrs) Client Location 2 7:00 AM-7:00 PM	Day Shift MF L4 (8 Hrs) Client Location 4 8:00 AM-4:00 PM		
Hall, James Security Officer Client Location 2		Night Shift L2 (12 Hrs) Client Location 2 8:00 PM-8:00 AM	Day Shift L2 (12 Hrs) Client Location 2 7:00 AM-7:00 PM		Day Shift L2 (12 Hrs) Client Location 2 7:00 AM-7:00 PM	Night Shift L2 (12 Hrs) Client Location 2 7:00 PM-7:00 AM
Munoz, Nikkie Security Officer Client Location 2		Day Shift L2 (12 Hrs) Client Location 2 7:00 AM-7:00 PM	Night Shift L2 (12 Hrs) Client Location 2 7:00 PM-7:00 AM		Day Shift L2 (12 Hrs) Client Location 2 7:00 AM-7:00 PM	Night Shift L2 (12 Hrs) Client Location 2 7:00 PM-7:00 AM

4)TrackSmart

TrackSmart Scheduling allows you to schedule in advance.

TrackSmart can be accessed from any device, anywhere, anytime.

Workers can make changes to their shifts without your input.

Downtown Diner Day Week Month  Print

Files Employees Set
Dashboard | **Schedule** | Time Clock | Reports

July 2015

1 2 3 4 5 6 7
8 9 10 11 12 13 14
15 16 17 18 19 20 21
22 23 24 25 26 27 28
29 30 31


[TODAY]

Copy Schedule

Auto Schedule

Publish

14 shifts unpublished

VIEWS  Refresh

☒ All Shifts
☐ My Shifts
☐ Unscheduled Shifts
☐ Scheduled Shifts
☐ Shifts on Hold

Jul 22 - 28 2015

Open Schedule Clear Schedule

Wed 7/22	Thu 7/23	Fri 7/24	Sat 7/25	Sun 7/26	
10A-6P 6 Gregg Johnson Henry O'Hare Jennifer Taylor Jonathan Gold Jordan Labelle Mark Rogers Weekday Daytime	10A-6P 6 Gregg Johnson Henry O'Hare Hilary Felizo Jessica Hanover Jonathan Gold Mark Rogers Weekday Daytime	10A-6P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Weekday Daytime	8A-2P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Weekend Morning	8A-2P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Nathan Landow Weekend Morning	10A-6P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Weekday Daytime
6P-11P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Weekday Evening	6P-11P 8 Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Nathan Landow Weekday Evening	6P-2A 8 Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Nathan Landow Weekend Evening	6P-2A 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Nathan Landow Weekend Evening	6P-11P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Sunday Evening	6P-11P 8 Gregg Johnson Henry O'Hare Hilary Felizo Jennifer Taylor Jessica Hanover Jonathan Gold Jordan Labelle Mark Rogers Weekday Evening

5)SmartLinx

SmartLinx provides notifications when shifts free up, and any changes in the schedule can be sent via text, email, push notification, or in-app notification. SmartLinx is often used to put the focus back on the patients.

Time & Attendance > Daily Punches

From: 04/08/2018 To: 05/01/2018 **Apply**

Exceptions	Name	Date	Position	Badge	In	Out
Missing Punch	EWING, BILL J	04/10/2018	CNA	7891	06:00 AM	10:00 AM

Add Punch **Save** **Reset** **Punch Log** **Missing Punch**

Display	Type	Timeclock	Actual time	Rounded time
Edited	Out	Celestial CC	04/10/2018 11:00 PM	

+ Missing Punch	EWING, BILL J	04/19/2018	CNA	7891	02:00 PM	
+ Missing Punch	EWING, BILL J	04/21/2018	CNA	7891	07:00 AM	
+ Missing Punch	EWING, BILL J	04/22/2018	CNA	7891	02:00 PM	
+ Missing Punch	EWING, BILL J	04/21/2018	CNA	7891	07:00 AM	
+ Missing Punch	EWING, BILL J	04/22/2018	CNA	7891	02:00 PM	
+ Missing Punch	EWING, BILL J	04/23/2018	CNA	7891	02:00 PM	
+ Missing Punch	EWING, BILL J	04/24/2018	CNA	7891	06:00 AM	
+ Missing Punch	Arroyo, Ebelia	04/18/2018	LPN	8509	12:30 AM	01:00 AM
+ Misplaced Punch	Mau, Crystal	05/05/2018	Director of Nursing / Assistant	1234	02:00 PM	

Literature review of academic publications

Paper(1)

In recent years, NRP has been studied widely. In a review paper on the subject, Burke et al. categorize research according to solution methods, constraints and performance measures. They provide tables with information on expertise, substitutability, the data that were used (i.e., real case or theoretical), the planning period, etc.

De Causmaecker and Vanden Berghe (Causmaecker and Berghe 2011) have extended a notation for classifying the NRP, which could

be used for the personnel scheduling problem. This notation is based on the $\alpha/\beta/\gamma$ -notation (used in the field of resource-constrained project scheduling).

Ernst et al. categorize different sections of the workforce scheduling problem in their review paper.

Van den Bergh et al. present a review paper on personnel scheduling problems. Considering the solution method, they observe that the literature is closely linked to mathematical programming approaches and metaheuristics and the researchers pay more attention to decomposition algorithms and hybrid techniques.

It has been proved that the complexity of the personnel scheduling problem is NP-complete. Of course, not all job shift issues fall into this category, and the complexity of these issues depends on the constraints defined for the problem. Brunner et al.

and also, Osogami and Imai have argued that the difficulty of the problem depends on the limits of allocating particular shifts or working and non-working shifts. For example, the complexity of the problem by the shifts sequence constraint is NP-complete.

Brucker et al. (2011) are among the researchers who systematically deal with the complexity of personnel scheduling. In their research, based on the general model of the personnel scheduling problem, four polynomial solving modes are identified, two of which are related to the work shift scheduling problem.

Kellogg and Walczak (2007) point out in a paper that only a handful of these studies have come to the real world stage. From a few of these studies, computer programs have been developed to help the health system. The reason for this is that so far there has not been a general and flexible model for different situations and factors for the NRP. Attempts to provide a general model that understands all the real-world conditions and can be used by any hospital started many years ago.

Okada (1992) tried to provide a general model for the problem. Azaiez and Al Sharif (2005) developed a computerized nurse scheduling model at Riyadh Al-Kharj hospital in Saudi Eskandari and Ziarati (2008) used the theory of fuzzy sets to

model flexible constraints and uncertain data on the NRP.

This linear model is designed to meet the need for workforce satisfaction, with fuzzy restrictions for a one-week program at Namazi Hospital of Shiraz, which is the largest hospital in the south of Iran.

Parr and Thompson (2007) solved instances of two NRP models (SAWing and Noising) with simulated annealing and demonstrate that “Noising” produces better schedules.

Glass and Knight (2010) looked at the structure of the problem of nurse scheduling, and by categorizing the problem into four sets, they proposed solutions to reduce the dimensions of the solution space.

One of the features of their work is the use of the obtained solution to solve the problem for the next period. Pham et al. (2012) solved the problem with a multi objective

approach. Before these people, this problem has been investigated with various objective functions and methods used in multi-objective approaches, and most research is continuing with new objective functions.

Huangetal. (2014) categorized constraints and solved a problem with a large number of nurses, using an evolutionary algorithm. According to Huang’s paper, the number of nurses and the length of the planning horizon increase the complexity of the problem.

Santos et al. (2016) used the concept of cutting in integer programming to solve the problem innovatively.

Awadallah et al. (2015) solved some instances using a hybrid algorithm. This solving method utilizes the combination of two evolutionary metaheuristic algorithms, a local search to improve the quality of solutions and create equilibrium between exploration and exploitation factors.

Maass et al. (2017) investigated and modeled the nurses’ absence and demand under uncertainty and used the genetic algorithm to solve the instances of the problem.

In this research, the absence of nurses has been considered an effective factor in the scheduling program. By analyzing the effect of the reserved nurse on the scheduling stability, Ingelsand Maenhout(2015) proposed an approach to improve the stability of a scheduling program.

Bagheri et al. (2016) proposed two-stage stochastic programming for a real case. In this study, the Sample Average Approximation (SAA)approach was used to minimize the assignment costs.

Recently, Turhan and Bilgen (2020) studied the nurse rostering problem. They proposed a new heuristic approach, namely fix and relax and fix and optimize algorithms, and integrated them with mixed-integer programming as a problem solving method.

In addition to these researches, there have been many other studies, most of which are based on improving, expanding and reviewing these papers. On the other hand, most researchers focus on the problem in a certain condition, and fewer studies are in the field of uncertainty.

For this reason, research on uncertainty and modeling, taking into account real world conditions, provides a suitable platform for researchers interested in working in this field.

Paper(2)

In an influential early overview, Warner (1976), distinguishes 3 major areas of manpower decision research: staffing, scheduling and reallocation of nurses. Five different criteria are defined for the scheduling part of the problem:

- coverage: how different the required and the scheduled number of people for a task are.

- quality: how fair schedules are, what the work stretch length is.

- stability: how the nurses perceive the schedules (in terms of consistency, predictable on/off days and weekend work).

- flexibility: how well the system can adapt to changes in the environment.

- cost: how many resources are consumed in making the decision: e.g. personnel manager's time or computer time.

It is very interesting to combine these criteria for evaluating schedules since they address more than computable standards. From a general hospital scheduling point of view, it makes sense to take such an interpretation of cost (to generate the schedule) into account. However, it would also make sense to add other criteria (like 'personnel cost', for example) to the list. Nearly all the criteria are very hard to measure. Warner compares three scheduling approaches against these 5 criteria:

- In the Traditional Approach, the schedules are generated by hand. This policy is flexible, which is the only advantage with respect to the criteria.

- Cyclical Scheduling generally provides good schedules but it cannot easily address personal requests.

- Computer Aided Traditional Scheduling enables a fast and more complete search for good schedules. The advantages of this approach are high with respect to all the criteria considered

Warner's overview is oriented towards techniques for determining desired/required staffing levels, which are also briefly discussed

Fries (1976), presents a bibliography of early methods for personnel rostering in healthcare institutions. Many of these early approaches rely on manual procedures, following a set of arbitrary rules. They are too restricted to be directly applicable for large modern hospitals with today's complexity. However, there is always the

possibility of hybridizing early approaches (or some features of early approaches) with more sophisticated modern techniques to produce even better methods.

Tien and Kamiyama (1982), present a list of personnel scheduling algorithms, which are not restricted to healthcare. Many of them are based on trial and error methods. Tien and Kamiyama concentrate on the hospital scheduling situation in the United States. A particularly interesting contribution is that they decompose the 'manpower' scheduling problem into five separate stages: determination of temporal manpower requirements, total manpower requirement, recreation blocks, recreation/work schedule, and shift schedule. Stages 1 and 2 are management decisions (also called the 'manpower allocation problem'), which belong to the long-term staffing part of the problem. Both stages consider the definition of hospital requirements and the selection of resources. Stages 3 to 5 include the entire short-term timetabling part of the problem, that takes preferences and constraints on personal schedules into account. Tien and Kamiyama were able to classify many papers in their 5 stage model, some covering a number of stages simultaneously. However, we believe that this division is often too simplified to capture all the problem specific features of modern nurse rostering problems.

Sitompul and Randhawa(1990),concentrate on financial cost. The goal is to reduce the personnel cost. Characteristics of manpower scheduling in hospitals are fluctuating demand, human effort (which cannot be inventoried), and critical customer convenience, while the schedules are subject to different kinds of constraints. They define four stages in nurse scheduling:

- Determine a set of feasible schedules that satisfy the constraints.
- Select the best schedule in terms of cost, coverage, and/or other criteria.
- Fine tune to accommodate changes.
- Make specific shift assignments.

It is interesting to compare schedules with respect to the perceived quality by the personal members instead of using the violation of constraints as a criterion. In practice, it often makes no sense to separate specific shift assignments from the schedule design because assignment to different people really influences the quality of the schedule. Sitompul and Randhawa advocate the approach of tackling staffing and rostering at the same time. They argue that separating the rostering from management decisions leads to sub-optimal schedules. From a theoretical

point of view, this is absolutely true. However, we believe that a general scheduling procedure would not work without significant changes in working practices, for the following reasons:

- Even though there is a high fluctuation in patient needs, it is not recommended to shift personnel around the hospital each time the request does not match the available staff. This would be the consequence if the problem were looked at from a purely global point of view.

- People prefer to express personal preferences with respect to work and free time. These preferences differ from month to month. Planners only seem to grant personal wishes if they know the people in person.

- The problems are nearly all over-constrained and far too complex to find an optimal solution in a reasonable amount of time. Splitting them up cannot lead to optimality either but it certainly results in less complex subproblems.

Warner, Keller, and Martel (1990), discuss patient-oriented and employee-oriented issues in nurse management. The latter are divided into several 'chronological' areas. The most interesting one is called short range scheduling and staffing. It includes the weekly, daily, or shift by shift adjustments to the long range schedule. The paper contains a description of the history of computerized nurse scheduling in the US. The article also introduces a nurse scheduling system called ANSOS (Automated Nurse Scheduling Office System) which will be discussed in more detail in Section 3. Warner et al. demonstrate a deep appreciation for the difficulties involved in tackling real world nurse rostering problems. Bradley and Martin (1990), distinguish three basic decisions in hospital personnel scheduling: staffing, personnel scheduling and allocation (as introduced by Warner, 1976). The first problem consists of determining the long-term number of personnel that have to be employed. The number of personnel is expressed in terms of full time equivalents and is supposed to be sufficient to cover holiday periods (annual leave), training and further education. Hiring part-time nurses and allowing flexible work (or permitting the definition of different work agreements) facilitates a closer match between the personnel demands and the effective hours worked. Staffing decisions are influenced by the stochastic nature of personnel requirements and personnel capabilities. The second phase in Bradley and Martin's decision scheme is the conversion of the expected daily work force into precise assignments i.e. personnel rostering. Better schedules can be generated if the problem allows for differentiation between days of the week and seasonal variations. Since schedules are generated before the actual patient needs are known, the personnel manager

or scheduler has to anticipate the personnel requirements. The third phase (allocation) consists of assigning the scheduled personnel to actual work sites. It enables the hospital to correct schedules as fluctuations in the demands occur. The time horizon for the allocation phase is typically very short (varying from a few hours to a couple of days).

Bradley and Martin present a useful classification of schedules both formally and from a solution method view point, just like Sitompul and Randhawa(1990) do. These can be summarized as :exact cyclical, heuristic cyclical, exact non-cyclical, and heuristic non-cyclical. Siferd and Benton (1992), present an excellent review of the factors influencing hospital staffing and scheduling in the United States. A survey among hospital managers reveals the complexity of the problem. The work first discusses the staffing history in which cost reduction became more and more important. In the second part of the work, short-term personnel scheduling is discussed, in which various constraints on the nurses' schedules are taken in to account. There searches collected data from 31 different hospitals and, in total, 348 wards. Decentralised manual scheduling was the most common approach, often performed in co-operation with a large number of people per ward. The questioned hospitals worked with different skill categories for personnel. Personnel shortage is often solved by allowing overtime (sometimes leading to working days of 12 or 16 hours) and by using personnel from other wards. Full time work seemed to be more popular than any kind of part time work. It was also rather rare to have nurses doing both day and night shifts. In some cases the night work was carried out by a special group of personnel (but this does not generally hold). A large number of personnel is assigned to a set shift in practice. Most shifts have fixed start and end times. 50% of the hospitals work with three start times for day shifts on weekdays and 30% have 5 different start times. Most hospitals seem to work with stricter rules (e.g. in 93% of the cases, either there are no 'split' shifts, or, people are working the same days every week). This is an excellent paper which fully illustrates the difficulty of nurse rostering problems in practice

Hung (1995), collected 128 articles on nurse scheduling, from the 60's up until 1994, and he presents a brief overview. Most papers study the experience of new work week arrangements. The overview is just a bibliographic selection but it can be useful for collecting literature from a variety of research domains. Cheang et al. (2003), present a survey of recent models and approaches for the nurse rostering problem. The paper does not attempt to critically evaluate the area but, as the authors say, its purpose is to provide a brief overview of the area. The emphasis of the paper is on briefly covering mathematical programming methods, different

kinds of constraint programming techniques and metaheuristics. Ernst et al. (2004), present a very comprehensive overview of the literature on staff scheduling and rostering. They concentrate on staff scheduling that they describe in general rather than concentrating on nurse rostering in particular. They have split the paper into three main parts:

- definitions, classification of personnel scheduling problems
- a classification of the literature into application areas
- solution methods, with comments on applicability.

Ernst et al. describe the personnel scheduling and rostering problem in detail. An extensive list of terminology and problem characteristics is introduced. They deliberately do not distinguish between rostering and scheduling. However, in a very interesting section about common decompositions of the problem, one of the described proposals is to present demand modelling as a separate module. Many of the characteristics that they describe apply to staff scheduling problems other than nurse rostering. When discussing their classification by application area, most attention is paid to crew scheduling and rostering. They consider this to be the best covered application domain. However, one of their categories specifically discusses health care systems and it is mainly concerned with nurse scheduling and rostering. Ernst et al. discuss, more or less in historical order, nurse rostering

approaches as they have been reported upon in the literature. They point out that mathematical programming and metaheuristic approaches are by far the most investigated techniques, in the literature on personnel scheduling. They point out that metaheuristics are promising for very difficult problems and for real world problems for which optimal solutions cannot be obtained with exact approaches. Ernst et al. highlight a few areas for improvement in the personnel scheduling and rostering domain:

- widening the applicability by generalizing models and methods,
- gaining more efficiency through a better integration of the problem steps,
- increasing the accessibility by integrating the tools in Enterprise Resource Planning systems,
- catering for individual preferences,
- generalizing the rostering algorithms.

There are a few PhD dissertations on the topic of hospital scheduling. Most of them belong to the staffing domain. They are briefly summarized in Appendix A. It is clear that nurse scheduling problems have been addressed across a wide spectrum of research articles. It is also clear that some papers in the literature have tackled simplified problems and have developed methods which cannot be directly applied to hospitals. Even in the early days of automated nurse rostering (Warner, 1976), there was an awareness of requirements other than just measurable optimization objectives. Automated schedule generation needs to be flexible, to consider personal preferences, patient satisfaction, and human effort (Sitompul and Randhawa, 1990; Tien and Kamiyama, 1982). In particular, Siferd and Benton started to show the way by highlighting the complexity of the real-world problem. Too many scientific articles in the past have not addressed this complexity. As we shall see, this is a theme which will recur in this paper and, indeed, it is one of our main conclusions. The early papers on nurse scheduling research often considered the entire process of staffing, assigning people to wards, and short-term timetabling as a whole. The most recent overviews started to decompose the problem.

Paper (3)

When referring to the literature on nurse rostering and scheduling, one can see that the problem is extensively studied. present in-depth review. They describe the methodology, models, and algorithms. A wide variety of methods have been used for nurse scheduling: mathematical programming, constraint programming, heuristics and meta-heuristics, hybrid methods, and simulation.

Different objectives are considered in the literature:

- to decrease manual scheduling;
- to increase demand coverage in terms of workforce size and also according to required skills;
- to maximize nurse preferences;
- to obtain equity between the schedules.

Researchers agree that although nurse scheduling is a well-studied problem, its practical solution and the implementation at the institution are still problematic. They emphasize that better solutions are obtained when the specific features of each application are included. are among the few researchers who focus on developing generic models and algorithms for the nurse scheduling problem.

Most studies are application-focused and use approaches such as tabu search , genetic algorithms , learning methodologies , scatter search , combinations , or even mathematical programming . They deal with the constraints by penalizing their violation in the objective function. It is difficult to find feasible solutions, and in numerous applications, the quota requirement constraint cannot be satisfied; see e.g. Therefore, some researchers introduce an acceptable shortage or surplus that allows flexibility in the quota requirement. In, the demand constraint and respect of preferences are relaxed and a Lagrangian-based heuristic is used.

Three studies are particularly pertinent. introduce an assignment-type model for the scheduling problem. They consider a set of objectives consisting of the formal objectives of the problem as well as a set of constraints. They use a tabu search where at each iteration, two solutions are compared by considering their objectives in a lexical order. This prioritization of objectives is central in scheduling. In , a multi-objective approach is introduced that differentiates between hard and soft constraints. formalize the nurse scheduling problem using directly the rosters (alternating between work days and rest days) in the model. A non-optimal solution is generated by solving the mathematical model and a post-optimization phase using tabu search is performed. solve the nurse scheduling problem in a Hong Kong

emergency department with a two-phase heuristic implemented in Excel. They build a feasible planning which is then improved by a local search taking into account soft constraints. However, the nurse scheduling problem for an emergency department is a particular case as the work environment is very dynamic.

We propose solving the nurse scheduling problem for both regular and float team using a scientific method based on operations research tools, simple and easy to implement at no extra cost. Indeed, one objective is to implement our method in a spreadsheet; nursing units already use Excel. Furthermore, as note, one of the reasons that even approaches based on practical studies are not implemented is the use of complicated technology. Solutions based on free software such as COIN-OR are therefore not suitable. We address directly this issue with our approach. Because application-based approaches are more suitable for implementation in hospitals, we focus on a specific application in the constrained context of Quebec. To summarize, our objectives are threefold: we first conduct a practical study of the process of nurse scheduling in two different large size teaching hospitals, we then introduce a procedure based on local search that can be easily implemented at no extra cost. In addition of being user-friendly, it aims for standardization and efficiency. We then conduct an analysis on the performance of the tool. We primarily focus on the practical implementation of the proposed approach rather than the optimality of the solution.

This paper is organized as follows. The next section introduces new heuristics as well a description of the transferable prototypes. The results and discussions section shows the benefits of the proposed approaches in terms of process and scheduling method and we close with concluding remarks.

Paper (4)

There is a large body of research literature relating to nurse rostering. The survey papers by Ernst et al., 2004a, Ernst et al., 2004b and Burke et al. (2004) give a detailed overview of the literature up to 2005. However, the latter concludes that only a few papers are based on real world data or address the development of rostering systems for implementation in hospitals. A recent paper, Burke et al. (2008), was jointly authored by researchers at Nottingham University and at ORTEC,² a major supplier of rostering software. This paper presents a practical, heuristic-based methodology and compares results from variable neighborhood local search heuristics with those obtained using ORTEC's Harmony software on a range of problem instances. Since the context is the same of the recent article by Burke et al. in this journal, we omit a detailed literature review in order to avoid unnecessary repetition.

Our main research interest is in employee rostering for call centers. In this domain, the application of Mixed Integer Programming (MIP) techniques is more common than the use of heuristics. The traditional approach to call centre rostering is to decompose the problem into two stages. First, a set of weekly *tours* are scheduled, where each tour consists of a set of attendance details; days on and off duty, daily shift start and finish times, and even lunch and coffee break timings. At a second stage, each tour is assigned to an individual employee, taking account of availability and preferences. The problem is complicated by the large number of feasible shifts, which may start or finish in any reasonable quarter-hour interval, unlike in nurse rostering where generally only a handful of shifts are considered. The most comprehensive tour scheduling model in the call center literature is that of Brusco and Jacobs (2000). This model allows the specification of a *bandwidth* parameter which limits the variation in shift start-times within a working week. A major problem is that this approach does not distinguish between forward and backward rotation. For example, a typical restriction in practice is that a day shift cannot immediately follow a night shift, as indicated by constraint H5 in Appendix A. If this condition is imposed through a bandwidth restriction, then the same constraint prevents a night shift from appearing *anywhere* in the same tour as a day shift. Hence, bandwidth has little relevance in the context of nurse rostering, where

specific rules relating to night shift working (for example, constraints H3, H5, H10, S10) are typically imposed. In the rostering literature relating to healthcare, Isken (2004) developed a tour scheduling model which treated start time variations in the same way as Brusco and Jacobs (2000), albeit with a different formulation. The Isken model therefore suffers from the same drawback.

We have developed an alternative tour scheduling model, presented in our paper Glass and Knight (2008). We recognize that while it is preferable to schedule the same shift start time (or later) on consecutive working days, there is usually no objection to any shift change following a period of one or two days off-duty.

An important issue in both call center and nurse rostering is the continuity from one rostering period to the next. The nurse rostering benchmark instances are designed only to produce rosters for an isolated period, applying penalties in accordance with the convention that all potential violations are counted at the beginning of the period, and ignored at the end. We recognize that the benchmark instances are intended as a basis for comparison between alternative rostering methodologies, and that the consideration of an isolated rostering period serves this purpose. However, in a practical environment, information relating to one rostering period is carried forward to the next, creating additional considerations of “continuity”.

Our contribution to the mathematical nurse rostering literature is as follows:

- 1) analysis of a benchmark problem which identifies a lower bound on the solution, and which brings to light aspects of the problem structure which we are later able to exploit in order to radically reduce the size of solution space;
- 2) improved results for four related benchmark problem instances, by providing an optimal solution to each instance, within a practical execution time, using an MIP approach;
- 3) improvements in handling shift changes within an MIP, extended from the context of call center rostering to nurse rostering; and
- 4) a methodology for handling continuity between rostering periods.

The paper is organized in two parts. In the first part we analyze a class of four benchmark problem instances initially specified by ORTEC, each of which involves a single rostering period. We give a structural analysis of the interrelationship between constraints leading to a lower bound on penalty costs for one of the problems. We also provide optimal solutions for the four problems and compare our results to those obtained previously.

In the second part of this paper we propose a more flexible approach, handling those requirements which relate to the continuity between rostering periods. We are aware that these issues are well understood and taken account of in practice. Indeed, the benchmark problems originate from practice. Our contribution is to formalize the continuity goals. We illustrate how our approach achieves these objectives by producing a solution which continues from one month to the next.

Paper (5)

The present study describes hospital nurses' job satisfaction. The purpose of this literature review was to create a concept of nurses' job satisfaction that is based on systematically collected and analyzed data from earlier research. This review provides answers to the following questions: what factors influence nurses' job satisfaction positively, and what evokes job satisfaction among nurses working in hospitals?

Job satisfaction can be defined as 'the degree of positive affect towards a job or its components' (Adams & Bond 2000). Adams and Bond (2000) describe job satisfaction theories as discrepancy theories (examining the extent to which employee needs or wants are satisfied within the workplace), equity theories (highlighting social comparisons in the evaluation of job rewards) and expectancy theories (focusing on employee motivation). Job satisfaction can be seen as a positive concept describing work attitudes in particular. The viewpoint of this article is positive, meaning that job satisfaction is observed from the viewpoint of positively affecting factors and promotion abilities. This has a connection with the philosophy of positive psychology (Seligman & Csikszentmihalyi 2000, Sheldon & King 2001), which involves scientific study of ordinary human strengths and virtues. According to Sheldon and King (2001), 'positive psychology revisits the average person, with an interest in finding out what works, what is right, and what is improving'.

There are several theoretical models of work stress (Karasek & Theorell 1990, Warr 1990, Siegrist 1996) and some of them have also been studied among health care professionals (De Jonge & Schaufeli 1998, Bakker *et al.* 2000). Work stress is a very well-understood phenomenon, whereas the positive viewpoint needs to be studied and understood more deeply. Based on that, this article aims to find answers to the question 'what creates job satisfaction' instead of asking 'what undermines job satisfaction'.

Original papers were analyzed using content analysis. According to Polit and Hungler (1999) content analysis is 'the process of organizing and integrating narrative, qualitative information according to emerging themes and concepts; classically, a procedure for analyzing written or verbal communications in a systematic and objective fashion, typically with the goal of quantitatively measuring variables'. Content analysis is a method to analyze documents systematically and objectively. This method makes it possible to describe a phenomenon in an abstract and conceptual form. The analysis progressed inductively, finding answers to the following research questions how is nurses' job satisfaction studied from the positive viewpoint and what factors influence nurses' job satisfaction positively. While analyzing the data, original papers were read and significant themes and points were marked. All main findings from the viewpoint of job satisfaction were included. During the phase that followed, all papers were presented in a table form and the

factors found that influenced job satisfaction positively were written down in lists. The factors were then categorized so that similar factors were combined into the same categories and further into three main categories. All categories were named according to their content.

The aim of the present study was to produce knowledge about nurses' job satisfaction and the factors influencing it that is as valid and truthful as possible. The validity of this review is linked to research ethics, and considering both of these, the following have been the critical phases of the study: succeeding in the electronic data search, selection of original studies and carrying out the analysis process.

In the search for original studies, various comprehensive databases were used. Cinahl and Medline are most likely to be useful to nurse researchers; in addition, PsycINFO covering psychology and related disciplines is one of the most important databases in this area (Polit & Hungler 1999). ABI/Inform, in turn, covers articles and abstracts, for example, in administrative sciences, and was chosen to expand and complement the other databases. Original papers were searched together with an information worker to ensure that the search was carried out accurately.

Original papers should be chosen according to certain and limited criteria and also using two independent researchers (Glasziou *et al.* 2001). It has to be noted that in this process papers were selected by one researcher: validity was ensured because the criteria were clear, and it was possible and economical for one person to select the studies. Original papers were selected according to limited criteria and the aim was to find high-quality studies (peer-reviewed articles, highly regarded publications, assessing the scientific quality of every study) that ensure validity and a high level of results in this review.

The data were analyzed exactly and conscientiously using the method of scientific analysis (content analysis). To demonstrate validity, progress of the process from the search for original papers to the end of the analysis is described as exactly as possible. Good scientific practice and high ethical quality were taken into consideration in every phase of the research process.

Proposed Solution

Main Functionalities

In the Nurse Assignment phase, the goal is to solve the second nurse scheduling problem: determining which specific nurses should work each shift.

In nurse scheduling problem (NSP), nurses are assigned into a set of shifts and rest days in a timetable called nurse roster in such a way that several constraints are satisfied.

In the “Nurse Planning” phase, the first problem of nurse scheduling is meeting the demand and variability in demand of the OR while minimizing cost.

Schedulers, who typically are head or chief nurses in the units, must assign nurses to each shift according to numbers and skill levels required while at the same time balancing the workload among the nurses involved and considering staff nurses’ preferences such as providing requested days-off. A nurse roster is a timetable consisting of shift assignments and rest days of nurses working at a hospital.

In nurse scheduling, the ultimate aim is to create high quality timetables, taking well-being of nurses as a basis without discarding the concerns of employers.

Easily plan and allocate shifts for your nurses.

Nurses are allowed to actively impact their schedule by notifying managers of future absences, restrictions or shift preferences.

Nurses are allowed to request a swap or replacement.

System should attach forms and checklists for nurses to complete every shift task to perfection.

Nurses are allowed to see the entire schedule.

Nurses are allowed to see her schedule only.

Nurses can look at their payroll.

Nurses can update their data.

Nurses can request track and manage time off request.

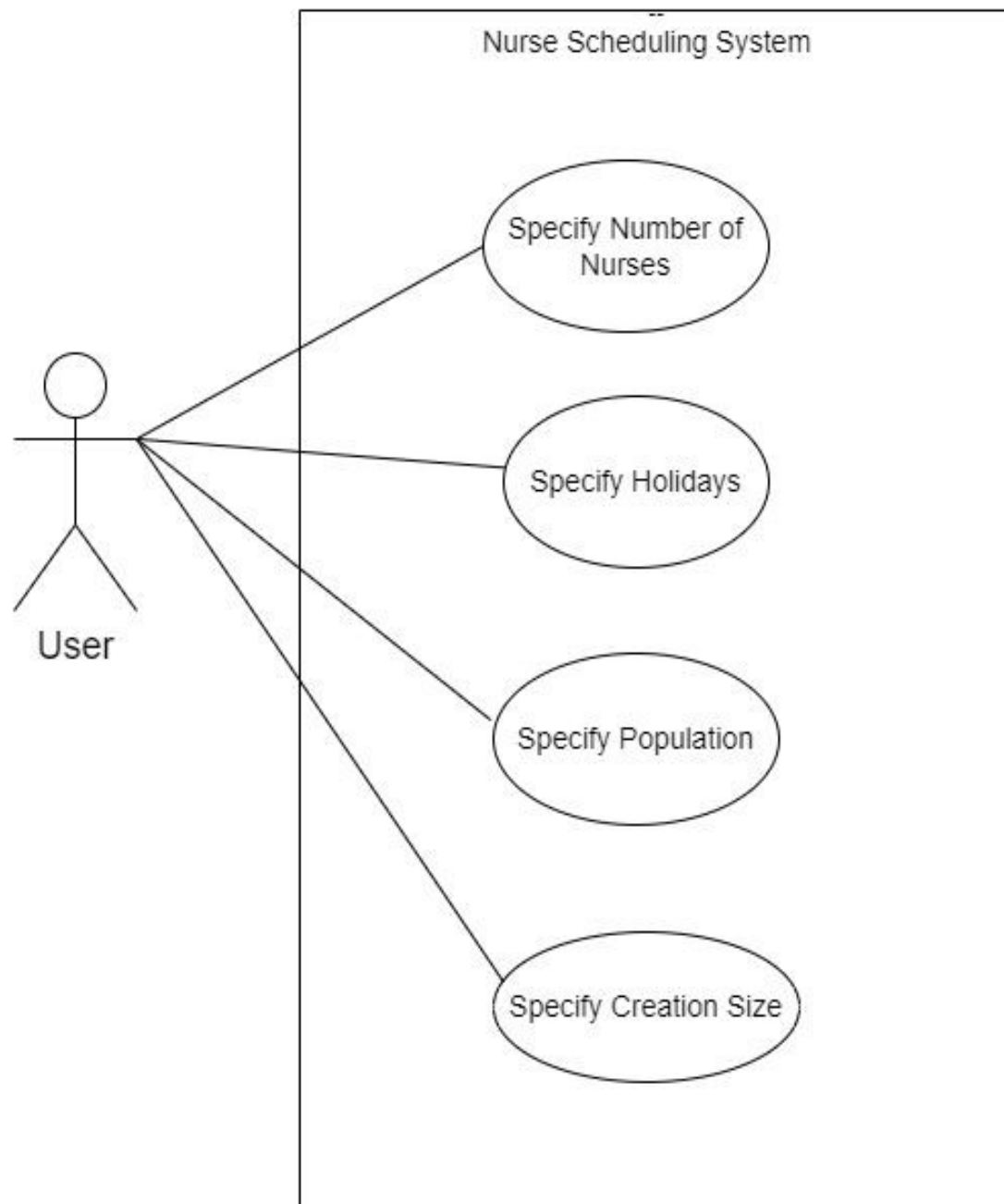
Nurses are allowed to see her schedule only.

Nurses can look at their payroll.

Nurses can update their data.

Nurses can request, track, and manage time off requests and vacation time.

USE CASE DIAGRAM



Applied Algorithms

Differential evolution algorithm

DE algorithm has several versions. The version that has been used for the problem is “Classic DE” (DE/Random/1/bin). The base vector is chosen randomly. “1” in this notation is the number of vector difference added to it, and therefore the number of parameters donated by the mutant vector intimately follows a binomial distribution. The reason for choosing this random search version is the proper convergence of the algorithm for a discrete state.

This algorithm is suitable for continuous spaces, but other versions for use in discrete spaces are also provided. An example of its use in discrete space is integer programming. In this algorithm, solutions (individuals) are represented as matrices, and the distances and locations of these matrices are the information that the algorithm needs in each iteration to use its operators for search. According to the algorithm name, this algorithm uses the difference (distance) of the solutions to find a direction to move to the optimal point. Differential evolution algorithm has three important operators: Mutation (generate a mutant vector), Crossover (develop trial vector) and Selection (select individuals to create a trial vector and the best member to survive for each generation).

Fitness Calculation

To evaluate each nurse, a fitness function denoted by F is calculated for each nurse. All nurses are assigned the same fitness function. F consists of three factors that are described in the following subsections

1-Fitness of the night shift pattern, in respect to it's order and length: F

To evaluate the number of the consecutive night shift patterns (as a vector value), four valid patterns and their corresponding penalty values are assigned. These patterns and their corresponding penalty values are shown in table 3. Selected length and order of F is observed, and corresponding fitness values are assigned. The more is the gap from the basic night patterns, the more penalty is charged.

Table 3: Pre-assigned fitness values of working-patterns

l: late night

n: Night

d: Day

h: Holiday

Working Pattern	Assigned Penalty F
l l n n	0
l l n	-40
l n n	-40
l n	-60
Others	-120

Pattern 1

2-Number of consecutive night shifts, in respect to its length: F

The aim of this evaluation factor is to observe the number of consecutive night shift patterns, regardless of their order. F is formalized according to the following equation:

$$F_i = \alpha F_i^p + \beta F_i^c + \gamma F_i^d$$

Where, counting from the beginning of the month, c_k represents the length of the k the night shift pattern. In words, while the number of the consecutive night shifts are less or equal to four consecutive shifts, no penalty is assigned, otherwise corresponding penalty values are assigned. 3- The interval between night shifts : F

$$F_i^d = - \sum_j (d_j - 11)^2$$

Where, D_j represents either of the j th number of the consecutive day shifts or day off s, counting from the beginning of the month. In words, if the interval between night shifts (i.e., either of the consecutive day shifts or day off) is exactly 11 days, it is considered as an ideal interval, and thus no penalty is charged. Otherwise, a penalty according to F_i^d is assigned. 4.4 Overall fitness of individual nurse: F
Overall fitness of individual nurses, F_i is calculated according to equation 7

$$F_i = \alpha F_i^p + \beta F_i^c + \gamma F_i^d$$

F_i ; are parameters, representing specific criterion of the hospitals, chief nurses, etc.
After all, the objective function for the entire schedule is formalized as:

Maximize:

$$avg = \frac{1}{N} \sum_{i=1}^N F_i$$

Minimize:

$$\sqrt{\frac{1}{N} \sum_{i=1}^N F_i^2 - \frac{1}{N^2} \left(\sum_{i=1}^N F_i \right)^2}$$

Subject to: DAILY restrictions

It is desired to attain maximization of the average of the fitness of all nurses, and minimization of the variance of all nurses. A schedule with $avg = 0$ and $dev = 0$ represents the desired absolute solution to the problem. Moreover, for individual nurses $F_i = 0$ represents the best fitness value. Since it is very difficult to predict whether there ever exists a solution to a schedule with the values of $avg = 0$ and $dev = 0$, thus it is desired to attain the values of avg and dev as close to zero as possible.

4.5 Multi-agent approach for NSP In order to compare search performance of CGA with another evolutionary approach for NSPs, a brief overview of a multi-agent approach for solution of the same model of NSP is described.

Mutation

The DE mutation operator produces a pilot vector for each individual of the population by mutating a target vector and a weighted difference. For each parent ($X_o(t)$), the mutant vector is generated in this way:

$$\vec{u}_o(t) = X_{o1}(t) + (X_{o2}(t) - X_{o3}(t))$$

β is a scaling factor and controls the degree of different effects. Several methods have been proposed for creating the mutant vector.

Crossover

The DE crossover operator uses a discrete combination of mutant vector and parent vector for child production. Crossover is done in this way:

$$X'_o = \begin{cases} u_o(t), & o \in J \\ X_o(t), & \text{otherwise} \end{cases}$$

Several methods are used to determine the J set. The most commonly used method is the binomial crossover. The cutting points are randomly selected from the set of possible points of the cut $\{1, 2, \dots, n_x\}$, where n_x is the dimension of the problem. In this algorithm, Pr is to be used for specifying the cutting points. A low Pr value means that more cutting points are selected. This means that more elements are taken from the mutant vector for the production of children. A high Pr value means no point can be selected, and then the child will be the same as the parents. For this reason, to differentiate at least one element from the child with his parents, the set of cutting points, J, is initialized to contain a random point j^* .

Selection

The selection operator is used in two parts of the DE algorithm. First, this operator decides who should participate in the mutation process to create a trial vector.

Random selection is often used for individual selection in producing differential vectors, and secondly, for selection of the best member to survive (to determine which parent or child will survive) . In some algorithms, only age determines which individuals survive and come to the next generation (i.e., the simple GA). When only trial vectors are allowed to participate in the next generation, potentially the best-so-far solution will be lost. Retaining the best-so-far solution is one of the tasks of the selection operator in the DE algorithm and known as elitism

The operators used in the algorithm produce infeasible chromosomes. For this reason, the feasibility of the solutions must first be checked. The following four methods are used to deal with infeasible solutions in metaheuristic algorithms:

1-Preserving Strategies.

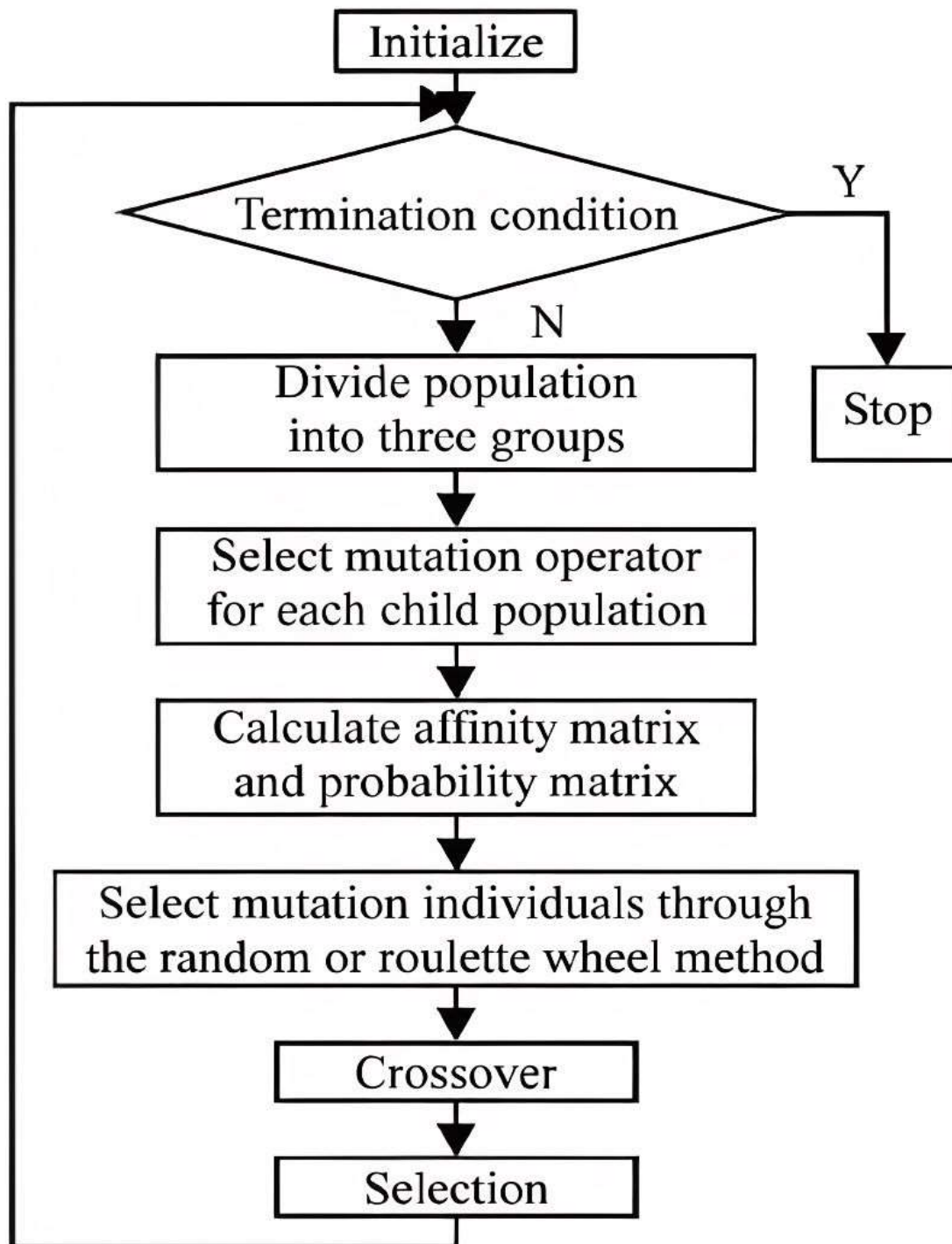
2-Reject Strategies.

3-Repairing Strategies

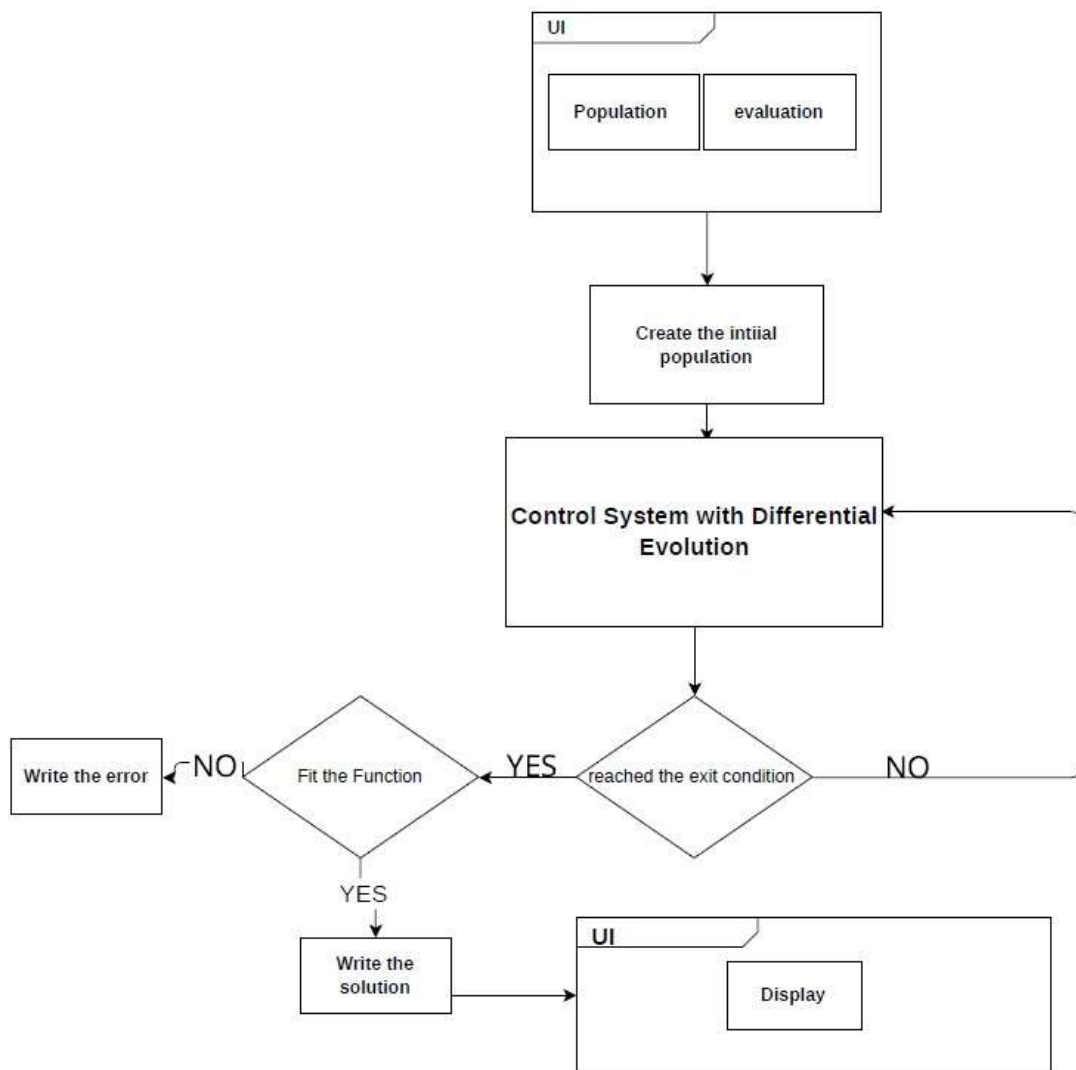
4-Penalizing Strategies.

In the research, the Reject strategy and the Penalizing strategy have been used. By generating any new solution using the crossover operator, the feasibility of the solution should be checked during the algorithm. If the solution is feasible, the algorithm calculates its fitness function. Otherwise, the algorithm rejects the solution and again generates a new solution. For flexibility of the algorithm in cases where violations of the limitations are defined, the infeasible solution with better fitness function from previous solutions is accepted (survived) with a penalty. (This penalty is added to the calculated fitness value in the objective function). The flowchart of the proposed DE algorithm.

Flow Chart describe the Differential Evolution



Block Diagram describe the Differential Evolution Algorithms



Experiments & Results

In this section, by solving the examples of the problem, the performance of the proposed algorithm can be examined.

Parameter setting

One of the ways to improve the process is to select the levels of factors in a way that minimizes the impact of disruptive factors. In this case, the goal is to find the optimal interplay between control factors and disruptive factors to reduce the impact of disturbances. The method used to control the parameters of the DE algorithm in this paper is the Taguchi experimental design method.

Taguchi designs provide a mechanism to recognize control factors that reduce variability in a process or production by decreasing the effects of uncontrollable factors.

Uncontrollable factors (noise factors) are those parameters that cannot be controlled during the process or product use but can be controlled during experiments. Higher values of the signal-to-noise ratio (S/N) identify control factor settings that minimize the effects of the noise factors (Khurana and Banerjee 2018). For the experimental design, three instances are solved with the parameters setting of the DE algorithm for the main parameters was performed at five initial levels presented in the next Table , per ten runs, with the goal of minimizing the

Table 5 Suggested levels for DE factors

Parameters	Symbol	Suggested levels		
Population size	<i>Pop size</i>	50	70	90
Recombination probability (Probability of accepting a mutated gene)	<i>Pr</i>	0.2	0.4	0.6
Beta factor	<i>B</i>	0.1	0.15	0.2

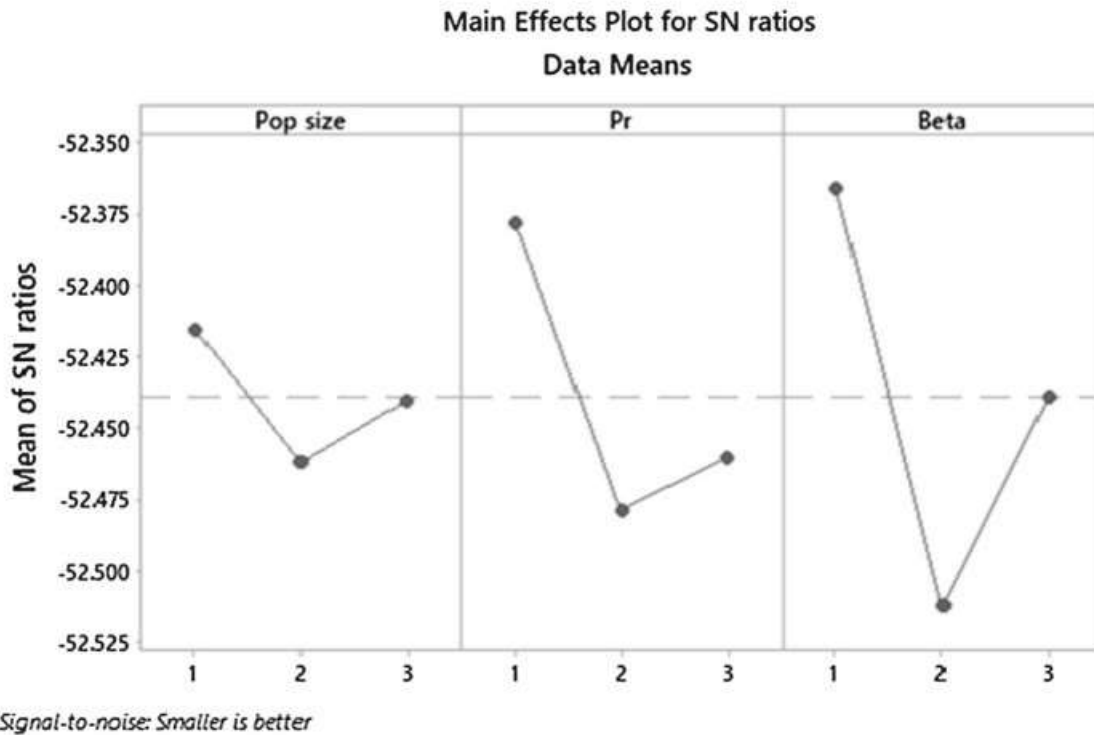


Fig. 15 Taguchi signal-to-noise ratio (S/N) chart plotted by Minitab for DE parameters

average of the test. By examining the input values, it was found that the effect of the changes of the three parameters is greater than the other two main parameters in the problem solving, and the values of those parameters were considered constant, i.e., the number of iterations (50) and the size of the population (100). In the following, each instance was executed ten times for three factors on three levels. Due to the equivalent of multiplication of the number of examples, effective parameters and parameter levels ($3 \times 3 \times 3$) equal to 27 designs were generated. Each of the 27 designs was executed ten times, and the mean of the data was analyzed with Minitab 17 by L27(33) Taguchi Orthogonal Arrays. The analysis result is presented in Fig. 15.

Comparative study

To verify the model and proposed DE algorithm, five simple instances of the problem in small dimension (with only one expertise and one scenario) was written in IBM ILOG CPLEX Optimization Studio v12.6 (for exact solution) and MATLAB R2015b –Academic User (for DE & GA solution comparison) run on a PC with a Windows 10 (64-bit Operating System), 16.00 GB (RAM), Intel(R) Core™ i7-4790K @4.00GHz processor. The reason for the selection of the genetic algorithm for comparing the results is the similarity of these two algorithms and its research validity based on its being the most used by researchers to compare with other algorithms. Due to the complexity of the problem, the CPLEX software was able to solve three examples with exact methods in reasonable memory capacity. The other CPLEX limit is due to the number of variables and problem constraints. For 13 examples, according to the DOE results, proposed DE algorithm (population size 100, number of iteration 50, Recombination probability 0.6 and Scaling factor 0.2) and GA (population size 100, number of iteration 50, Recombination probability 0.2 with roulette wheel and two-point crossover) were run ten times. The average processing time and the best of the obtained results (Z) of the algorithms are given in Table 6. In this table, the best solution is bold and the best solving time is underlined. As can be seen in Table 6, by increasing the number of nurses, the number of shiftdays, and nurse expertise types, solving time increases exponentially. This increase is due to the enlargement of the dimensions of the problem as well as the increase in the number of constraints that the problem has to meet. As the population size grows, better exploration of the solution space occurs, and by increasing the number of iterations, the convergence of the algorithm increases to the optimal solution. Due to the limitation of the processing time, as well as the memory, there must be a balance between the quality of the solution, the time, and the volume of the calculations. For the comparison of the DE algorithm presented with the Ga algorithm, 25 instances with three scenarios and 15 instances with four, five and six scenarios were executed ten times, and the average processing time and the minimum objective function value (Z) for the algorithms are given in Tables 7 and 8. A graph of the average processing time and minimum objective function value of the DE algorithm and GA are presented in Fig. 16. In Fig. 16, comparing the processing time for two algorithms, despite the similarity of the two algorithms to each other, the solving time of the genetic algorithm (the red points) increased with

the increasing complexity of the problem compared to the DE algorithm (the blue points) except in two problems. This difference ranged from a few tenths of a second to a few seconds in simpler examples to 177 seconds in the final example. Another criterion for examining two algorithms is the quality of the solutions of these two algorithms regarding the best solution obtained from each one that is presented in Fig. 17. The five instances with the features presented in Table 9 are solved with the number of different scenarios (one, three, four, five, and six scenarios/the points respectively in the two following charts are marked blue, red, black, orange and green) with the algorithm, and the least time for each problem is shown in Fig. 18. From the diagram, it can be concluded that the larger number of scenarios leads to an increase in the average processing time of the algorithm. The difference between the time of the single scenario and the multi-scenario is because the algorithm needs more time to solve the problem and to examine the constraints under the scenarios that were added to the model. Thus, the solving time

Table 6 Comparison of minimum objective function value and average processing time for DE, GA and CPLEX (with one scenario)

Instance (f = 1)	Num of nurses	Expertise types	Shift- types	Planning horizon (day)	DE		GA		CPLEX	
					Z	Time (s)	Z	Time (s)	Z	Time (s)
Small dimension										
1	5	1	2	3	1254	1.83	1254	2.06	1254	<u>0.63</u>
2	7	1	2	3	1618	2.92	1618	3.23	1618	<u>0.82</u>
3	10	1	3	4	295	7.48	300	8.67	293	<u>1.67</u>
4	12	1	3	4	3070	<u>16.4</u>	3510	17.23	–	–
5	15	1	3	7	9950	<u>20.69</u>	1279	22.67	–	–
Large dimension										
6	15	1	3	15	2402	<u>26.34</u>	2546	31.52	–	–
7	21	1	3	30	2840	<u>47.74</u>	3493	48.48	–	–
8	30	1	3	15	2006	38.31	2737	<u>34.69</u>	–	–
9	(21 + 30)	2	3	30	6185	<u>107.28</u>	7270	147.46	–	–
10	(30 + 15)	2	3	30	8255	<u>122.85</u>	8312	151.62	–	–
11	(42 + 30)	2	3	15	12,196	<u>90.94</u>	12,881	177.75	–	–
12	(42 + 30 + 30)	3	3	30	16,738	<u>285.32</u>	19,253	492.7	–	–
13	(50 + 50)	3	3	30	18,263	<u>305.14</u>	22,658	564.89	–	–

is proportional to the number of scenarios considered by the decision-maker and the number of uncertainty factors applied in the model. Figure 19 is drawn based on the minimum objective function value obtained from the solutions for ten execution times. As the number of considered scenarios increases, the objective function value of the best solution will not be better. By increasing the number of scenarios, the cost gap increases. For the first instance, the cost difference between a problem with a scenario and with six scenarios is about 294 units. The difference in cost in the fifth example reaches 3520 monetary units. If there are several active constraints in different scenarios, the algorithm will consider the worst possible scenario and will base its calculation on that. In the proposed algorithm, the conditions governing the system (i.e. the special conditions of nurses, labor law constraints, the age and professional limitations of nurses) can be applied by coefficients to the constraints and cost parameters of the objective function. This weighting can be considered from the importance of its limitations.

Table 7 Comparison of minimum objective function value and average processing time for DE and GA (with three scenarios)

Instance (f = 3)	Num of nurses	Expertise types	Shift- types	Planning horizon (day)	DE		GA	
					Z	Time (s)	Z	Time (s)
1	15	1	3	7	1294	<u>52.86</u>	1405	58.67
2	15	1	3	15	2450	<u>68.19</u>	2897	75.69
3	30	1	3	15	2047	<u>100.43</u>	2892	112.48
4	21	1	2	15	2586	101.72	2474	<u>98.78</u>
5	30	1	2	45	4928	<u>112.26</u>	5487	124.60
6	21	1	3	30	2879	<u>125.90</u>	3653	139.74
7	21	1	3	45	3732	<u>137.66</u>	3429	152.80
8	30	1	3	30	4963	<u>160.67</u>	5146	176.36
9	42	1	3	45	6398	224.20	7143	<u>216.34</u>
10	50	1	3	15	9528	<u>238.70</u>	9604	245.16
11	(42 + 30)	2	3	15	12,444	<u>242.53</u>	14,997	269.20
12	50	1	2	30	7106	<u>262.03</u>	8112	279.42
13	50	1	3	30	8495	<u>278.96</u>	9668	283.41
14	(21 + 30)	2	3	30	6309	<u>276.65</u>	7415	318.18
15	(30 + 15)	2	3	30	8521	<u>328.68</u>	8478	364.83
16	60	1	3	30	12,408	<u>384.64</u>	14,651	414.08
17	70	1	2	45	16,177	<u>475.35</u>	16,507	499.84
18	70	1	3	15	11,591	<u>482.19</u>	15,826	529.27
19	90	1	2	30	17,543	<u>501.26</u>	17,845	540.69
20	90	1	2	45	18,852	<u>508.98</u>	19,432	522.85
21	90	1	3	30	15,894	<u>554.83</u>	16,120	626.77
22	100	1	3	15	18,537	<u>728.85</u>	19,069	842.97
23	(42 + 30 + 30)	3	3	30	18,093	<u>767.36</u>	19,138	881.76
24	100	1	2	45	20,173	<u>771.01</u>	20,195	912.24
25	(50 + 50)	2	3	30	18,798	<u>794.78</u>	19,086	946.54

Advantages

Evolution isn't a mechanism. It's an effect. Evolution is what happens when the environment changes and some animals can survive the change and others can't.

If there's an advantage in the process, it's that individuals have slight variations in their DNA. That means if the environment gets hotter, the animals best adapted to the cooler temperature will die off but some better able to handle heat will survive. Or if the environment gets wetter, a different set of animals will die and a different set will thrive. No matter how the environment changes, if there is variation in the DNA, it's more likely that some of the group will die and some will survive.

Sometimes the environment changes too drastically and whole groups are wiped out. The DNA doesn't have enough variation to work in the new environment.

It's hard to discuss evolution without making it sound like it's a thing with a goal. It isn't. Language just lacks words to describe mechanisms that act randomly but give the illusion of intelligent choice. We say water runs down hill. We say it gathers in valleys to form lakes. When humans run and gather they've decided to. But no learned person presumes water decides to run and gather.

What can help if when you read the word "evolution" is to replace it with "change". Evolution is more complex than just change. It's the cumulative effect of three or more different changes that give the illusion of choice. But, substituting the word change can help the mind stop thinking of evolution as a thing making choices.

What are the advantages of change? None! Sometimes change can be bad, sometimes change can be good.

One difference between random change and evolution is the general trend with evolution is towards living things that are more complex. Change in the environment + variations in DNA + death to the unfit = survivors that are usually a bit more complex. A one celled creature that divides but — because of some random change in its genes — stays stuck together has some extra instructions that one that separates doesn't. It's more complex.

(Sometimes less complex is more advantageous. More complex takes more energy and is more prone to damage. That's why creatures evolving in caves become less complex. Eyes are expensive and prone to damage. The more complex structures that aren't needed create a disadvantage. (Though their feelers are more complex.) But, on the whole, greater complexity more often creates more of an advantage than less complexity.)

If that random sticking makes the cells better able to survive, the single cells won't be able to compete as well. Death will remove the genes that tell the creatures to separate. (But, since the changes are random, it could have ended up being a disadvantage.)

The effect of two stuck cells thriving in an environment better than single cells gives the illusion that something knew how to improve that cell for that environment. But what's creating the illusion is death. Death wipes out all the random changes that don't work as well.

If we could see all the changes we could see the randomness. We'd see fatter and thinner cells, bigger and smaller cells, cells with thinner walls and thicker walls, lighter and darker cells. If a random change ups the cells ability to get food and survive the environment, it thrives. If a random change lowers the cells ability to get food and survive the environment, death removes it. Death removes the changes that don't work for the environment the cells are in. What's left behind is the illusion that something is making choices to create a better creature.

Do you know why every previous decade had better music than the current decade? That's because we're always living in an age of a few good songs mixed with crap filler songs. Over time the crap filler songs die off leaving behind just the good stuff. Some decades are more innovated but that's because — no surprise — the environment is more chaotic. The dying off of the songs that can't survive creates the illusion that past decades had better song writers.

Evolutionary change works the same way. Except that it isn't humans deciding what differences work and what don't. It's death.

Disadvantages

A differential df at a point is an approximation to the change in the value of $f(x)$ as given by the tangent line to the curve $y=f(x)$. If the slope of the curve changes rapidly near the point, approximation by differentials will not be very effective. They will also not be effective at a substantial distance from the point, unless the slope is a constant.

$df(x,dx) = f'(x).dx$, where dx is an arbitrary change in the value of x .

Similarly for a function $f(x,y)$, $df(x,y,dx,dy)$ is defined as $(\partial f/\partial x).dx + (\partial f/\partial y).dy$, where dx and dy are arbitrary changes in the values of x and y respectively.

Future Work:

Nurses who received high penalty schedules in previous iterations of the schedule construction will be considered early in the current iteration, which reduces the chance of having trouble with generating their individual schedule later on in the process. The approach is simple and efficient. It is also observed that the greedy local search carried out between the schedule constructions for each nurse greatly improved the roster constructed. This indicates that the approach can be easily adapted for hybridization with other techniques (i.e. exact methods and meta-heuristics). It will be particularly interesting to perform a study using exact methods to obtain the best combinations for high quality schedules or rosters.

Operating on sequences, rather than on individual shifts by meta-heuristics, is also an interesting direction for future work.

1-Optimize the payment for the nurses.

2-Optimize the payment/shifts for all staff.

Development Platform

a- programming language:

Python

b- libraries:

numpy

c- IDE:

1- Visual studio code

2- jupyter