Examen Final de Matemáticas III

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1.- Use fracciones parciales para encontrar la sig. transformada inversa de Laplace.

$$\mathcal{L}^{-1}\left\{\frac{11s^2-10s+11}{(s^2+1)(s^2-2s+5)}\right\} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2-2s+5}$$

$$11s^2-10s+11 = (As+B)(s^2-2s+5)+(Cs+D)(s^2+1)$$
$$= As^3-2As^2+5As+Bs^2-2Bs+5B+Cs^3+Cs+Ds^2+D$$
$$= s^3(A+C)+s^2(-2A+B+D)+s(5A-2B+C)+(5B+D)$$

Ecuaciones $\longrightarrow$

$A+C=0$ --- (1) $\longrightarrow$ $A=-C$ (1)

$-2A+B+D=11$ --- (2)

$5A-2B+C=-10$ --- (3)

$5B+D=11$ --- (4)

Sust. B en (2)

$\cancel{-2A}+5+\cancel{2A}+D=11$

$\underline{\underline{D=6}}$

Sust. (1) en (3)

$5(-C)-2B+C=-10$

$-4C-2B=-10$

$C=\dfrac{10-2B}{4}$

Sust. C en (3)

$5A-2B+2.5-0.5B=-10$

$-2.5B=-12.5-5A$

$B=5+2A$

Sust. D en (4)

$5B+D=11$

$5B+6=11$

$5B=5$

$\underline{\underline{B=1}}$

Sust. B y D en (4)

$5B+D=11$

$5(5+2A)+6=11$

$25+10A=5$

$A=\dfrac{-20}{10}=\underline{\underline{-2}}$

Sust. A en (1)

$A+C=0$

$-2+C=0$

$\underline{\underline{C=2}}$

Sust. Valores $\rightarrow$ $\dfrac{-2s+1}{s^2+1}+\dfrac{2s+6}{s^2-2s+5}$ $\longrightarrow$ Desarr. $\dfrac{2s+6}{s^2-2s+5}=2\cdot\dfrac{s+1}{(s-1)^2+4}+8\cdot\dfrac{1}{(s-1)^2+4}$

$$\therefore \mathcal{L}^{-1}\left\{\frac{-2s+1}{s^2+1}+2\cdot\frac{s-1}{(s-1)^2+4}+8\cdot\frac{1}{(s-1)^2+4}\right\}$$

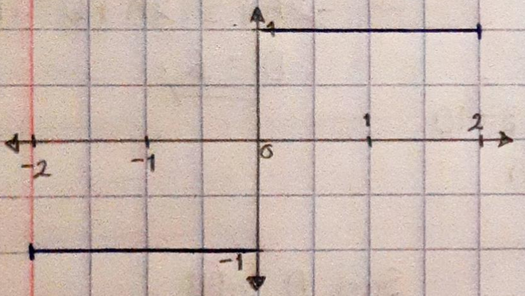
$$\mathcal{L}^{-1}\left\{\frac{2s}{s^2+1} + \frac{1}{s^2+1} + 2\cdot\frac{s-1}{(s-1)^2+4} + 8\cdot\frac{1}{(s-1)^2+4}\right\}$$

$$= -2\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\} + 8\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+4}\right\}$$

$$= \underline{\underline{-2\cos(t) + \operatorname{sen}(t) + 2e^t\cos(2t) + 4e^t\operatorname{sen}(2t)}} \;//$$

2.- Grafique la función sig. y encuentre su serie de Fourier

$$f(x) = \begin{cases} = -1; & -2 < x < 0 \\ = 1; & 0 < x < 2 \end{cases} \qquad \text{Periodo} = 4 \;\therefore\; L = 2$$



$$a_0 = \frac{1}{2}\int_{-2}^{0} -\,dx + \frac{1}{2}\int_{0}^{2} dx$$

$$a_0 = -\frac{1}{2}x\Big|_{-2}^{0} + \frac{1}{2}x\Big|_{0}^{2} = 0 - \cancel{\frac{2}{2}} + \cancel{\frac{2}{2}} + 0 = \underline{\underline{0}}\;/$$

$$a_n = \frac{1}{2}\int_{-2}^{0} -\cos\left(\frac{n\pi x}{2}\right)dx + \frac{1}{2}\int_{0}^{2}\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= -\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{n\pi}\operatorname{sen}\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{-2}^{0} + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{n\pi}\operatorname{sen}\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{0}^{2}$$

$$= \frac{1}{n\pi}\left[-\cancel{\operatorname{sen}\left(\frac{n\pi 0}{2}\right)}^{0} + \cancel{\operatorname{sen}\left(\frac{\pi n 2}{2}\right)}^{0}\right] + \left[\cancel{\operatorname{sen}\left(\frac{\pi n 2}{2}\right)}^{0} - \cancel{\operatorname{sen}\left(\frac{n\pi 0}{2}\right)}^{0}\right]$$

$$\underline{\underline{a_n = 0}}\;/$$

$$b_n = \frac{1}{2}\int_{-2}^{0} -\operatorname{sen}\left(\frac{n\pi x}{2}\right)dx + \frac{1}{2}\int_{0}^{2}\operatorname{sen}\left(\frac{n\pi x}{2}\right)dx$$

$$b_n = \left[\frac{1}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\right]_{-2}^{0} + \left[-\frac{1}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\right]_{0}^{2}$$

$$= \frac{1}{n\pi}\left[\cos(0) - \cos(-n\pi)\right] - \frac{1}{n\pi}\left[\cos(n\pi) - \cos(0)\right]$$

$$= \frac{1}{n\pi}\left(1 - (-1)^n\right) - \frac{1}{n\pi}\left((-1)^n - 1\right)$$

$$= \frac{1}{n\pi} - \frac{(-1)^n}{n\pi} - \frac{(-1)^n}{n\pi} + \frac{1}{n\pi}$$

$$= \frac{2}{n\pi} - \frac{2(-1)^n}{n\pi} = \frac{2}{n\pi}\left(1 - (-1)^n\right)$$

Serie de Fourier:

$$f(x) = \sum_{n=1}^{\infty} \frac{2\left(1 - (-1)^n\right)}{n\pi} \operatorname{sen}\left(\frac{n\pi x}{2}\right)$$