Answers: Chapter 1 - Binomial Distributions

Pearson Edexcel International A Level Statistics 2

Instructions: Answer all questions, showing clear working where appropriate. You may use a calculator, and refer to binomial cumulative distribution tables where applicable.

1. **Understanding the Binomial Distribution Definition and Conditions** A random variable X is said to follow a binomial distribution, denoted as $X \sim B(n, p)$.

   (a) The four conditions that a random variable must satisfy for it to be modeled by a binomial distribution are:
   - There is a **fixed number of trials**, n.
   - Each trial has only **two possible outcomes** (success or failure).
   - The probability of success, p, is **constant** for each trial.
   - The trials are **independent** of each other.

   (b) The number of defective electronic components can be modeled by a binomial distribution because there is a fixed number of trials ($n = 20$ components), each trial has two outcomes (defective or not defective), and the probability of a defective component is assumed to be constant for each component. The random selection implies that the trials are independent.

2. **Calculating Probabilities using the Binomial Probability Formula** Let X be a random variable such that $X \sim B(10, 0.3)$.

   (a) The binomial probability formula is $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$.

$$
\begin{aligned}
P(X = 4) &= \binom{10}{4} (0.3)^4 (1 - 0.3)^{10-4} \\
&= 210 \times (0.3)^4 \times (0.7)^6 \\
&= 210 \times 0.0081 \times 0.117649 \\
&\approx 0.2001
\end{aligned}
$$

   (b) For a multiple-choice test with 8 questions and 4 options per question, the number of trials is $n = 8$ and the probability of a correct guess is $p = \frac{1}{4} = 0.25$. Let $Y \sim B(8, 0.25)$.

$$
\begin{aligned}
P(Y = 3) &= \binom{8}{3} (0.25)^3 (1 - 0.25)^{8-3} \\
&= 56 \times (0.25)^3 \times (0.75)^5 \\
&= 56 \times 0.015625 \times 0.2373 \\
&\approx 0.2076
\end{aligned}
$$

3. **Working with Cumulative Probabilities** A biased coin is tossed 15 times, with $p = 0.6$ for a head. Let $H \sim B(15, 0.6)$.

(a) The probability of obtaining no more than 7 heads is $P(H \leq 7)$. Using a binomial cumulative distribution table or calculator, we find:

$$P(H \leq 7) \approx 0.2131$$

(b) The probability of obtaining at least 10 heads is $P(H \geq 10)$. We use the identity $P(H \geq 10) = 1 - P(H \leq 9)$. From the cumulative distribution table or calculator, $P(H \leq 9) \approx 0.8752$.

$$P(H \geq 10) = 1 - P(H \leq 9) = 1 - 0.8752 = 0.1248$$

4. **Calculating Mean and Variance of a Binomial Distribution** For a random variable $X \sim B(25, 0.4)$.

   (a) The mean (expected value) is $E(X) = np$.

   $$E(X) = 25 \times 0.4 = 10$$

   (b) The variance is $Var(X) = np(1-p)$.

   $$Var(X) = 25 \times 0.4 \times (1 - 0.4) = 25 \times 0.4 \times 0.6 = 6$$

   The standard deviation is the square root of the variance.

   $$\text{Standard Deviation} = \sqrt{Var(X)} = \sqrt{6} \approx 2.449$$

5. **Problem-Solving and Real-World Applications** A manufacturer states that 15% of its light bulbs are faulty. A quality control inspector randomly selects 12 light bulbs.

   (a) The distribution of the number of faulty light bulbs, F, is a binomial distribution with parameters $n = 12$ and $p = 0.15$.

   $$F \sim B(12, 0.15)$$

   (b) We need to find the probability that there are exactly 2 faulty light bulbs, $P(F = 2)$.

   $$\begin{aligned} P(F = 2) &= \binom{12}{2}(0.15)^2(0.85)^{10} \\ &= 66 \times 0.0225 \times 0.19687 \\ &\approx 0.2924 \end{aligned}$$

6. **Finding Unknown Parameters** A random variable X follows a binomial distribution $B(n, p)$.

   (a) Given that $E(X) = 6$ and $n = 20$:

   $$E(X) = np \implies 6 = 20p \implies p = \frac{6}{20} = 0.3$$

(b) Given that $E(X) = 4.8$ and $Var(X) = 2.88$:

$$E(X) = np = 4.8 \quad (1)$$

$$Var(X) = np(1-p) = 2.88 \quad (2)$$

Substitute (1) into (2):

$$4.8(1-p) = 2.88$$
$$1 - p = \frac{2.88}{4.8} = 0.6$$
$$p = 1 - 0.6 = 0.4$$

Substitute $p = 0.4$ back into (1):

$$n(0.4) = 4.8 \implies n = \frac{4.8}{0.4} = 12$$

The values are $n = 12$ and $p = 0.4$.

7. **Comprehensive Problem-Solving / Justification (Exam-style)** A market research firm conducts a telephone survey, with the probability of a completed survey being $p = 0.2$. A researcher makes 15 calls.

(a) Let S be the number of completed surveys. Two assumptions needed to model S using a binomial distribution are:

- **Independence of trials:** Each phone call is independent of the others. This is a reasonable assumption as the result of one call is unlikely to influence the outcome of another.
- **Constant probability of success:** The probability of a person answering and completing the survey is constant for each call. This is reasonable if the calls are made to a random sample under similar conditions.

(b) We need to find $P(3 \leq S < 6)$, which is equivalent to $P(S \leq 5) - P(S \leq 2)$. For $S \sim B(15, 0.2)$: Using a cumulative distribution table or a calculator:

$$P(S \leq 5) \approx 0.9389$$

$$P(S \leq 2) \approx 0.6482$$

$$P(3 \leq S < 6) = 0.9389 - 0.6482 = 0.2907$$