

## Classwork: Chapter 1 - Binomial Distributions

### Pearson Edexcel International A Level Statistics 2

**Instructions:** Answer all questions, showing clear working where appropriate. You may use a calculator, and refer to binomial cumulative distribution tables (if provided, e.g., on pages 139-143 of the Student Book [1, 2]) for cumulative probabilities where applicable.

1. **Understanding the Binomial Distribution Definition and Conditions** A random variable  $X$  is said to follow a binomial distribution, denoted as  $X \sim B(n, p)$ .
  - (a) State the four conditions that a random variable must satisfy for it to be modeled by a binomial distribution [28 (point 1)].
  - (b) A company manufactures electronic components. A sample of 20 components is randomly selected and tested for defects. Explain why the number of defective components in this sample can be modeled by a binomial distribution, stating any assumptions made [17 (Q3, Q7a, Q8a), 22 (Q9a), 28 (point 1)].
2. **Calculating Probabilities using the Binomial Probability Formula** Let  $X$  be a random variable such that  $X \sim B(10, 0.3)$ .
  - (a) Calculate  $P(X = 4)$  using the binomial probability formula, showing your working [15, 16 (Example 1a)].
  - (b) A student guesses the answers to a multiple-choice test with 8 questions, each having 4 options, only one of which is correct. Let  $Y$  be the number of correct answers. Find the probability that the student guesses exactly 3 questions correctly [16 (Example 1b)].
3. **Working with Cumulative Probabilities** A biased coin is tossed 15 times. The probability of landing a head in a single toss is 0.6. Let  $H$  be the number of heads obtained.
  - (a) Find the probability of obtaining no more than 7 heads, i.e.,  $P(H \leq 7)$  [19 (phrase interpretation), 21 (Q1a)].
  - (b) Find the probability of obtaining at least 10 heads, i.e.,  $P(H \geq 10)$ . You may use the identity  $P(H \geq x) = 1 - P(H \leq x - 1)$  and binomial cumulative distribution tables or a calculator function [19 (phrase interpretation), 21 (Q2b)].
4. **Calculating Mean and Variance of a Binomial Distribution** For a random variable  $X \sim B(25, 0.4)$ .
  - (a) Calculate the mean (expected value) of  $X$ ,  $E(X)$ , using the appropriate formula [23 (Example 7), 28 (point 3)].
  - (b) Calculate the variance of  $X$ ,  $Var(X)$ , and the standard deviation of  $X$  [23 (Example 7), 28 (point 3)].
5. **Problem-Solving and Real-World Applications** A manufacturer states that 15% of its light bulbs are faulty. A quality control inspector randomly selects 12 light bulbs from a large batch.

- (a) State the distribution of the number of faulty light bulbs,  $F$ , in the sample, including its parameters [17 (Q5a)].
  - (b) Find the probability that there are exactly 2 faulty light bulbs in the sample [17 (Q5b)].
6. **Finding Unknown Parameters** A random variable  $X$  follows a binomial distribution  $B(n, p)$ .
- (a) Given that  $E(X) = 6$  and  $n = 20$ , find the value of  $p$  [24 (Q2)].
  - (b) Given that  $E(X) = 4.8$  and  $Var(X) = 2.88$ , find the values of  $n$  and  $p$  [25 (Q5)].
7. **Comprehensive Problem-Solving / Justification (Exam-style)** A market research firm conducts a telephone survey. From past experience, the probability that a randomly chosen person will answer the phone and complete the survey is 0.2. A researcher makes 15 calls.
- (a) Let  $S$  be the number of people who answer the phone and complete the survey. State two assumptions that are necessary to model  $S$  using a binomial distribution and explain why these assumptions might be reasonable in this context [17 (Q3, Q7a, Q8a), 22 (Q9a), 28 (point 1)].
  - (b) Find the probability that the researcher completes at least 3 but fewer than 6 surveys, i.e.,  $P(3 \leq S < 6)$  [19 (phrase interpretation), 21 (Q2c)].