Session 8: Differentiation Practice - Logarithmic Functions

November 14, 2024

Differentiate the following functions:

1. $y = 10^x$
   $\frac{dy}{dx} = 10^x \ln(10)$
2. $y = 2^{x^2+3x}$
   $\frac{dy}{dx} = 2^{x^2+3x} \ln(2) \cdot (2x+3)$
3. $y = 5^{\sin(x)}$
   $\frac{dy}{dx} = 5^{\sin(x)} \ln(5) \cdot \cos(x)$
4. $y = e^{2x} + 10^{3x}$
   $\frac{dy}{dx} = 2e^{2x} + 3 \cdot 10^{3x} \ln(10)$
5. $y = x^2 \cdot 3^x$
   $\frac{dy}{dx} = 2x \cdot 3^x + x^2 \cdot 3^x \ln(3)$
6. $y = \frac{2^x}{x+1}$
   $\frac{dy}{dx} = \frac{2^x \ln(2) \cdot (x+1) - 2^x}{(x+1)^2}$
7. $y = 7^{\sqrt{x^2+1}}$
   $\frac{dy}{dx} = 7^{\sqrt{x^2+1}} \ln(7) \cdot \frac{x}{\sqrt{x^2+1}}$
8. $y = \log_{10}(2^x)$
   $\frac{dy}{dx} = \frac{1}{2^x \ln(10)} \cdot 2^x \ln(2) = \frac{\ln(2)}{\ln(10)}$
9. $y = 3^{x^3+2x} \cdot 5^{x^2-1}$
   $\frac{dy}{dx} = 3^{x^3+2x} \ln(3) \cdot (3x^2+2) \cdot 5^{x^2-1} + 3^{x^3+2x} \cdot 5^{x^2-1} \ln(5) \cdot 2x$

10. $y = (\sin(x))^{2^x}$ (Hint: Use logarithmic differentiation)
    $\ln(y) = 2^x \ln(\sin(x))$
    $\frac{1}{y}\frac{dy}{dx} = 2^x \ln(2) \ln(\sin(x)) + 2^x \cdot \frac{\cos(x)}{\sin(x)}$
    $\frac{dy}{dx} = (\sin(x))^{2^x} \left[2^x \ln(2) \ln(\sin(x)) + 2^x \cot(x)\right]$
11. $y = \ln(x^2 + 1)$
    $\frac{dy}{dx} = \frac{2x}{x^2+1}$
12. $y = \log_2(5x^3 - 2x)$
    $\frac{dy}{dx} = \frac{1}{(5x^3-2x)\ln(2)} \cdot (15x^2 - 2)$
13. $y = x^2 \ln(x)$
    $\frac{dy}{dx} = 2x \ln(x) + x^2 \cdot \frac{1}{x} = 2x \ln(x) + x$
14. $y = \frac{\ln(x)}{x+1}$
    $\frac{dy}{dx} = \frac{\frac{1}{x}\cdot(x+1) - \ln(x)}{(x+1)^2} = \frac{x+1-x\ln(x)}{x(x+1)^2}$
15. $y = \ln(\sin(x))$
    $\frac{dy}{dx} = \frac{1}{\sin(x)} \cdot \cos(x) = \cot(x)$
16. $y = \ln(\sqrt{x^2 + 4})$
    $\frac{dy}{dx} = \frac{1}{\sqrt{x^2+4}} \cdot \frac{x}{\sqrt{x^2+4}} = \frac{x}{x^2+4}$
17. $y = e^x \ln(x)$
    $\frac{dy}{dx} = e^x \ln(x) + e^x \cdot \frac{1}{x} = e^x \left(\ln(x) + \frac{1}{x}\right)$
18. $y = \ln(x^3 + 3x^2 + 3x + 1)$
    $\frac{dy}{dx} = \frac{3x^2+6x+3}{x^3+3x^2+3x+1} = \frac{3(x+1)^2}{(x+1)^3} = \frac{3}{x+1}$
19. $y = \ln(\sec(x) + \tan(x))$
    $\frac{dy}{dx} = \frac{1}{\sec(x)+\tan(x)} \cdot (\sec(x)\tan(x) + \sec^2(x)) = \sec(x)$
20. $y = \ln|\ln(x)|$
    $\frac{dy}{dx} = \frac{1}{\ln(x)} \cdot \frac{1}{x} = \frac{1}{x\ln(x)}$
21. $y = \sqrt{\ln(x)}$
    $\frac{dy}{dx} = \frac{1}{2\sqrt{\ln(x)}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln(x)}}$
22. $y = \ln(\arctan(x))$
    $\frac{dy}{dx} = \frac{1}{\arctan(x)} \cdot \frac{1}{1+x^2} = \frac{1}{(1+x^2)\arctan(x)}$

23. $y = x\ln(x) - x$

    $\frac{dy}{dx} = \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x)$

24. $y = \ln(x^2 + 2x + 1)$

    $\frac{dy}{dx} = \frac{2x+2}{x^2+2x+1} = \frac{2(x+1)}{(x+1)^2} = \frac{2}{x+1}$

25. $y = [\ln(x)]^{\sin(x)}$ (Hint: Use logarithmic differentiation)

    $\ln(y) = \sin(x)\ln(\ln(x))$

    $\frac{1}{y}\frac{dy}{dx} = \cos(x)\ln(\ln(x)) + \sin(x) \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x}$

    $\frac{dy}{dx} = [\ln(x)]^{\sin(x)}\left[\cos(x)\ln(\ln(x)) + \frac{\sin(x)}{x\ln(x)}\right]$