Binomial Distribution Solutions

Solutions

1. **Basic Formula Usage** The binomial probability formula is given by $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$, where n is the number of trials, k is the number of successes, and p is the probability of success on a single trial.

   a) Here, $n = 5$, $k = 3$, and $p = 0.6$. $P(X = 3) = \binom{5}{3}(0.6)^3(1 - 0.6)^{5-3} = 10 \times (0.6)^3 \times (0.4)^2 = 10 \times 0.216 \times 0.16 = 0.3456$.

   b) Here, $n = 10$, $k = 7$, and $p = 1/4 = 0.25$. $P(X = 7) = \binom{10}{7}(0.25)^7(1 - 0.25)^{10-7} = 120 \times (0.25)^7 \times (0.75)^3 \approx 120 \times 0.000061 \times 0.421875 \approx 0.00309$.

   c) Here, $n = 8$, $k = 1$, and $p = 0.02$. $P(X = 1) = \binom{8}{1}(0.02)^1(1 - 0.02)^{8-1} = 8 \times 0.02 \times (0.98)^7 \approx 0.16 \times 0.8681 \approx 0.1389$.

2. **Cumulative Probabilities**

   a) We need to find $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$. Here, $n = 6$ and $p = 0.3$. $P(X = 0) = \binom{6}{0}(0.3)^0(0.7)^6 = 1 \times 1 \times 0.117649 = 0.117649$. $P(X = 1) = \binom{6}{1}(0.3)^1(0.7)^5 = 6 \times 0.3 \times 0.16807 = 0.302526$. $P(X = 2) = \binom{6}{2}(0.3)^2(0.7)^4 = 15 \times 0.09 \times 0.2401 = 0.324135$. $P(X \leq 2) = 0.117649 + 0.302526 + 0.324135 = 0.74431$.

   b) We need to find $P(X \geq 10) = P(X = 10) + P(X = 11) + P(X = 12)$. Here, $n = 12$ and $p = 0.75$. $P(X = 10) = \binom{12}{10}(0.75)^{10}(0.25)^2 = 66 \times 0.0563 \times 0.0625 \approx 0.2323$. $P(X = 11) = \binom{12}{11}(0.75)^{11}(0.25)^1 = 12 \times 0.0423 \times 0.25 \approx 0.1269$. $P(X = 12) = \binom{12}{12}(0.75)^{12}(0.25)^0 = 1 \times 0.0317 \times 1 \approx 0.0317$. $P(X \geq 10) = 0.2323 + 0.1269 + 0.0317 \approx 0.3909$.

   c) We need to find $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$. Here, $n = 9$ and $p = 0.15$. $P(X = 0) = \binom{9}{0}(0.15)^0(0.85)^9 \approx 0.2316$. $P(X = 1) = \binom{9}{1}(0.15)^1(0.85)^8 \approx 0.3679$. $P(X = 2) = \binom{9}{2}(0.15)^2(0.85)^7 \approx 0.2597$. $P(X < 3) = 0.2316 + 0.3679 + 0.2597 = 0.8592$.

3. **Mean and Variance** The mean of a binomial distribution is $E(X) = \mu = np$ and the variance is $Var(X) = \sigma^2 = np(1-p)$. The standard deviation is $\sigma = \sqrt{np(1-p)}$.

   a) Here, $n = 50$ and $p = 0.8$. Mean: $\mu = 50 \times 0.8 = 40$. Variance: $\sigma^2 = 50 \times 0.8 \times (1 - 0.8) = 50 \times 0.8 \times 0.2 = 8$.

b) Here, $n = 150$ and $p = 0.4$. Mean: $\mu = 150 \times 0.4 = 60$. Standard Deviation: $\sigma = \sqrt{150 \times 0.4 \times (1 - 0.4)} = \sqrt{150 \times 0.4 \times 0.6} = \sqrt{36} = 6$.

c) Here, $n = 1000$ and $p = 0.01$. Mean: $\mu = 1000 \times 0.01 = 10$. Variance: $\sigma^2 = 1000 \times 0.01 \times (1 - 0.01) = 1000 \times 0.01 \times 0.99 = 9.9$.