Optimal Investment Policy in an Open Economy Feldman Model

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Abstract

This paper explores the optimal investment policy for an open economy version of the Feldman model [9], attempting to find what an optimal national investment policy could look like. The study aims to extend the research conducted by C. Ghate et al [4] by extending their results into an open economy model as proposed by Harris [5]. The methodology involves finding optimal values for various parameters through a simulation on an annual basis using the MATLAB computing system. Two main findings are reported: (1) it takes approximately 10 years to eliminate the trade-off between consumption and investment, and (2) Increased investment in the export sector plays a large role in eliminating this trade off. Overall, this paper suggests that an increased proportion of investment going into both the export and heavy industry greatly increase living standards.

1 Introduction and Motivation

A dilemma that often faces developing countries was investment policy with regards to heavy industry, what should be preferred in investment decisions: consumption or heavy industry? Amidst this debate, one figure stands out, that of Grigory Feldman, a Russian mathematician who had developed a model to understand the dynamics of the relationship between heavy industry and consumption. What Feldman discovered was that while large investments in heavy industry could result in a short term decrease in living standards,it offsets in the long term, largely due to the huge expansion of the manufacturing sector which feeds into the production of consumer goods. The Feldman model raises two interesting questions: firstly, what investment patterns are necessary into each sector in order for consumption to be increasing? and secondly, what trade-offs exists between short and long term living standards based on this investment pattern? We aim to do this by exploring the Harris model which generalizes the results of the Feldman model to an open economy.

2 Literature Review

Research on the Feldman model is scant to say the least. The first mention of the model in the English speaking world appears to be in Harrod Domar's book *Essays on The Theory of Economic Growth*. However, it would appear that Domar's exposition of the model is inaccurate, but he is rather conflating Feldman's model with Mahalanobis'. The first exposition of the model in mainstream economics is in Robert Allen's book *Farm To Factory: A Reinterpretation of the Soviet Industrial Revolution*, which gives the clearest exposition of the model. In contrast to the Feldman model, the Mahalanobis model has drawn more attention from the literature, the classic references for which are Ghate et al [4], which was the first to apply optimization techniques on this model. Consequently, our paper is the first to apply these optimization techniques to an open economy model.

3 Historical Background

Before divulging into the model, a brief history of early Soviet Russia is needed in order to understand both the historical and intellectual context into which Feldman's ideas were born.

By 1921, the Bolsheviks had seized control of most of the Russian Empire, however, they inherited a country on the brink of total economic collapse. Due to war communism - wherein the government forcibly took the produce of the peasantry, the urban industrial economy had almost disappeared in the years between 1917 and 1920; the production of pig iron had decreased by 96%, cigarettes by 78%, and by 1921, grain output was 56% below 1913 levels [3]. This, along with the alienation of the peasantry, would result in the creation of the NEP (New Economic Policy), in which the state retained many of the heavy industries it had taken over during war communism, but rather liberalized trade, agriculture, and most soft industry. Its results were immediately noticeable, as economic indicators rebounded to 1913 levels by the late 1920's. However, the situation the NEP produced was inadequate for one of the main objectives of the Bolshevik Revolution, which was rapid industrialization, which the NEP failed in largely due to 2 reasons:

1. When deciding investment under the NEP, businesses would only look towards the profitable enterprises to invest in, ignoring potential benefits for the overall economy when invested in less profitable sectors.

2. Profit based enterprises could not hire workers that did not cover their cost, i.e they would not hire if the worker's marginal output did not cover his wage, this could be mitigated under a planned system, which did not care for profit instead caring more about maximizing output, thus growing the economy through employment expansion.

It is here that the so called "Great Debate" emerges all to answer one question: What is the optimal policy for ensuring rapid industrialization? The political division in Soviet politics of both the right and left opposition emerge from this central question, with figures like Bukharin representing the right opposition, Trotsky and Preobrazhensky representing the left opposition, and Stalin representing the center.

The reasons for wanting rapid industrialization are numerous, but amongst the most important was to achieve Russian industrial and financial Independence from the west, and creating a competent enough national defence industry to defend against potential attacks from Europe, to quote the proclamation of the XV Party Congress

> Taking into account the possible military attack on the... state... it is necessary in the Five Year Plan to pay maximum attention to the fastest possible development of those sectors of the economy and of industry which play the main role in supplying defense and in the economic stability of the country in wartime. Not only planning and economic organs, but, most importantly, the whole party must pay unflagging attention to the issues of defense in connection with compiling the Five Year Plan [6]

Thus, many Russian economists and mathematicians took it as their job to derive a model that could achieve the goals of the party congress, and with the relatively liberal NEP years, it is here that Feldman emerges on the scene.

4 Theoretical Model

Grigory Feldman was a Russian mathematician working at GOSPLAN (the Soviet planning agency) in 1928, the same year the resolution for the creation of the five year plans was passed. He would publish two articles in the journal *Planoveo Khoziaistvo*. In the articles, he developed a mathematical model for capital investment with the aim of answering the following question: Was there a a trade off between rapid accumulation of the capital stock and living standards? The surprising answer was no, high investment could be accompanied with increasing standards of living at the cost of a short term decrease in living standards.

Feldman starts off with Marx's division of the economy into two sectors, consumer goods and producer goods (it is worth noting that this division is the only thing Feldman inherits from Marx). Consumer goods include anything that helps sustain workers such as food, clothing, or general consumables, while producer goods is anything that can be used to expand the existing capital stock or be consumed in other non productive sectors such as the military, housing, or healthcare.

The essential difference between the two is that the consumer goods sector cannot expand using its own product, but instead relies on the producer goods sector's output to expand its own production, to quote Feldman

> Thus defined, sector B (consumer goods) possesses the remarkable property of being capable of existence without sector A (producer goods), but only for purposes of simple reproduction. Thus, starting from an analysis of what is required for a more precise division of output-from the viewpoint of determining the value of consumer goods required to satisfy the existing level of needs-we have arrived at a confirmation of the above idea: that production must be divided into sector B, capable of maintaining consumption at a given level even with a cessation of the inflow of producers' and consumers' goods from sector A to be added to the capital

of sector B, and sector A, which provides both sector B and itself with all the capital required for expansion of reproduction. [9]

$$y_t^p = \alpha K_t^p \tag{1}$$

$$y_t^c = \beta K_t^c \tag{2}$$

the previous two equations represent the output for each sector with y_t^p representing producer goods and y_t^c representing consumer goods at time t, while αK_t^p is the producer goods capital stock at time t, and βK_t^c represents the capital stock of the consumer goods. α and β are the coefficients that relate capital to production. The model is attempting to understand how the capital stock of producer goods affects the capital stock of consumer goods, we can then denote λ to be the amount of the producer's capital stock that is reinvested in it, so we can write the investment of each in the following way.

$$I_t^p = \lambda y_t^p \tag{3}$$

$$I_t^c = (1-\lambda) y_t^c \tag{4}$$

we can then rewrite with all of the previous 4 equations to get the capital stock of each

$$K_t^p = (1-d) K_{t-1}^p + I_t^p \tag{5}$$

$$K_t^c = (1-d) K_{t-1}^c + I_t^c \tag{6}$$

where d is the rate of depreciation of the previous year's capital to which we simply add the investment of the current year in order to reach this year's capital stock for both.

If we solve this difference equation we reach the following solution

$$K_t^p = \frac{(1-d)}{(1-\lambda\alpha)} K_{t-1}^p \tag{7}$$

$$K_t^c = (1-d) K_{t-1}^c + \frac{(1-\lambda)(1-d)\alpha}{(1-\lambda\alpha)} K_{t-1}^p \tag{8}$$

The most important thing to note is that the capital stock of the consumer sector is largely dependant on what is *not* reinvested in the producer stock along with the actual size of the producer capital stock, meaning that, over the long term, as the producer capital stock grows - even if a lower percentage is invested in the consumer capital stock - so too will the consumer capital stock. Thus it can be stated that larger investments in heavy industry will actually increase standards of living over the long term, but likely stagnate or decrease it over the short term.

This then raises a very important question with regards to the behaviour of λ and its effects on consumption, that is, what is the optimal path for λ that maximizes consumption? We will first extend the model to include an open economy and then derive the necessary parameters for the new open economy model.

4.1 Harris Model

One of the major problems with the Feldman model is that it assumes a closed economy, which was a realistic assumption to make during the five year plans due to the economic and political embargo on the USSR, but cannot work in the case of most other economies, especially developing countries, which often depend on exports to increase their national wealth. Thankfully, there have already been multiple attempts to turn the model into an open economy model, the one we will use is the formulation created by Donald J Harris [5], which goes as follows:

Suppose the economy is made up of three sectors: investment, consumption, and an export commodity - note that investment is analogous to producer goods, and consumption to consumer goods. What Harris essentially does is create a third sector in the economy, which only exports some commodity to the rest of the world, while only importing capital machinery, assuming that trade is perfectly balanced.

He starts off by listing the production relations of each sector, which are written as:

$$X = a_x K_x \tag{9}$$

$$I = a_i K_i \tag{10}$$

$$C = a_c K_c \tag{11}$$

where a is the capital-output coefficients of the corresponding sectors and K represents the capital in each of the 3 sectors.

Several assumptions are worth noting:

1. Once capital is installed, it cannot be transferred between sectors
2. The model assumes no government spending or taxation
3. There are no foreign transfers
4. Foreign trade is merely an exchange of the commodity of the export sector and imports consisting only of capital equipment
5. There is a surplus amount of rural labor (perhaps the most crucial assumption)

We then follow a similar scheme and derivation to the one used by Feldman, we reach the following system of differential equations:

$$\dot{K}_x = \epsilon_x X(t) - \delta K_x(t) \tag{12}$$

$$\dot{K}_i = \lambda I(t) + \epsilon_i X(t) - \delta K_i(t) \tag{13}$$

$$\dot{K}_c = (1-\lambda) I(t) + \epsilon_c X(t) - \delta K_c(t) \tag{14}$$

where the ϵ's are the allocation coefficients of imported capital into each of the sectors, i.e what percentage of the imported capital goes into which sector and δ is the depreciation rate. It then logically follows that:

$$\epsilon_x + \epsilon_c + \epsilon_i = 1$$

and similar to the Feldman model, $0 \leq \lambda \leq 1$ where, again, λ represents the percentage of producer goods that is reinvested in the producer goods sector. Since the Harris model is based on a systaem of first-order linear ODEs, an analytic solution for the following system of equations can be obtained using differential methods. The solution is as follows:

$$K_x(t) = K_x(0)e^{gt}, \; g = \frac{\epsilon_x}{a_x} - \delta \tag{15}$$

$$K_i(t) = Ae^{rt} + Be^{gt}, \; r = \frac{\lambda}{a_i} - \delta \tag{16}$$

$$K_c(t) = He^{-\delta t} + \frac{1-\lambda}{\lambda}Ae^{rt} + Ge^{gt}, \; r = \frac{\lambda}{a_i} - \delta \tag{17}$$

where

$$B = \frac{\epsilon_i}{(g-r)a_x}K_x(0)$$

$$A = K_i(0) - B$$

$$G = \frac{1-\lambda}{\epsilon_x}\frac{a_x}{a_i}B + \frac{\epsilon_c}{\epsilon_x}K_x(0)$$

$$H = K_c(0) - \frac{1-\lambda}{\lambda}A - G$$

5 Empirical Application of the Model

In order to present a general idea for investment policy in a developing country, we acquired data from India starting from 1960, largely due to it being the only developing country having the required data needed in order to formulate how much capital falls under each sector and the relevant capital-output coefficients. We follow a similar division of the economy to that used by Robert Allen [3] in order to determine what capital falls under which sector; while this division may not be 100% accurate, but it provides a rough image of what a developing country's capital stock will look like, with most of the capital being in the consumer goods sector, in particular the agricultural sector.

Using this data, we can then get a rough idea of mainly two things: firstly, the behavior of λ that ensures consumption is maximized, and secondly, to which sector does the imported capital go to. The results will allow us to deepen our understanding of the mechanisms behind sound investment policy for growing economies. In our simulation, we used the following values, the sources for which can be found in the bibliography [10] [11] [4]:

- $K_x(0) = \$3,196,500,000$
- $K_i(0) = \$2,957,654,938$
- $K_c(0) = \$8,106,067,072$

- $\delta = 0.05$

We assumed that the $\delta = 0.05$ by convention found in majority of literature.

As can be seen from figures 1-5 in the appendix, certain interesting patterns start to emerge. Firstly, when it pertains to the ϵ's, we find that for the first 10 years, the ϵ's are constant with $\epsilon_c = 1$ and the rest are near 0, as the aim is the maximization of consumption; however, starting with the 10th year, results take on a more consistent pattern, as ϵ_i and ϵ_c both start to consistently decrease while ϵ_x increases massively and $\lim_{t\to\infty} \epsilon_x$=1 What this essentially tells us is that not only is investing in the export sector is crucial to increasing national income, it will also increase standards of living in the long run. We already see much empirical evidence to support this, as we find that countries such as South Korea and Japan having grown massively during the 20th century largely due to their careful planning of the export commodity sector. Secondly, we find that λ behaves in a similar way to ϵ_x, as starting with the 20th year, it also tends towards 1 as it time increases indefinitely. As noted by Dr. Harris, if $g > r$, then the growth of the model will be dominated by the export sector. If the inverse occurs, then the growth in the economy will be dominated by r and λ. If they are both equal, then the economy will follow a balanced growth path in each sector.

6 Discussion and Topics for Further Research

There are many limitations to the current model with regards to an accurate representation of the real world. Firstly, the model assumes that there are no shocks to capital accumulation or markets that can happen, which is all the more unrealistic when creating an open economy extension, as there can be demand shocks (positive or negative) or economic recessions that can affect the export of the commodity and import of capital goods. A further interesting question is at what point does the developing country no longer need to import capital, but instead become a capital *exporter*?

To incorporate a shock into the model, we can modify the equations for capital accumulation to include a stochastic element. For example, we could modify the law of motion of K_x as follows:

$$\dot{K}_x = \epsilon_x X(t) - dK_x(t) + \eta_x(t) \tag{18}$$

where $\eta_x(t)$ is a stochastic error term that captures the effect of the shock on the accumulation of the export sector. We could further assume that:

$$\eta_x(t) \sim N(0, \sigma_x^2) \tag{19}$$

Secondly, the model assumes that the capital-output coefficients are constant, which is a very unrealistic assumption. This had already been pointed out by Mahalanobis [8], an Indian statistician who had developed a similar model to Feldman's when working on India's five year plans. Instead, Mahalanobis suggested that the capital-output ratio behaves parabolically. Furthermore, under a more planned environment, it is likely that there can be deliberate attempts to increase the capital-output coefficients for a specific industry such as the export industry.

The capital-output coefficient can be understood to be a relationship between GDP per capita, the level of industrialization, and worker's wage per week, as suggested by Korayem [7]. It would take on the general form for under-developed countries :

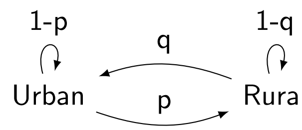
$$1.703 - 0.4568x_1 + 0.1899x_2 + 0.0766x_3 \tag{20}$$

where x_1 is the rate of growth of GDP, x_2 is industry as a % of GDP, and x_3 the wage per week in USD. Such a formula would not only provide time varying coefficients, but would also directly link the effects production to the output coefficients, providing us with much more realistic outcomes, further ensuring the accuracy of the model. It is worth noting that time varying coefficients will cause the system of ODEs to become non-linear, and thus an analytic solution might be harder to obtain. Moreover, The model would need for the capital-output coefficients to remain roughly constant as soon as the economy reaches a more advanced level, as highlighted by Kaldor's facts, which state that in developed economies, the ratio of labor and capital's contribution to growth remains constant after a certain stage [2]. Another interesting extension of this would be to find the relevant capital-output coefficients for *each sector* and turning all the Ks into a vector and all the as into a diagonal matrix, looking something closer to this:

$$K \cdot A = \begin{bmatrix} K_1 \\ K_2 \\ \vdots \\ K_n \end{bmatrix} \begin{bmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_n \end{bmatrix} = \begin{bmatrix} K_1 a_1 \\ K_2 a_2 \\ \vdots \\ K_n a_n \end{bmatrix}$$

Where K_n denotes an industry of the economy and a_n the corresponding coefficient. This would not only provide us with a fuller idea of how the economy will change, but will also show us which sectors are worth investing in more than others and how the overall composition of the economy changes. Unfortunately, due to the lack of available data and computational power to process each sector and capital-labor coefficient, it is not feasible to formulate the model in this form.

Thirdly, another limitation for the model is that it does account for rural-urban migration, a deciding factor for the process of capital accumulation. Part of the reason for the economic success of the Soviet Union from 1930-1970 was the large inflow of rural peasants into the urban industrial economy; this allowed the Soviet Union to open factories at an unprecedented rate due to there always being laborers for the jobs. However, as this rural migration reaching a stagnant point, what is known in mainstream economics as the Lewis Turning Point, economic growth began to take a downturn, and more focus was spent on qualitative rather than quantitative growth. Therefore, the dynamics of rural-urban migration are worth investigating in order to determine what effect it may have on the capital accumulation process and what potential remedies there may be to avoid the pitfalls of the Lewis Turning Point. This could be achieved using Markov chains and finding the stationary distribution (some of which have already been estimated for India [1] and China [12]), looking something like this



where p and q are the transition probabilities.

Finally, the model does not take resource constraints into account, which would again set limits for how much growth can be produced in any sector, limiting it by its exploitation rate or its import price, along with other logistical and financial constraints. This could also play into providing an idea of which sectors are worth more investment, as some sectors may work better for the natural endowment of certain countries. This is particularly useful as the growth exhibited by the model shows a similar pattern to what is known in microeconomics as a ponzi game, in which a representative agent can accumulate infinite debt in order to maximize their utility [2]. A similar issue arises in the growth of each sector, as there are no physical or financial limits to how much it can grow by. This opens a discussion of other potential algorithms that may play a better role, such as for instance setting an average growth target over a certain period (say 5 years)

7 Conclusion

In conclusion, the open economy extension of the Feldman model presents us with interesting results as to what the optimal path for increasing living standards is. While the model may still miss many crucial factors in the development process such as rural-urban migration, the relationship between the capital-output coefficient and the production process, and production shocks, it provides us with a clear blueprint for the development process and a preliminary framework to mitigate the limitations listed earlier.

References

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8 Appendix

MATLAB Code

The MATLAB code can be found here in this GitHub Repository.

Figures

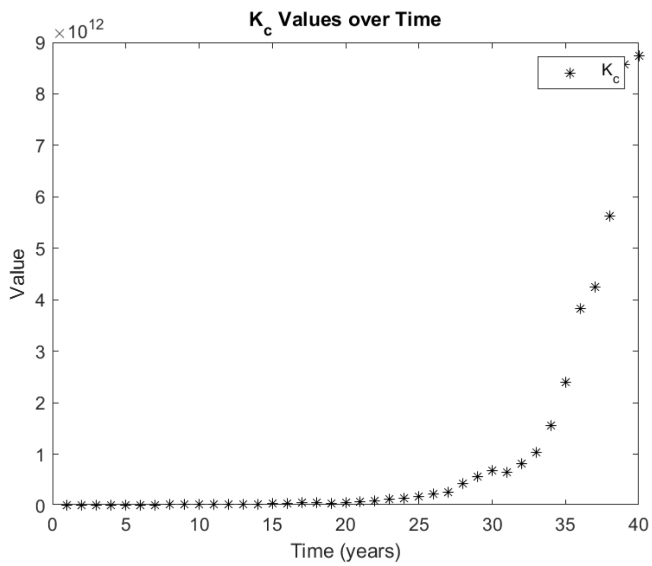

Figure 1: K_c over time

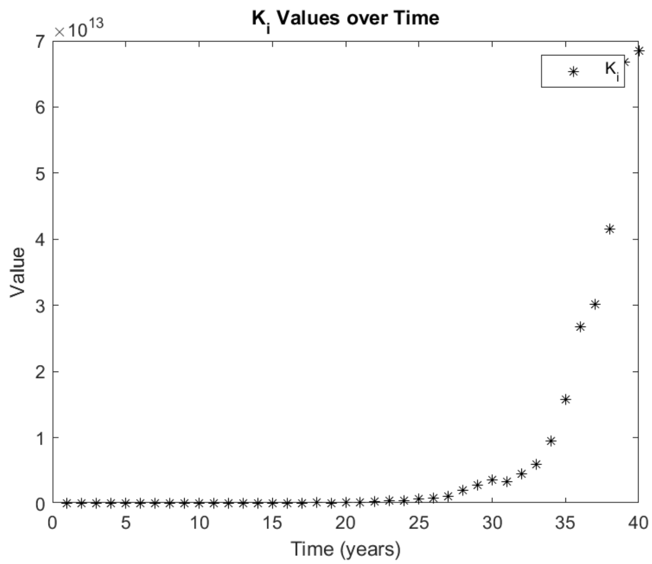

Figure 2: K_i over time

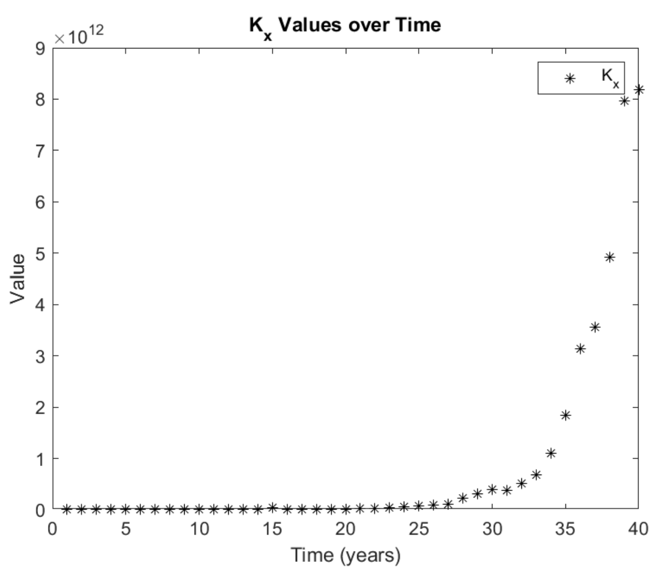

Figure 3: K_x over time

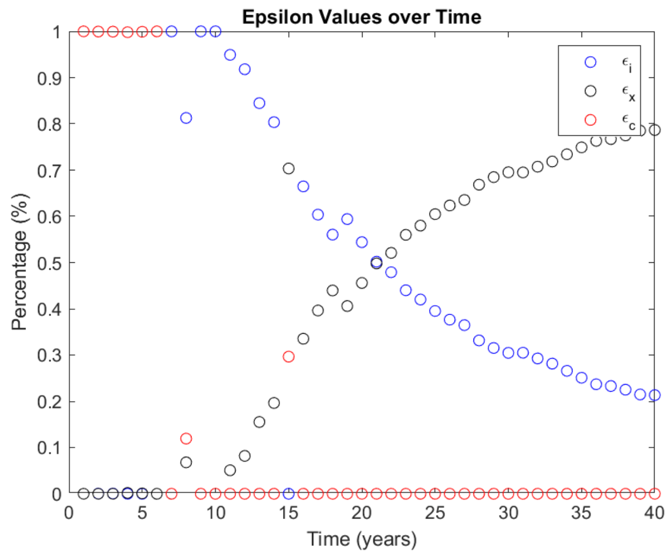

Figure 4: ϵs over time

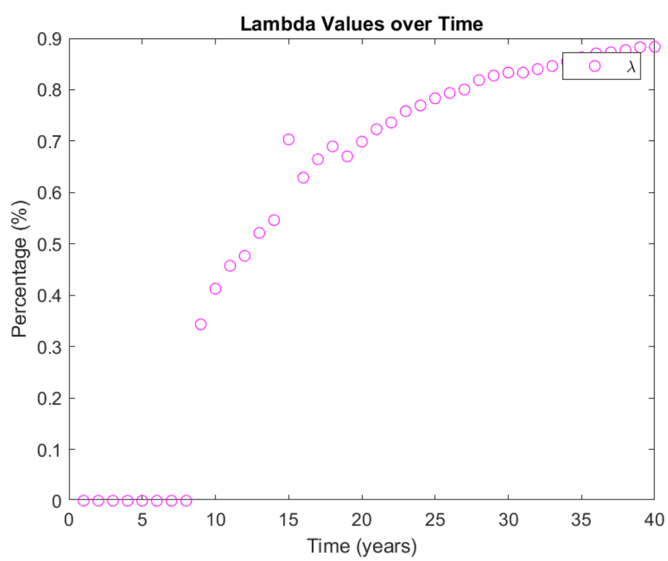

Figure 5: λ over time