Course Name
Fall 2025

Assignment 00

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Due: Date Here

Q1

Solution.

a) First we show that if u is a solution to $u_t + uu_x = 0$, then $w = u^2$ is a smooth solution of (2).

$$\begin{aligned} u_t + uu_x &= 0 \\ (2u)u_t + (2u)u_x &= 0 \\ 2uu_t + 2uu_x &= 0 \\ \left(u^2\right)_t + \left(\frac{2}{3}(u^2)^{\frac{3}{2}}\right)_x &= 0 \quad \text{Since } u \text{ is smooth.} \\ w_t + \left(\frac{2}{3}w^{\frac{3}{2}}\right)_x &= 0 \end{aligned}$$

In the other direction, we start with $w = u^2$ as a solution to $w_t + \left(\frac{2}{3}w^{\frac{3}{2}}\right)_x = 0$.

$$\begin{aligned} w_t + \left(\frac{2}{3}w^{\frac{3}{2}}\right)_x &= 0 \\ (u^2)_t + \left(\frac{2}{3}u^3\right)_x &= 0 \\ 2uu_t + 2u^2u_x &= 0 \quad \text{Since } u \text{ is smooth.} \\ 2u(u_t + uu_x) &= 0 \end{aligned}$$

Thus we have two cases. If $u = 0$, then trivially, (1) holds. Otherwise if $u > 0$, then we can divide by $2u$ giving us that,

$$u_t + uu_x = 0$$

as required. This concludes the proof in both directions.

b) Figures can be added as follows:

Convergence Table for Lax-Wendroff Method

N	dt	Error	Rate
20	0.05000	3.81099e-02	NaN
40	0.02500	9.65411e-03	1.98095
80	0.01250	2.42033e-03	1.99594
160	0.00625	6.05469e-04	1.99908

Convergence Table for Upwind Method

N	dt	Error	Rate
20	0.05000	2.19454e-01	NaN
40	0.02500	1.16176e-01	0.91761
80	0.01250	5.98359e-02	0.95723
160	0.00625	3.03737e-02	0.97819

Convergence Table for Lax-Friedrichs Method

N	dt	Error	Rate
20	0.05000	5.25876e-01	NaN
40	0.02500	3.09733e-01	0.76370
80	0.01250	1.68991e-01	0.87408
160	0.00625	8.83820e-02	0.93512