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OPTIMUM SYNTHESIS OF RIGID MECHANISMS USING A DYNAMIC ANT-SEARCH METHOD WITH SENSITIVITY ANALYSIS

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ABSTRACT

Four-bar linkages are commonly used mechanisms in various mechanical systems and components. Several techniques for optimum synthesis of planar mechanisms have been suggested in literature such as the Genetic, Tabu, Simulated Annealing, Swarm-Based and many other algorithms. This paper covers optimization of four-bar mechanisms with path generation tasks using a Dynamic Ant Search (DAS) algorithm. Unlike the Modified Ant Search (MAS) technique [1] where ants unanimously moved between the exploration and exploitation phases, in the proposed algorithm, each ant is free to travel between the two aforementioned phases independent of other ants and as governed by its own pheromone intensity level. Moreover, sensitivity analysis is conducted on the design parameters to determine their corresponding neighborhood search boundaries and thus improve the search while in the exploitation mode. These implemented changes demonstrated a remarkable impact on the optimum synthesis of mechanisms for path generation tasks. A briefing of the MAS based algorithm is first presented after which the proposed modified optimization technique and its implementation on four-bar mechanisms are furnished. Finally, three case studies are conducted to evaluate the efficiency and robustness of the proposed methodology where the performances of the obtained optimum designs are benchmarked with those previously reported in literature.

Nomenclature

AS	Ant Search
MAS	Modified Ant Search
DAS	Dynamic Ant Search
ACO	Ant Colony Optimization
GA	Genetic Algorithm
P	Non-change probability function
p	Vector of Parameters
d	Drive link length
c	Coupler link length
f	Follower link length
g	Ground link length
l	Coupler link's left side length
r	Coupler link's right side length
τ	Pheromone trail intensity
α	Pheromone evaporation rate
E	Structural Error

Introduction

Determining the parameters of a four-bar mechanism to achieve a certain path specified by a set of points is called path generation task [2, 3]. Different techniques have been used

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throughout the literature; graphical-based methods [4], analytical-based methods [5], and optimization techniques [1]. The synthesis of a four bar mechanism for a given path trajectory becomes complicated as the number of points to be traversed increases where optimization methods are usually used [1, 6, 7]. Researchers have used different optimization methods for the path generation problems of four bar planar mechanisms. Early work in literature focused on local search methods like Angeles et al. [8] who used an unconstrained nonlinear least-square method for 4R planar mechanism optimization. Later, global search optimization techniques gain popularity for their ability to search among all possible solutions, and to converge to the one with the minimum error. They can be used for different applications, including mechanism synthesis. These search techniques are classified into two types; probabilistic and deterministic techniques. The former include methods such as modified ant search [1], ant colony optimization (ACO) [3], genetic algorithms [9, 10], and tabu search [11], while the latter are reflected in gradient-based algorithms [12]. The various aforementioned algorithms have been applied to several types of design optimization problems. For example, Batista et al. [3] used the basic ACO method in topology optimization problems. Laribi et al [7] used a genetic algorithm search combined with a fuzzy logic control method for path generation mechanism synthesis problems. Kaveh et al. [9, 10] used the genetic algorithm, hybridized with the indirect shared memory of hybrid ant strategies and the trail update strategy of the ACO, for structural optimization purposes. Wu et al. [13] used a modified ACO algorithm for the same purpose, in which they added the MAX-MIN ant system and the pheromone trail centralization in order to avoid premature convergence. Ebrahimi and Payvandy [14] went on using the imperialistic competitive algorithm and the parallel simulated annealing, which are heuristic optimization algorithms, for synthesis of path generating four-bar mechanisms with dimensional constraints. Acharyya and Mandal [15] worked on optimizing four-bars with given coupler curves using the particle swarm optimization, the genetic algorithm, and the differential evolution performance (DE). Moreover, some researchers went for different methods, such as the graphical and analytical techniques. For example, Brunnthaler et al [16] used the three-dimensional projective kinematic image space of planar Euclidean displacements for the synthesis of planar four-bars with given path trajectory. Hassan [17] worked on forming a mathematical model for simple path generation four-bar mechanisms. Todorov [18] used a mathematical method that applies the Chebyshev's Alternation Theorem [19] on the Freudenstein's Equation [5], for the synthesis of four-bar mechanisms.

This paper aims to propose a Dynamic Ant Search (DAS) technique for the optimization of 4R four bar mechanisms synthesis with path generation task. In contrast to the technique presented by Diab and Smaili [1] which was referred to as Modified Ant Search (MAS) where all ants coexisted at one phase, the parameters (i.e. ants) of the 4R mechanisms in this work can travel between the two phases, from exploration to exploitation and vice versa, in an independent manner. Thus, the entire colony of ants is not forced to stay at the same phase. Moreover, a sensitivity study is done on the optimization parameters to aid the searching process of the exploitation phase. Accordingly, the sensitivity of any design parameter determines the range of allowable change in its value and the time spent by the corresponding ants in the exploitation phase.

Mechanism Parameters and Design Constraints

Any four-bar rigid mechanism can be fully represented by the set of nine parameters $p = (s_x, s_y, d, f_x, f_y, l_x, l_y, r_x, r_y)$ shown in Figure 1. p_i represents the *i-th* entry of the parameters vector p.

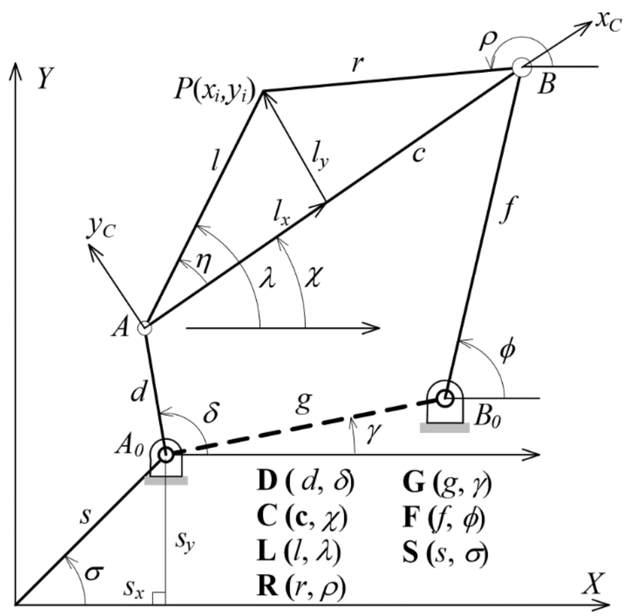

FIGURE 1. Four-bar design parameters

The driver's link length is represented by d, the follower's link projections are f_x and f_y, the driver-ground pivot coordinates are s_x and s_y, coupler's left-side link projections are l_x and l_y projected on the coupler link local frame $x_c y_c$, and coupler's right-side link projections are r_x and r_y projected on the global frame *XY*. Any other remaining parameter could be deduced by using the appropriate loop closure equations.

In most of the cases, the mechanism under study is designed to make a complete rotation of its input, thus it must satisfy Grashof's Criteria [20].

In addition, the mechanism link lengths must fall within a range of acceptable dimensions and must ensure that the mechanism is able to be assembled at all positions. All of these conditions increase the complexity of the problem, and limit the feasible solutions.

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Cost Function

Optimum synthesis of planar mechanisms is carried out by determining the link lengths so that the coupler point traverses the closest possible trajectory to the desired one. Thus, the error function must describe the difference in the location between the actual generated points and the desired ones. However, different error functions are used by researchers throughout the literature. The most commonly used cost functions are stated in equations 1 through 4.

The first error function used in [1, 21, 22] is given by:

$$E = \sum_{i=1}^{n}\left(\left(x_{id} - x_{ig}\right)^2 + \left(y_{id} - y_{ig}\right)^2\right) \tag{1}$$

where the error is the summation of the square of the distances between the coupler points and the desired points; n represents the number of the specified desired points; x_{id} and y_{id} represent the desired points coordinates; and x_{ig} and y_{ig} represents the generated points coordinates.

The second error function used in [23] is obtained by normalizing the first error function and given by:

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[\left(x_{id} - x_{ig}\right)^2 + \left(y_{id} - y_{ig}\right)^2]} \tag{2}$$

The third error function used in [24] is given by:

$$E = \sum_{i=1}^{n}(\xi_i{}^2 + \eta_i{}^2) + \sum_{j=1}^{m} k_j re_j \tag{3}$$

where m is the number of restrictions for the problem, re is the restriction, and k is the value of the penalty associated to each restriction violation. The variables ξ_i and η_i are given as $\left|y_{ig} - y_{id}\right| - d_y$ and $\left|x_{ig} - x_{id}\right| - d_x$ respectively where d_x and d_y are the allowed deviations of the point i.

The above cost function is used in the controlled deviation methods, and is divided into two parts, the first part represents the sum of the square of the distances between the desired and generated points, whereas the second part represents penalties for violating restrictions, such as negative link lengths.

The fourth error function presented in [14] is given by:

$$E = W_1 \sum_{i=1}^{n}[\left(x_{ig} - x_{id}(\theta_{2i})\right)^2 + \left(y_{ig} - y_{id}(\theta_{2i})\right)^2] + W_2(f_1 + f_2) \tag{4}$$

where $x_{ig}(\theta_{2i})$ and $y_{ig}(\theta_{2i})$ are the x and y coordinates of the generated point i at the crank angle θ_{2i}. The parameters f_1 and f_2 can be defined as:

$$f_1 = [\sum_{i=1}^{n} gt(L_x, x_{id}(\theta_{2i}))|x_{id}(\theta_{2i}) - L_x| + gt(x_{id}(\theta_{2i}), U_x)|x_{id}(\theta_{2i}) - U_x|]$$
$$f_2 = [\sum_{i=1}^{n} gt(L_y, y_{id}(\theta_{2i}))\left|y_{id}(\theta_{2i}) - L_y\right| + gt(y_{id}(\theta_{2i}), U_y)\left|y_{id}(\theta_{2i}) - U_y\right|] \tag{5}$$

Where $gt(a,b) = \begin{cases} 1 \; if \; a > b \\ 0 \; if \; a \le b \end{cases}$

L_x and U_x are the lower and upper limits of the workspace in the x-direction respectively; L_y and U_y are the lower and upper limits of the workspace in the y-direction respectively; W_1 and W_2 are the weights of the equation parts, however according to [14], W_1 and W_2 must be chosen such that both parts of the equation share an equal effect in finding the optimized solution. The first part of the cost function is for calculating the structural error between the generated and the target points, whereas the second part of the cost function is for adding a penalty for violating the workspace limits.

In this paper, the cost function represented in equation 2 is adopted for solving the presented case studies. It is worth noting that all constraints are embedded within the code where non-Grashof mechanisms are automatically rejected. In addition, all link lengths are limited to prescribed dimension ranges and the optimized mechanism coupler point is forced to traverse the given order of points.

Modified Ant Search (MAS) Algorithm

Ants live in colonies, in which work is collaborative while surfing the environment in search of food. However, ants lead each other towards the shortest path linking their nest to a food source by depositing a chemical called pheromone. The amount of the deposited pheromone depends on the followed trajectories, where larger amounts of pheromone are deposited over shorter paths [25, 26]. Moreover, a pheromone is a temporary chemical as it gets evaporated over time. From this behavior, researchers had inspired an optimization technique called the Ant Colony Optimization (ACO), and developed several modified techniques afterwards.

In the AS optimization technique, each parameter previously introduced in Fig. 1 is represented by an ant, and an equal amount of pheromone is assigned initially for all the ants. As the search iterations proceed, the values of these parameters vary while trying to approach the coupler's curve to the desired path. Through these iterations, new pheromones are deposited on the trajectory of ants that are contributing to a reduction in the error. However, the amount of the newly deposited pheromones is proportional to the generated error values, in which the highest pheromone values are deposited by ants that contributed most to the reduction of error. The amount of the

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newly deposited pheromones is calculated based on the following relation:

$$\Delta\tau_i(t) = \frac{E_t - E_{t+1}}{E_t} \tag{6}$$

where $\Delta\tau_i(t)$ is the pheromone deposited by ant i, E_t and E_{t+1} are the errors generated at iterations t and $t+1$ respectively. $\Delta\tau_i(t)$ ranges ideally from 0 if the error value did not decrease, to 1 if the error at the iteration $t+1$ was reduced to zero.

On the other hand, pheromones evaporate continuously as already discussed before. In this work, the pheromone evaporation rate is set at $\alpha = 0.8$. Thus the pheromone value for an ant at an iteration $t+1$ can be calculated as given in equation 7:

$$\tau_i(t+1) = (1-\alpha)\tau_i(t) + \Delta\tau_i(t) \tag{7}$$

To determine whether the parameter's value is going to be changed in the current iteration t, a non-change probability parameter presented in equation 8 is evaluated and compared to a random number "*rand*" generated between 0 and 1.

$$P_i(t) = \frac{\tau_i(t)}{\sum_{i=1}^{n}\tau_i(t)} \tag{8}$$

If "*rand*" is greater than the non-change parameter P_i, the corresponding parameter *i* will be randomly changed else *i* will remain unchanged.

Later, Diab and Smaili [1] introduced the Modified Ant Search (MAS) method. In this optimization algorithm, the searching process is divided into two phases, the exploration phase and the exploitation phase. All ants start in the exploration phase initially, shift into the exploitation phase after minimizing the error into a relatively small value, and stay in the exploitation unless no further improvement is occurring. However, during the whole search process, all the ants must remain in the same searching phase at a specific time step.

Dynamic Ant Search (DAS) Algorithm

The Dynamic Ant Search (DAS) algorithm is an upgrade of the MAS technique and can be divided into two main stems: dynamic exploration/exploitation technique and the sensitivity analysis technique.

Sensitivity Analysis

The same change in different parameters yields a different change in the generated coupler's curve and in the structural error according to the sensitivity of the changed parameter. For this, sensitivity analysis is performed to determine the level of sensitivity of each parameter. Sensitivity analysis has been defined in two different ways throughout the literature [12]. It's defined as the variation of the output with respect to a slight change in the input's value as in [27] or as the ratio of the change of the cost function to a small variation in the input value as in [28]. In this paper, the sensitivity of a parameter *i* taking the value *j* is calculated based on equation 9.

$$Sensitivity(i) = \left|\frac{E_{p_i=j} - E_{p_i=j+h}}{h}\right| \tag{9}$$

where $E_{p_i=j}$ is the objective function value when the parameter *i* acquires the value *j (i.e.* $p_i = j$*)*, and h is the small increment on which the sensitivity is tested. No direct relation specifying the value of h exists, however it must be a small percentage of the studied parameter's value and in this work it's chosen to be 5% of *j*.
The sensitivity of each of the nine parameters is represented as a percentage of that of the parameter with the highest sensitivity. This mapping is represented by equation 10:

$$SP(i) = round\left(\left|\frac{Sensitivity(i)\times 100}{Sensitivity(i_m)}\right|\right) \tag{10}$$

where $SP(i)$ is the sensitivity percentage of the parameter *i*, and p_m is the parameter having the highest sensitivity.

Dynamic Exploration/Exploitation Search

Two searching modes are used while searching for the optimum solution, an exploration phase and an exploitation phase. The exploration mode is the phase where all design parameters are free to change their values randomly between a fixed pre-defined global upper and lower boundaries according to equation 11.

$$p_i(t+1) = rand \times \left(U_{p_i} - L_{p_i}\right) + L_{p_i} \tag{11}$$

where U_{p_i} and L_{p_i} are the upper and lower limits of the parameter i respectively, and p_i is the value taken by that parameter at a specific time.

The exploitation mode is the phase where design parameters are set to surf their local surrounding domains for better solutions. However, the local domain of each ant in the exploitation phase is determined according to a sensitivity study for each of the parameters as presented in equation 12:

$$p_i(t+1) = p_i(t) + (-1)^{round(rand)} \times \gamma \times p_i(t) \tag{12}$$

The second part of equation 10 is the change in the parameter's value between iterations t and $t+1$. γ is the portion of change in the parameters value, and is computed as in equation 13.

$$\gamma = rand \times \frac{100 - SP(i)}{100} \times \eta \tag{13}$$

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where η is a decimal constant that specifies the maximum allowable range of change in the exploitation phase; however, there is no direct relation for specifying η.

The pheromone value is the only factor that determines the searching mode of the ant, in which an ant starts initially in the exploration mode having an initial pheromone value, and moves to the exploitation mode when its pheromone value goes under a specific pheromone threshold, denoted by β; whereas it returns to the exploration mode after completing a number of searches in the exploitation mode, called the exploitation searches (ES), which is related to the sensitivity percentage $SP(p)$ according to equation 14.

$$ES = ceil\left(\frac{SP(p)}{10}\right) \tag{14}$$

where "$ceil(A)$" is a function that rounds up A to the nearest integer greater than or equal to A. Thus the value of ES is bounded as follows:

$$0 \leq ES \leq 10 \tag{15}$$

Therefore, according to equation 15, any ant can spend a maximum of 10 iterations in the exploitation phase.

However, after finishing the exploitation searches specified by ES, the ant returns to the exploration mode and the pheromone value of the ant resets to its initial value.

Figure 2 presents a detailed flow chart of the proposed DAS algorithm.

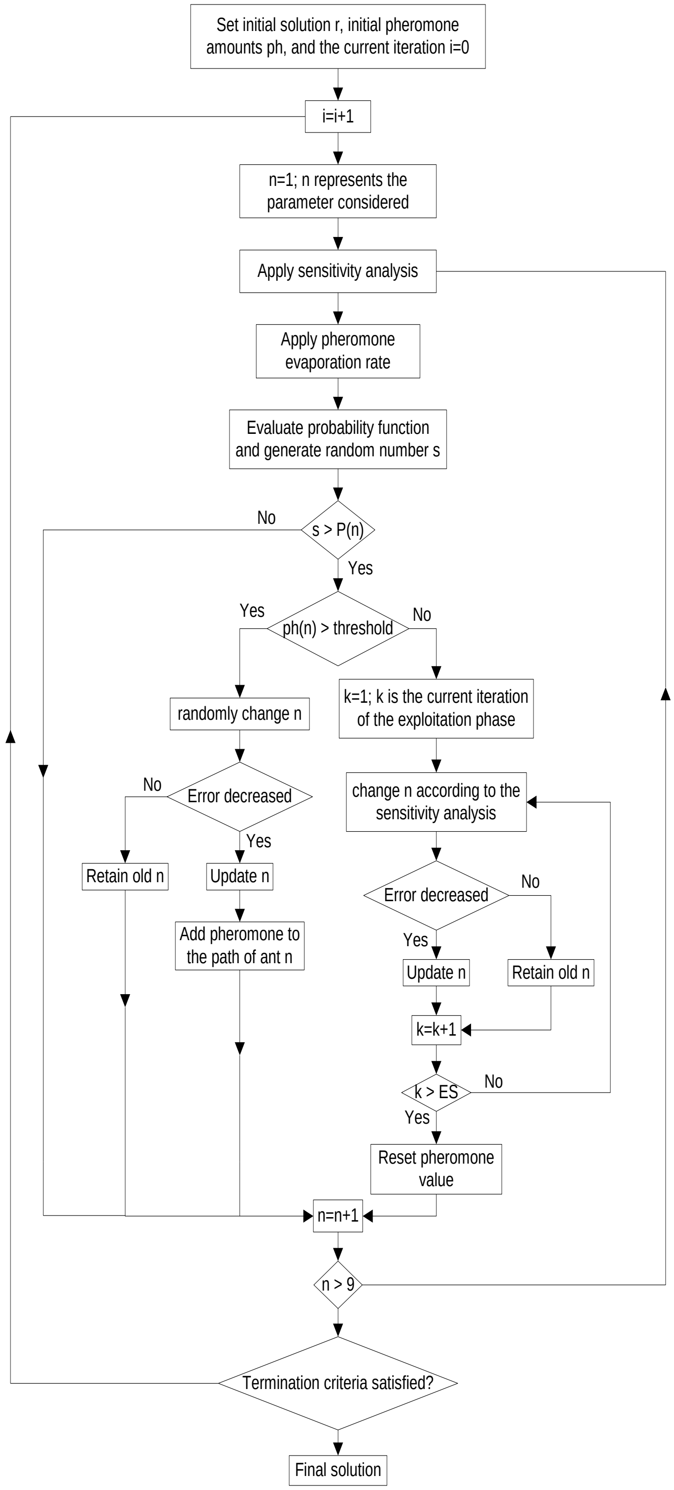

FIGURE 2. Flow chart of the proposed algorithm (DAS)

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Case Studies

The three case studies presented in this section were performed to evaluate the proposed DAS optimization methodology. The maximum range of change in the exploitation phase is chosen to be 10% of the parameters' values; for this, the variable η shown in equation 13 is set to 0.1. Moreover, the pheromone threshold β is set to be 6.4×10^{-6}, in which the search process for ant i switches from the exploration mode into the exploitation mode if the pheromone value τ_i at any instant drops below this pheromone threshold.

Case Study I

The first example is a path consisting of 25 desired points, without prescribed timing. Coordinates of the target points are presented in Table 1. Table 2 presents a comparison between the parameters of the optimized mechanism and the structural error obtained using DAS and those obtained using the MAS algorithm [1] and the Genetic Algorithm - Fuzzy Logic (GA-FL) combined approach [7]. Figure 3 shows the generated curve along with the optimized four-bar mechanism. Figure 4 presents the generated coupler curves by the DAS, MAS [1], and GA-FL [7].

TABLE 1. Desired points coordinates for case study 1

Point	x_{id}	y_{id}	Point	x_{id}	y_{id}	Point	x_{id}	y_{id}
1	7.03	5.99	10	4.04	1.67	19	4.07	6.4
2	6.95	5.45	11	3.76	1.22	20	4.53	6.75
3	6.77	5.03	12	3.76	1.97	21	5.07	6.85
4	6.4	4.6	13	3.76	2.78	22	5.05	6.84
5	5.91	4.03	14	3.76	3.56	23	5.89	6.83
6	5.43	3.56	15	3.76	4.34	24	6.41	6.8
7	4.93	2.94	16	3.76	4.91	25	6.92	6.58
8	4.67	2.6	17	3.76	5.47			
9	4.38	2.2	18	3.8	5.98			

TABLE 2. Generated solution by various optimization techniques for case study 1

Parameter	GA-FL [7]	MAS [1]	DAS
g	9	8.2255	10.4067
d	3.01	1.8789	2.9353
c	8.8	5.333	10.1923
f	8.8	4.9055	7.608
l	11.1	10.0015	13.7342
s_x	-2.4	-4.285	-4.428
s_y	-4	2.1186	-5.7265
η	321	3.4711	-31.4119
γ	-28	19.57	35.9222
E	0.902	0.8492	0.1814

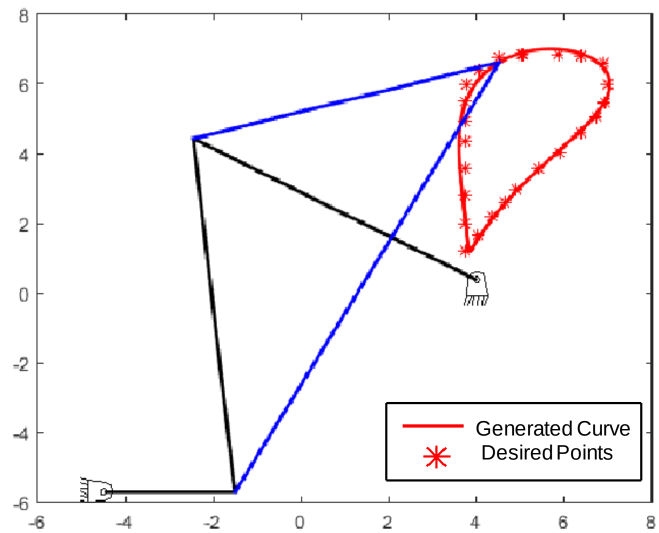

FIGURE 3. Coupler curve generated along with the four-bar mechanism of case study 1

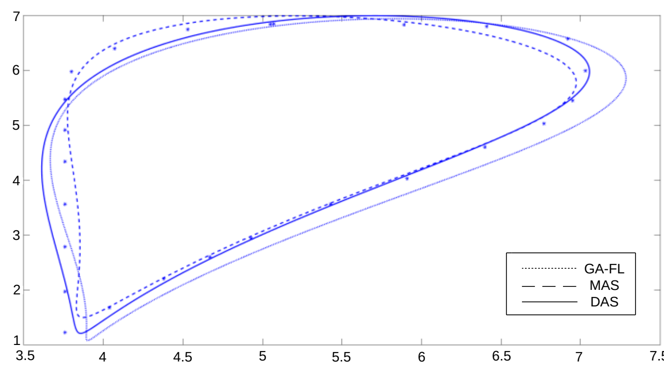

FIGURE 4. Coupler curves generated by the GA-FL *[7]*, MAS *[1]*, and the DAS algorithms for case study 1

Case Study II

In this example, the synthesis problem of 6 target points arranged in a vertical straight line without prescribed timing is presented. Moreover, limitations on the mechanism design parameters are applied where the coordinates of the desired points and the limits of the design variables are presented in Tables 3 and 4 respectively. Table 5 presents the parameters of the optimized mechanism and its structural error and compares them to the solution generated while using the Deferential Evaluation (DE) algorithm [23] and the Imperialist Competitive Algorithm (ICA) [14]. Figure 5 shows the generated curve with the optimized four-bar mechanism. Figure 6 presents the generated curves by the DAS, and the DE [23].

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TABLE 3. Desired points' coordinates for case study 2

Desired Point	1	2	3	4	5	6
x_{id}	20	20	20	20	20	20
y_{id}	20	25	30	35	40	45

TABLE 4. Parameters' limits for case study 2

Parameter	g	d	c	f	l	x_0	y_0	η	γ
Lower Limit	5	5	5	5	0	-60	-60	0	0
Upper Limit	60	60	60	60	85	60	60	2π	2π

TABLE 5. Generated solution by various optimization techniques for case study 2

Parameter	DE [23]	ICA [14]	DAS
g	35.0207	60	17.9405
d	6.4042	17.511	6.8363
c	31.6072	60	23.5399
f	50.5995	33.2268	23.5379
l	46.4613	60	47.0469
s_x	60	60	60
s_y	18.0779	-1.4551	14.6612
η	63.3978	241.5189	0.0123
γ	0	53.3034	89.9746
E	0.0151	2.00E-03	4.02E-05

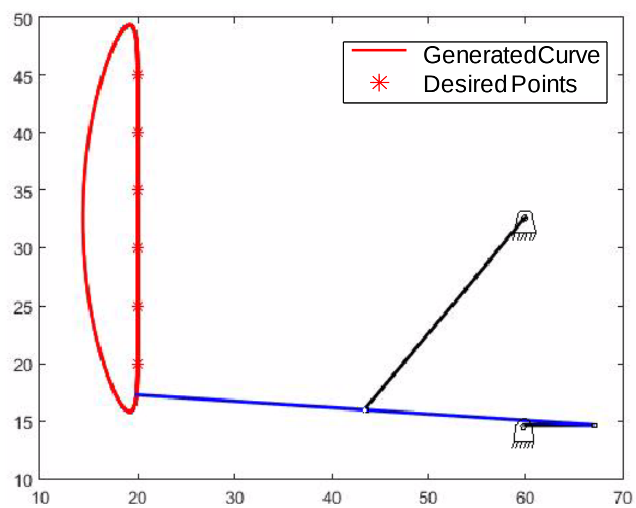

FIGURE 5. Coupler curve along with the four-bar mechanism of case study 1

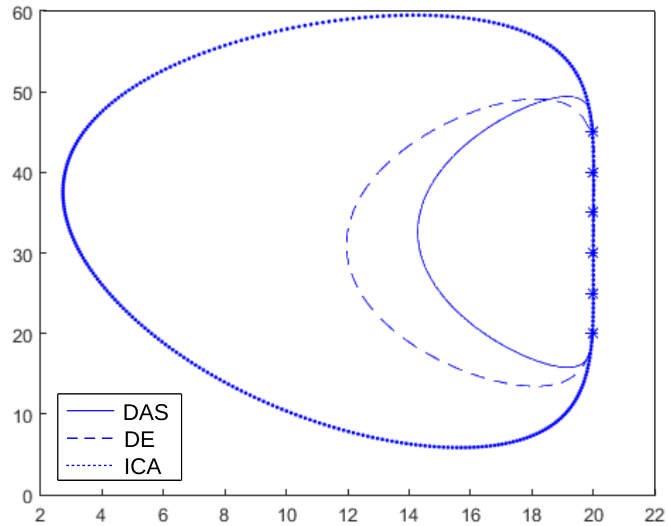

FIGURE 6. Coupler curves generated by the DAS, ICA [14], and the DE [23] algorithms for case study 2

Case Study III

This example is a path consisting of 8 target points, without a prescribed input angle. The coordinates of the target points are presented in Table 6. Mechanism design restrictions are applied, the mechanism parameters limits are found in Table 7. The searching process is limited to 600 iterations. Table 8 shows the parameters of the optimized mechanism along with the structural error and compares them to those generated by the Particle Swarm Optimization (PSO) algorithm [20]. Figure 7 shows the generated coupler curve along with the mechanism. Figure 8 presents the coupler curves of the DAS and the PSO [20].

TABLE 6. Desired points coordinates for case study 3

Point	1	2	3	4	5	6	7	8
x_{id}	2.6	2.3	2	1.7	1.4	1	2	3
y_{id}	1.6	1.6	1.6	1.6	1.6	1.3	0.7	1.3

TABLE 7. Parameters' limits for case study 3

Parameter	g	d	c	f	r_5	r_6	x_0	y_0	γ
Lower Limit	0.1	0.1	0.1	0.1	0.1	0.1	-4	-4	-90
Upper Limit	5	5	5	5	4	4	4	4	90

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TABLE 8. Generated solution by various optimization techniques for case study 3

Parameter	PSO [20]	DAS
g	3.787	2.7391
d	0.6991	0.6312
c	2.243	3.3517
f	3.9742	4.6379
l_x	0.6845	1.7251
l_y	1.5014	0.7615
s_x	2.9689	2.7424
s_y	2.464	0.9423
γ	89.96583	1.3791
E	4.33E-03	1.61E-03

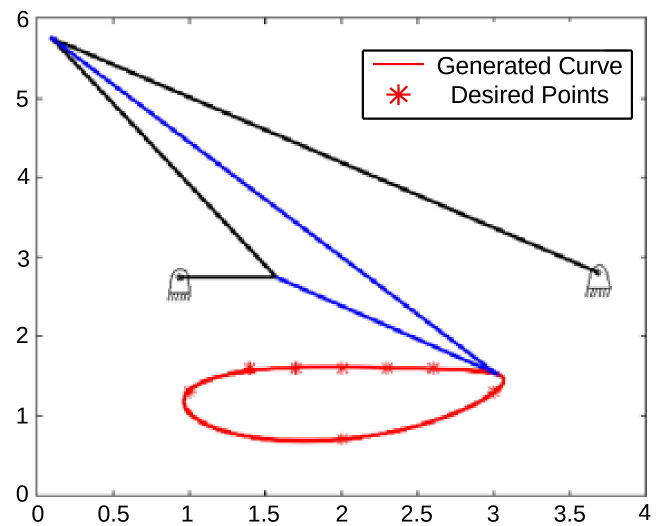

FIGURE 7. Coupler curve along with the four-bar mechanism of case study 3

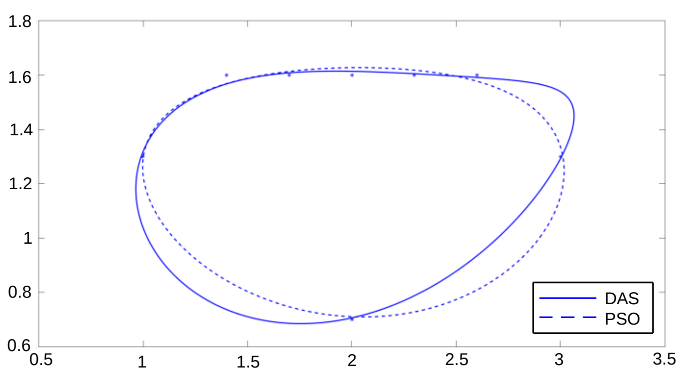

FIGURE 8. Coupler curves generated by the DAS and the PSO [20] algorithms for case study 3

9. Conclusion

In this paper, a Dynamic Ant Search (DAS) method was proposed for solving four-bar synthesis problems with path generation tasks. In the newly proposed methodology, ants where free to switch between both exploration and exploitation phases in an independent manner. Moreover, sensitivity analysis was performed to determine the maximum allowable change of the variables while in the exploitation phase, and to determine the exploitation search trials. The three case studies conducted showed a remarkable improvement in solving 4R mechanism synthesis problems with path generation tasks. As a future work, an improvement in the topic could be achieved by combining the proposed algorithm with a gradient based method.

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