GUC
German University in Cairo
Faculty of Engineering and Material Science
Department of Mechatronics Engineering

Reinforcement Learning and Optimal Control (MCTR1024)

Assignment #3: Model Predictive Control on an Inverted Pendulum

Due Date: Sunday, 25-May-2025

This assignment is groups of 2 students.

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Please note that cheating will not be tolerated and that it is your responsibility to ensure the genuineness of your work.

Problem: Inverted Pendulum on a Cart:

The inverted pendulum is considered a benchmark in robotics and control since it is used as a testbed for various controllers.

Following is the linearized state space representation of the system around the unstable equilibrium point:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

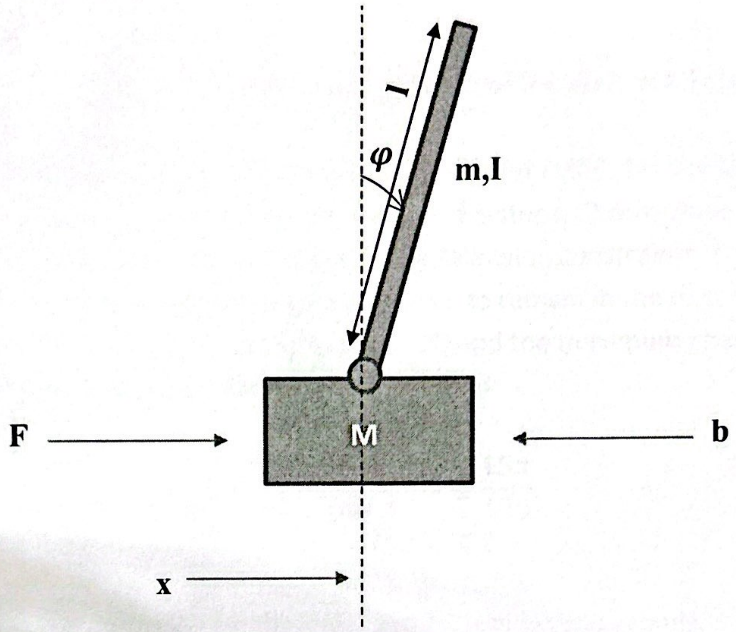
The parameters of the model are given in the following table:

Parameter	Symbol	Value	Unit
Cart Mass	M	0.5	kg
Pendulum Mass	m	0.2	kg
Pendulum Length	l	0.3	m
Pendulum Inertia	I	0.006	Kgm^2
Cart Damping Coefficient	b	0.1	Ns/m
Gravitational Field Strength	g	9.81	m/s^2

Substituting:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.675 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.21 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1.8181 \\ 0 \\ 4.545 \end{bmatrix}$$

Pass to the code.



Submit the assignment in this form.

Required:

1. Obtain the discrete state space representation of the system with a sampling time of 0.1 sec.
2. Use CasADi to design a Model Predictive Controller (MPC) for the discrete-time linearized system. Use a prediction horizon of 5 steps. Choose your own weighting matrices. The system should adhere to the following constraints. The maximum angle of the pendulum is 15 degrees in order to remain in the linear region. The maximum force the cart can withstand is 2N and the maximum change in this force per time step is 0.1N per sampling time interval.

$$N = 5$$
$$\frac{-15\pi}{180} \leq \theta \leq \frac{15\pi}{180}$$
$$-2 \leq u \leq 2$$
$$-0.1 \leq \Delta u \leq 0.1$$

3. Simulate your controller on the linearized system for 50 seconds.

$$\Delta x_0 = \begin{bmatrix} -0.4 & \frac{14\pi}{180} & 0 & 0 \end{bmatrix}^T$$

Plot the response of the system outputs and control effort.

Note that:

- You should use Python for this assignment.
- You should show all your steps clearly and submit all the codes used and the results.

Sequence of sol:

① Discretize the continuous dynamics using Scipy in Python

② Formulate and solve the MPC using Casadi in Python.

③ Simulate & plot the results.

Submission Guidelines:

- Submit a zip file with the following:
    - The assignment (in this form) along with all resulting figures and the solution steps.
    - A ".ipynb" Python notebook containing all the code.
- Your submission will be through the following link:

  https://forms.gle/MRmh9hm35SBeEDvd7
- The submission deadline is on Sunday, 25-May-2025, at 23:59.
- Please note that cheating will not be tolerated and that it is your responsibility to ensure the genuineness of your work.

Best of Luck!