Management-IS project

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Project: Networks Reliability

Reliability is the quality of being trustworthy or performing consistently well. This performance may concern different aspects of a system's operation. In the case of transportation networks, two main reliability performances are studied: travel time reliability and connectivity reliability.

Connectivity reliability concerns the accessibility performances in the network, while travel time reliability studies the variability in travel time. It argues that traffic congestion is common in many cities, and in these cities, drivers are accustomed to congestion. They expect and plan for some delay, particularly during peak driving times. They need to adjust their schedules or budget extra time to allow for traffic delays.

The objective of this project is to study different ways to measure travel time reliability and to analyze the best way to take into account to guarantee being on time in most instances, despite the congestion. In paper [1], authors studied the impact of a management strategy, namely on-ramp metering, on travel time reliability. They used different reliability indicators to measure travel time reliability before and after the application of the ramp metering strategy. In your project, you will apply these indicators to a public transportation network and analyze their objective.

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Part 1: Main indicators

1. Read the paper and analyze the different indicators :

1.1 Standard Deviation (STD):

Calculates the distribution of travel times around the mean and indicates the variability.

$$STD = \sqrt{\frac{1}{N-1}\Sigma_N(TT_i - M)^2}$$

1.2 Coefficient of Variation (COV):

The ratio of the standard deviation to the mean, showing relative variability. The coefficient of variation is written as follows:

$$COV = \frac{STD}{M}$$

1.3 Buffer Time (BT):

This is the extra time travelers should allow for the average or expected travel time to ensure a higher likelihood of arriving on time. Typically, BT is set to a value that

ensures on-time arrival at least 95% of the time, and is calculated as the difference between the 95th percentile travel time and the average travel time.

1.4 Buffer Index (BI):

This is a corresponding metric that measures the ratio of slack time to average driving time, providing a standardized measure of the extra time required compared to typical driving conditions.

$$BI = \frac{TT_{95} - M}{M}$$

1.5 Misery Index (MI):

This is a metric used in transportation to measure how bad the travel experience is for the unluckiest travelers compared to the average traveler. It is calculated by taking the average travel time of the 20% of trips with the most delays, subtracting the average travel time of all trips, and dividing by the average travel time. The index helps understand the extent of extreme travel delays a small number of travelers may experience.

$$MI = \frac{M|_{TT_i > TT_{80}} - M}{M}$$

1.6 Probabilistic Measures:

In transportation, measuring the probability that a travel time falls within a certain range of average travel times. They help differentiate between reliable and unreliable travel times. For example, a policy goal could stipulate that 95% of travel times should not exceed a certain threshold that deviates from the median. formula

Calculate the probability that the travel time exceeds the median value by some magnitude β.
The range can be adjusted based on political goals or standards.

$$\Pr\left(TT_i \geq \beta + TT_{50}\right)$$

1.7 Skewness :

It measures the asymmetry of the travel time distribution. The larger the skewness value, the higher the probability of extreme travel times and the lower the reliability of travel times.

$$\lambda^{skew} = \frac{TT_{90} - TT_{50}}{TT_{50} - TT_{10}}$$

1.8 Width :

It measures how spread out the travel time distribution is around the median. Larger values mean greater spread, indicating greater variability in travel times.

$$\lambda^{var} = \frac{TT_{90} - TT_{10}}{TT_{50}}$$

1.9 Index (UIr) :

This metric combines skewness and breadth to provide a single measure that illustrates the probability of extreme travel times and their dispersion relative to the median, independent of location-specific travel time characteristics.

$$UI_r = \frac{\lambda^{var} \ln(\lambda^{skew})}{L_r}$$

2. Load the matrix containing 50 **T**ravel **T**imes of the bus network in "TT_link.mat" *(based on "Route.mat" used in tutorials).* This matrix provides the travel time (in minutes) between two consecutive bus stops on a given bus route when a bus route is linking these stops.

2.1 Load The Data :

```
>> cd Documents
>> load('TT_link.mat')
```

2.2 Size of The Data :

```
>> size(mat_data.TT_link)
ans =

    15    15    50
```

3. Compute the shortest travel time using the Dijkstra algorithm from any origin to any destination of the network to build a "TT_OD" matrix for the 50 samples of the TT_link 3D-Matrix.

3.1 The Result of computing The shortest travel time using Dijkstra :

```
>> TT_OD = zeros(size(TT_link));  % Replace TT_matrix with TT_link
>> for sample = 1:size(TT_link, 3)  % Replace TT_matrix with TT_link
    % Call dijkstra function for the current sample
    [shortest_paths, total_times] = dijkstra_algorithm(TT_link(:, :, sample), 1);
    % Assign the total_times to the TT_OD matrix for the current sample
    TT_OD(:, :, sample) = total_times;
end
>> disp(TT_OD(:, :, 1));
Columns 1 through 13:

         0    3.7800    7.0300    9.4300   15.8400   21.4100    2.2200    6.1800   25.3000   26.7800   14.0300    5.6900   31.0100
    3.4600         0    3.2500    5.6500   12.0600   17.6300    5.6800    2.4000   21.5200   23.0000   17.4900    5.5500   27.2300
    7.0400    3.5800         0    2.4000   10.5700   16.1400    9.2600    2.8600   20.0300   21.5100   21.0700    9.1300   25.7400
    9.9100    6.4500    2.8700         0    8.1700   13.7400   12.1300    5.7300   17.6300   19.1100   23.9400   12.0000   23.3400
   14.8900   11.4300   11.8500    9.0500         0    5.5700   17.1100    9.5500    9.4600   10.9400   28.9200   16.9800   15.1700
   21.4600   18.0000   18.4200   15.6200    6.5700         0   23.6800   16.1200   16.0300   17.5100   35.4900   23.5500    9.6000
    1.9600    5.7400    8.9900   11.3900   17.8000   23.3700         0    8.1400   27.2600   28.7400   11.8100    3.4700   32.9700
    5.3400    1.8800    2.3000    4.7000    9.6600   15.2300    7.5600         0   19.1200   20.6000   19.3700    7.4300   24.8300
   25.7500   22.2900   22.7100   19.9100   10.8600   16.4300   27.9700   20.4100         0    2.6700   39.7800   27.8400   26.0300
   24.4700   21.0100   21.4300   18.6300    9.5800   15.1500   26.6900   19.1300    1.9000         0   38.5000   26.5600   24.7500
    9.9100   13.6900   16.9400   19.3400   25.7500   31.3200    7.9500   16.0900   35.2100   36.6900         0   11.4200   40.9200
    4.7500    3.9900    7.2400    9.6400   16.0500   21.6200    2.7900    6.3900   25.5100   26.9900   14.6000         0   31.2200
   35.4100   31.9500   32.3700   29.5700   20.5200   13.9500   37.6300   30.0700   29.9800   31.4600   49.4400   37.5000         0
   18.0300   14.5700   10.9900   13.3900   21.5600   27.1300   20.2500   13.8500   31.0200   32.5000   32.0600   20.1200   36.7300
    9.7000    8.9400   12.1900   14.5900   21.0000   26.5700    7.7400   11.3400   30.4600   31.9400   19.5500    4.9500   36.1700

Columns 14 and 15:

   18.8900   10.9300
   15.1100   10.7900
   11.8600   14.3700
   14.7300   17.2400
   23.7100   22.2200
   30.2800   28.7900
   20.8500    8.7100
   14.1600   12.6700
   34.5700   33.0800
   33.2900   31.8000
   28.8000   16.6600
   19.1000    5.2400
   44.2300   42.7400
         0   25.3600
   24.0500         0
```

3.2 Script Dijkstra_algorithm :

dijkstra_algorithm.m

```
function [shortest_paths, total_times] = dijkstra_algorithm(graph, start_node)
    % Implementation of Dijkstra's algorithm

    n = size(graph, 1);
    shortest_paths = zeros(n, n);
    total_times = inf * ones(n, n);

    for i = 1:n
        [shortest_paths(:, i), total_times(:, i)] = dijkstra_2(graph, i);
    end
end


####Implementation of Dijkstra2 that helps dijkstra_algorithm using graph and start_node

function [shortest_path, total_time] = dijkstra_2(graph, start_node)
    n = size(graph, 1);
    unvisited_nodes = 1:n;
    total_time = inf * ones(n, 1);
    total_time(start_node) = 0;
    shortest_path = zeros(n, 1);

    while ~isempty(unvisited_nodes)
        [~, current_node] = min(total_time(unvisited_nodes));
        current_node = unvisited_nodes(current_node);
        unvisited_nodes(unvisited_nodes == current_node) = [];

        neighbors = find(graph(current_node, :) > 0);

        for neighbor = neighbors
            alt_time = total_time(current_node) + graph(current_node, neighbor);

            if alt_time < total_time(neighbor)
                total_time(neighbor) = alt_time;
                shortest_path(neighbor) = current_node;
            end
        end
    end
end
```

4. Compute all the key performance indicators given in the paper *(named COV, TT10, TT50, TT80, TT90, TT95, λ^{var}, λ^{dev} , UI, BI, PTI, MI, Pr(TTi>B+TT50) in the paper)*. Where TT80/ TT90, TTk… is the 80th / 90th / kth ... percentile, also known as percentile score or centile.

A 5% percentile could be chosen to define the *freeflow Travel time.* To define the route length L

let consider a 15km/h mean speed together with the free flow travel time.

4.1 Computing all indicators :

- COV :

```
>> COV
COV =

 Columns 1 through 14:

       0   0.1429   0.0933   0.0768   0.0886   0.0751   0.1329   0.0984   0.0896   0.0788   0.1089   0.1117   0.0765   0.0956
  0.1533        0   0.1347   0.0998   0.1058   0.0839   0.1162   0.1465   0.0987   0.0864   0.0910   0.1403   0.0838   0.1157
  0.1132   0.1397        0   0.1440   0.1085   0.0913   0.0990   0.1360   0.1000   0.0865   0.0796   0.0893   0.0845   0.1518
  0.0933   0.1056   0.1396        0   0.1259   0.1025   0.0840   0.0927   0.1101   0.0935   0.0767   0.0758   0.0921   0.1245
  0.1011   0.1217   0.1025   0.1521        0   0.1583   0.0928   0.1491   0.1359   0.1192   0.0812   0.0946   0.1189   0.0801
  0.0809   0.0913   0.0770   0.1038   0.1398        0   0.0752   0.1058   0.1129   0.0962   0.0703   0.0799   0.1555   0.0660
  0.1337   0.0931   0.0728   0.0633   0.0751   0.0668        0   0.0720   0.0814   0.0714   0.1274   0.1476   0.0709   0.0848
  0.1075   0.1449   0.1369   0.0972   0.1373   0.1009   0.0910        0   0.1110   0.0934   0.0809   0.0936   0.0920   0.1237
  0.0751   0.0810   0.0768   0.0943   0.1323   0.1033   0.0721   0.0888        0   0.1447   0.0670   0.0754   0.0790   0.0578
  0.0820   0.0888   0.0803   0.0891   0.1360   0.1135   0.0778   0.0999   0.1500        0   0.0689   0.0814   0.0923   0.0620
  0.1159   0.0964   0.0815   0.0708   0.0588   0.0537   0.1398   0.0848   0.0652   0.0584        0   0.1162   0.0632   0.0684
  0.1159   0.1501   0.1012   0.0802   0.0899   0.0764   0.1559   0.1086   0.0869   0.0776   0.0976        0   0.0787   0.0884
  0.0581   0.0607   0.0614   0.0756   0.0960   0.1273   0.0557   0.0678   0.0861   0.0782   0.0548   0.0582        0   0.0524
  0.0918   0.1114   0.1413   0.1127   0.0906   0.0770   0.0853   0.1190   0.0801   0.0716   0.0745   0.0913   0.0756        0
  0.0991   0.1043   0.0901   0.0790   0.0746   0.0672   0.1182   0.0815   0.0746   0.0671   0.0909   0.1487   0.0709   0.0772

 Column 15:

  0.0903
  0.0935
  0.0726
  0.0693
  0.0776
  0.0665
  0.1012
  0.0741
  0.0656
  0.0710
  0.0925
  0.1466
  0.0495
  0.0765
       0
```

- TT10 :

```
>> TT10
TT10 =

 Columns 1 through 13:

         0    2.8300    6.2300    8.6500   13.7050   19.5250    1.9450    5.2200   22.0300   24.0050   10.1950    5.0300   29.8750
    2.8150         0    2.9450    5.0300   10.1600   15.9900    4.9950    1.8550   18.2500   20.4450   13.5200    3.9050   26.3300
    5.7100    2.8450         0    1.9100    9.0100   14.2200    8.0200    1.9750   17.5150   19.2500   17.0600    7.2900   24.8450
    8.0650    5.0350    1.8850         0    6.7250   12.3550   10.2450    4.1550   15.2750   17.1650   19.0700    9.5300   22.8850
   13.5400   10.1550    9.2750    6.7800         0    4.8000   15.9400    7.8700    7.8900    9.3000   24.7050   14.8200   15.1750
   19.2700   15.7750   14.8000   12.6400    4.7350         0   21.8050   13.5050   13.3900   15.1000   30.4500   20.3250    9.4150
    1.9600    5.2100    8.6550   11.1300   15.8800   21.7350         0    7.6050   24.4300   26.5100    7.9250    2.9000   32.8100
    5.1000    1.9650    1.9900    4.2250    7.9150   13.2400    7.4150         0   16.2600   18.3300   15.8300    6.3700   24.3300
   23.2650   19.5700   18.0800   15.8400    8.0000   13.6450   25.5050   17.4950         0    1.9550   34.0450   24.5050   24.7500
   23.7950   20.8700   19.8600   17.7850    9.5400   15.0000   26.0650   18.1900    1.8900         0   35.9300   25.3950   25.5550
   10.0500   13.5300   17.1150   19.5750   25.4000   31.1950    7.6200   15.8250   33.9300   36.1600         0   10.8200   41.4950
    5.0800    3.7500    7.1650    9.5250   14.6450   20.1300    2.8300    6.0500   23.3500   25.3700   11.5100         0   31.1600
   31.0150   27.8000   26.3350   24.3150   16.1400   10.0500   33.2650   25.4800   24.4050   26.2350   42.0050   32.6050         0
   15.0700   11.5250    8.0100   10.3650   18.0850   24.2900   17.3850   10.3500   27.0100   28.8150   26.5500   16.2550   34.7450
    9.3950    8.0600   11.3000   13.6500   19.2350   24.7900    6.9350   10.2000   27.5900   29.7400   15.8900    3.7600   35.4300

 Columns 14 and 15:

   14.7450    9.4300
   11.1950    8.7800
    7.6300   12.1150
    9.8800   14.5300
   18.1600   19.5900
   24.2100   25.7350
   16.9400    7.0700
   10.0200   11.2300
   28.3100   29.4000
   29.3650   30.3400
   26.3150   15.8050
   15.7750    3.8950
   35.9150   37.3250
         0   21.0300
   20.2750         0
```

- TT50 :

```
>> TT50
TT50 =

 Columns 1 through 13:

         0    3.6700    7.1800    9.7150   15.5700   21.5500    2.3650    6.0650   25.4950   26.4950   12.1950    5.8600   33.3200
    3.4500         0    3.6700    6.0400   12.1550   17.6850    5.8500    2.2950   21.6100   22.6550   15.7350    4.8900   29.9950
    7.1100    3.5550         0    2.4150   10.1900   16.2450    9.3700    2.4750   20.0500   21.0650   19.4200    8.3600   28.0200
    9.4850    5.8900    2.2850         0    8.1000   14.1100   11.8550    4.7600   17.8950   18.9400   21.6900   10.7600   25.7700
   15.4750   11.9800   10.5650    8.8700         0    5.6750   18.0200    9.5100    9.8200   10.9550   27.9450   16.9750   17.6550
   21.6500   18.1350   16.5600   14.7850    6.1800         0   24.0700   15.9150   15.8750   16.9800   34.1350   22.9250   12.0800
    2.3950    6.0950    9.6900   12.1000   17.9000   23.9250         0    8.3500   27.7600   28.8300    9.9200    3.3750   35.6400
    5.9450    2.5450    2.4950    4.8450    9.8350   15.6600    8.2450         0   19.5400   20.5300   18.2850    7.2450   27.5250
   25.6150   21.9450   20.8200   19.1150   10.2150   16.0100   27.8700   19.4700         0    2.3700   37.9350   26.9550   27.9150
   27.0100   23.4600   21.7600   20.0200   11.3050   16.4700   29.2100   21.1900    2.2950         0   38.9100   28.3350   29.4800
   11.9750   15.7450   19.0550   21.4350   27.1800   33.4450    9.6250   17.8650   37.2350   38.3900         0   13.0850   45.1450
    5.9000    4.7450    8.2450   10.6950   16.5700   22.5900    3.5300    6.8400   26.3000   27.4950   13.6200         0   34.3150
   34.1700   30.4100   29.1050   26.9950   18.2850   11.9900   36.5650   28.1900   27.9600   29.0600   46.8200   35.0800         0
   16.9250   13.5400   10.0800   12.3550   19.9650   26.2550   19.5700   12.5650   29.9200   31.2100   29.1850   18.3200   38.1050
   10.7450    9.5400   12.9400   15.4500   21.2650   27.4350    8.3200   11.8700   31.0300   32.2700   18.2250    4.6100   39.2750

 Columns 14 and 15:

   16.8250   10.8200
   13.1100    9.8600
    9.7250   13.2500
   12.0000   15.4650
   20.1100   21.4850
   26.3450   27.6950
   19.2000    8.5850
   12.0450   12.1950
   30.0100   31.7350
   31.3300   33.1500
   29.1850   18.1100
   17.9250    5.1350
   38.1650   40.1150
         0   23.6100
   22.6000         0
```

- TT80 :

```
>> TT80
TT80 =

 Columns 1 through 13:

         0    4.1600    7.9150   10.2850   16.8050   22.6700    2.6850    6.4050   27.3300   28.5850   13.6150    6.4450   35.7000
    4.1150         0    4.0800    6.3800   13.1050   19.2500    6.4600    2.6700   23.6700   24.8100   16.9300    5.4850   31.8350
    7.8550    4.0250         0    2.7050   11.5250   17.6500   10.2200    2.7100   21.8300   22.7500   20.6200    8.9700   30.6250
   10.1700    6.4950    2.6400         0    9.0750   15.5300   12.7550    5.1100   19.6800   20.7750   23.2150   11.2950   28.4500
   16.7600   13.5500   11.6200    9.9600         0    6.9100   19.3800   11.1950   11.0250   11.9650   29.4850   18.2800   20.0300
   23.1950   19.8000   17.9050   16.3250    6.8500         0   25.6400   17.1750   17.1800   18.6600   35.8400   24.6000   13.8850
    2.7150    6.6000   10.1750   12.5600   19.2550   25.1500         0    8.8700   29.8900   30.8700   10.9250    3.9650   38.0700
    6.6050    2.7900    2.8500    5.2300   10.9650   16.6550    9.0000         0   21.3950   22.6600   19.4350    7.8650   29.8300
   27.3200   23.4700   21.9150   20.2100   11.2150   17.5250   29.7250   20.9150         0    2.7900   39.8850   28.4950   29.8450
   28.4300   24.7300   23.4950   21.5400   12.9050   19.3650   30.9450   22.6550    2.6650         0   41.0650   30.0700   31.8200
   13.2100   16.7300   20.5200   22.8450   28.9250   35.0150   10.8100   19.2100   39.3850   40.5350         0   14.3650   48.0450
    6.6050    5.4450    9.0300   11.2800   17.8650   24.0850    4.1400    7.7550   28.5000   29.9900   14.5000         0   36.9250
   35.7250   31.9250   30.4500   28.6350   19.9150   13.9950   38.1100   29.6100   30.0950   31.4750   48.4100   37.1800         0
   18.4200   14.8200   11.3300   13.7350   21.9200   27.6850   20.8850   13.8650   32.3100   32.6800   31.3000   20.1100   40.8500
   11.4700   10.2850   14.2200   16.5150   22.7850   29.0650    9.0250   12.6700   33.4200   33.8050   19.6950    5.4200   41.9600

 Columns 14 and 15:

   18.6750   11.6200
   14.9000   10.4350
   11.0450   14.2500
   13.4600   16.6600
   21.7550   23.3450
   27.8100   29.7950
   20.9800    9.0700
   13.6850   12.8500
   32.0450   33.2700
   33.3450   35.0650
   30.2000   19.2950
   19.6000    5.7300
   40.3550   41.7150
         0   24.7800
   24.3700         0
```

- TT90 :

```
>> TT90
TT90 =

 Columns 1 through 13:

         0    4.2850    8.1100   10.5250   17.2600   23.8950    2.8250    6.6100   27.9250   29.9250   13.9650    6.6700   37.0150
    4.3050         0    4.2000    6.7650   13.3800   19.8300    6.9200    2.7750   24.5250   26.2800   17.5550    5.7450   32.8050
    8.0750    4.2350         0    2.8050   11.9650   18.4250   10.6150    2.8650   22.6900   23.9650   21.2650    9.3400   31.3050
   10.4800    6.6950    2.8150         0    9.4650   16.0400   12.9400    5.2100   20.5950   21.7850   23.7450   11.7500   29.4750
   17.6200   14.0750   11.9200   10.3800         0    7.3050   20.0050   11.6950   11.5450   13.2400   30.4000   19.0950   20.6750
   23.8700   20.4900   18.3800   16.8600    7.1450         0   26.4350   17.9650   18.5300   19.2200   36.9450   25.3150   14.5050
    2.8750    6.7350   10.4750   12.9050   19.5550   26.1350         0    9.1700   30.3500   32.2650   11.5750    4.3100   39.3900
    6.9500    2.9100    2.9100    5.5500   11.2750   17.9400    9.4300         0   22.2250   23.7400   20.0100    8.1500   30.6350
   27.9200   24.5600   22.4950   20.6550   11.7200   17.8450   30.5550   22.1300         0    2.8850   40.5850   29.9000   30.7450
   29.5600   26.1900   24.5300   22.5400   13.8550   20.1700   32.1600   23.9650    2.7750         0   42.1650   30.9950   32.4450
   13.4550   17.5200   21.4700   23.6850   29.5400   35.7050   11.2200   19.7000   40.1600   42.2100         0   15.0250   49.0300
    6.8500    5.7500    9.4850   11.7950   18.5300   24.7200    4.3800    8.0800   29.1300   30.6500   15.1050         0   37.7750
   36.1150   32.8050   30.8250   29.5400   20.5000   14.3550   38.4350   30.5300   30.9400   32.1650   49.2000   37.4600         0
   19.1350   15.4150   11.9050   14.1400   22.8150   29.2400   21.3100   14.2550   33.4700   33.8850   32.0500   20.4650   42.2500
   12.1600   10.7100   14.5350   16.8300   23.4400   29.5800    9.7750   12.8850   34.3100   35.3200   20.2150    5.8800   42.5300

 Columns 14 and 15:

   18.8650   11.9950
   15.1900   11.1100
   11.8250   14.6750
   14.1450   17.2500
   22.3750   23.8700
   28.5500   30.2250
   21.3650    9.4850
   14.0850   13.5850
   32.5200   34.2300
   34.2000   36.0000
   31.2000   20.3350
   20.0500    5.8850
   41.2400   42.9550
         0   25.2850
   25.0650         0
```

- TT95:

```
>> TT95
TT95 =

 Columns 1 through 13:

         0    4.4700    8.3100   10.8600   17.3400   24.3900    2.9200    6.9300   28.4600   30.2400   14.0300    7.0400   37.8200
    4.3200         0    4.3600    6.9700   13.8300   20.7100    7.0800    2.8500   25.0100   26.5300   17.6000    5.7900   34.2000
    8.2000    4.2700         0    2.9400   12.2200   18.9600   10.9400    2.9200   23.6500   25.5800   21.4400    9.4800   32.1700
   10.7600    6.8000    2.8700         0   10.2300   16.4600   13.1000    5.3300   21.3100   22.6900   23.8500   12.0000   29.8600
   18.3800   14.5000   12.0800   10.4700         0    7.4100   20.8200   11.8200   11.7300   13.4200   31.4200   19.6500   21.4900
   24.4900   20.6800   18.7200   17.0100    7.3400         0   26.8600   18.4100   18.8000   20.3600   37.7000   26.3000   14.7200
    2.9300    6.8700   10.7500   13.0700   19.7400   26.7000         0    9.3100   31.0900   32.7300   11.6300    4.3900   39.7200
    7.0100    2.9800    2.9300    5.6900   11.6200   18.4300    9.8300         0   22.8400   24.0100   20.1400    8.5200   31.4500
   29.2700   25.5000   22.7100   20.7800   11.8300   18.3300   31.7900   22.8400         0    2.9500   42.3100   30.6600   31.7800
   30.9600   27.0800   24.8300   22.9000   14.0900   20.4100   33.2200   25.0700    2.9100         0   44.5900   32.7700   34.1800
   14.1500   18.0400   21.8900   24.1900   29.8900   35.8500   11.5100   20.2300   40.8500   42.9900         0   15.2900   49.7500
    7.0800    5.8200    9.6200   12.0600   18.8200   25.2900    4.4200    8.3900   30.0300   31.2000   15.3100         0   38.8600
   36.3600   33.5500   31.5500   29.8900   20.6000   14.4400   38.8400   30.8200   31.9700   34.2000   49.4400   38.8800         0
   19.2500   15.7900   11.9400   14.3700   23.4000   29.8700   21.7800   14.4100   33.8300   36.7600   32.3800   21.0000   43.4200
   12.5800   10.9800   14.7500   17.0800   23.8300   29.7100   10.0500   13.1500   34.7800   36.6900   20.4900    5.9400   43.7600

 Columns 14 and 15:

   19.4300   12.2800
   15.8300   11.2400
   11.9000   14.9600
   14.4500   17.6000
   22.9100   24.6000
   29.7600   30.9200
   22.0700    9.7800
   14.1900   13.8100
   33.3700   36.1900
   35.0500   36.5500
   32.6800   20.8500
   20.4100    5.9500
   41.5000   43.3000
         0   25.7400
   25.3200         0
```

- lambda_var :

```
>> lambda_var
lambda_var =

 Columns 1 through 14:

      NaN    0.4469    0.2897    0.2275    0.2335    0.2258    0.4123    0.2819    0.2522    0.2353    0.3145    0.3430    0.2384    0.2785
   0.4362       NaN    0.3856    0.3212    0.3019    0.2669    0.3564    0.4336    0.3128    0.2686    0.2593    0.3855    0.2624    0.3535
   0.3502    0.4008       NaN    0.4265    0.3150    0.2918    0.3116    0.3818    0.3060    0.3005    0.2255    0.2620    0.2614    0.4391
   0.2841    0.2997    0.4311       NaN    0.4327    0.2909    0.2408    0.2468    0.3372    0.2917    0.2204    0.2296    0.2707    0.3808
   0.3128    0.3627    0.2655    0.4160       NaN    0.4599    0.2708    0.4154    0.3910    0.3761    0.2403    0.2845    0.3577    0.2362
   0.2411    0.2705    0.2367    0.2956    0.4215       NaN    0.2100    0.3082    0.3408    0.3098    0.2124    0.2606    0.4392    0.2107
   0.4050    0.2724    0.2162    0.1603    0.2156    0.2075       NaN    0.2042    0.2399    0.2157    0.3735    0.4415    0.1939    0.2672
   0.3213    0.3988    0.3768    0.3024    0.3767    0.3314    0.2929       NaN    0.3367    0.2767    0.2357    0.2968    0.2587    0.3462
   0.2344    0.2702    0.2224    0.2584    0.3749    0.2926    0.2255    0.2745       NaN    0.4198    0.2179    0.2283    0.2518    0.1686
   0.2653    0.2647    0.2284    0.2555    0.4025    0.3285    0.2450    0.3247    0.4444       NaN    0.2226    0.2603    0.2926    0.1815
   0.3424    0.2864    0.2506    0.2153    0.1652    0.1392    0.4042    0.2466    0.1858    0.1779       NaN    0.3416    0.1829    0.2181
   0.3390    0.4362    0.2978    0.2370    0.2520    0.2284    0.4504    0.3421    0.2540    0.2120    0.2790       NaN    0.2244    0.2586
   0.1564    0.1891    0.1792    0.2065    0.2439    0.3661    0.1525    0.1894    0.2706    0.2741    0.1588    0.1789       NaN    0.1463
   0.2470    0.3150    0.3899    0.3242    0.2662    0.2125    0.2246    0.3231    0.2279    0.2546    0.1998    0.2590    0.2277       NaN
   0.2964    0.3061    0.2666    0.2220    0.2161    0.1793    0.3744    0.2485    0.2317    0.2154    0.2524    0.4729    0.2121    0.2232

 Column 15:

   0.2634
   0.2495
   0.2147
   0.1985
   0.2332
   0.1872
   0.3157
   0.2116
   0.2140
   0.1873
   0.2786
   0.4002
   0.1489
   0.1995
      NaN
```

- lambda_skew :

```
>> lambda_skew
lambda_skew =

 Columns 1 through 14:

     NaN   0.7321   0.9789   0.7606   0.9062   1.1580   1.0952   0.6450   0.7013   1.3775   0.8850   0.9759   1.0726   0.9808
  1.3465      NaN   0.7310   0.7178   0.6140   1.2655   1.2515   1.0909   0.8676   1.6403   0.8217   0.8680   0.7667   1.0862
  0.6893   0.9577      NaN   0.7723   1.5042   1.0765   0.9222   0.7800   1.0414   1.5978   0.7818   0.9159   1.0346   1.0024
  0.7007   0.9415   1.3250      NaN   0.9527   1.0997   0.6739   0.7438   1.0305   1.6028   0.7844   0.8049   1.2842   1.0118
  1.1085   1.1479   1.0504   0.7225      NaN   1.8629   0.9543   1.3323   0.8938   1.3807   0.7577   0.9838   1.2177   1.1615
  0.9328   0.9979   1.0341   0.9674   0.6678      NaN   1.0442   0.8506   1.0684   1.1915   0.7626   0.9192   0.9099   1.0328
  1.1034   0.7232   0.7585   0.8299   0.8193   1.0091      NaN   1.1007   0.7778   1.4806   0.8296   1.9684   1.3251   0.9580
  1.1893   0.6293   0.8218   1.1371   0.7500   0.9421   1.4277      NaN   0.8186   1.4591   0.7026   1.0343   0.9734   1.0074
  0.9809   1.1011   0.6113   0.4702   0.6795   0.7759   1.1353   1.3468      NaN   1.2410   0.6812   1.2020   0.8942   1.4765
  0.7932   1.0541   1.4579   1.1275   1.4448   2.5170   0.9380   0.9250   1.1852      NaN   1.0923   0.9048   0.7554   1.4606
  0.7688   0.8014   1.2448   1.2097   1.3258   1.0044   0.7955   0.8995   0.8850   1.7130      NaN   0.8565   1.0644   0.7021
  1.1585   1.0101   1.1481   0.9402   1.0182   0.8659   1.2143   1.5696   0.9593   1.4847   0.7038      NaN   1.0967   0.9884
  0.6165   0.9176   0.6209   0.9496   1.0326   1.2191   0.5667   0.8635   0.8383   1.0991   0.4943   0.9616      NaN   1.3667
  1.1914   0.9305   0.8816   0.8970   1.5160   1.5191   0.7963   0.7630   1.2199   1.1169   1.0873   1.0387   1.2336      NaN
  1.0481   0.7905   0.9726   0.7667   1.0714   0.8110   1.0505   0.6078   0.9535   1.2055   0.8522   1.4941   0.8466   1.0602

 Column 15:

   0.8453
   1.1574
   1.2555
   1.9091
   1.2586
   1.2908
   0.5941
   1.4404
   1.0685
   1.0142
   0.9653
   0.6048
   1.0179
   0.6492
      NaN
```

- UIr :

```
>> UIr
UIr =

 Columns 1 through 10:

         NaN  -4.2363e-02  -1.8744e-03  -1.7936e-02  -5.8788e-03   6.4741e-03   9.2542e-03  -3.8543e-02  -1.5029e-02   1.1890e-02
   3.4492e-02          NaN  -3.6732e-02  -3.0676e-02  -3.7632e-02   1.2283e-02   1.9727e-02   1.1759e-02  -7.4639e-03   2.0968e-02
  -3.4636e-02  -5.2618e-03          NaN  -3.1749e-02   3.2869e-02   4.2065e-03  -6.2262e-03  -2.9572e-02   2.0856e-03   2.2214e-02
  -2.6860e-02  -5.4905e-03   3.6889e-02          NaN  -8.0719e-04   5.4052e-03  -2.3452e-02  -2.2775e-02   1.7035e-03   2.1709e-02
   8.5651e-03   1.5216e-02   3.9689e-03  -3.8955e-02          NaN   5.5926e-02  -3.1239e-03   3.7149e-02  -7.3746e-03   1.9137e-02
  -4.4599e-03  -1.7442e-04   2.4131e-03  -2.8250e-03  -4.3492e-02          NaN   2.2388e-03  -1.5543e-02   3.7873e-03   8.5617e-03
   1.0597e-02  -2.6841e-02  -1.8177e-02  -8.6117e-03  -1.0983e-02   3.6876e-04          NaN   6.1054e-03  -1.0126e-02   1.3357e-02
   1.4808e-02  -5.6162e-02  -2.2487e-02   1.1191e-02  -2.7696e-02  -3.8604e-03   2.5735e-02          NaN  -1.1320e-02   1.6489e-02
  -1.2048e-03   7.9096e-03  -3.3281e-02  -5.6174e-02  -3.7030e-02  -1.4513e-02   7.0615e-03   2.5481e-02          NaN   1.4298e-02
  -1.6339e-02   4.2371e-03   2.6184e-02   8.8335e-03   3.7845e-02   5.9266e-02  -3.8688e-03  -7.8905e-03   1.2682e-02          NaN
  -2.3923e-02  -1.9287e-02   1.6689e-02   1.1806e-02   1.1907e-02   1.2065e-04  -2.2814e-02  -8.1400e-03  -3.8123e-03   1.5106e-02
   1.3259e-02   1.3265e-03   1.2509e-02  -4.2125e-03   1.1602e-03  -6.4311e-03   2.1579e-02   4.8077e-02  -1.7715e-03   1.3220e-02
  -2.0112e-02  -4.9424e-03  -2.5964e-02  -3.0750e-03   2.0017e-03   1.4177e-02  -2.1369e-02  -8.6682e-03  -8.0171e-03   4.0862e-03
   1.1495e-02  -6.8969e-03  -1.4935e-02  -1.0152e-02   2.8305e-02   1.7369e-02  -1.2620e-02  -2.7248e-02   7.6102e-03   4.4401e-03
   3.7049e-03  -2.1874e-02  -2.2557e-03  -1.6993e-02   3.8099e-03  -7.3448e-03   4.5550e-03  -3.8575e-02  -1.8534e-03   6.3506e-03

 Columns 11 through 15:

  -6.0291e-03  -2.1117e-03   2.0743e-03  -9.5737e-04  -8.7423e-03
  -7.9925e-03  -1.3772e-02  -8.6550e-03   5.1738e-03   7.2038e-03
  -8.7135e-03  -5.8095e-03   1.1056e-03   1.8532e-04   9.6500e-03
  -8.4005e-03  -1.2577e-02   8.4079e-03   7.9051e-04   2.5354e-02
  -1.0462e-02  -1.1760e-03   8.7500e-03   6.2626e-03   1.0593e-02
  -9.0356e-03  -5.5404e-03  -5.1464e-03   1.2033e-03   9.4398e-03
  -1.0951e-02   7.5466e-02   6.7769e-03  -2.0316e-03  -3.2470e-02
  -1.3054e-02   2.5251e-03  -8.6613e-04   4.5239e-04   1.5249e-02
  -1.3124e-02   1.0606e-02  -3.4986e-03   1.1633e-02   2.8009e-03
   3.0830e-03  -6.5751e-03  -1.0190e-02   1.2171e-02   5.2299e-04
          NaN  -1.3355e-02   1.4168e-03  -1.3658e-02  -1.9436e-03
  -1.5380e-02          NaN   2.5713e-03  -5.3549e-04  -3.9744e-02
  -1.7560e-02  -1.7672e-03          NaN   8.0938e-03   5.2257e-04
   2.6234e-03   2.4849e-03   5.9357e-03          NaN  -1.7021e-02
  -6.3327e-03   4.7927e-02  -4.3873e-03   2.3111e-03          NaN

>>
```

- BI:

```
>> BI
BI =

 Columns 1 through 12:

       NaN   0.224322   0.149600   0.129579   0.114625   0.135158   0.231340   0.160687   0.125666   0.134743   0.153555   0.200750
   0.228250        NaN   0.218694   0.168836   0.161619   0.161200   0.202527   0.228660   0.156168   0.153569   0.122549   0.200796
   0.160520   0.203291        NaN   0.232394   0.180106   0.164319   0.159365   0.201547   0.177730   0.192685   0.115087   0.132562
   0.146339   0.158591   0.236749        NaN   0.250887   0.166764   0.114249   0.121916   0.190224   0.177441   0.106842   0.122439
   0.181522   0.204419   0.143182   0.209566        NaN   0.249747   0.161403   0.220822   0.206046   0.209837   0.133576   0.165425
   0.131889   0.141331   0.124513   0.154300   0.207197        NaN   0.118850   0.167984   0.189407   0.185610   0.115457   0.146419
   0.223995   0.137229   0.117626   0.088786   0.099918   0.118278        NaN   0.113556   0.123486   0.127097   0.187826   0.257303
   0.174283   0.215136   0.181356   0.169386   0.201034   0.181092   0.178657        NaN   0.177259   0.156139   0.110805   0.171263
   0.144925   0.156578   0.103724   0.113326   0.181960   0.150082   0.137983   0.159933        NaN   0.218102   0.121496   0.141066
   0.150194   0.157265   0.132342   0.144028   0.240208   0.180437   0.134268   0.191370   0.243377        NaN   0.141034   0.161159
   0.188436   0.159860   0.144204   0.124238   0.088516   0.073720   0.209974   0.131862   0.098549   0.115123        NaN   0.175774
   0.188318   0.239089   0.162594   0.131311   0.133544   0.122404   0.238720   0.195736   0.140576   0.126549   0.146027        NaN
   0.073466   0.105276   0.092361   0.108211   0.124737   0.180201   0.071685   0.100817   0.140100   0.162957   0.074012   0.105293
   0.132060   0.170740   0.201376   0.165999   0.153073   0.139085   0.124141   0.165028   0.126930   0.171223   0.110204   0.146977
   0.175130   0.162618   0.132716   0.108558   0.116159   0.089108   0.208601   0.117836   0.119192   0.130927   0.131644   0.251264

 Columns 13 through 15:

   0.130535   0.150985   0.133364
   0.147566   0.196505   0.147665
   0.138705   0.232828   0.121238
   0.145176   0.206862   0.123667
   0.200800   0.133059   0.126745
   0.230029   0.131568   0.107728
   0.108160   0.145232   0.155537
   0.140675   0.169557   0.127697
   0.138856   0.103929   0.136563
   0.168251   0.109859   0.101109
   0.096883   0.135361   0.159867
   0.126403   0.138493   0.196702
       NaN   0.076943   0.078504
   0.136947        NaN   0.105623
   0.115007   0.116678        NaN
```

- PTI :

```
>> PTI
PTI =

 Columns 1 through 14:

   1.0000   0.0000   0.0000  -0.0000  -0.0000  -0.0000   0.0000   0.0000   0.0000  -0.0000  -0.0000   0.0000   0.0000  -0.0000
        0   1.0000   0.0000  -0.0000  -0.0000  -0.0000   0.0000   0.0000        0        0        0        0        0        0
   0.0000        0   1.0000  -0.0000   0.0000   0.0000  -0.0000   0.0000   0.0000  -0.0000   0.0000   0.0000        0        0
   0.0000  -0.0000   0.0000   1.0000   0.0000   0.0000  -0.0000  -0.0000   0.0000  -0.0000   0.0000   0.0000   0.0000  -0.0000
  -0.0000   0.0000  -0.0000        0   1.0000   0.0000   0.0000   0.0000  -0.0000  -0.0000  -0.0000  -0.0000  -0.0000  -0.0000
  -0.0000   0.0000   0.0000  -0.0000        0   1.0000   0.0000   0.0000  -0.0000   0.0000  -0.0000  -0.0000  -0.0000  -0.0000
  -0.0000   0.0000   0.0000  -0.0000   0.0000  -0.0000   1.0000   0.0000   0.0000  -0.0000   0.0000   0.0000   0.0000   0.0000
   0.0000  -0.0000   0.0000  -0.0000   0.0000   0.0000   0.0000   1.0000        0        0        0        0        0        0
   0.0000  -0.0000   0.0000  -0.0000   0.0000   0.0000  -0.0000   0.0000   1.0000  -0.0000   0.0000   0.0000        0        0
  -0.0000   0.0000  -0.0000        0   0.0000   0.0000  -0.0000   0.0000  -0.0000   1.0000  -0.0000   0.0000   0.0000   0.0000
   0.0000  -0.0000   0.0000   0.0000   0.0000  -0.0000  -0.0000  -0.0000  -0.0000   0.0000   1.0000   0.0000   0.0000  -0.0000
   0.0000  -0.0000   0.0000  -0.0000  -0.0000  -0.0000   0.0000   0.0000  -0.0000   0.0000   0.0000   1.0000   0.0000  -0.0000
   0.0000  -0.0000   0.0000  -0.0000   0.0000  -0.0000   0.0000  -0.0000  -0.0000   0.0000  -0.0000  -0.0000   1.0000  -0.0000
  -0.0000   0.0000   0.0000  -0.0000   0.0000  -0.0000   0.0000  -0.0000  -0.0000   0.0000   0.0000  -0.0000   0.0000   1.0000
  -0.0000   0.0000        0        0   0.0000  -0.0000   0.0000  -0.0000   0.0000  -0.0000  -0.0000  -0.0000   0.0000  -0.0000

 Column 15:

  -0.0000
        0
        0
        0
   0.0000
   0.0000
  -0.0000
        0
        0
  -0.0000
  -0.0000
        0
   0.0000
        0
   1.0000
```

- MI :

```
>> MI
MI =

 Columns 1 through 10:

   4.3222e+00  -9.9701e-01  -4.7959e-02   1.4627e-01  -1.1905e-01   1.0751e-01  -3.8201e-01   5.1456e-01  -2.8397e-01   2.4687e-01
  -1.4627e-01   3.7596e+00   7.4817e-02   2.0313e-01  -2.9119e-02   3.8414e-02   3.2240e-01  -2.2318e-01  -2.7252e-01   2.7422e-01
   2.6143e-03  -2.4356e-01   3.7172e+00   9.6261e-02  -5.6677e-02   8.6576e-02   6.9466e-02   1.4921e-01  -2.8995e-01   3.4699e-01
  -1.6252e+00   4.3272e-01   1.3785e-01   3.7562e+00   5.4239e-01  -6.2068e-01   1.4648e+00   6.3657e-02  -1.1279e-01   2.0591e-01
   5.9503e-01  -6.2514e-01  -3.2647e-02   1.5204e-01   3.7488e+00  -3.4652e-02  -4.5582e-01   1.7811e-01  -4.5935e-01   4.7046e-01
   2.5611e+00  -3.5536e-01  -4.3008e-01  -1.4242e-02   6.9143e-02   3.2456e+00  -4.0029e+00   2.1715e-01   2.6192e-01  -3.0208e-01
   1.3417e+00  -1.8284e+00  -1.4950e-01   3.1033e-01  -2.3314e-01   5.2870e-01   2.4224e+00   7.9442e-01  -4.2036e-01   4.5269e-01
   9.3790e-01  -1.2657e-01  -2.2272e-01   2.0430e-01  -1.5958e-01   7.0469e-02  -1.3826e+00   3.7695e+00  -4.8601e-02   1.2932e-02
   6.1723e-01  -3.3124e-02   1.2100e+00  -2.0962e+00   2.4340e-01   2.0822e-01  -6.9030e-01   3.2696e-01   3.7916e+00   2.4213e-02
   1.9194e+00  -3.7150e+00  -3.2706e-02   2.0099e-01   8.8382e-02  -2.3418e-01  -1.2174e+00   1.9891e+00  -1.6777e-02   3.7348e+00
   1.5967e+00   3.7814e-01  -3.8213e+00   2.1418e+00   7.5727e-02  -8.5792e-01  -2.9184e+00   1.0375e+00  -5.6524e-01   3.9800e-01
  -3.6650e-02   7.7154e-02   4.6023e-02   8.0831e-03   4.3159e-02   6.6039e-03   5.4434e-03  -2.3088e-02   1.9515e-03  -3.1535e-02
  -4.2741e-01   3.6262e-01  -6.3662e-01   3.9908e-01  -4.8587e-02  -9.2345e-02   1.7968e-01   2.6969e-01   1.4405e-01  -6.7217e-02
   2.2262e-01  -5.3450e-01   5.2387e-01  -2.1452e-01  -9.6261e-02   2.8689e-01   7.1435e-01  -1.1767e-01  -4.4659e-01   5.5714e-01
   3.0751e-01  -7.2423e-01  -1.4630e-01   2.8354e-01   6.7489e-02  -6.3886e-02   1.1744e-01   2.0870e-01  -8.6261e-01   8.4644e-01

 Columns 11 through 15:

  -2.2476e-02   3.6395e-01  -1.2721e-02  -5.2497e-02   2.0765e-02
  -5.8801e-02  -1.7940e-01  -3.7536e-02  -2.7866e-02   1.0474e-01
  -5.2082e-02   8.5216e-02  -9.3960e-02   3.3318e-02  -3.9744e-02
  -6.8843e-02  -4.0688e-01   7.8429e-02   6.5541e-02   3.6331e-03
  -3.2061e-02   3.8922e-01   2.9610e-02  -6.5244e-02  -9.3318e-02
   3.1873e-01   1.8333e+00   4.5359e-01  -8.3793e-02  -1.7469e-01
   1.8403e-01   8.4772e-01  -4.3040e-01  -1.2502e-03   1.1842e-01
   7.2522e-02   5.1054e-01   5.2006e-02  -5.9439e-02   1.0250e-01
  -4.0878e-02   3.2945e-01   3.3915e-03  -3.2484e-02  -2.4413e-02
  -2.4592e-02   1.0754e+00   1.1437e-01  -1.0226e-01  -2.4539e-02
   4.0343e+00   1.3468e+00   4.4532e-01   2.3651e-01   4.7302e-02
   3.0815e-02   3.7783e+00  -1.0814e-02  -3.2504e-02  -3.0945e-04
  -3.9058e-02  -2.5650e-01   3.8663e+00   5.8782e-02   1.5509e-01
  -6.5100e-01  -9.3242e-02  -8.5580e-02   3.8819e+00   9.6164e-02
  -1.0513e-01   1.9329e-01  -3.3776e-02  -2.0108e-02   3.7798e+00
```

- Pr(TTi>B+TT50) :

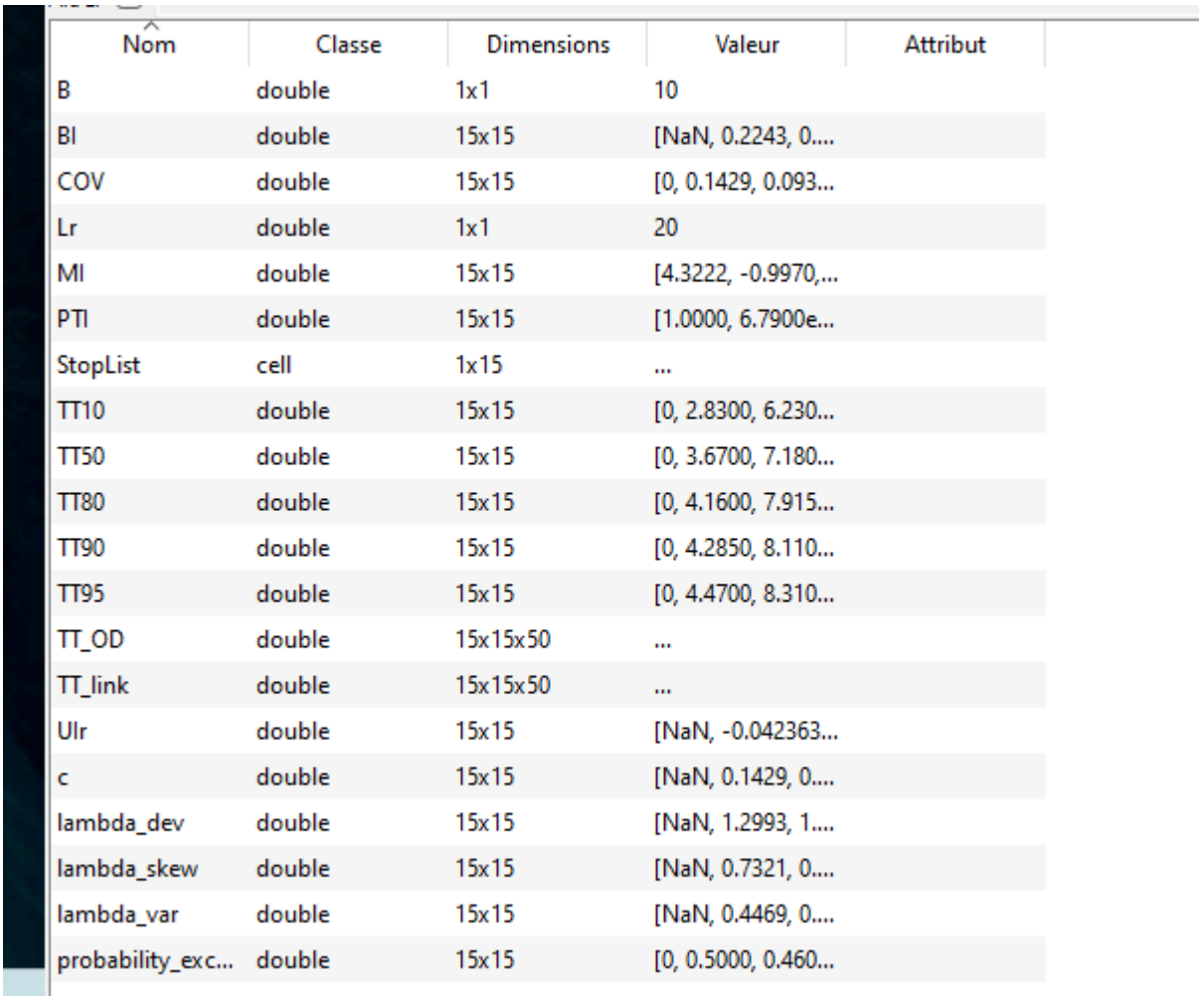
```
>> probability_exceeding
probability_exceeding =

 Columns 1 through 14:

        0   0.5000   0.4600   0.4200   0.4400   0.4400   0.4400   0.4400   0.4800   0.4600   0.4800   0.4600   0.4800   0.4600
   0.4400        0   0.4400   0.4800   0.4800   0.4800   0.5000   0.4800   0.4800   0.4800   0.5000   0.4800   0.5000   0.5000
   0.5000   0.4600        0   0.4400   0.4400   0.4800   0.4800   0.4400   0.4600   0.5000   0.4600   0.4800   0.5000   0.4800
   0.5000   0.4400   0.4400        0   0.4600   0.4600   0.4800   0.4600   0.5000   0.4800   0.4800   0.4600   0.4800   0.4600
   0.5000   0.5000   0.4800   0.5000        0   0.5000   0.4800   0.4800   0.4800   0.4800   0.4600   0.4600   0.5000   0.4800
   0.4800   0.5000   0.4800   0.5000   0.4800        0   0.5000   0.5000   0.4800   0.4800   0.5000   0.4800   0.5000   0.4800
   0.4200   0.4800   0.4400   0.4600   0.5000   0.5000        0   0.4400   0.5000   0.4800   0.4800   0.4600   0.4800   0.4800
   0.4400   0.4200   0.4800   0.4400   0.5000   0.4800   0.4400        0   0.5000   0.4800   0.4600   0.4800   0.5000   0.5000
   0.4800   0.4400   0.4800   0.5000   0.4800   0.4600   0.4800   0.4800        0   0.4800   0.4600   0.4600   0.4800   0.4600
   0.4600   0.5000   0.5000   0.4800   0.4600   0.4800   0.4600   0.4800   0.4800        0   0.5000   0.5000   0.5000   0.4800
   0.4800   0.4800   0.5000   0.5000   0.4400   0.5000   0.5000   0.4800   0.4800   0.4800        0   0.4600   0.4600   0.4800
   0.4800   0.5000   0.4600   0.4600   0.4800   0.4800   0.4600   0.4800   0.5000   0.5000   0.4800        0   0.4800   0.4800
   0.5000   0.4800   0.5000   0.5000   0.5000   0.4800   0.4800   0.5000   0.4600   0.4600   0.4800   0.4800        0   0.5000
   0.4800   0.4800   0.4600   0.4800   0.4800   0.4600   0.5000   0.4400   0.5000   0.4600   0.5000   0.5000   0.5000        0
   0.4800   0.5000   0.4800   0.4800   0.5000   0.4800   0.4600   0.4200   0.5000   0.5000   0.4800   0.4800   0.4800   0.4800

 Column 15:

   0.5000
   0.4400
   0.4800
   0.4800
   0.5000
   0.4800
   0.4600
   0.4800
   0.4600
   0.5000
   0.5000
   0.4600
   0.5000
   0.4800
        0
```

4.2 Scripts :

We used a function to calculate all indicators, naming the calculator indicator, as you can see in this picture.

```
function[COV, TT10, TT50, TT80, TT90, TT95, lambda_var, lambda_skew, UIr,BI,PTI, MI, probability_exceeding] = calculate_indicator
    % Input: TT_link 3D matrix (assumed to be of dimensions num_stops x num_stops x num_samples)
    %        B: Constant for Pr(TTi > B + TT50)
    %        Lr: Route length

    % Compute COV
    B = 0.05;  % Buffer time as a fraction of travel time (5%)
    m_speed = 15;  % Mean speed in km/h

############################################################

    COV = std(TT_OD, 0, 3) ./ mean(TT_OD, 3);
    COV(isnan(COV)) = 0;

############################################################

    % Compute Percentiles
    TT10 = prctile(TT_OD, 10, 3);
    TT50 = prctile(TT_OD, 50, 3);
    TT80 = prctile(TT_OD, 80, 3);
    TT90 = prctile(TT_OD, 90, 3);
    TT95 = prctile(TT_OD, 95, 3);

    TT_freeflow=TT95
##############################################

    lr = m_speed*mean(TT_freeflow/60);


    lambda_var = (TT95 - TT10) ./ TT50;
    lambda_skew = (TT90 - TT50) ./ (TT50 - TT10);

#########################################################

    UIr = (lambda_var.*log(lambda_skew))./lr;

#################################################
    BT = TT95 - mean(TT_OD, 3);
    BI = BT ./ mean(TT_OD, 3);


    TT_new = TT_OD > TT80;
    TT_0000 = TT_OD .* TT_new;

    PTI = TT95 / TT_freeflow;

    MI = (TT80 - mean(TT_0000, 3)) / mean(TT_0000, 3);

    probability_exceeding = sum(TT_OD > (B + TT50), 3) / size(TT_OD, 3);
end
```

4.3 List of Variable :



Nom	Classe	Dimensions	Valeur	Attribut
B	double	1x1	10	
BI	double	15x15	[NaN, 0.2243, 0....	
COV	double	15x15	[0, 0.1429, 0.093...	
Lr	double	1x1	20	
MI	double	15x15	[4.3222, -0.9970,...	
PTI	double	15x15	[1.0000, 6.7900e...	
StopList	cell	1x15	...	
TT10	double	15x15	[0, 2.8300, 6.230...	
TT50	double	15x15	[0, 3.6700, 7.180...	
TT80	double	15x15	[0, 4.1600, 7.915...	
TT90	double	15x15	[0, 4.2850, 8.110...	
TT95	double	15x15	[0, 4.4700, 8.310...	
TT_OD	double	15x15x50	...	
TT_link	double	15x15x50	...	
Ulr	double	15x15	[NaN, -0.042363...	
c	double	15x15	[NaN, 0.1429, 0....	
lambda_dev	double	15x15	[NaN, 1.2993, 1....	
lambda_skew	double	15x15	[NaN, 0.7321, 0....	
lambda_var	double	15x15	[NaN, 0.4469, 0....	
probability_exc...	double	15x15	[0, 0.5000, 0.460...	

5. Comment key performance indicators results. Which index will you use if you were a regular user compared to an occasional user :

5.1 COV :

The COV value is a measure of relative change in travel time. Lower COV values indicate more consistent and reliable travel times, while higher values indicate more unpredictability. Policy makers and travelers can use these results to understand and improve transport network reliability and identify routes that may require targeted interventions to reduce variability.

5.2 TT10 / TT50 / TT80 / TT90 / TT95 :

TT10 and TT50 (10th percentile and 50th percentile) may be of interest to the average user who values consistency and can plan trips around these more frequent travel times.

TT80, TT90 and TT95 represent longer journey times and, while less common, are crucial to understanding possible delays. Regular users can use them to understand worst-case scenarios.

5.3 BI / BT :

The buffer index is a measure of the extra time that must be planned for a trip to ensure on-time arrival, taking into account normal traffic fluctuations. High values in the matrix represent routes that require travelers to allow a significant amount of extra time due to higher variability, indicating that these routes are less reliable. Conversely, lower values indicate a more reliable route with less extra time. This information is critical for network users and planners to understand and mitigate the unpredictability of travel times.

5.4 MI :

The misery index quantifies how late the worst-off travelers (the first 20% of travel time) are compared to the average traveler. Positive values indicate "pain" or higher levels of delay, while negative values may indicate data anomalies or calculation errors, since one would not expect negative travel times in a standard travel time reliability analysis. This index can be used to identify which parts of the network are susceptible to extreme delays and can be targeted for improvements to increase overall reliability.

5.5 PR :

These values represent a probabilistic measure of travel time reliability. Most values are between 0.4 and 0.6, indicating that there is a 40-60% chance that the travel time is longer

than the median value plus some buffer. This data is vital for planning and risk assessment, showing the likelihood of delays on different routes or times.

5.6 Conclusion :

As a regular user who generally values consistency and planning based on predictable travel patterns, I rely on metrics like the Buffer Index (BI) and Planning Time Index (PTI). These indices give me an idea of the extra time it may take to ensure on-time arrival.

As a casual user, I would probably be more interested in probabilistic indicators such as the Misery Index (MI) and Pr(TTi>B+TT50). MI would tell me how bad the worst-case scenario might be, and the probability indicator would tell me the likelihood of significant delays, which was crucial when I wasn't familiar with the usual traffic patterns.

Part 2: Value of time

1. Considering values of time (VOT) given on Table 1 and using the VOT for the case "without a specific purpose" in the Île-de-France area, calculate the maximum and minimum time-savings benefits in the worst and best situation in the samples of the TT_OD matrix compared to the initial conditions.

1.1 Result :

- Max Savings (Worst Travel Time) :

VOT_max_savings [15x15 double]

	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-8.5348	-8.0532	-6.9407	-5.8045	-9.4497	-8.7467	-5.0549	-4.865	-7.5819	-8.7856	-3.5682	-6.7023	-7.8432	
2	-9.2017	-8.7514	-7.5896	-6.5389	-8.7875	-9.4629	-5.7931	-5.5945	-6.9094	-8.9699	-4.2	-7.4081	-8.0256	
3	-9.899	-9.4402	-7.9629	-6.8125	-8.1187	-9.4288	-6.0895	-5.8967	-6.2092	-8.3106	-4.5752	-8.0512	-7.3815	
4	-9.4649	-9.899	-8.36	-7.2181	-7.6466	-8.9946	-6.4989	-6.3004	-5.7779	-7.8546	-5.0027	-7.619	-6.9607	
5	-7.8916	-8.2137	-9.899	-8.8208	-6.4752	-8.0921	-8.0332	-7.8176	-4.5895	-6.6738	-6.5446	-6.0781	-5.8169	
6	-6.7526	-7.0898	-8.7248	-9.899	-5.3257	-6.8752	-6.8828	-6.6728	-3.4134	-5.5433	-7.6038	-4.8934	-4.637	
7	-8.0579	-7.6	-6.498	-5.3533	-9.899	-8.3125	-4.6246	-4.4213	-8.0142	-9.2577	-3.1274	-6.261	-8.2679	
8	-9.425	-8.9785	-8.0304	-6.9236	-8.3325	-9.899	-6.1864	-5.9983	-6.4248	-8.5225	-4.6693	-7.6105	-7.5819	
9	-5.9432	-6.2671	-7.9582	-6.8571	-4.6037	-6.1997	-9.899	-9.4487	-2.6913	-4.7775	-4.5952	-4.1971	-3.8694	
10	-5.7646	-6.0952	-7.7511	-6.7697	-4.3491	-5.8729	-9.4629	-9.899	-2.5061	-4.5153	-4.2978	-3.9463	-3.6005	
11	-6.2786	-5.8263	-4.7348	-3.5445	-8.0702	-6.5046	-2.8244	-2.6049	-9.899	-7.4129	-1.3215	-4.3539	-6.4581	
12	-8.3325	-7.867	-6.7507	-5.6069	-9.2283	-8.5994	-4.902	-4.6749	-7.3112	-9.899	-3.3792	-6.4933	-8.9234	
13	-4.3691	-4.77	-6.4248	-7.6209	-2.9517	-4.5429	-4.5866	-4.3776	-1.0032	-3.2338	-9.899	-2.6477	-2.2772	
14	-7.9838	-7.5616	-6.1057	-4.9106	-6.1807	-7.5116	-4.2142	-3.9691	-4.3539	-6.4182	-2.659	-9.899	-5.4131	
15	-7.4404	-6.9635	-5.8586	-4.6864	-8.3182	-7.6437	-4.0033	-3.7677	-6.4363	-9.0231	-2.4367	-5.605	-9.899	

- Min Savings (Best Travel Time) :

VOT_min_savings [15x15 double]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0.6973	1.3642	1.8458	2.9583	4.0945	0.44935	1.1523	4.8441	5.034	2.3171	1.1134	6.3308	3.1967	2.0558
2	0.6555	0	0.6973	1.1476	2.3095	3.3602	1.1115	0.43605	4.1059	4.3045	2.9897	0.9291	5.699	2.4909	1.8734
3	1.3509	0.67545	0	0.45885	1.9361	3.0866	1.7803	0.47025	3.8095	4.0023	3.6898	1.5884	5.3238	1.8478	2.5175
4	1.8021	1.1191	0.43415	0	1.539	2.6809	2.2525	0.9044	3.4001	3.5986	4.1211	2.0444	4.8963	2.28	2.9383
5	2.9403	2.2762	2.0074	1.6853	0	1.0783	3.4238	1.8069	1.8658	2.0815	5.3095	3.2253	3.3545	3.8209	4.0821
6	4.1135	3.4456	3.1464	2.8092	1.1742	0	4.5733	3.0239	3.0162	3.2262	6.4856	4.3557	2.2952	5.0056	5.2621
7	0.45505	1.1581	1.8411	2.299	3.401	4.5457	0	1.5865	5.2744	5.4777	1.8848	0.64125	6.7716	3.648	1.6312
8	1.1296	0.48355	0.47405	0.92055	1.8687	2.9764	1.5666	0	3.7126	3.9007	3.4741	1.3765	5.2298	2.2885	2.3171
9	4.8669	4.1696	3.9658	3.6319	1.9408	3.0419	5.2953	3.6993	0	0.4603	7.2077	5.1215	5.3038	5.7019	6.0297
10	5.1319	4.4574	4.1344	3.8038	2.1479	3.1293	5.5499	4.0261	0.43605	0	7.3929	5.3837	5.6012	5.9527	6.2985
11	2.2753	2.9916	3.6204	4.0727	5.1642	6.3545	1.8288	3.3943	7.0747	7.2941	0	2.4862	8.5775	5.5451	3.4409
12	1.121	0.90155	1.5666	2.032	3.1483	4.2921	0.6707	1.2996	4.997	5.2241	2.5878	0	6.5198	3.4057	0.97565
13	6.4923	5.7779	5.53	5.129	3.4741	2.2781	6.9473	5.3561	5.3124	5.5214	8.8958	6.6652	0	7.2513	7.6218
14	3.2158	2.5726	1.9152	2.3475	3.7934	4.9885	3.7183	2.3874	5.6848	5.9299	5.5451	3.4808	7.24	0	4.4859
15	2.0416	1.8126	2.4686	2.9355	4.0404	5.2127	1.5808	2.2553	5.8957	6.1313	3.4628	0.8759	7.4623	4.294	0

2. Analyze and comment results linked to the monetary waste of time due to reliability of travel time.

For minimum savings: These values represent the best case, with an improvement in travel time compared to the baseline. Positive values represent time savings, which translates into monetary savings based on VOT.

To maximize savings: Values may represent the worst-case scenario, where travel time is longer than the baseline. Negative values may indicate lost time and financial losses due to reduced reliability.

Positive minimum savings value: Indicates increased reliability and more efficient travel times, thus saving time and cost for travelers.

Negative maximum savings value: Indicates that when travel times are unpredictable and longer than expected, reliability is reduced, resulting in higher costs.