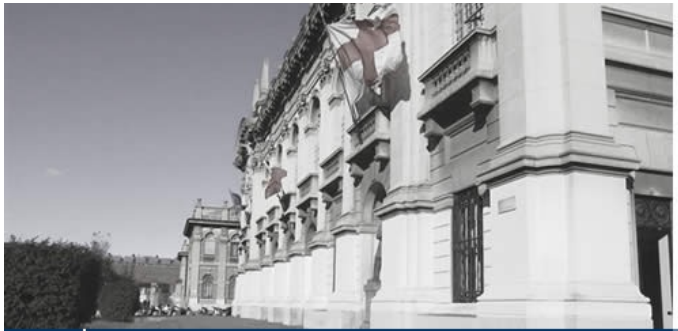

POLITECNICO DI MILANO

Computing Infrastructures





Operational laws

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Credits: Raffaela Mirandola, Jane Hilston, Ed Lazowska, Marco Gribaudo, John Zahorian

Model Definition

Existing system

System Design

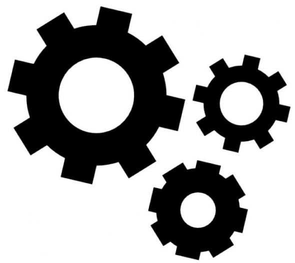
Parameterize the Model

Performance Model

Define the goal

Create the Model

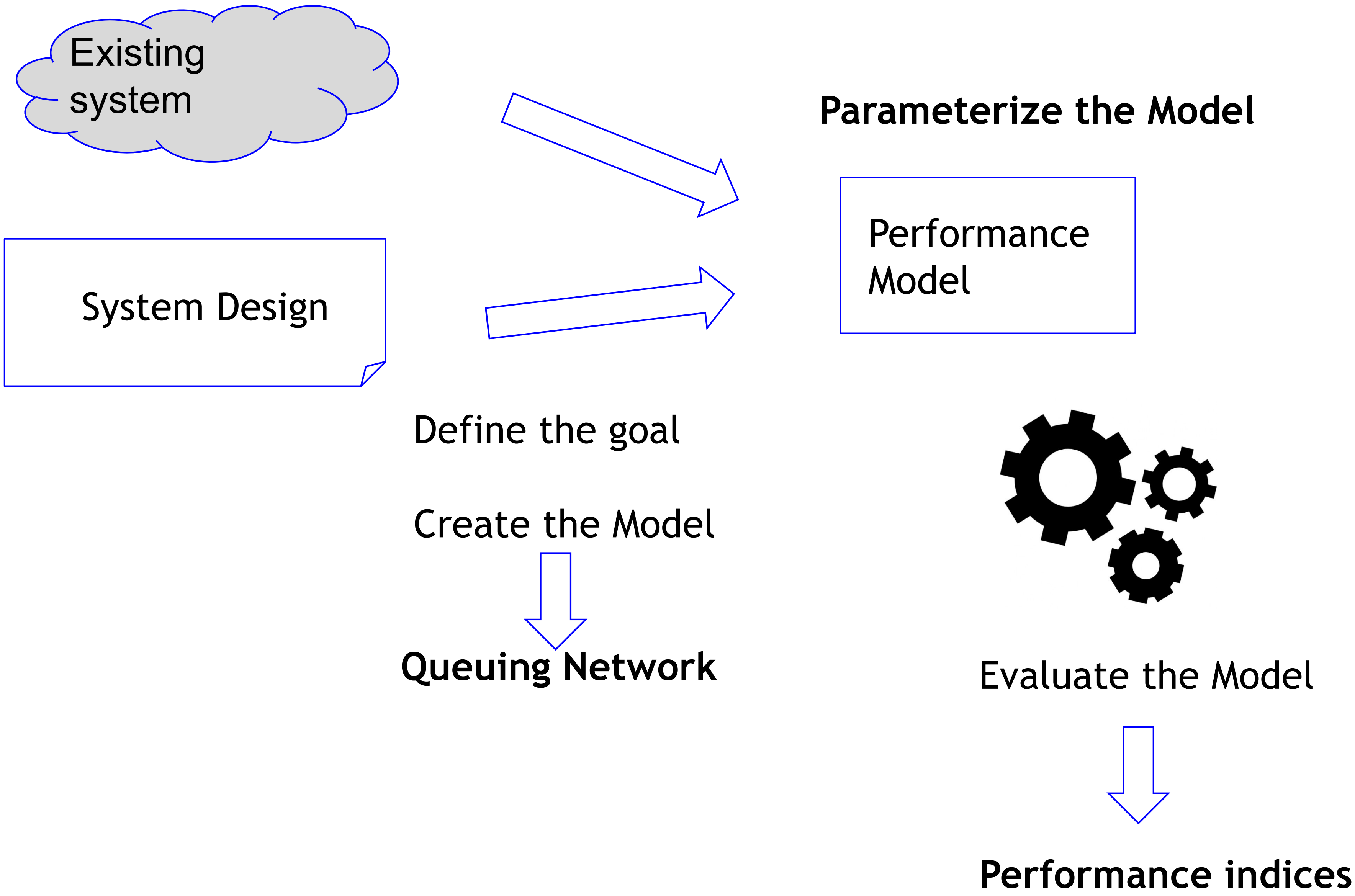
Queuing Network



Evaluate the Model

Performance indices

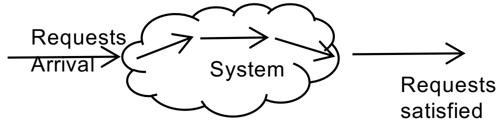
Model Definition

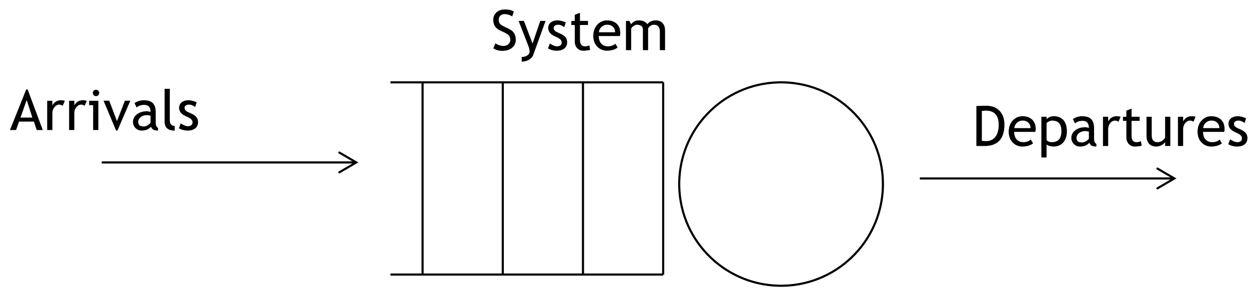


Operational laws

- Operational laws are simple equations which may be used as an abstract representation or model of the average behaviour of almost any system
- The laws are very general and make almost no assumptions about the behaviour of the random variables characterising the system
- Another advantage of the laws is their simplicity:
  - they can be applied quickly and easily

Operational laws



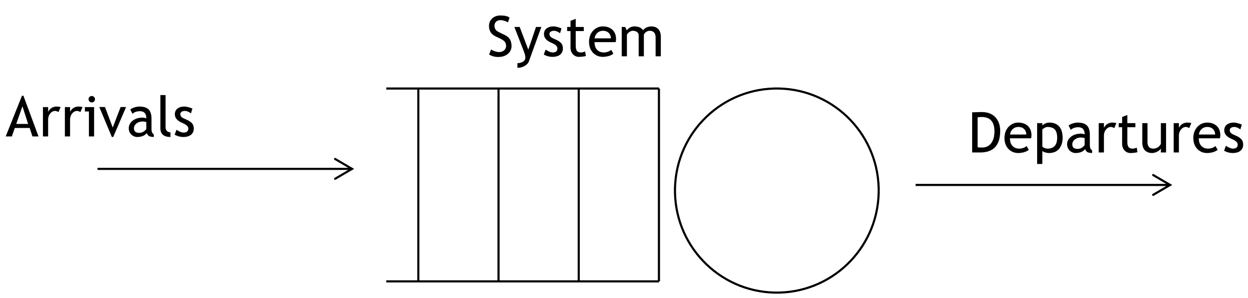


- Operational laws are based on observable variables - values which we could derive from watching a system over a finite period of time
- We assume that the system receives requests from its environment
- Each request generates a job or customer within the system
- When the job has been processed the system responds to the environment with the completion of the corresponding request

Observations and measurements

If we observed such an abstract system we might measure the following quantities:

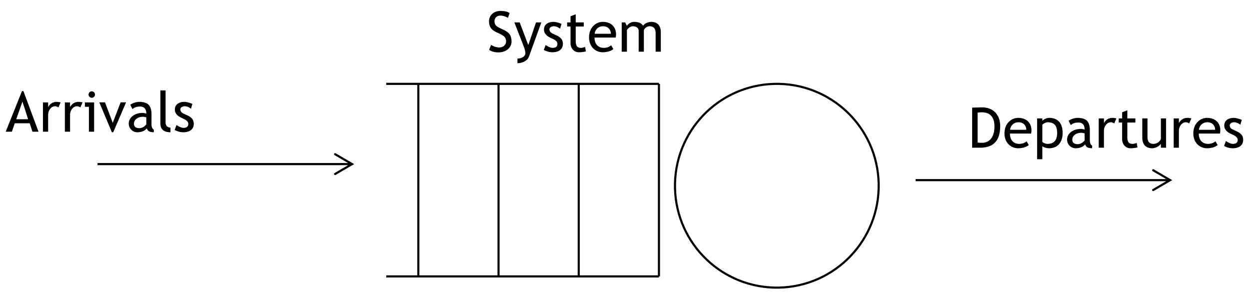
- T, the length of time we observe the system
- A, the number of request arrivals we observe
- C, the number of request completions we observe
- B, the total amount of time during which the system is busy ($B \le T$)
- N, the average number of jobs in the system



Four important quantities

From these observed values we can derive the following four important quantities:

- $\lambda = A/T$, the arrival rate
- $X = C\ /T$, the throughput or completion rate
- $U = B/T$, the utilisation
- $S = B/C$, the mean service time per completed job

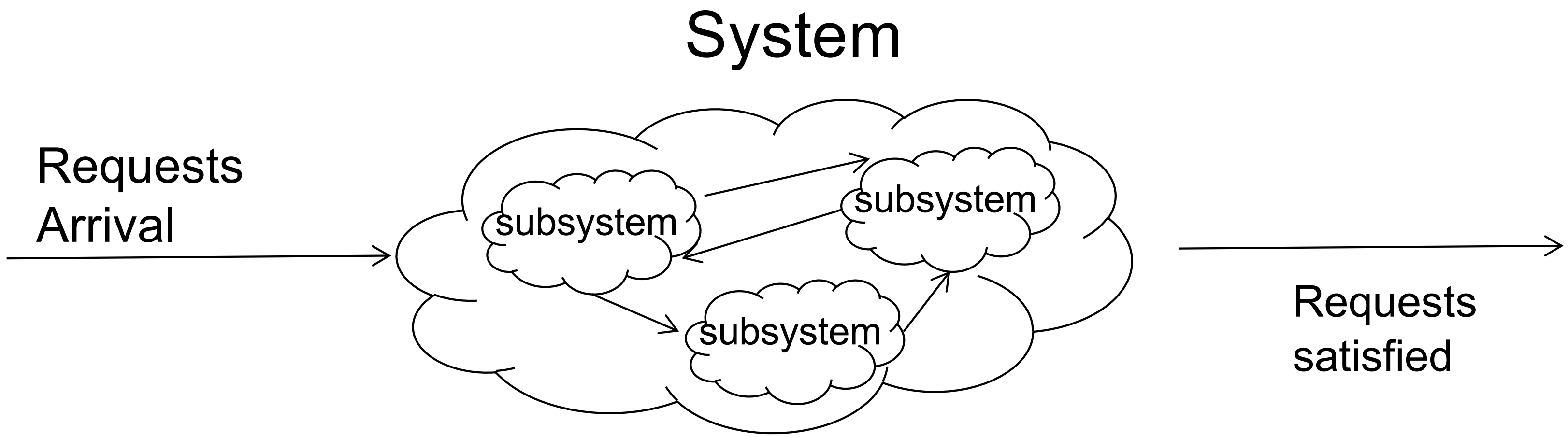


Job flow balance

- We will assume that the system is job flow balanced
  - This means that the number of arrivals is equal to the number of completions during an observation period, i.e., $A = C$
- This is a testable assumption because an analyst can always test whether the assumption holds
  - It can be strictly satisfied by careful choice of measurement interval
- Note that if the system is job flow balanced the arrival rate will be the same as the completion rate, that is:

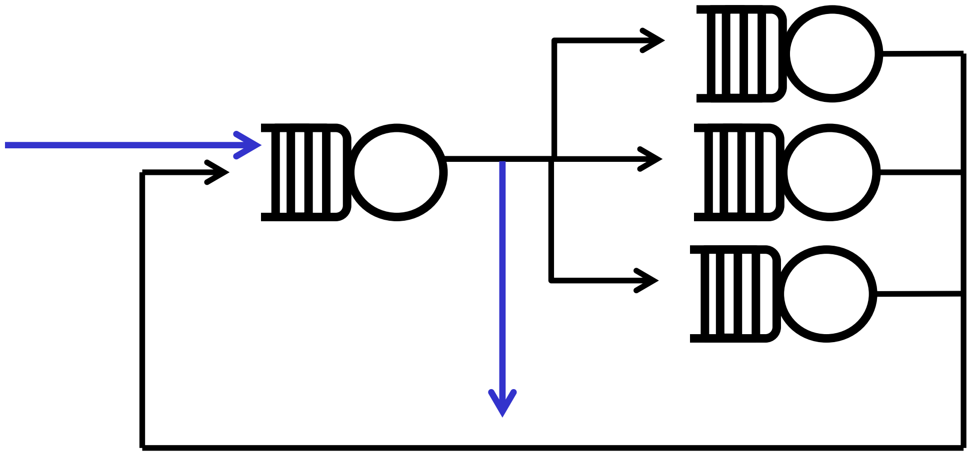
$$\boxed{\lambda = X}$$

Operational laws



- A system may be regarded as being made up of a number of devices or resources
- Each of these may be treated as a system in its own right from the perspective of operational laws
- An external request generates a job within the system; this job may then circulate between the resources until all necessary processing has been done; as it arrives at each resource it is treated as a request, generating a job internal to that resource

Operational laws



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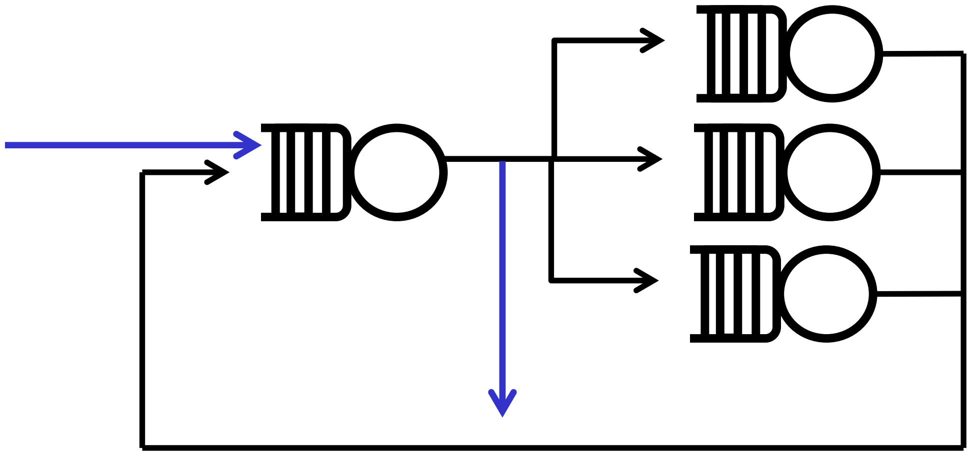
Operational laws



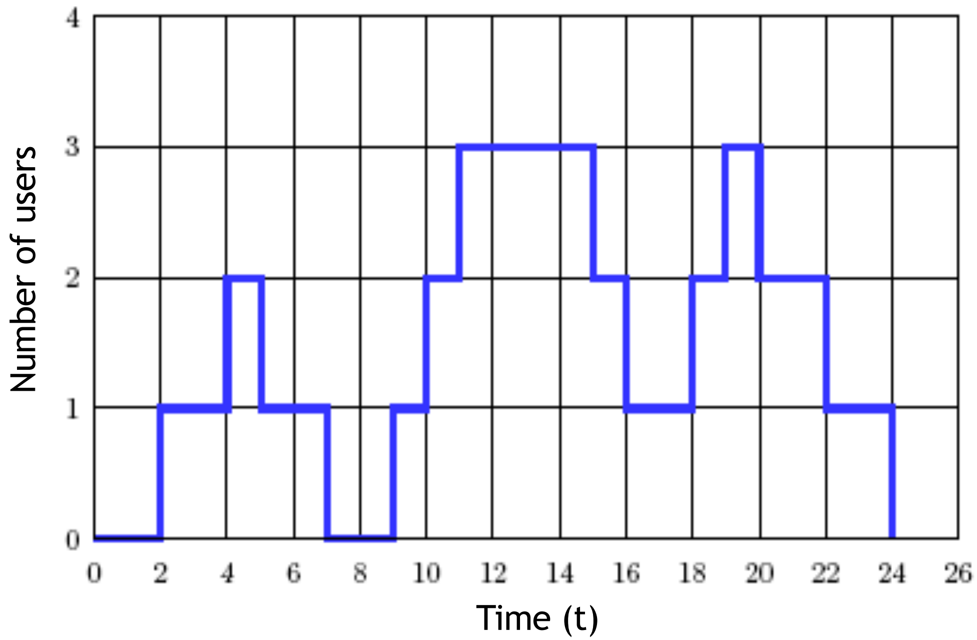
- T, the length of time we observe the system
- A_k, the number of request arrivals we observe for resource k
- C_k, the number of request completions we observe at resource k
- B_k, the total amount of time during which the resource k is busy ($B_k \leq T$)
- N_k, the average number of jobs in the resource k (queueing or being served)

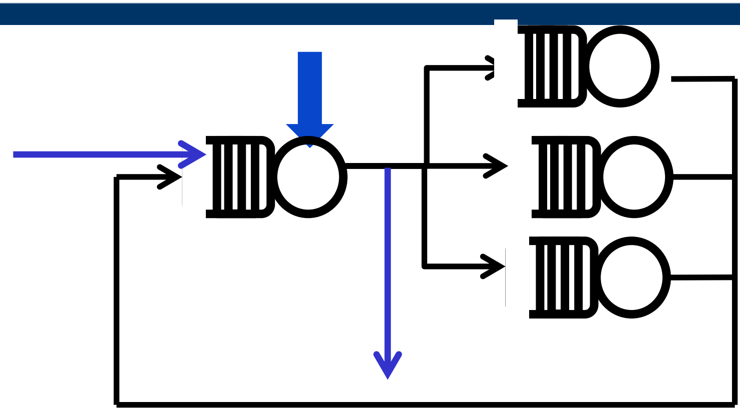
Four important quantities

From these observed values we can derive the following four important quantities for resource k:

- $\lambda_k = A_k/T$, the arrival rate
- $X_k = C_k\ /T$, the throughput or completion rate
- $U_k = B_k/T$, the utilisation
- $S_k = B_k/C_k$, the mean service time per completed job

Example





Let us observe the *k*-th resource and show in the graph the total number of users in k (both waiting for service and actually served)

T = 26 s

Arrivals number

$A_k = 7$

Completions number

$C_k = 7$

Arrival rate: $\lambda_k = A_k/T = 7/26$ req/s

Throughput: $X_k = C_k/T = 7/26$ req/s

Utilization: $U_k = B_k/T = 20/26 = 0.77 = 77\%$

Average service time: $S_k = B_k/C_k = 20/7$ s

Utilization law

Let us recall the following definitions for a resource k:

Throughput: $X_k = C_k / T$

Service time $S_k = B_k / C_k$

Utilization: $U_k = B_k / T$

From:

$$X_k = C_k / T$$

we can derive (utilization law):

$$\boxed{U_k = X_k S_k}$$

Utilization law

Let us recall the following definitions for a resource k:

Throughput: $X_k = C_k/T$

Service time $S_k = B_k/C_k$

Utilization: $U_k = B_k/T$

From:

$$X_k\, S_k = C_k/T \;*\; B_k/C_k = B_k/T = U_k$$

we can derive (utilization law):

$$\boxed{U_k = X_k S_k}$$

Example

- Let us consider a resource k serving 40 requests/s, each of them requiring on average 0.0225 s

- From the utilization law we have:

$$U_k=X_kS_k$$

- Let us consider a gas station serving 2 cars per minute dedicating to each of them 12 s for refuelling

- From the utilization law we have:

  $U=XS=$

Example

- Let us consider a resource k serving 40 requests/s, each of them requiring on average 0.0225 s

- From the utilization law we have:

$$U_k=X_kS_k=40 \times 0.0225 = 0.9 \dashrightarrow 90\%$$

- Let us consider a gas station serving 2 cars per minute dedicating to each of them 12 s for refuelling

- From the utilization law we have:

  U=XS=2 cars/min x 12s/car = 2 cars/min x 0.2 min/cars = 0.4 -->40%

Little Law

Little's law:

$$\boxed{N = XR}$$

Little law can be applied to the entire system as well as to some subsystems

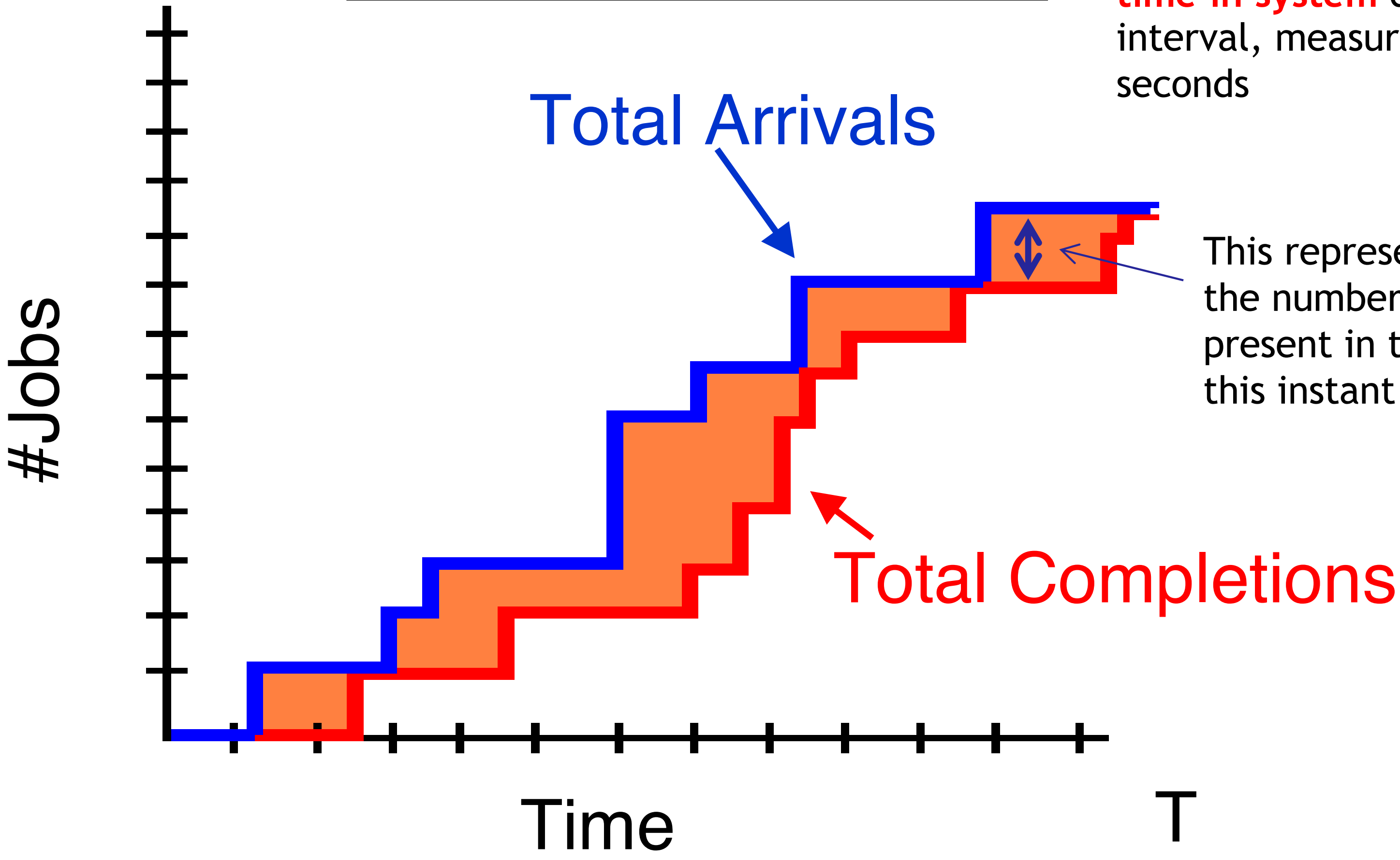
N = average number of requests in the system

If the system throughput is *X* requests/sec, and each request remains in the system on average for R seconds, then for each unit of time, we can observe on average XR requests in the system

Derivation of Little Law

W = total time in system
(in job-seconds)

the area W between the arrival and completion functions represents the **accumulated time in system** during that interval, measured in request-seconds



Derivation of Little Law

W denotes the accumulated time in system (jobs- sec)

if there are 3 requests in the system during a 2 second period, then W is 6 request-seconds

N the average number of requests in the system is: $N=W/T$

R the average system residence time per request is: $R=W/C$

We can write:

$$N = W/T$$

$$N = XR$$

Derivation of Little Law

W denotes the accumulated time in system (jobs- sec)

if there are 3 requests in the system during a 2 second period, then W is 6 request-seconds

N the average number of requests in the system is: N=W/T

R the average system residence time per request is: R=W/C

We can write:

N = W/T = C/T * W/C = X*R, so

$$N = XR$$

Example

- Consider a disk that serves 40 requests/seconds (X = 40 req/s) and suppose that on average there are 4 requests (N = 4) present in the disk system (waiting to be served or in service)
- Little's law tells us that the average time spent at the disk by a request must be
- If we know that each request requires 0.0225 seconds of disk service we can then deduce that the average waiting time (time in the queue) is

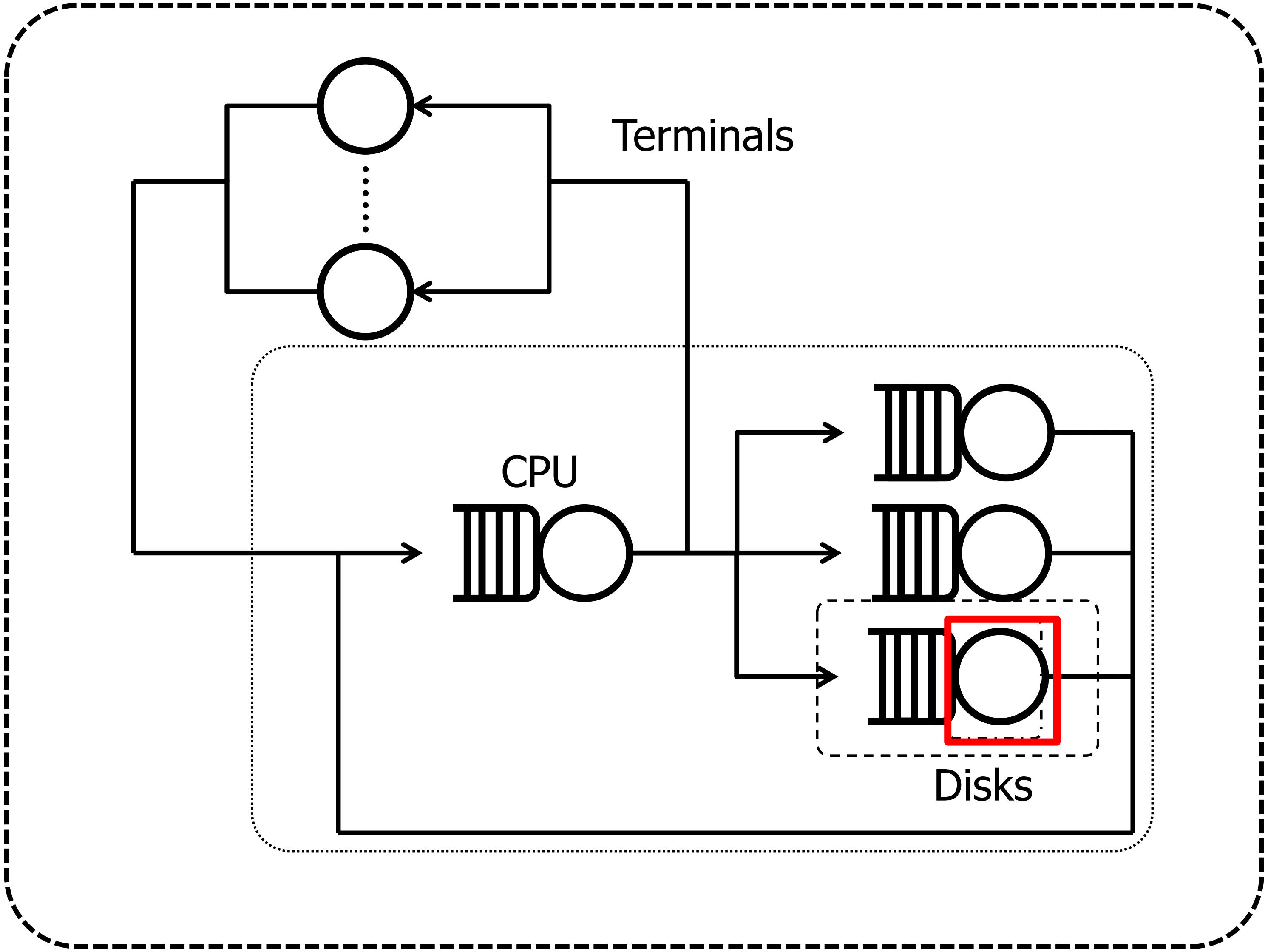
Example

- Consider a disk that serves 40 requests/seconds (X = 40 req/s) and suppose that on average there are 4 requests (N = 4) present in the disk system (waiting to be served or in service)

- Little's law tells us that the average time spent at the disk by a request must be 4/40 = 0.1 seconds

- If we know that each request requires 0.0225 seconds of disk service we can then deduce that the average waiting time (time in the queue) is 0.0775 seconds

Application of Little's Law at different levels



Little's Law, level 1

It can be applied to the single *server* Disk (*without the queue*) (Red Box)

$N_{(1)}$ in this case represents the percentage of time in which the server Disk is busy, so it corresponds to U_{disk}

$R_{(1)}$ represents the requests average service time

$X_{(1)}$ represents the rate of serving requests

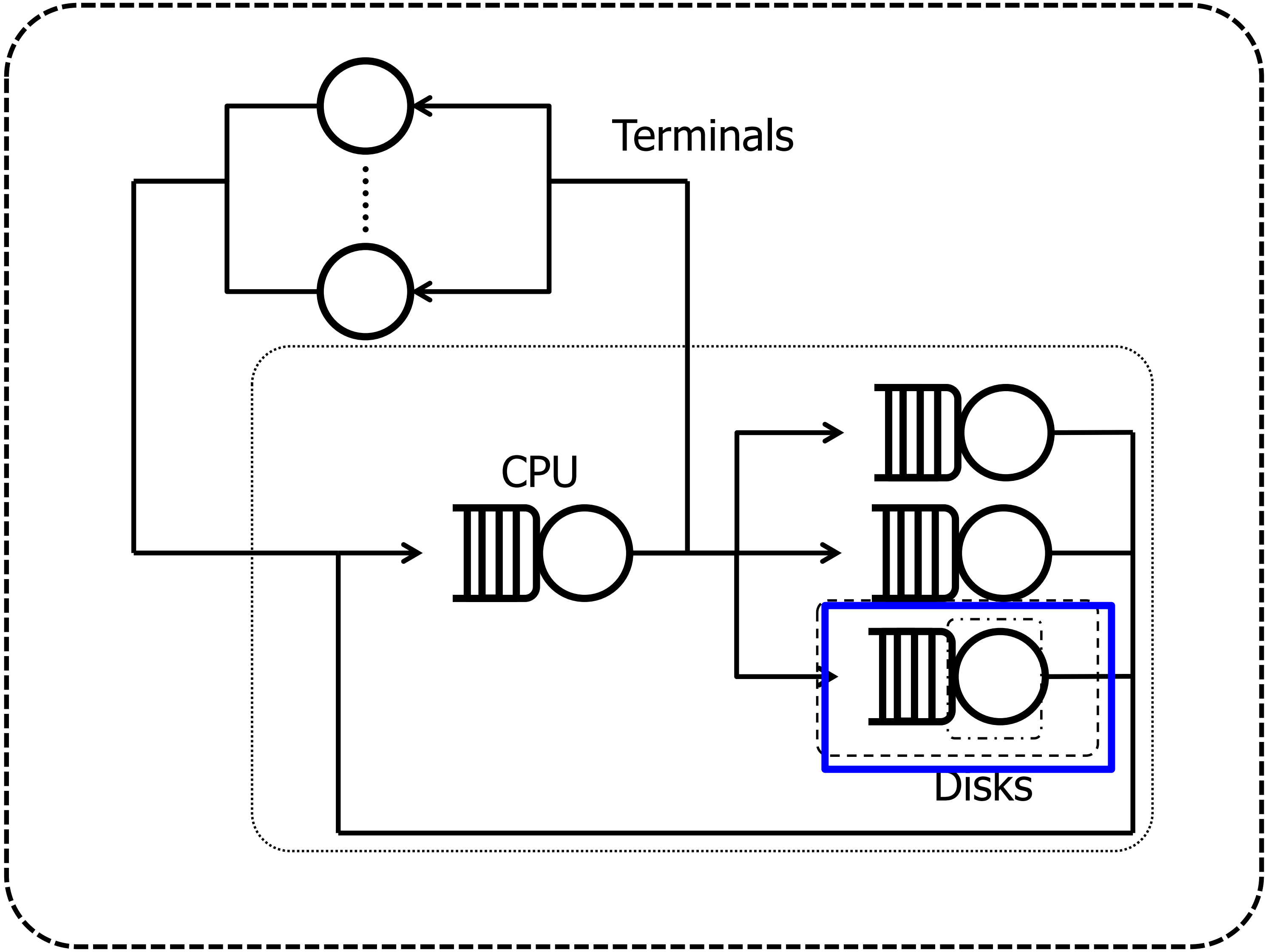
Let us apply Little law:

$X_{(1)}$ =40 req/sec, S =$R_{(1)}$ =0.0225 sec,

Little law $N_{(1)} = X_{(1)} R_{(1)} = 0.9$

$U_{(1)} = X_{(1)} S$, $U_{(1)} = 90\%$

Application of Little's Law at different levels



Little's Law, level 2

Let us include now the queue (Blue Box)

$N_{(2)}$ is the number of users in the service center (waiting + in service)

$R_{(2)}$ is the time spent in the service center (waiting time + service time)

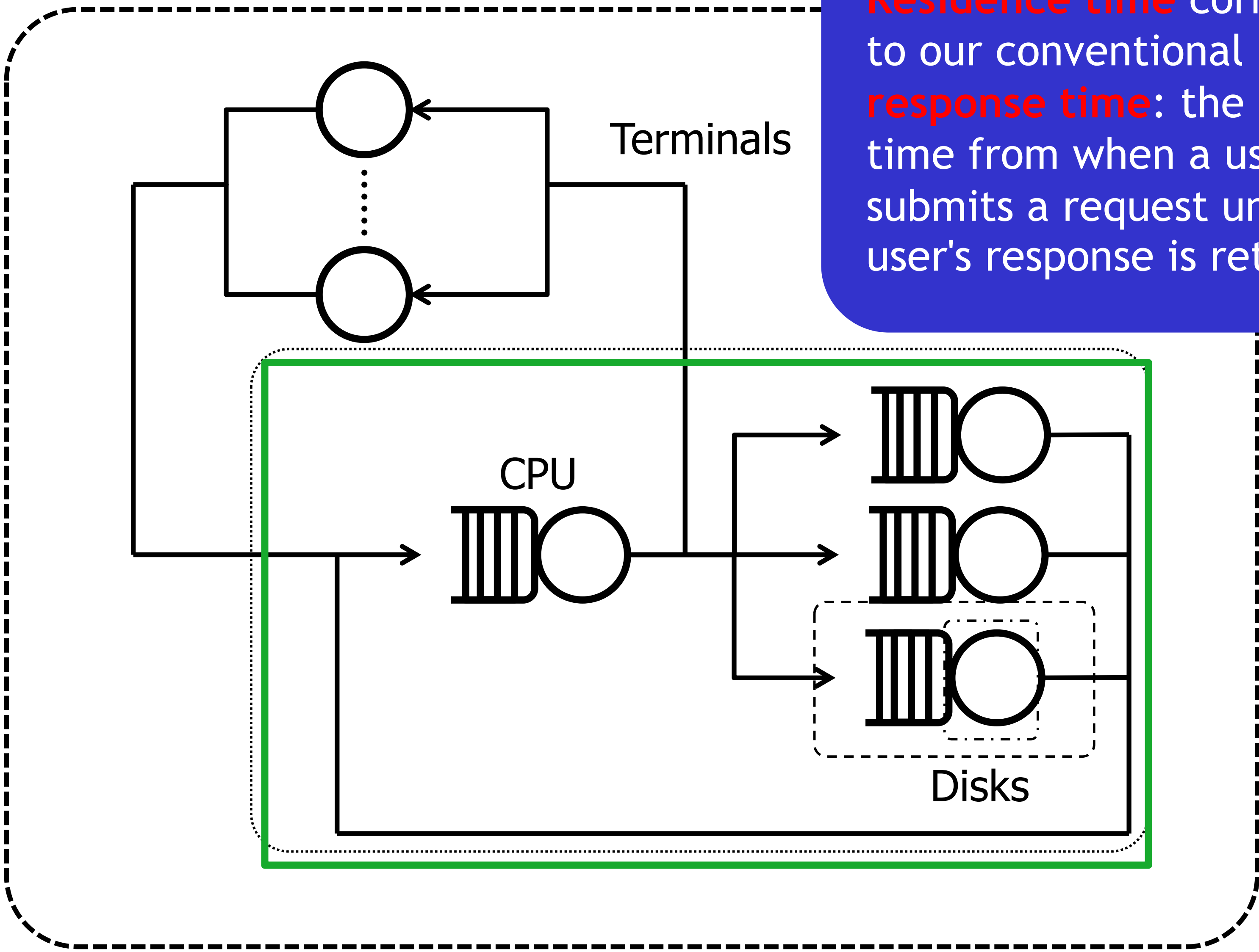
$X_{(2)}$ is the throughput of the server Disk and corresponds to $X_{(1)}$

We know that $X_{(1)}=X_{(2)} = 40$ req/s.
Let us suppose to have obtained, through observations, the following information: $N_{(2)} = 4$.

From Little's law we have: $R_{(2)} = N_{(2)}/X_{(2)} = 0.1$ s.

Since $R_{(1)} = 0.0225$ s, we can compute the waiting time as:
$R_{(2)} - R_{(1)} = 0.0775$ s.

Application of Little's Law at different levels



Residence time corresponds to our conventional notion of response time: the period of time from when a user submits a request until that user's response is returned

Little's Law, level 3

Let us consider the central subsystem (Green Box)

$N_{(3)}$ represents the total number of users in the subsystem (e.g., requests of web pages/s)

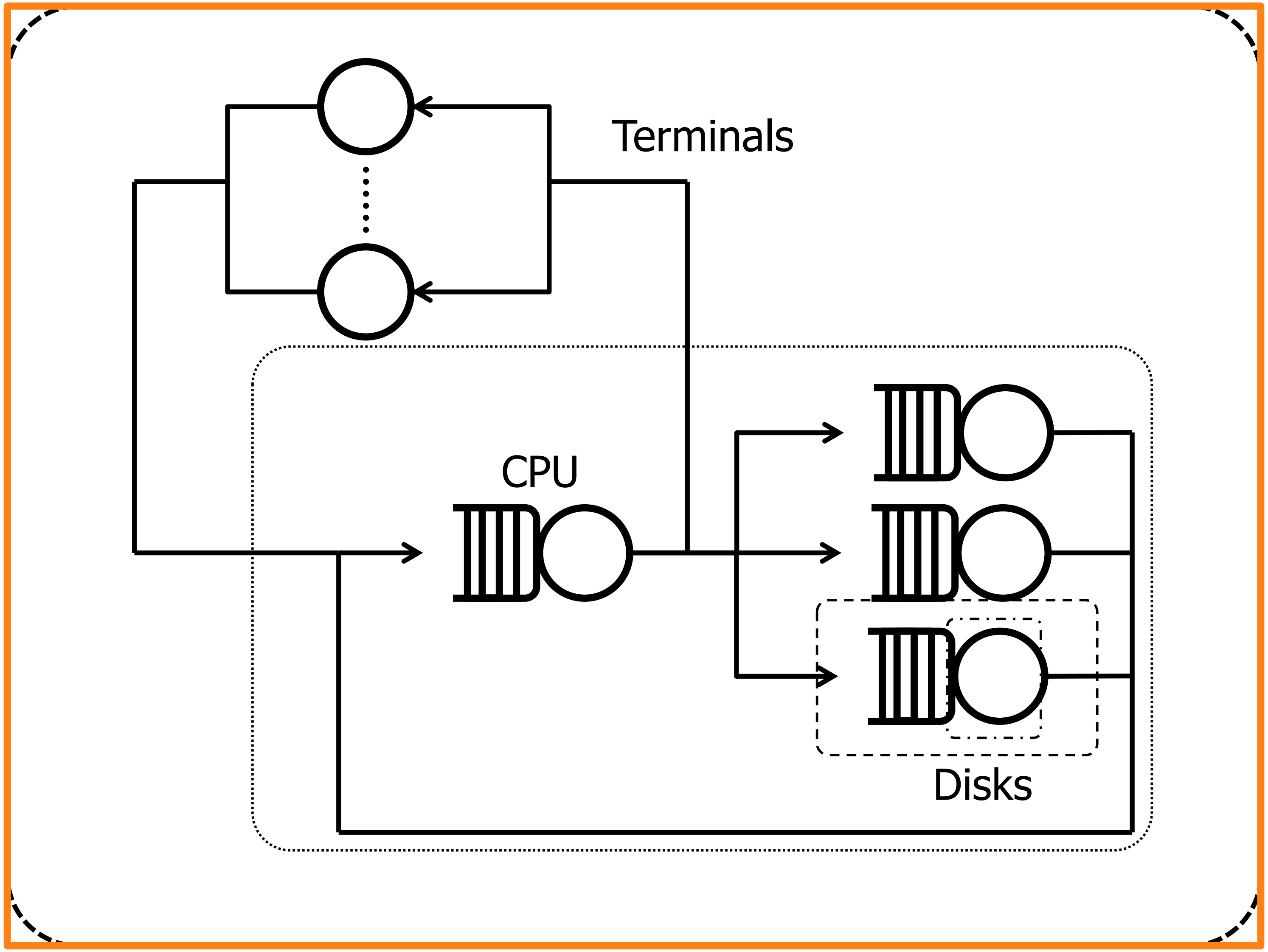
$R_{(3)}$ represents the average time spent in the subsystem by each request

$X_{(3)}$ is the subsystem throughput (e.g., number of web pages/s)

If from observations we know that $X_{(3)} = 0.5$ job/sec and requests number is $N_{(3)} = 7.5$

From Little law we have that
$R_{(3)} = N_{(3)}/X_{(3)}=15$ s

Application of Little's Law at different levels



Little's Law, level 4

Let us now consider the complete system (Orange Box)

$N_{(4)}$ is the total number of users in the system (which is fixed since we have a closed system)

$R_{(4)}$ is the total amount of time spent in service, waiting and at the "terminals" client side (think time, e.g., the time a user spend to read a web page and to elaborate a request)

$X_{(4)}$ is the rate with which the requests reach the systems from the terminals client and it corresponds to $X_{(3)}$

Let us suppose that there are $N_{(4)} = 10$ users, and the think time is $Z = 5$ s.
We know that the time spent in the system is $R_{(3)} = 15$ s.
Now, since $R_{(4)} = R_{(3)} + Z$, we can derive from Little law that
$N_{(4)} = X_{(4)}(R_{(3)} + Z)$,
and we can compute
$X_{(4)} = 0.5$ job/s

Interactive Response Time Law

- Back when most processing was done on shared mainframes, think time, Z, was quite literally the length of time that a programmer spent thinking before submitting another job

- More generally in interactive systems, jobs spend time in the system not engaged in processing, or waiting for processing: this may be because of interaction with a human user, or may be for some other reason

- The think time represents the time between processing being completed and the job becoming available as a request again

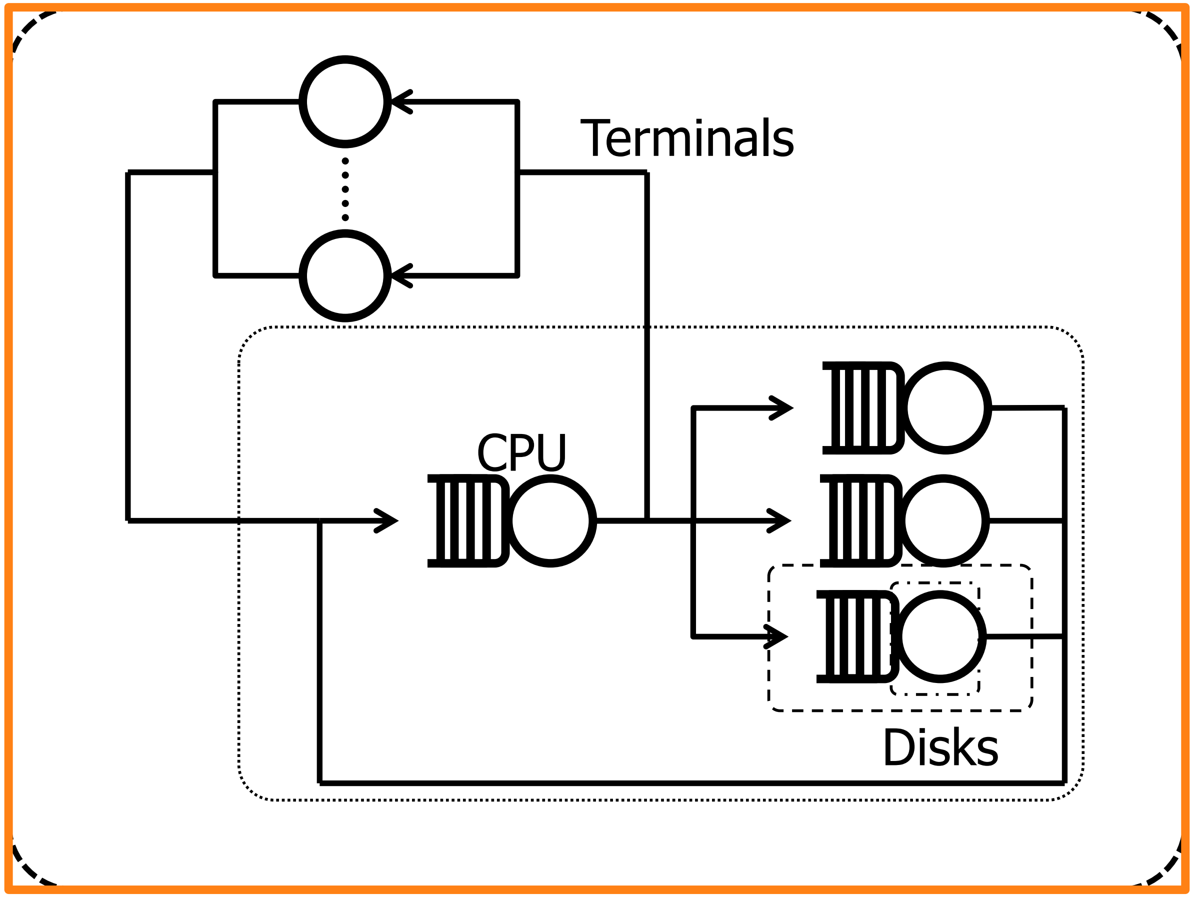
Example

For example, if we are studying a cluster of workstations with a central file server to investigate the load on the file server, the think time might represent the average time that each workstation spends processing locally without access to the file server.

At the end of this non-processing period (from the file server point of view) the job generates a fresh request.

Interactive Response Time Law

$N_{(4)}$ = «ready» + «not ready» (Z) users
$R_{(4)} = R_{(3)} + Z$
$N = X_{(3)}\,(R_{(3)} + Z) = X\,(R+Z)$



Interactive Response Time Law

- Interactive Response Time Law:

$$\boxed{R = N/X - Z}$$

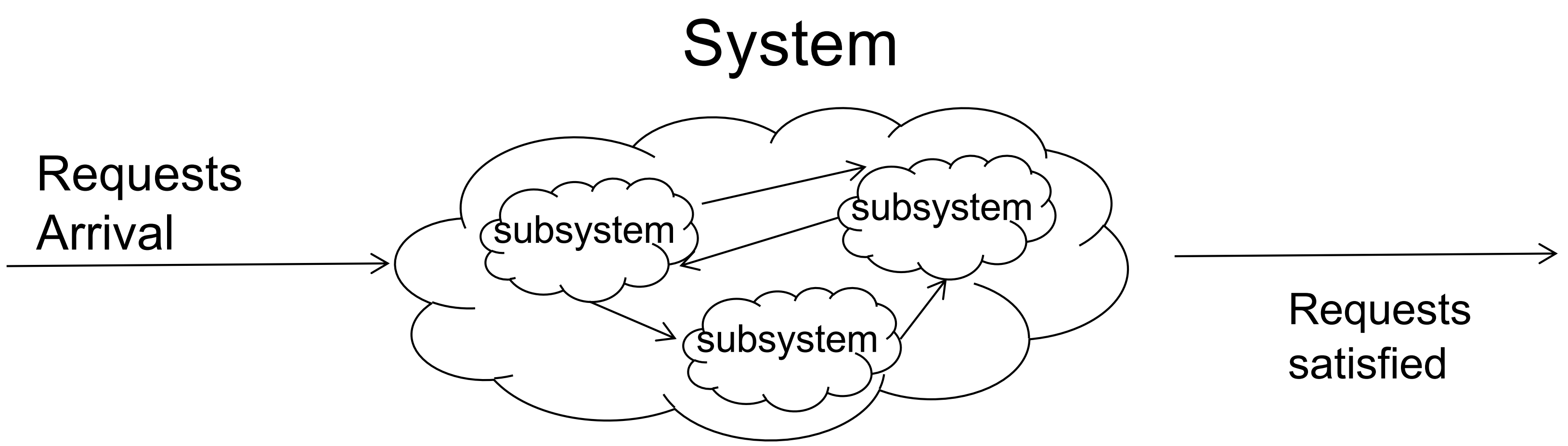
- The response time in an interactive system is the residence time minus the think time

- Note that if the think time is zero, Z = 0 and R = N/X , then the interactive response time law simply becomes Little's Law

Interactive Response Time Law: Example

- Suppose that the library catalogue system has 64 interactive users connected via Browsers, that the average think time is 30 seconds, and that system throughput is 2 interactions/second.
- What is the response time?
- The interactive response time law tells us that the response time must be 64/2 − 30 = 2 seconds

Operational laws



- In an observation interval we can count not only completions external to the system, but also the number of completions at each resource within the system

- *Let C_k* be the number of completions at resource *k*

- We define the visit count, V_k , of the k-th resource to be the ratio of the number of completions at that resource to the number of system completions $V_k = C_k/C$

Visit count: example

For example, if, during an observation interval, we measure 10 system completions and 150 completions at a specific disk, then on the average each system-level request requires 15 disk operations.

Visits

Note that:

- If $C_k > C$, resource k is visited several times (on average) during each system level request. This happens when there are loops in the model
- If $C_k < C$, resource k might not be visited during each system level request. This can happen if there are alternatives (i.e. caching of disks)
- If $C_k = C$, resource k is visited (on average) exactly once every request

Forced Flow Law

The forced flow law captures the relationship between the different components within a system. It states that the throughputs or flows, in all parts of a system must be proportional to one another.

$$\boxed{X_k = V_k X}$$

The throughput at the k-th resource is equal to the product of the throughput of the system and the visit count at that resource.

Rewriting $C_k = V_k C$ and applying $X_k = C_k / T$, we can derive the forced flow law:

$$C_k = V_k C$$

$$C_k / T = V_k C / T$$

$$X_k = V_k X$$

Forced Flow Law example

Consider a robotic workcell within a computerised manufacturing system which processes widgets.

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Thus the press throughput is 4 widgets per minute.

Forced Flow Law example

Consider a robotic workcell within a computerised manufacturing system which processes widgets.

Suppose that processing each widget requires 4 accesses to the lathe and 2 accesses to the press.

We know that the lathe processes 8 widgets in a minute and we want to know the throughput of the press.

The throughput of the workcell will be proportional to the lathe throughput, i.e. $X = X_{lathe}/V_{lathe} = 8/4 = 2$ wid/min.

The throughput of the press will be $X_{press} = X * V_{press} = 2 * 2 = 4$ wid/min.

Thus the press throughput is 4 widgets per minute.

Utilisation Law

- If we know the amount of processing each job requires at a resource then we can calculate the utilisation of the resource

- Let us assume that each time a job visits the k-th resource the amount of processing, or service time it requires is S_k

- Note that service time **is not** necessarily the same as the response time of the job at that resource: in general a job might have to **wait** for some time before processing begin

- The total amount of service that a system job generates at the k-th resource is called the service demand, D_k:

$$D_k = S_k V_k$$

Utilisation Law

- The utilisation of a resource, the percentage of time that the k-th resource is in use processing to a job, is denoted U_k.

- Utilisation Law:

$$U_k = X_k S_k$$

- The utilisation of a resource is equal to the product of: 1) the throughput of that resource and the average service time at that resource, 2) the throughput at system level and the average service demand at that resource

$$\boxed{U_k = D_k X}$$

Utilisation Law

- The utilisation of a resource, the percentage of time that the k-th resource is in use processing to a job, is denoted U_k.

- Utilisation Law:

$$U_k = X_k S_k = (X V_k)\ S_k = D_k X$$

- The utilisation of a resource is equal to the product of: 1) the throughput of that resource and the average service time at that resource, 2) the throughput at system level and the average service demand at that resource

$$\boxed{U_k = D_k X}$$

Visits and demands

Note that:

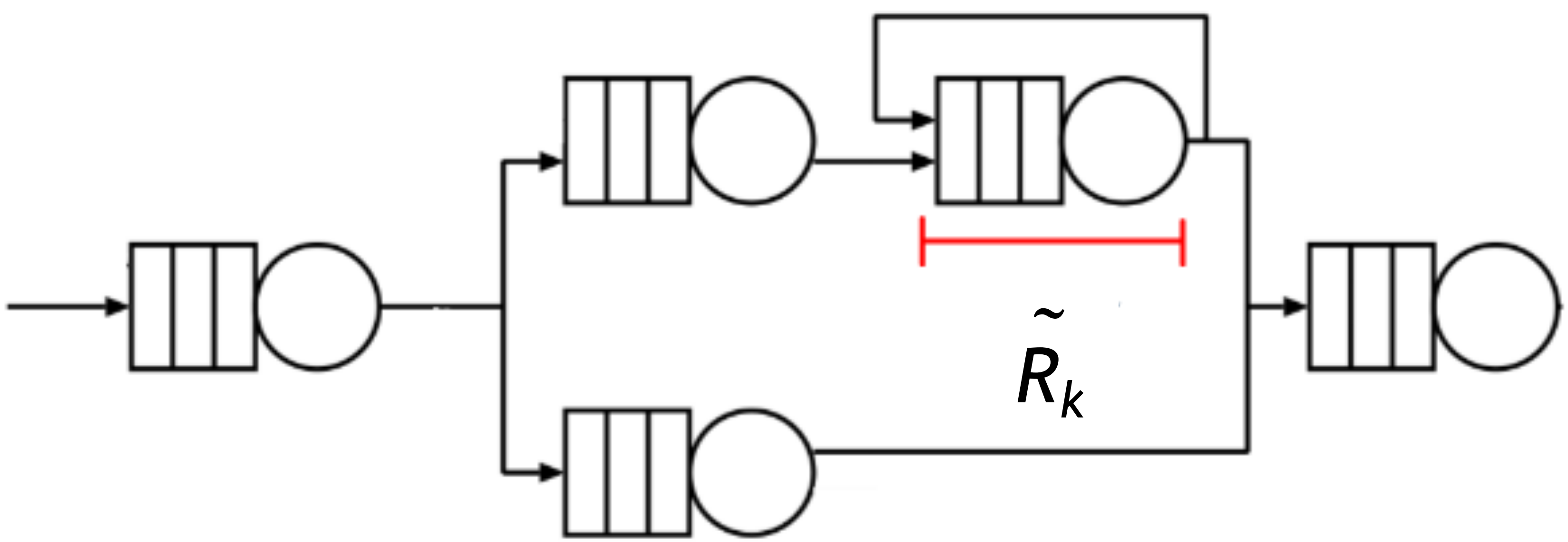
- Average service time S_k accounts for the average time that a job spends in station k when IT IS SERVED
- Average service demand D_k accounts for the average time a job spends in station k during ITS STAYING IN THE SYSTEM. As seen for the visits, depending on the way in which the jobs move in the system, the demand can be less than, greater than or equal to the average service time of station k

Response and Residence times

- When considering nodes characterized by visits different from one, we can define two permanence times:
  - *Response Time* $\tilde{R}_k$
  - *Residence Time* R_k

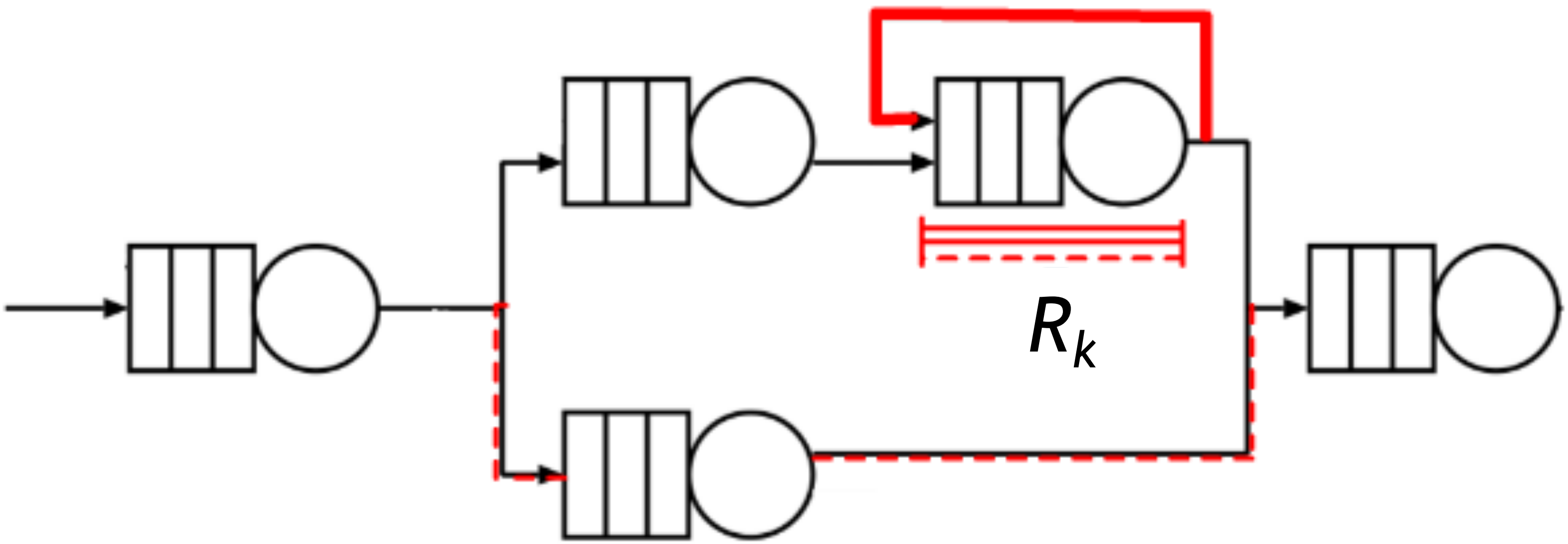
Response and Residence times

The *Response Time* $\tilde{R}_k$ accounts for the average time spent in station k, when the job enters the corresponding node (i.e., time for the single interaction, e.g. disk request):



Response and Residence times

The *Residence Time* R_k accounts instead for the average time spent by a job at station *k* during the staying in the system: it can be greater or smaller than the response time depending on the number of visits.



Response and Residence times

Note that there is the same relation between *Residence Time* and *Response Time* as the one between *Demand* and *Service Time*

$$D_k = v_k \cdot S_k$$
$$R_k = v_k \cdot \widetilde{\mathrm{R}}_k$$

Also note that for single queue open system, or tandem models, $v_k = 1$. This implies that average service time and service demand are equal, and response time and residence time are identical

$$v_k = 1 \quad \Rightarrow \quad \begin{array}{l} D_k = S_k \\ R_k = \widetilde{\mathrm{R}}_k \end{array}$$

General Response Time Law 1/2

- One method of computing the mean response time per job in a system is to apply Little's Law to the system as a whole

- However, if the mean number of jobs in the system, N, or the system level throughput, X, are not known an alternative method can be used

- Applying Little's Law to the k-th resource we see that $N_k = X_k\tilde{R}_k$, where N_k is the mean number of jobs at the resource and $\tilde{R}_k$ is the average time spent at the resource (for the single interaction at the k-th resource level, e.g. disk request)

- From the Forced Flow Law we know that $X_k = XV_k$. Thus we can deduce that:

$$N_k/X = V_k\tilde{R}_k = R_k$$

General Response Time Law 2/2

The total number of jobs in the system is clearly the sum of the number of jobs at each resource, i.e. $N = N_1 + \ldots + N_M$ if there are M resources.

From Little's Law $R = N/X$ and so, from $R_k = N_k/X = V_k\tilde{R}_k$

General Response Time Law

$$R = \sum_k V_k \tilde{R}_k = \sum_k R_k \quad , \quad R_k = V_k \tilde{R}_k \quad \forall\, k$$

- The average response time of a job in the system is the sum of the product of the average time for the individual access at each resource and the number of visits it makes to that resource
- The average response time of a job in the system is the sum of the resources residence time

General Response Time Law: Example

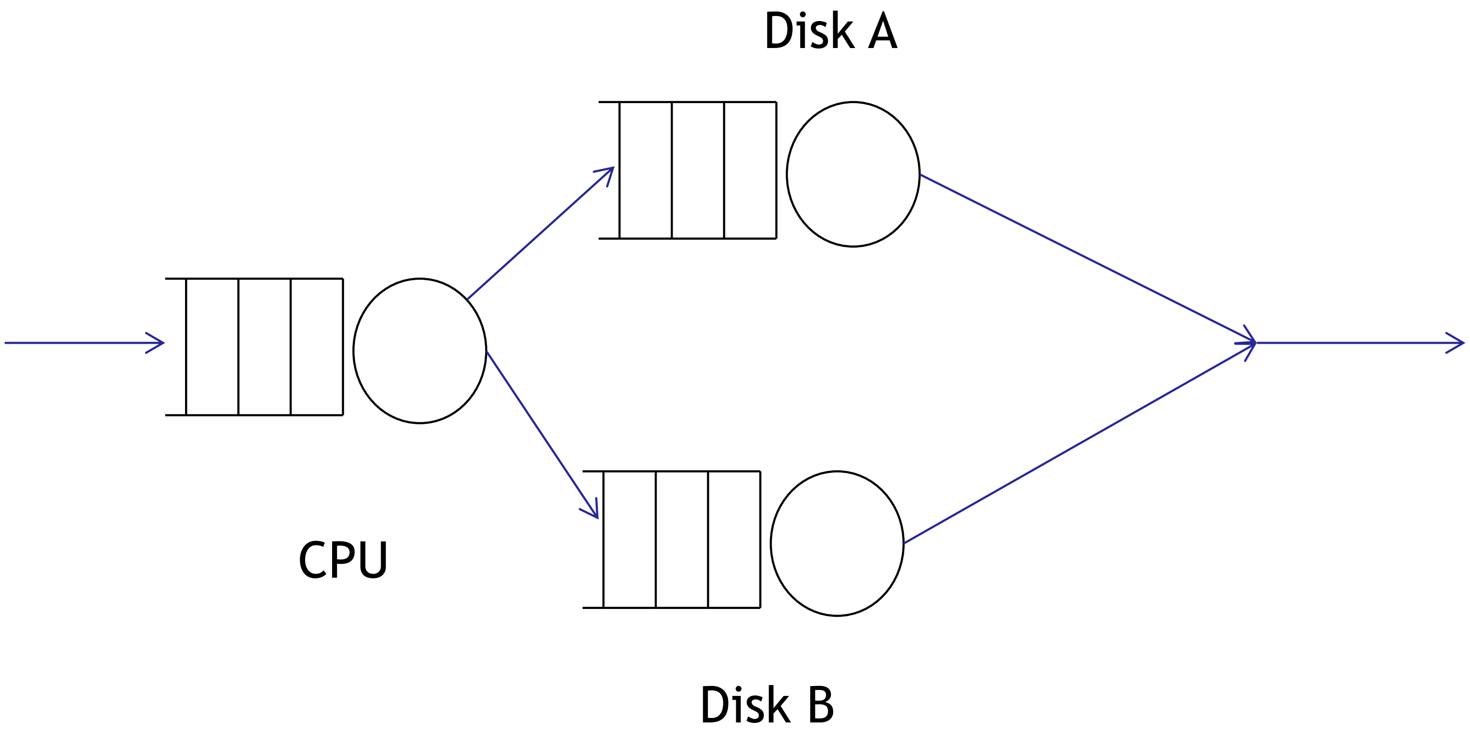
A program running on a computer server requires 126 bursts of CPU time and makes 75 I/O requests to disk A and 50 I/O requests to disk B.

On average each CPU burst requires 30 milliseconds (waiting + processing time).

Monitoring has shown that the throughput of disk A is 15 requests per second and the average number in the buffer is 4 whilst at disk B the throughput is 10 requests per second and the average number in the buffer is 3.

What is the system mean response time?

Model derivation and parameterization



V_{CPU}= 126
V_{DiskA}=75
V_{diskB}=50

X_{DiskA}=15 req/sec
X_{diskB}=10 req/sec

N_{DiskA}= 4+1
N_{diskB}=3+1

$\tilde{R}_{CPU}$= 30 msec

Model Evaluation

Using Little's Law we calculate the time spent at each disks (remembering that the number in the system is the number in the buffer +1):

- $\tilde{R}_{diskA}$

- $\tilde{R}_{diskB}$

Then

- $R = \tilde{R}_{CPU}V_{CPU} + \tilde{R}_{diskA}V_{diskA} + \tilde{R}_{diskB}V_{diskB}$

  $= 30 * 126 + (5000/15) * 75 + (4000/10) * 50$

  $= 3780 + 25000 + 20000$

  $= 48780$ ms

Model Evaluation

Using Little's Law we calculate the time spent at each disks (remembering that the number in the system is the number in the buffer +1):

- $\tilde{R}_{diskA} = N_{diskA}/X_{diskA} = (5/15)x1000 = 5000/15$ ms

- $\tilde{R}_{diskB} = N_{diskB}/X_{diskB} = (4/10)x1000 = 4000/10$ ms

Then

- $R = \tilde{R}_{CPU}V_{CPU} + \tilde{R}_{diskA}V_{diskA} + \tilde{R}_{diskB}V_{diskB}$

  $= 30 * 126 + (5000/15)*75 + (4000/10)*50$

  $= 3780 + 25000 + 20000$

  $= 48780$ ms

Operational laws

- Operational laws are simple equations which may be used as an abstract representation or model of the average behaviour of almost any system

- The laws are very general and make almost no assumptions about the behaviour of the random variables characterising the system

- Another advantage of the laws is their simplicity: this means that they can be applied quickly and easily

Operational laws summary

- T: observation interval
- $\lambda_k = A_k/T$, the arrival rate
- $X_k = C_k / T$, the throughput or completion rate
- $U_k = B_k/T$, the utilisation
- $S_k = B_k/C_k$, the mean service time per completed job
- $V_k = C_k/C$, visit count of the k-th resource
- $D_k = S_k V_k$, the total amount of service that a system job generates at the k-th resource

Operational laws summary

Utilisation Law:

$$U_k = X_k S_k \qquad U_k = D_k X$$

Little's law:

$$N = XR$$

Forced flow law:

$$X_k = V_k X$$

Response time law:

$$R = N/X - Z$$