

QUICK REVIEW

$S_c \rightarrow$ STORAGE CAPACITY $N \rightarrow$ NUMBER OF DISKS $\frac{1}{\text{MTTF}} = \text{FAILURE RATE}$

$S \rightarrow$ TRANSFER RATE SEQUENTIAL WORKLOAD $R \rightarrow$ RANDOM TRANSFER RATE

MTTDL \rightarrow MEAN TIME TO DATA LOSS MTTF \rightarrow MEAN TIME TO FAILURE

$D \rightarrow$ LATENCY TO ACCESS A SINGLE DISK MTBF \rightarrow MEAN TIME BETWEEN FAILURE

a) RAID 0



MTTR \rightarrow MEAN TIME TO RECOVER

$S_c = \sum_i C_i$ SUM OF ALL DISKS CAPACITY. IF C_i IS THE SAME $\forall i$,

$S_c = N \cdot C$ SEQUENTIAL READ/WRITE $N \cdot S$

RANDOM READ/WRITE $N \cdot R$ READ/WRITE D

RELIABILITY = 0 ONE DISK FAIL \Rightarrow EVERYTHING FAILS MTTDL = MTTF

b) RAID 1

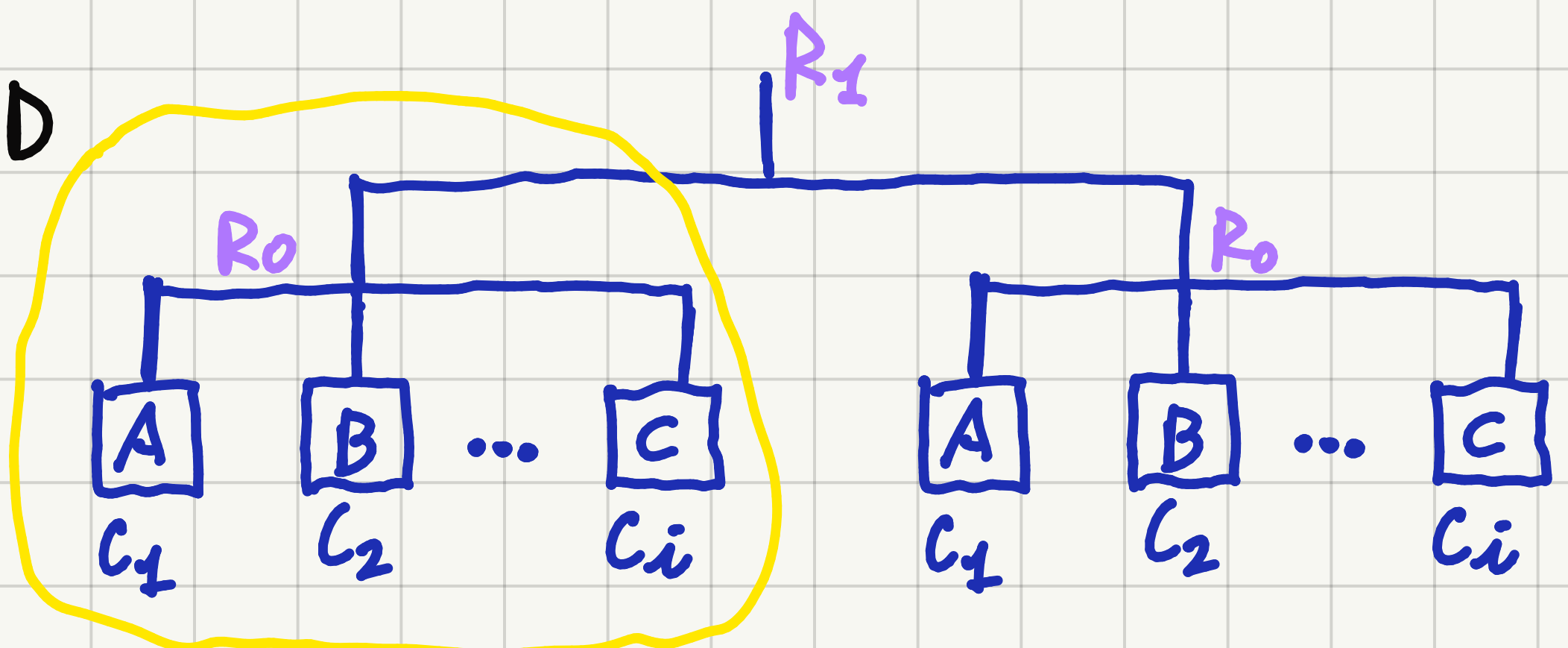


$S_c = C$ RELIABILITY = 1 (AT MOST $\frac{N}{2}$) RANDOM READ $N \cdot R$

SEQUENTIAL READ/WRITE $\frac{N}{2} \cdot S$ RANDOM WRITE $\frac{N}{2} \cdot R$

READ/WRITE D

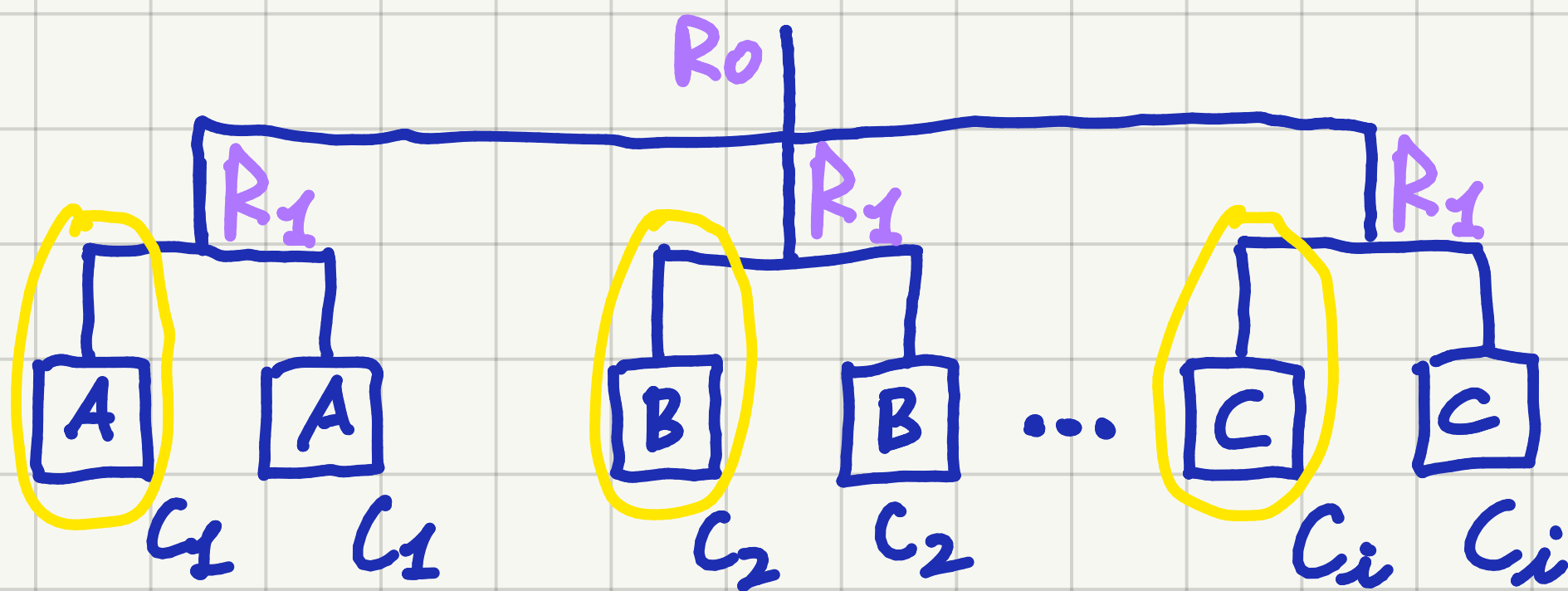
c) RAID 0+1



$S_c = \frac{1}{2} (\sum_i C_i)$ IF C_i IS THE SAME $\forall i$, $S_c = \frac{N}{2} C$

d) RAID 1+0

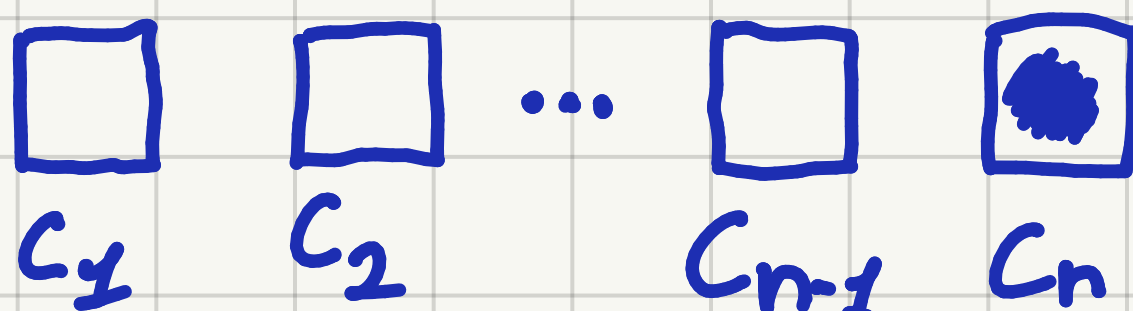
$$S_c = \frac{1}{2} (\sum_i C_i)$$



IF C_i IS THE SAME $\forall i$, $S_c = \frac{N}{2} C$

e) RAID 4

$$S_c = \sum_{i=1}^{N-1} C_i \quad \text{IF } C_i \text{ IS THE SAME } \forall i, S_c = (N-1)C$$



RELIABILITY: 1 FAILURE AT MOST

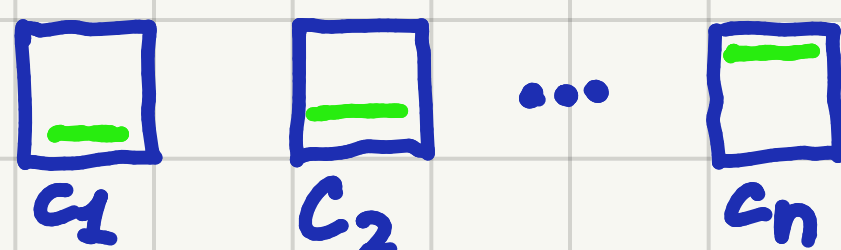
SEQUENTIAL READ/WRITE $(N-1) \cdot S$

READ D

RANDOM READ $(N-1) \cdot R$ RANDOM WRITE $\frac{R}{2}$ WRITE 2D

f) RAID 5

$$S_c = \sum_{i=1}^{N-1} C_i \quad \text{IF } C_i \text{ IS THE SAME } \forall i, S_c = (N-1)C$$



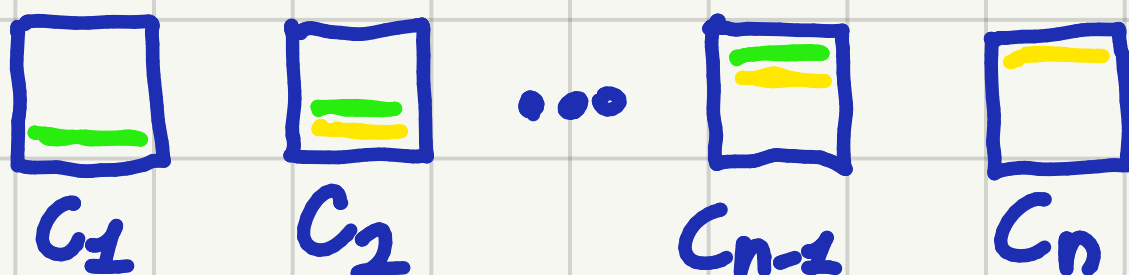
RELIABILITY: 1 FAILURE AT MOST

SEQUENTIAL READ/WRITE $(N-1) \cdot S$

READ D

RANDOM READ $N \cdot R$ RANDOM WRITE $\frac{N}{4} \cdot R$ WRITE 2D

g) RAID 6



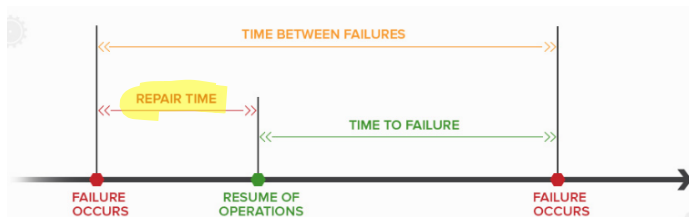
$$S_c = \sum_{i=1}^{N-2} C_i \quad \text{IF } C_i \text{ IS THE SAME } \forall i, S_c = (N-2) \cdot C$$

RAID Preliminaries

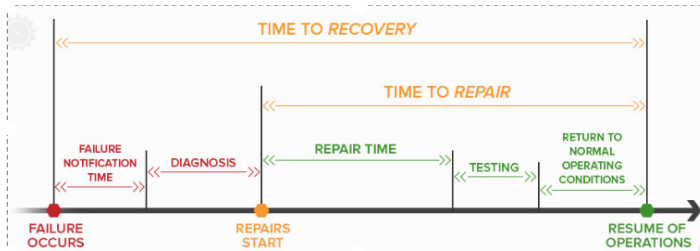
giovedì 19 marzo 2020 10:05

Definitions:

- **RAID** - Redundant Array of Independent Disks
- **MTTDL** - Mean Time To Data Loss
 - Mean time until drive failures cause data loss in the array
- **MTBF** - Mean Time Between Failures
 - is the mean time between hard disk failures.
- **MTTF** - Mean Time To Failure
 - mean time before a disk has a failure
 - $1/\text{MTTF}$ is called Failure Rate



- **MTTR** - Mean Time To Recover (or Repair)
 - Mean time to rebuild redundancy in the array.



Ex_01 – RAID Capacity

giovedì 19 marzo 2020 10:56

Consider to have 6 disks, each one with a capacity of 1TB.

What will be to total storage capacity of the system if they are in the following configurations?

- a. RAID 0
- b. RAID 1
- c. RAID 0+1
- d. RAID 1+0
- e. RAID 5
- f. RAID 6

$C_1 = 1\text{Tb}$ $N = 6$ S_c FOR THE VARIOUS CASE?

a) RAID 0

$$S_c = N \cdot C_1 = 6 \cdot 1\text{Tb} = 6\text{Tb}$$

b) RAID 1

$$S_c = 1\text{Tb}$$

c) RAID 0+1

$$S_c = \frac{N}{2} \cdot C_1 = \frac{6}{2} \cdot 1\text{Tb} = 3\text{Tb}$$

d) RAID 1+0

$$S_c = \frac{N}{2} \cdot C_1 = \frac{6}{2} \cdot 1\text{Tb} = 3\text{Tb}$$

e) RAID 5

$$S_c = (N-1) \cdot C_1 = 5\text{Tb}$$

f) RAID 6

$$S_c = (N-2) \cdot C_1 = 4\text{Tb}$$

Ex_02 – RAID 0

mercoledì 4 marzo 2020 11:46

Consider the following RAID 0 setup:

- $n = 5$ disks
- $MTTR = 8$ hours
- $MTTF(\text{one disk}) = 1600$ day

The **MTTDL** will be:

$n=5$ DISKS $MTTR=8h$ $MTTF$ (ONE DISK) $=1600$ DAYS $MTTDL?$

! SOMETIMES, $MTTDL$ IS CALLED $MTTF_{RAID}$ 0

$$MTTDL = \frac{MTTF}{n} = \frac{1600 \text{ DAYS}}{5} = 320 \text{ DAYS}$$

NOTICE THAT $MTTR$ IS AN USELESS INFORMATION

Ex_03 - RAID1

mercoledì 4 marzo 2020 11:33

Consider the following RAID 1 setup:

- $n = 2$ disks
- $MTTR = 8$ days
- $MTTF(\text{one disk}) = 1800$ day

The MTDDL will be:

$n=2$ DISKS $MTTR=8$ DAYS $MTTF(\text{ONE DISK})=1800$ DAYS $MTTDL?$

$$P(\text{FAIL RAID } 1) = P(1^{\text{st}} \text{ FAIL}) \cdot P(2^{\text{nd}} \text{ FAIL DURING REPAIR})$$

$$P(1^{\text{st}} \text{ FAIL}) = \frac{n}{MTTF} = \frac{2}{MTTF} \quad P(2^{\text{nd}} \text{ FAIL DURING REPAIR}) = \frac{1}{MTTF} \cdot MTTR = \frac{MTTR}{MTTF}$$

$$P(\text{FAIL RAID } 1) = \frac{2MTTR}{MTTF^2}$$

$$MTTDL = \frac{1}{P(\text{FAIL RAID } 1)} = \frac{MTTF^2}{2 \cdot MTTR} = \frac{1800^2 \cdot \text{DAYS}^2}{16 \text{ DAYS}} = 202500 \text{ DAYS}$$

Ex_04 – RAID 1+0

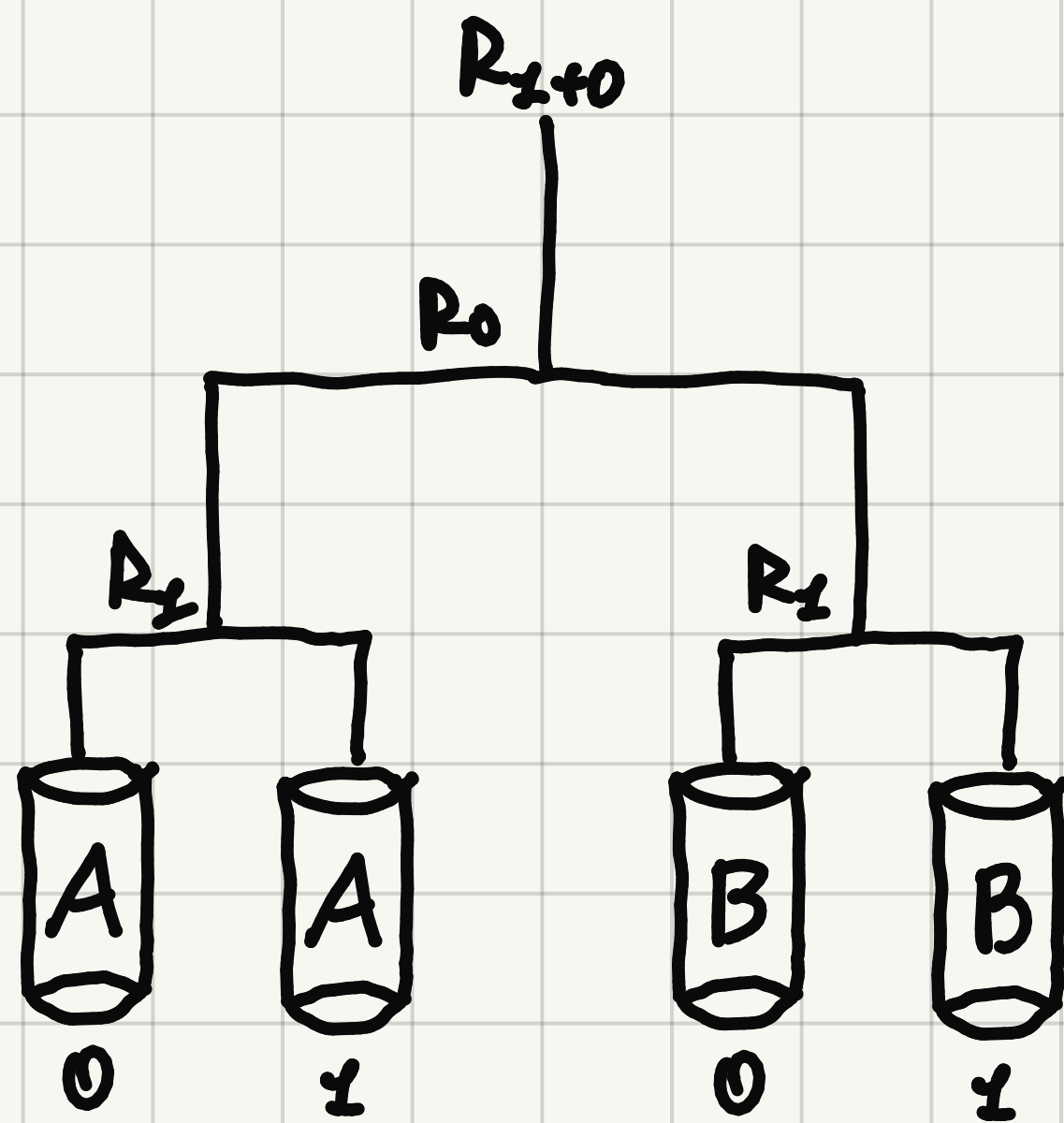
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Consider 2 groups (RAID 0) of 2 disks each (RAID 1), for a total of 4 disks in configuration RAID 1+0:

- $MTTR = 3$ days
- $MTTF(\text{one disk}) = 1400$ day

The **MTTDL** will be:

$N=4$ $MTTR=3$ DAYS $MTTF(\text{ONE DISK})=1400$ DAYS $MTTDL?$



$P(\text{FAIL RAID } 1+0) = P(1^{\text{ST}} \text{ FAIL}) \cdot P(2^{\text{ND}} \text{ FAIL DURING REPAIR})$

$$P(1^{\text{ST}} \text{ FAIL}) = \frac{N}{MTTF} = \frac{4}{MTTF} \quad \text{RAID } 1+0 \text{ STRUCTURE}$$

$$P(2^{\text{ND}} \text{ FAIL DURING REPAIR}) = \frac{1}{MTTF} \cdot MTTR = \frac{MTTR}{MTTF}$$

$$P(\text{FAIL RAID } 1+0) = \frac{4 \cdot MTTR}{MTTF^2}$$

$$MTTDL = \frac{1}{P(\text{FAIL RAID } 1)} = \frac{MTTF^2}{4 \cdot MTTR} = \frac{1400^2 \cdot \text{DAYS}^2}{4 \cdot 3} = 163333 \text{ DAYS}$$

Ex_05 – RAID 0+1

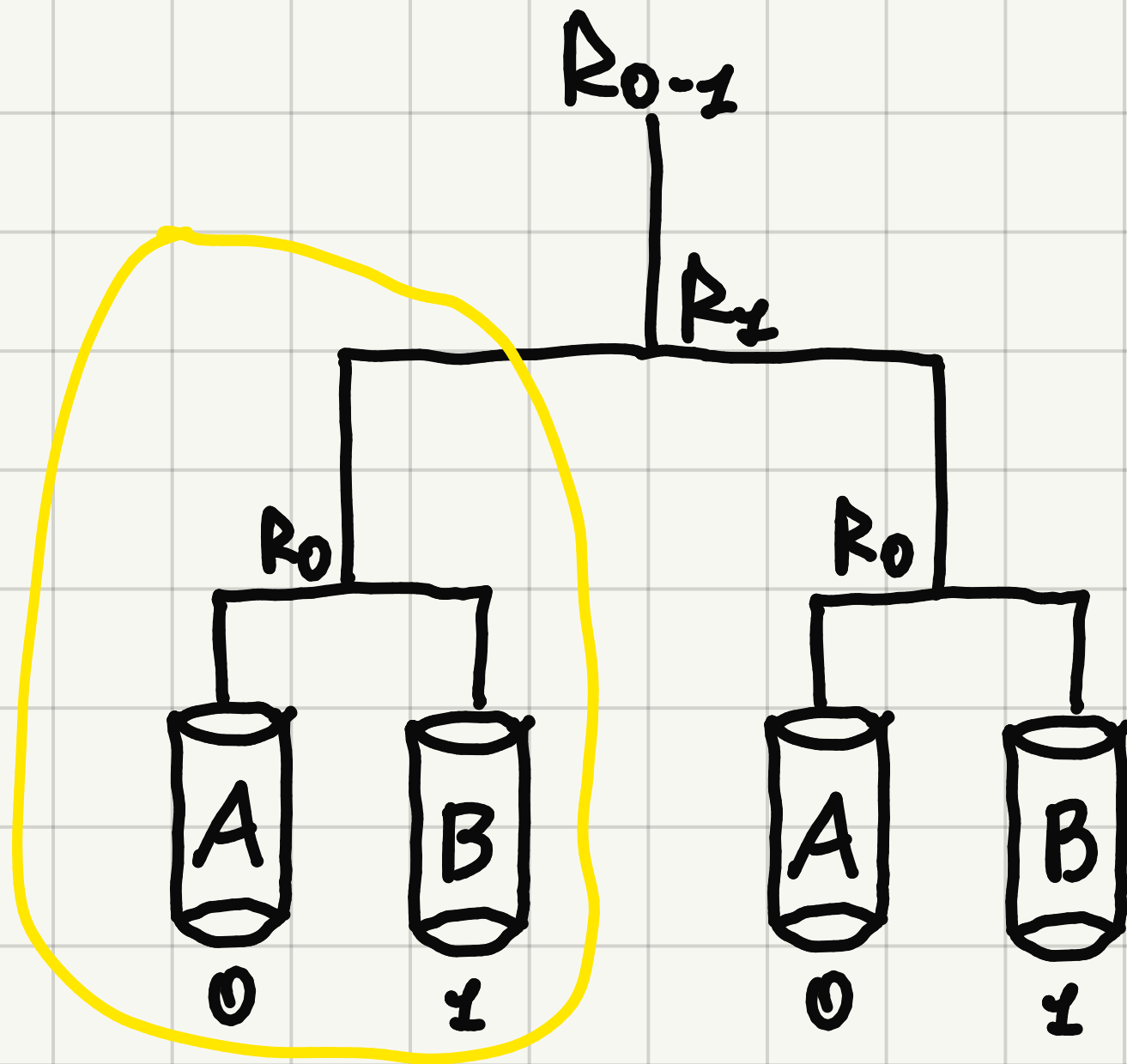
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Consider 2 groups (RAID 1) of 4 disks each (RAID 0), for a total of 8 disks in configuration RAID 0+1:

- MTTR \approx 4 days
- MTTF(one disk) \approx 2200 days

The MTDDL will be:

$N=8$ $MTTR=4$ DAYS $MTTF$ (ONE DISK)=2200 DAYS $MTDL$?



$P(\text{FAIL } 01) = P(1^{\text{st}} \text{ FAIL}) \cdot P(2^{\text{nd}} \text{ FAIL DURING REPAIR})$

$$P(1^{\text{st}} \text{ FAIL}) = \frac{N/2}{MTTF} \quad P(2^{\text{nd}} \text{ FAIL}) = \frac{N}{MTTF} \cdot MTTR$$

$$P(\text{FAIL } 01) = \frac{N^2}{2} \cdot \frac{MTTR}{MTTF^2}$$

$$MTDL = \frac{1}{P(\text{FAIL } 01)} = \frac{2 MTTF^2}{N^2 \cdot MTTR} = \frac{2 \cdot 2200^2}{8^2 \cdot 4} = 37813 \text{ DAYS}$$

Quiz_2 - RAID 0 + 1

mercoledì 4 marzo 2020 18:31

A system administrator has decided to use a stock of disks characterized by:

MTTF = 800 days

MTTR = 20 days

The target lifetime of the system is 3 years;

The maximum number of disks that could be used in RAID 01 to have a MTDDL larger than the system lifetime is:

1. No more than 58 disks
2. No more than 7 disks
3. At least 8 disks
4. No more than 6 disks
5. None of the other options

MTTF = 800 DAYS MTTR = 20 DAYS MTDL = 3 YEARS

$$P(\text{FAIL } 0\%) = P(1^{\text{st}} \text{ FAIL}) \cdot P(2^{\text{nd}} \text{ FAIL DURING REPAIR})$$

$$P(1^{\text{st}} \text{ FAIL}) = \frac{N/2}{\text{MTTF}} \quad P(2^{\text{nd}} \text{ FAIL}) = \frac{N}{\text{MTTF}} \cdot \text{MTTR}$$

$$P(\text{FAIL } 0\%) = \frac{N^2}{2} \cdot \frac{\text{MTTR}}{\text{MTTF}^2}$$

$$\text{MTDL} = \frac{1}{P(\text{FAIL } 0\%)} = \frac{2 \text{MTTF}^2}{N^2 \cdot \text{MTTR}} = \frac{2 \cdot 800^2}{N^2 \cdot 20} = 3 \cdot 365$$

$$\Rightarrow N = \sqrt{\frac{2 \cdot 800^2}{20 \cdot 3 \cdot 365}} = 7,6 \Rightarrow 4) \quad N \leq 6$$

Ex_06 – RAID 5

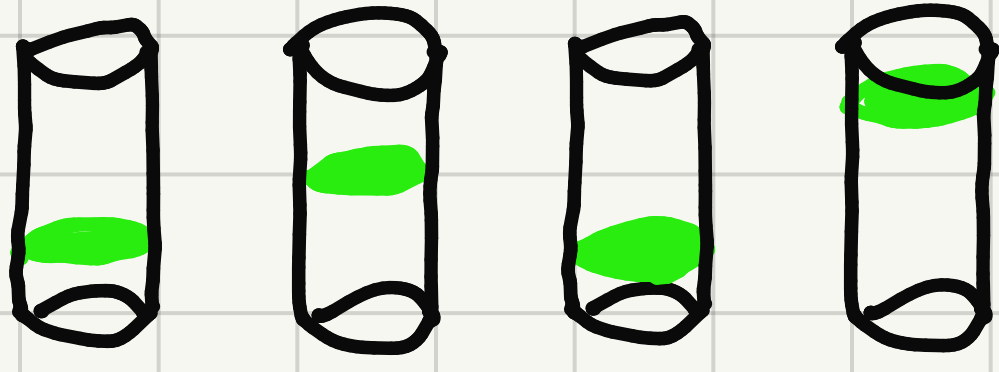
mercoledì 4 marzo 2020 11:10

Consider the following RAID 5 setup:

- $n = 4$ disks
- $MTTR = 3$ days
- $MTTF(\text{one disk}) = 1000$ day

The MTDDL will be:

$N=4$ $MTTR=3$ DAYS $MTTF(\text{ONE DISK})=1000$ DAYS $MTTL?$



$P(\text{FAIL } 5) = P(1^{\text{ST}} \text{ FAIL}) \cdot P(2^{\text{ND}} \text{ FAIL DURING REPAIR})$

$$= \frac{N}{MTTF} \cdot \frac{N-1}{MTTF} \cdot MTTR = \frac{N(N-1)MTTR}{MTTF^2}$$

$$MTTL = \frac{1}{P(\text{FAIL } 5)} = \frac{MTTF^2}{N(N-1)MTTR} = \frac{1000^2}{4 \cdot 3 \cdot 3} = 27778 \text{ DAYS}$$

Ex_07 – RAID 6

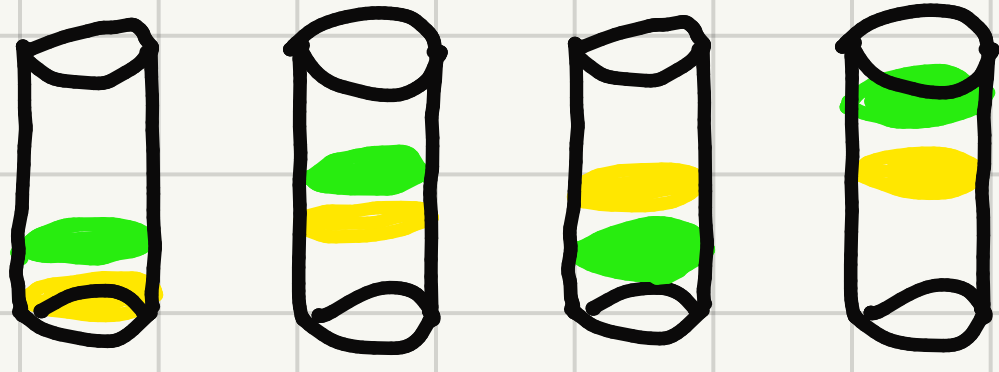
mercoledì 4 marzo 2020 11:11

Consider the following RAID 6 setup:

- $n = 5$ disks
- $MTTR = 2$ days
- $MTTF(\text{one disk}) = 1100$ day

The ~~MTT~~**MTTDL** will be:

$N=5$ $MTTR=2$ DAYS $MTTF(\text{ONE DISK})=1100$ DAYS $MTTL?$



$$P(\text{FAIL } 6) = P(1^{\text{ST}} \text{ FAIL}) \cdot P(2^{\text{ND}} \text{ FAIL}) \cdot P(3^{\text{RD}} \text{ FAIL})$$

$$= \frac{N}{MTTF} \cdot \frac{N-1}{MTTF} MTTR \cdot \frac{N-2}{MTTF} \frac{MTTR}{2} = \frac{N(N-1)(N-2)MTTR^2}{MTTF^3}$$

$$MTTL = \frac{1}{P(\text{FAIL } 6)} = \frac{2 \cdot MTTF^3}{N(N-1)(N-2)MTTR^2} = \frac{2 \cdot 1100^3}{5 \cdot 4 \cdot 3 \cdot 2^2} = 5545834 \text{ DAYS}$$

Storage Capacity

giovedì 6 maggio 2021 11:40

A Raid6 system uses eight 3TB drives to store data and the required parity bits. How many identical drives would be needed to assemble a RAID1+0 system of equivalent capacity? For the RAID1 configuration consider a single replica of data blocks.

RAID 6 $N=8$ $C_1=3Tb$ S_c ? IF RAID 1+0, N FOR SAME S_c ?

$$S_c = (N-2) \cdot C_1 = 18Tb$$

$$S_c^{1+0} = \frac{N}{2} \cdot C_1 = 18Tb \Rightarrow N = \frac{36}{3} = 12$$

Design Choice

giovedì 6 maggio 2021 11:41

You have been assigned from your boss to configure a storage system with 8 disks considering the maximum capacity and a constraint of MTTR > 8 years. If the MTTF of each disk is 2 years and the MTTR is 3 days, what type of configuration are you going to select:

- RAID 0
- RAID 0+1
- RAID 1+0
- RAID 5
- RAID 6

$N=8$ $MTTF$ (ONE DISK) = 2 YEARS $MTDL > 8$ YEARS $MTTR = 3$ DAYS

$$MTDL = \frac{1}{P(\text{FAIL})} > 8 \cdot 365 = 2920$$

RAID 0) $MTDL = \frac{MTTF}{2} = \frac{2}{8}$ YEARS NO

RAID 0+1) $MTDL = \frac{2 MTTF^2}{N^2 MTTR} = \frac{2 \cdot (2 \cdot 365)^2}{8^2 \cdot 3} = 5551$ DAYS ✓

RAID 1+0) $MTDL = \frac{MTTF^2}{N \cdot MTTR} = \frac{(2 \cdot 365)^2}{8 \cdot 3} = 22204$ DAYS ✓

RAID 5) $MTDL = \frac{MTTF^2}{N(N-1)MTTR} = \frac{(2 \cdot 365)^2}{8 \cdot 7 \cdot 3} = 3172$ DAYS ✓

RAID 6) $MTDL = \frac{2 MTTF^3}{N(N-1)(N-2)MTTR^2} = \frac{(2 \cdot 365)^2}{8 \cdot 7 \cdot 6 \cdot 3^2} = 70468$ DAYS ✓

RAID 5

SUMMARIZE

$$MTDL = \frac{1}{P(\text{FAIL})}$$

$$P(\text{FAIL RAID } 0) = \frac{N}{MTTF} \quad ! \quad \text{MTTR IS NOT INVOLVED}$$

$$P(\text{FAIL RAID } 1) = P(\text{FAIL } 1+0) = \frac{N \cdot \text{MTTR}}{\text{MTTF}^2}$$

$$P(\text{FAIL RAID } 0+1) = \frac{N^2}{2} \cdot \frac{\text{MTTR}}{\text{MTTF}^2}$$

$$P(\text{FAIL RAID } 5) = \frac{N(N-1)\text{MTTR}}{\text{MTTF}^2}$$

$$P(\text{FAIL RAID } 6) = \frac{N(N-1)(N-2)\text{MTTR}^2}{2\text{MTTF}^3}$$