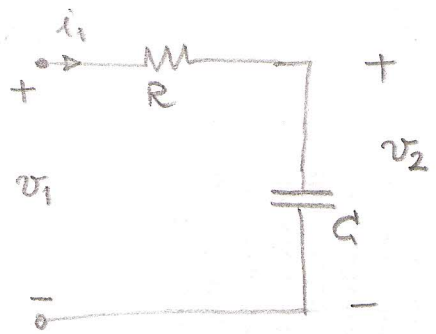


Circuito analogico Filtro PB a TC

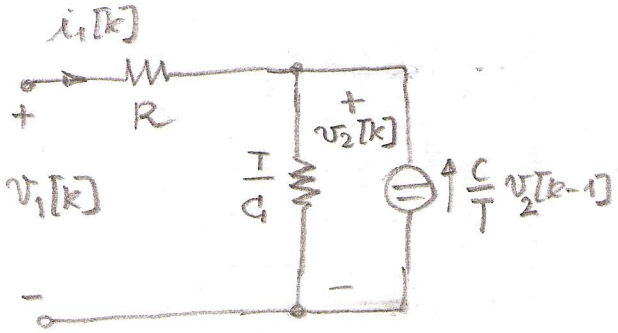
$$\begin{cases} v_1 = R i_1 + v_2 \\ i_1 = C \dfrac{dv_2}{dt} \end{cases}$$

$$v_1 = RC \frac{dv_2}{dt} + v_2$$

$$\begin{cases} v_1 = \text{input} \\ v_2 = \text{output} \end{cases}$$

+ C.i. (es $v_2(0) = v_{20}$) Rem: $RC = [\text{sec}]$

Circuito equivalente a TD (con C parallelo)



$$\begin{cases} v_1[k] = R i_1[k] + v_2[k] \\ v_2[k] = \dfrac{T}{C}\left(i_1[k] + \dfrac{C}{T} v_2[k-1] \right) \end{cases}$$

$$\begin{cases} i_1[k] = \dfrac{v_1[k] - v_2[k]}{R} \\ v_2[k] = \dfrac{T}{C} i_1[k] + v_2[k-1] \end{cases}$$

Approssimazioni TC → TD

$v_1(t) \longrightarrow v_1[k]$, $k \in \mathbb{Z}$

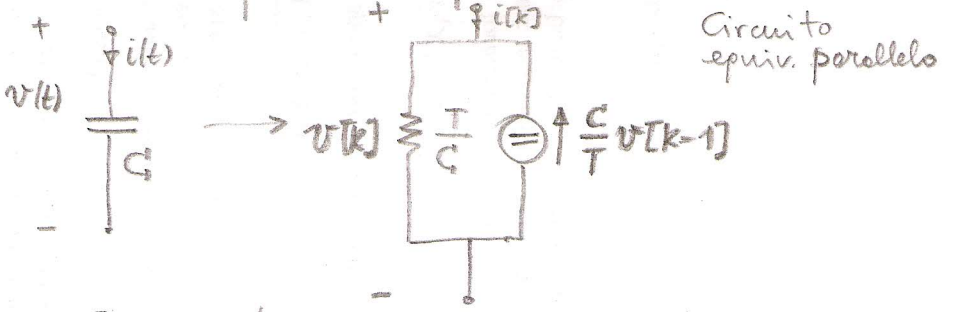
$v_2(t) \longrightarrow v_2[k]$,

$$C \frac{dv_2}{dt} \to C \frac{v_2[k] - v_2[k-1]}{T}, \quad T = \text{campionamento}$$

FILTRO PASSA-BASSO DEL I ORDINE A TEMPO DISCRETO

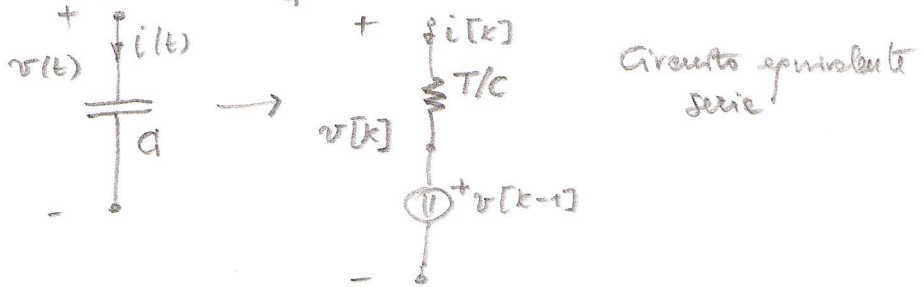
Corrente sul condensatore:

$$i[k] = \frac{C}{T} v[k] - \frac{C}{T} v[k-1]$$



Oppure, tensione sul condensatore:

$$v[k] = \frac{T}{C} i[k] + v[k-1]$$



$$v_2[k] = \frac{T}{C}\,\frac{v_1[k]-v_2[k]}{R} + v_2[k-1]$$

$$v_2[k] = \frac{T}{RC}v_1[k] - \frac{T}{RC}v_2[k] + v_2[k-1]$$

$$\left(1+\frac{T}{RC}\right)v_2[k] = \frac{T}{RC}v_1[k] + v_2[k-1]$$

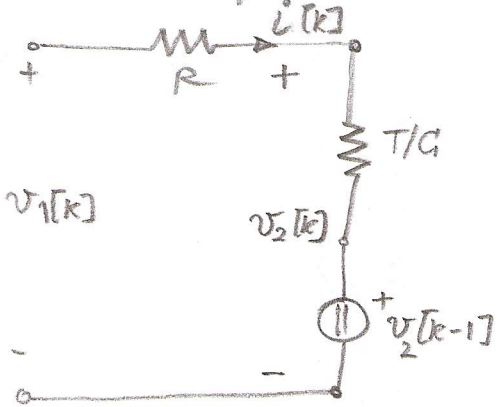
$$\frac{RC+T}{RC}v_2[k] = \frac{T}{RC}v_1[k] + v_2[k-1]$$

$$v_2[k] = \frac{RC}{RC+T}\,\frac{T}{RC}v_1[k] + \frac{RC}{RC+T}v_2[k-1]$$

$$\boxed{v_2[k] = a_1 v_2[k-1] + b_0 v_1[k]}$$ FPB a TD

$$\begin{cases} a_1 = \dfrac{RC}{RC+T} \\ b_0 = \dfrac{T}{RC+T} \end{cases}$$

Circuito equivalente con C serie:



$$\begin{cases} v_1[k] = R\, i[k] + v_2[k] \\ v_2[k] = \frac{T}{C}\, i[k] + v_2[k-1] \end{cases}$$

$$\begin{cases} i[k] = \dfrac{v_1[k] - v_2[k]}{R} \\ v_2[k] = \frac{T}{C}\, i[k] + v_2[k-1] \end{cases}$$

$$v_2[k] = \frac{T}{C}\,\frac{v_1[k] - v_2[k]}{R} + v_2[k-1]$$

$$v_2[k] = \frac{T}{RC} v_1[k] - \frac{T}{RC} v_2[k] + v_2[k-1]$$

$$\frac{T+RC}{RC} v_2[k] = v_2[k-1] + \frac{T}{RC} v_1[k]$$

$$v_2[k] = \underbrace{\frac{RC}{T+RC}}_{a_1} v_2[k-1] + \underbrace{\frac{T}{T+RC}}_{b_0} v_1[k]$$

$$\boxed{v_2[k] = a_1\, v_2[k-1] + b_0\, v_1[k]} \quad \text{FPB a TD}$$

NB $\quad a_1 + b_0 = 1 \Rightarrow \boxed{b_0 = 1 - a_1}$