Vettore di Poynting e conservazione della potenza 1/5

Consideriamo un fascio di onde piane che incidano su una superficie dielettrica e indichiamo con A la sezione di impatto del fascio sulla superficie.

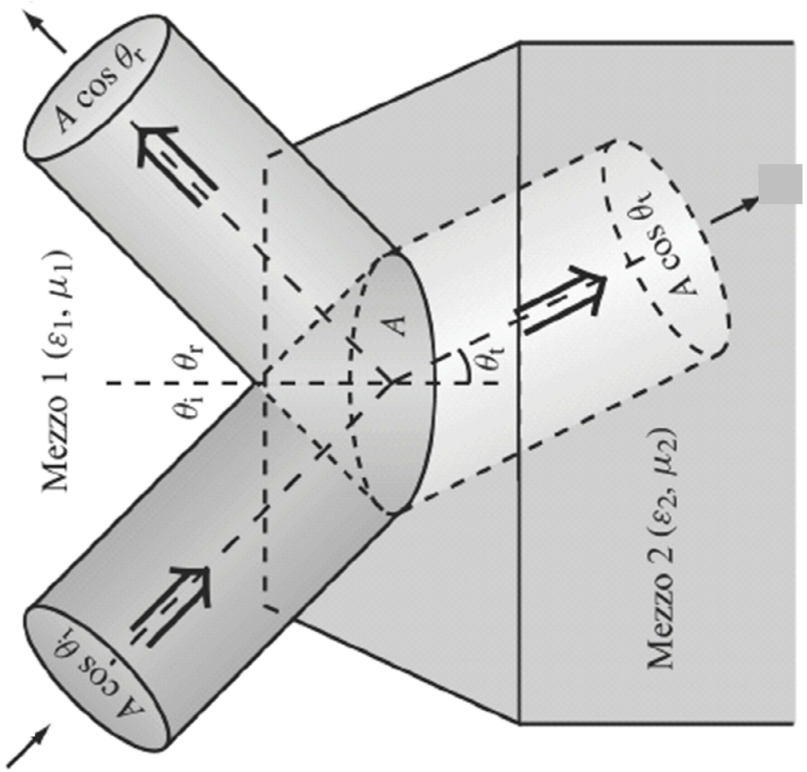
Le sezioni “oblique” ortogonali alla direzione di propagazione delle onde saranno A $\cos\theta_i$, A $\cos\theta_r$ e A $\cos\theta_t$.

Le potenze attive medie incidente riflessa e trasmessa valgono:

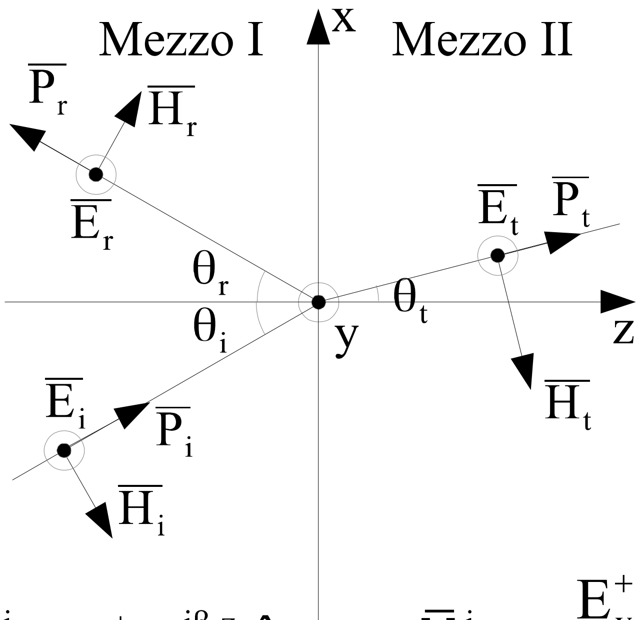
$$W_i = \frac{1}{2}\int_{A\cos\theta_i} \mathrm{Re}\left\{\bar{\mathbf{E}}_i \times \bar{\mathbf{H}}_i^*\right\} \cdot d\bar{\mathbf{S}}$$

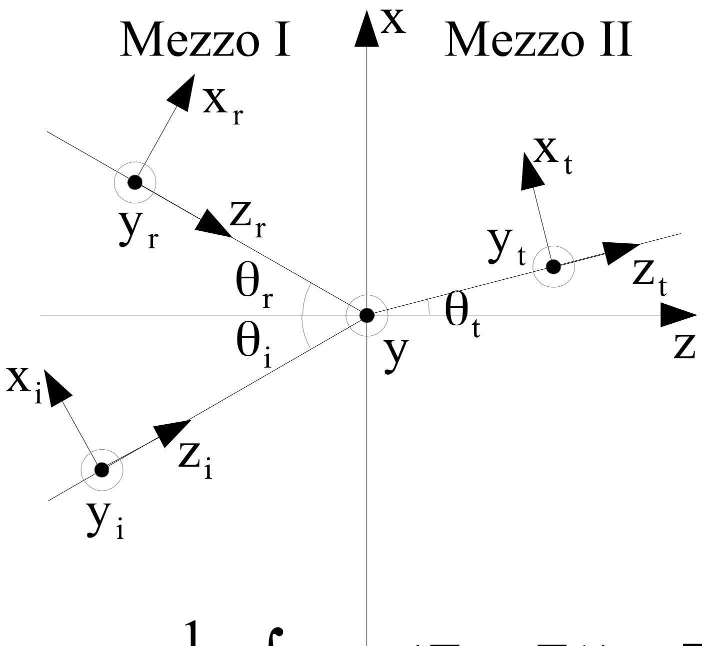
$$W_r = \frac{1}{2}\int_{A\cos\theta_r} \mathrm{Re}\left\{\bar{\mathbf{E}}_r \times \bar{\mathbf{H}}_r^*\right\} \cdot d\bar{\mathbf{S}}$$

$$W_t = \frac{1}{2}\int_{A\cos\theta_t} \mathrm{Re}\left\{\bar{\mathbf{E}}_t \times \bar{\mathbf{H}}_t^*\right\} \cdot d\bar{\mathbf{S}}$$



Vettore di Poynting e conservazione della potenza TE





$$\bar{\mathbf{E}}^i = E_y^+ e^{-j\beta_1 z_i}\hat{\mathbf{y}} \qquad \bar{\mathbf{H}}^i = -\frac{E_y^+}{\eta_1} e^{-j\beta_1 z_i}\hat{\mathbf{x}}_i$$

$$\bar{\mathbf{E}}^r = \rho^{TE} E_y^+ e^{j\beta_1 z_r}\hat{\mathbf{y}} \qquad \bar{\mathbf{H}}^r = \frac{\rho^{TE} E_y^+}{\eta_1} e^{j\beta_1 z_r}\hat{\mathbf{x}}_r$$

$$\bar{\mathbf{E}}^t = \tau^{TE} E_y^+ e^{-j\beta_2 z_t}\hat{\mathbf{y}} \qquad \bar{\mathbf{H}}^t = -\frac{\tau^{TE} E_y^+}{\eta_2} e^{-j\beta_2 z_t}\hat{\mathbf{x}}_t$$

$$W_i = \frac{1}{2}\int_{A\cos\theta_i} \mathrm{Re}\left\{\bar{\mathbf{E}}_i \times \bar{\mathbf{H}}_i^*\right\}\cdot d\bar{\mathbf{S}}_i$$

$$W_r = \frac{1}{2}\int_{A\cos\theta_r} \mathrm{Re}\left\{\bar{\mathbf{E}}_r \times \bar{\mathbf{H}}_r^*\right\}\cdot d\bar{\mathbf{S}}_r$$

$$W_t = \frac{1}{2}\int_{A\cos\theta_t} \mathrm{Re}\left\{\bar{\mathbf{E}}_t \times \bar{\mathbf{H}}_t^*\right\}\cdot d\bar{\mathbf{S}}_t$$

Vettore di Poynting e conservazione della potenza TE

$$W_i = \int_{A\cos\theta_i} \mathrm{Re}\left\{ \frac{E_y^+ e^{-j\beta_1 z_i}\hat{\mathbf{y}}}{2} \times \left(-\frac{E_y^+}{\eta_1} e^{-j\beta_1 z_i}\hat{\mathbf{x}}_i \right)^* \right\} \cdot d\bar{\mathbf{S}}_i = \int_{A\cos\theta_i} \frac{\left|E_y^+\right|^2}{2\eta_1} \hat{\mathbf{z}}_i \cdot d\bar{\mathbf{S}}_i = \frac{\left|E_y^+\right|^2}{2\eta_1} A\cos\theta_i$$

$$= \left|\bar{\mathbf{P}}_i\right| A\cos\theta_i$$

$$W_r = \int_{A\cos\theta_i} \mathrm{Re}\left\{ \frac{\rho^{TE} E_y^+ e^{j\beta_1 z_r}\hat{\mathbf{y}}}{2} \times \left(\frac{\rho^{TE} E_y^+}{\eta_1} e^{j\beta_1 z_r}\hat{\mathbf{x}}_r \right)^* \right\} \cdot d\bar{\mathbf{S}}_r = -\int_{A\cos\theta_i} \left|\rho^{TE}\right|^2 \frac{\left|E_y^+\right|^2}{2\eta_1} \hat{\mathbf{z}}_r \cdot d\bar{\mathbf{S}}_r$$

$$= \left|\rho^{TE}\right|^2 \frac{\left|E_y^+\right|^2}{2\eta_1} A\cos\theta_i = \left|\rho^{TE}\right|^2 \left|\bar{\mathbf{P}}_i\right| A\cos\theta_i \qquad d\bar{\mathbf{S}}_r = dS_r(-\hat{\mathbf{z}}_r)$$

$$W_t = \int_{A\cos\theta_t} \mathrm{Re}\left\{ \frac{\tau^{TE} E_y^+ e^{-j\beta_2 z_t}\hat{\mathbf{y}}}{2} \times \left(-\frac{\tau^{TE} E_y^+}{\eta_2} e^{-j\beta_2 z_t}\hat{\mathbf{x}}_t \right)^* \right\} \cdot d\bar{\mathbf{S}} = \int_{A\cos\theta_t} \left|\tau^{TE}\right|^2 \frac{\left|E_y^+\right|^2}{2\eta_2} \hat{\mathbf{z}}_t \cdot d\bar{\mathbf{S}}_t$$

$$= \left|\tau^{TE}\right|^2 \frac{\left|E_y^+\right|^2}{2\eta_2} A\cos\theta_t = \left|\tau^{TE}\right|^2 \frac{\left|E_y^+\right|^2}{2\eta_1} \frac{\eta_1}{\eta_2} A\cos\theta_t = \left|\tau^{TE}\right|^2 \frac{\eta_1}{\eta_2} \left|\bar{\mathbf{P}}_i\right| A\cos\theta_t$$

Vettore di Poynting e conservazione della potenza TE

$$W_i - W_r = |\mathbf{P}_i| A\cos\theta_i - |\rho^{TE}|^2 |\mathbf{P}_i| A\cos\theta_i = |\mathbf{P}_i| A\cos\theta_i (1 - |\rho^{TE}|^2) =$$

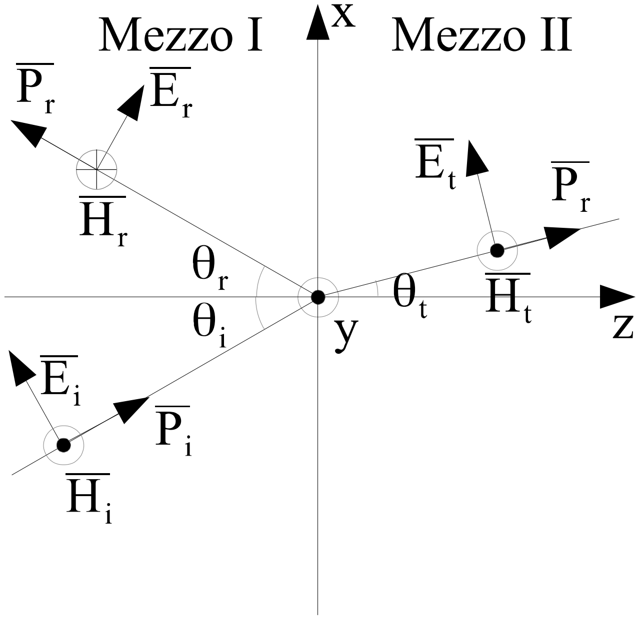
$$= |\mathbf{P}_i| A\cos\theta_i \frac{4\sqrt{\varepsilon_{r1}\varepsilon_{r2}}\cos\theta_i\cos\theta_t}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_i + \sqrt{\varepsilon_{r2}}\cos\theta_t\right)^2}$$

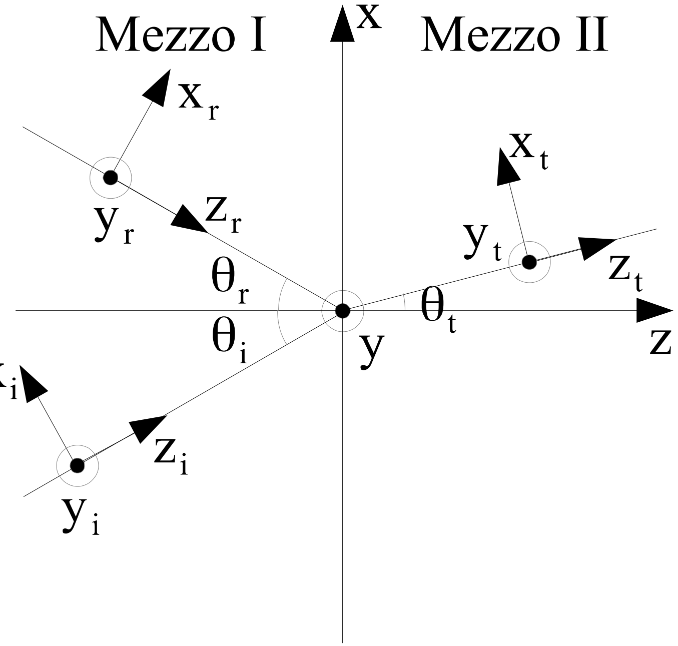
$$W_t = |\tau^{TE}|^2 \frac{\eta_1}{\eta_2} |\mathbf{P}_i| A\cos\theta_t = \frac{4\varepsilon_{r1}\cos^2\theta_i}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_i + \sqrt{\varepsilon_{r2}}\cos\theta_t\right)^2} \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}} |\mathbf{P}_i| A\cos\theta_t$$

$$= \frac{4\sqrt{\varepsilon_{r1}\varepsilon_{r2}}\cos^2\theta_i}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_i + \sqrt{\varepsilon_{r2}}\cos\theta_t\right)^2} |\mathbf{P}_i| A\cos\theta_t$$

$$W_i = W_r + W_t \qquad \textbf{N.B.}$$

Vettore di Poynting e conservazione della potenza TM





$$\bar{\mathbf{E}}^i = \eta_1 H_y^+ e^{-j\beta_1 z_i} \hat{\mathbf{x}}_i \qquad \bar{\mathbf{H}}^i = H_y^+ e^{-j\beta_1 z_i} \hat{\mathbf{y}}$$

$$\bar{\mathbf{E}}^r = \rho^{TM} \eta_1 H_y^+ e^{j\beta_1 z_r} \hat{\mathbf{x}}_r \qquad \bar{\mathbf{H}}^r = -\rho^{TM} H_y^+ e^{j\beta_1 z_r} \hat{\mathbf{y}}$$

$$\bar{\mathbf{E}}^t = H_y^+ \eta_1 \tau^{TM} \frac{\cos\theta_i}{\cos\theta_t} e^{-j\beta_2 z_t} \hat{\mathbf{x}}_t \qquad \bar{\mathbf{H}}^t = H_y^+ \tau^{TM} \frac{\eta_1 \cos\theta_i}{\eta_2 \cos\theta_t} e^{-j\beta_2 z_t} \hat{\mathbf{y}}$$

$$W_k = \frac{1}{2} \int_{A\cos\theta_k} \mathrm{Re}\left\{ \bar{\mathbf{E}}_k \times \bar{\mathbf{H}}_k^* \right\} \cdot d\bar{\mathbf{S}}_k \qquad k = i, r, t$$

Vettore di Poynting e conservazione della potenza TM

$$W_i = 0.5 \int_{A\cos\theta_i} \mathrm{Re}\left\{\eta_1 H_y^+ e^{-j\beta_1 z_i}\hat{\mathbf{x}}_i \times \left(H_y^+ e^{-j\beta_1 z_i}\hat{\mathbf{y}}\right)^*\right\}\cdot d\bar{\mathbf{S}}_i = 0.5 \int_{A\cos\theta_i} \eta_1 \left|H_y^+\right|^2 \hat{\mathbf{z}}_i \cdot d\bar{\mathbf{S}}_i$$

$$= 0.5\eta_1 \left|H_y^+\right|^2 A\cos\theta_i = \left|\bar{\mathbf{P}}_i\right| A\cos\theta_i$$

$$W_r = 0.5 \int_{A\cos\theta_i} \mathrm{Re}\left\{\rho^{TM}\eta_1 H_y^+ e^{j\beta_1 z_r}\hat{\mathbf{x}}_r \times \left(-\rho^{TM} H_y^+ e^{j\beta_1 z_r}\hat{\mathbf{y}}\right)^*\right\}\cdot d\bar{\mathbf{S}}_r = \qquad d\bar{\mathbf{S}}_r = dS_r(-\hat{\mathbf{z}}_r)$$

$$= -0.5 \int_{A\cos\theta_i} \left|\rho^{TM}\right|^2 \eta_1 \left|H_y^+\right|^2 \hat{\mathbf{z}}_r \cdot d\bar{\mathbf{S}}_r = 0.5\left|\rho^{TM}\right|^2 \eta_1 \left|H_y^+\right|^2 A\cos\theta_i = \left|\rho^{TE}\right|^2 \left|\bar{\mathbf{P}}_i\right| A\cos\theta_i$$

$$W_t = 0.5 \int_{A\cos\theta_t} \mathrm{Re}\left\{H_y^+\eta_1\tau^{TM}\frac{\cos\theta_i}{\cos\theta_t} e^{-j\beta_2 z_t}\hat{\mathbf{x}}_t \times \left(H_y^+\tau^{TM}\frac{\eta_1\cos\theta_i}{\eta_2\cos\theta_t} e^{-j\beta_2 z_t}\hat{\mathbf{y}}\right)^*\right\}\cdot d\bar{\mathbf{S}} =$$

$$= 0.5 \int_{A\cos\theta_t} \left|\tau^{TM}\right|^2 \eta_1 \left|H_y^+\right|^2 \frac{\eta_1\cos^2\theta_i}{\eta_2\cos^2\theta_t}\hat{\mathbf{z}}_t \cdot d\bar{\mathbf{S}}_t = \left|\tau^{TM}\right|^2 \frac{\eta_1\cos^2\theta_i}{\eta_2\cos^2\theta_t}\left|\bar{\mathbf{P}}_i\right| A\cos\theta_t =$$

$$= \left|\tau^{TM}\right|^2 \frac{\eta_1\cos^2\theta_i}{\eta_2\cos\theta_t}\left|\bar{\mathbf{P}}_i\right| A$$

Vettore di Poynting e conservazione della potenza TM

$$W_i - W_r = \left|\bar{\mathbf{P}}_i\right| A\cos\theta_i - \left|\rho^{TM}\right|^2 \left|\bar{\mathbf{P}}_i\right| A\cos\theta_i = \cos\theta_i (1 - \left|\rho^{TM}\right|^2) \left|\bar{\mathbf{P}}_i\right| A =$$

$$= \cos\theta_i \frac{4\sqrt{\varepsilon_{r1}\varepsilon_{r2}}\cos\theta_i\cos\theta_t}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_t + \sqrt{\varepsilon_{r2}}\cos\theta_i\right)^2} \left|\bar{\mathbf{P}}_i\right| A = \frac{4\sqrt{\varepsilon_{r1}\varepsilon_{r2}}\cos^2\theta_i\cos\theta_t}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_t + \sqrt{\varepsilon_{r2}}\cos\theta_i\right)^2} \left|\bar{\mathbf{P}}_i\right| A$$

$$W_t = \left|\tau^{TM}\right|^2 \frac{\eta_1\cos^2\theta_i}{\eta_2\cos\theta_t} \left|\bar{\mathbf{P}}_i\right| A = \frac{4\varepsilon_{r1}\cos^2\theta_t}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_t + \sqrt{\varepsilon_{r2}}\cos\theta_i\right)^2} \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}} \frac{\cos^2\theta_i}{\cos\theta_t} \left|\bar{\mathbf{P}}_i\right| A$$

$$= \frac{4\sqrt{\varepsilon_{r1}}\sqrt{\varepsilon_{r2}}\cos\theta_t\cos^2\theta_i}{\left(\sqrt{\varepsilon_{r1}}\cos\theta_t + \sqrt{\varepsilon_{r2}}\cos\theta_i\right)^2} \left|\bar{\mathbf{P}}_i\right| A$$

$$W_i = W_r + W_t \qquad \textbf{N.B.}$$

Angolo di Brewster

Esistono dei valori di angolo di incidenza per cui il coefficiente di riflessione vale zero?
Se tale angolo esiste, esso viene definito angolo di Brewster.
Per rispondere a questa domanda, bisogna analizzare separatamente le due polarizzazioni.

Angolo di Brewster: TM

$$\rho^{TM} = \frac{Z_2^{TM} - Z_1^{TM}}{Z_2^{TM} + Z_1^{TM}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0 \Leftrightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_i$$

$$\eta_2 \cos\vartheta_t = \eta_2 \sqrt{1 - \sin^2\vartheta_t} = \eta_2 \sqrt{1 - \frac{\varepsilon_{r1} \sin^2\vartheta_i}{\varepsilon_{r2}}} = \eta_1 \cos\theta_i$$

$$1 - \frac{\varepsilon_{r1} \sin^2\vartheta_i}{\varepsilon_{r2}} = \frac{\eta_1^2}{\eta_2^2} \cos^2\theta_i = \frac{\eta_1^2}{\eta_2^2}(1 - \sin^2\theta_i) = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}(1 - \sin^2\theta_i)$$

$$\Downarrow$$

$$1 - \frac{\varepsilon_{r1} \sin^2\vartheta_i}{\varepsilon_{r2}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}(1 - \sin^2\theta_i)$$

$$\sin^2\vartheta_i \left(\frac{\varepsilon_{r1}}{\varepsilon_{r2}} - \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \right) = 1 - \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

$$\sin^2\vartheta_i \frac{(\varepsilon_{r1} - \varepsilon_{r2})(\varepsilon_{r1} + \varepsilon_{r2})}{\varepsilon_{r1}\varepsilon_{r2}} = \frac{\varepsilon_{r1} - \varepsilon_{r2}}{\varepsilon_{r1}}$$

Angolo di Brewster: TM

$$\sin^2\theta_i \frac{(\varepsilon_{r1}-\varepsilon_{r2})(\varepsilon_{r1}+\varepsilon_{r2})}{\varepsilon_{r1}\varepsilon_{r2}} = \frac{\varepsilon_{r1}-\varepsilon_{r2}}{\varepsilon_{r1}} \Rightarrow \sin^2\theta_i = \frac{\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}} < 1 \longrightarrow \text{Esiste}$$

angolo di Brewster $\quad \theta_B^{TM} = \mathrm{Arc}\sin\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}} = \mathrm{Arc}\tan\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} \quad \exists \quad \forall \varepsilon_{r1}, \varepsilon_{r2}$

Angolo di Brewster: TE

$$\rho^{TM} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_2^{TE} + Z_1^{TE}} = \frac{\dfrac{\eta_2}{\cos\theta_t} - \dfrac{\eta_1}{\cos\theta_i}}{\dfrac{\eta_2}{\cos\theta_t} + \dfrac{\eta_1}{\cos\theta_i}} = 0 \Leftrightarrow \frac{\eta_2}{\cos\theta_t} = \frac{\eta_1}{\cos\theta_i}$$

$$\boldsymbol{\eta_2 \cos\vartheta_i} = \eta_1 \cos\theta_t = \eta_1\sqrt{1-\sin^2\theta_t} = \eta_1\sqrt{1-\frac{\varepsilon_{r1}\sin^2\theta_i}{\varepsilon_{r2}}}$$

$$\Downarrow$$

$$\boldsymbol{\eta_2 \cos\vartheta_i} = \eta_1\sqrt{1-\frac{\varepsilon_{r1}\sin^2\theta_i}{\varepsilon_{r2}}}$$

$$\boldsymbol{\frac{\eta_2^2}{\eta_1^2}\cos^2\vartheta_i = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}(1-\sin^2\vartheta_i)} = 1-\frac{\varepsilon_{r1}\sin^2\theta_i}{\varepsilon_{r2}} \Rightarrow \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = 1$$

Quindi l’angolo di Brewster per la polarizzazione TE esiste soltanto se i due mezzi dielettrici sono uguali. Questa risposta è ovvia e quindi possiamo asserire che per la polarizzazione TE non esiste l’angolo di Brewster.

http://micro.magnet.fsu.edu/primer/java/scienceopticsu/polarizedlight/brewster/index.html

angolo di Brewster

Angolo di riflessione totale o angolo critico

Esistono dei valori di angolo di incidenza per cui il modulo del coefficiente di riflessione vale uno e quindi tutta la potenza incidente viene riflessa?

Se tale angolo esiste, esso viene definito angolo critico o di riflessione totale.

$$\rho^{TM} = \frac{Z_2^{TM} - Z_1^{TM}}{Z_2^{TM} + Z_1^{TM}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \Rightarrow \left|\rho^{TM}\right| = 1 \Leftrightarrow \eta_2 \cos\theta_t = 0 \Rightarrow \theta_t = \pi / 2$$

$$\rho^{TE} = \frac{Z_2^{TE} - Z_1^{TE}}{Z_2^{TE} + Z_1^{TE}} = \frac{\dfrac{\eta_2}{\cos\theta_t} - \dfrac{\eta_1}{\cos\theta_i}}{\dfrac{\eta_2}{\cos\theta_t} + \dfrac{\eta_1}{\cos\theta_i}} \Rightarrow \left|\rho^{TE}\right| = 1 \Leftrightarrow \frac{\eta_2}{\cos\theta_t} \to \infty \Rightarrow \theta_t = \pi / 2$$

Quindi

$$\sqrt{\varepsilon_{r1}} \sin\theta_i = \sqrt{\varepsilon_{r2}} \sin\theta_t = \sqrt{\varepsilon_{r2}} \Rightarrow \theta_c = \mathrm{Arc}\sin\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} \quad \exists \Leftrightarrow \varepsilon_{r1} > \varepsilon_{r2}$$

Angolo di riflessione totale o angolo critico

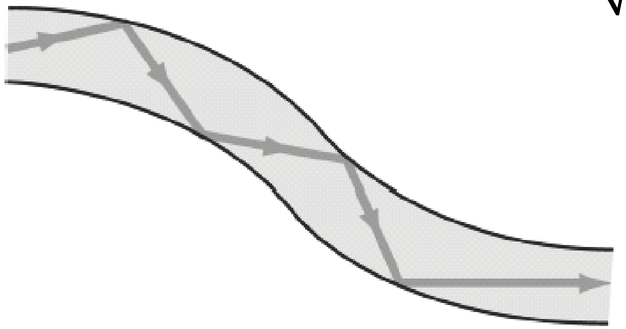
Quindi, esiste un angolo critico soltanto se l'onda passa da un mezzo più denso a un mezzo meno denso.
E' possibile dimostrare che se l'angolo di incidenza supera questo valore il coefficiente di riflessione continua ad avere modulo unitario e quindi tutta la potenza viene riflessa.
Quindi per avere riflessione totale basta che sia

$$\theta_i > \theta_c$$

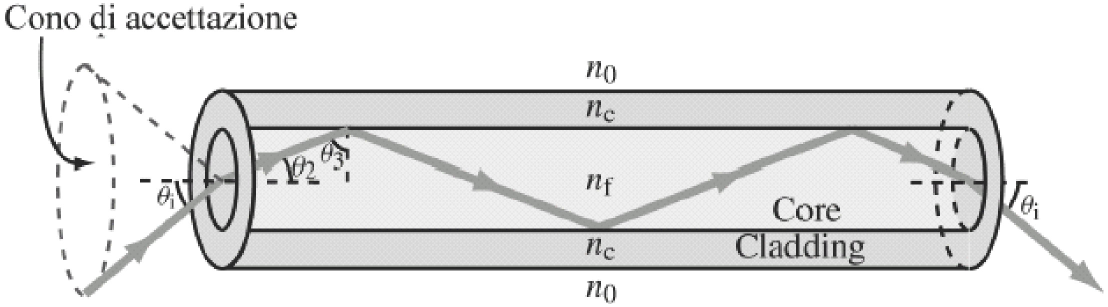
incidenza obliqua
http://www.amanogawa.com/archive/Oblique/Oblique.html

Applicazioni dell’angolo critico: fibre ottiche

$n=\sqrt{\varepsilon_r}$: indice di rifrazione



(a) Fibra ottica



(b) Riflessioni interne successive

$$\theta_3 > \theta_c \qquad \theta_c = \mathrm{Arc}\sin\sqrt{\frac{\varepsilon_{rc}}{\varepsilon_{rf}}} = \mathrm{Arc}\sin\frac{n_c}{n_f} \qquad \theta_2 = \frac{\pi}{2} - \theta_3$$

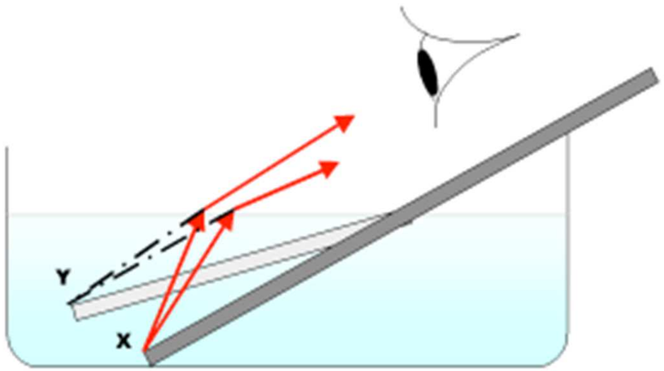
$$\text{Snell}: \quad n_i \sin\theta_i = n_f \sin\theta_2 \quad (n_i = 1) \quad \sin\theta_i = n_f \sin(\pi/2 - \theta_3) = n_f \cos\theta_3$$

$$\sin\theta_i = n_f \cos\theta_3 = n_f\sqrt{1-\sin^2\theta_3} = n_f\sqrt{1-\sin^2\theta_c} = n_f\sqrt{1-\frac{n_c^2}{n_f^2}} = \sqrt{n_f^2 - n_c^2}$$

cono di accettazione: $\theta_i \le \mathrm{Arc}\sin\sqrt{n_f^2 - n_c^2}$ $\quad$ es. $n_c = 1.49 \quad n_f = 1.52 \quad \theta_i \le 17.5°$

Applicazioni della rifrazione:

la penna si piega !!!!



arcobaleno

Applicazioni della rifrazione:

http://micro.magnet.fsu.edu/primer/java/scienceopticsu/refraction/refractionangles/index.html

rifrazione della luce

http://micro.magnet.fsu.edu/primer/java/scienceopticsu/lenses/magnify/index.html

lenti

http://micro.magnet.fsu.edu/primer/java/scienceopticsu/prism/index.html

rifrazione di un prisma