Barnacles Mating Optimizer: An Evolutionary Algorithm for Solving Optimization

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***Abstract* — A new evolutionary algorithm called Barnacles Mating Optimizer (BMO) to solve optimization problems is presented in this paper. BMO is inspired from the behavior of barnacle' mating in nature. They are known as micro-organisms that existed since Jurassic times and classified as hermaphroditic micro-organisms. They have a unique feature which is they own long penises that can be said as the longest among micro-organisms, relatively to size of their body. To show the effectiveness of proposed BMO in solving optimization problems, a set of 23 mathematical functions are used to test the characteristic of BMO in finding the optimal solutions especially in unimodal, multimodal and composite test functions. Comparisons with other evolutionary and swarm algorithms also will be presented.**

Keywords— BMO, evolutionary algorithm, optimization, swarm intelligence

I. INTRODUCTION

One of the active researches since the last decade in the various engineering and computer applications fields is the invention of new algorithms based on the nature especially to solve the problems of optimization. The proposed nature-based algorithms can be classified into four categories namely evolutionary, swarm, physics and human behavior based algorithms. Evolutionary algorithms are basically referring to the process of breeding or off-spring generations from the parents in order to obtain the optimal results depending on the satisfaction of termination criterion. Among the popular algorithms under this category are Genetic Algorithm (GA) [1, 2], Evolutionary Programming (EP) [3], Differential Evolution (DE) [4], and many more.

Swarm based algorithms on the other hand, mimic the social behavior of group of animals whether for food foraging, the movement of the swarm animals or to find a mate. Particle Swarm Optimization (PSO) [5], Artificial Bee Colony (ABC) [6], Grey Wolf Optimizer (GWO) [7] and Firefly Algorithm (FA) [8] are among the popular algorithms fall in this category. Physics-based algorithms are the algorithms that use the concept of physical rules and natural phenomena in solving the optimization problem such as Gravitational Search Algorithm (GSA) [9], Lightning Search algorithm (LSA) [10], Multi-Verse Optimizer (MVO) [11] as well as Black Hole Algorithm (BH) [12]. The fourth category is called human-based algorithms due to mimic the human behavior in life such as learning, competition socializing etc. Among the algorithms in this category are Harmony Search Algorithm (HSA) [13] and Teaching-Learning Based Optimization (TLBO) [14].

One factor that drives the invention of new algorithms based on nature by the researchers is the no free lunch (NFL) theorem. The NFL has stated that no algorithms can solve and perform well in all optimization problems. One algorithm may be good in solving on a certain set of optimization problem but that particular algorithm also might be worst when applied in other set of problems [15].

A new evolutionary algorithm is presented in this paper that mimicking the barnacles' mating process namely barnacles mating optimizer (BMO). The paper is organized as follows: section 2 presents the BMO concept and the algorithm's development is presented in Section 3. This is follows by the results and discussion in Section 4 and Section 5 concludes the paper.

II. BMO CONCEPT

A. Inspired from Barnacles

Barnacles are known as hermaphroditic micro-organisms which they own male and female reproduction organs. They also are known as sessile organisms since they are usually permanently attached to something such as rocks, corals and even to the ships. One of the unique behaviors of barnacles is they have long penises for mating to overcome their sessile lifestyle and on top of that, they are probably having the longest penis to body size ratio of the organisms [16]. Their penises can stretch to up to seven to eight times the length of their bodies.

To copulate, a functional male reproduction organ search for a partner by randomly within its penis range and deposits its sperm into the partner's mantle cavity. For the isolated barnacles, sperm-cast process is occurred. Sperm-cast mating is the mating process where the sperms released in the water fertilize the eggs. BMO development is inspired by these behaviors in solving the optimization problems.

B. *Off-spring's generation*

In BMO, the production of new off-springs is depending on the mating process of barnacles. For this, the concept of Hardy-Weinberg principle is adopted. The simplest case with two alleles of parents denoted as *Mum* (M) with frequencies $f(M) = q$ and *Dad* (D) with the frequencies $f(D) = p$ as depicted in Fig. 1. From this figure, it can be seen that $p^2 + 2pq + q^2 = 1$. It also can be noted that $p + q = 1$, hence for the simplification, the production of new off-springs are referring to the q and p of the barnacles' parents.

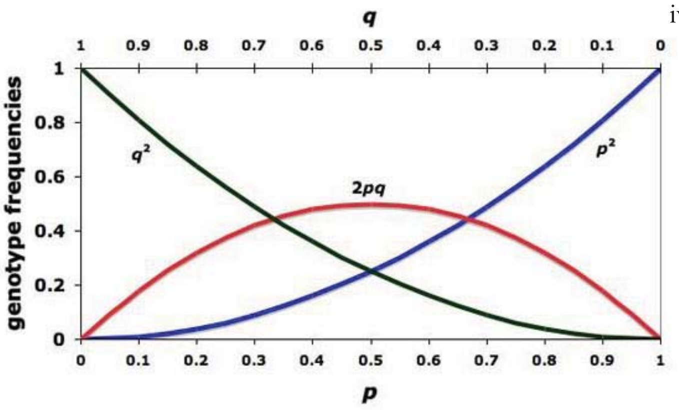


Fig. 1. The Hardy-Weinberg principle [17].

III. Development of BMO

A. *Initialization*

The first step of BMO is the initialization. Similar with other evolutionary algorithms such as GA and DE, BMO works in population based. The candidate of solutions can be represented such as follows:

$$X = \begin{bmatrix} x_1^1 & \cdots & x_1^N \\ \vdots & \ddots & \vdots \\ x_n^1 & \cdots & x_n^N \end{bmatrix} \tag{1}$$

where N and n are the number of control variables to be optimized and the number of population or barnacles, respectively. The next step is to evaluate the vector X to obtain the required output or objective. It followed by the sorting process by referring to the evaluation (objective function) so that the current best solution is located at the top of matrix X.

B. *Selection process*

One of the important steps in BMO is the selection process. To simplify the algorithm, the following assumptions have been made:

i. The selection process is made in random but limited to the length of its penis, *pl.*

ii. Every barnacle contributes and receives its sperm from other barnacles due to hermaphroditic behavior. However, in BMO, at one particular time, it is assumed that each barnacle can fertilized only by one barnacle even though in [16] stated that the more than one male can copulate to a single female barnacle. This is to ensure the simplicity of the proposed algorithm.

iii. At the certain iterations, if the algorithm selects the same barnacle, the self-mating process assumed to be happened. Nevertheless, self-mating is very rare according to [18], thus it will not be considered in the proposed BMO. When this situation is happened, no new off-springs will be produced.

iv. If at certain iterations, the selection for mating is out of range of *pl* that has been set, it is assumed that the new off-springs are produced by sperm-cast mating process.

The selection process is exhibited in Fig. 2. From this figure, barnacle #1 select barnacle #7 to be mated. For this example, *pl* is set to 7, which means that barnacle # 1 is able to be mated among the barnacles #2 until #7 only. If more than that, the new off-springs are produced by sperm-cast mating. This impose the exploration ability of the proposed BMO that will be presented later. Worth to mention here that this situation just to show how the BMO is developed using the ranking or sorting schemes and not related to the distance or location among the barnacles. The selection process for barnacles to find its mate are using the following expressions:

$$barnacle\_m = randperm(n) \tag{2}$$

$$barnacle\_d = randperm(n) \tag{3}$$

where n is the total of barnacles, *barnacle_m* and *barnacle_d* are the potential parents to be mated. These equations show that the selection is performed in random which is fulfill the first assumption.

C. *The production of new off-springs*

The production of the new off-springs in BMO is referring to the p and q concept of Hardy-Weinberg that been discussed previously. The following equations are used in BMO to generate the new off-springs from the barnacles' parents:

$$x_i^{N\_new} = p x_{barnacle\_d}^{N} + q x_{barnacle\_m}^{N} \quad for \quad k \leq pl \tag{4}$$

$$x_i^{N\_new} = rand() \times x_{barnacle\_m}^{N} \quad for \quad k > pl \tag{5}$$

where p, is the normal distributed random number, $q = (1 - p)$, $x^N{}_{barnacle\_m}$ and $x^N{}_{barnacle\_d}$ are the variables selected from eqns. (2) and (3) for barnalce's *Mum* and *Dad* respectively and k = |*barnacle_d-barncle_m*|. From eqns. (4) and (5), p and q can be represented as inheritance behavior or characteristic from respective barnacles' parents. For example, let p = 0.55, which means that the new off-spring inherits 55% and 45% of the Dad's and Mum's characters, respectively.

From eqns. (4) and (5), it also can be noted that p is the normalized distributed random number which may give a negative value. Thus, there is probabilities of the off-springs inherits a negative behavior of the parents as well as the other inherits magnifying the parent's behavior or character. For example, let p = -0.55, which means that -55% of the Dad's characters influence the new off-spring and that new off-spring inherits 155% of Mum's characters. In optimization, (4) can be treated as exploitation process while (5) is the exploration process.

The flowchart of BMO is depicted in Fig. 3. A set of random solutions is created initially followed by the new barnacles' off- springs generation using eqns. (4) and (5). From the evaluation process, the current best solution is recorded. To control the expansion of the solution matrix from the size of population, a new set of off-springs with the evaluation results are combined with the parents. Then, by referring to the evaluation results, the sorting process is performed. Only top half of the combined matrix (parents and off-springs) are selected and the bottom half of poor results are eliminated. So that the size of population is maintained.

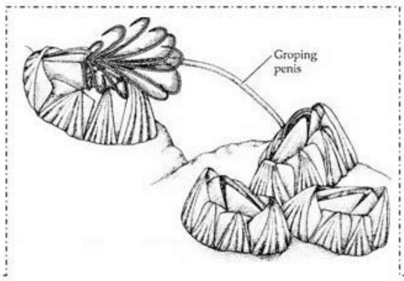
IV. RESULTS AND DISCUSSION

To evaluate the performance of proposed BMO, the 23 benchmark functions namely F-1 to F-23 [19] have been used.

The comparison with other evolutionary algorithms: GA and DE as well as with the swarm optimization algorithm viz. PSO has been performed to show the effectiveness of proposed BMO. These 23 benchmark functions can be classified into three groups viz. unimodal functions namely as F-1 to F-7, multi-modal functions namely as F-8 to F-13 as well as composite test functions namely as F-14 to F-23. The set up for all simulations are: 30 search agents and 500 iterations and all test functions were run for 30 free running times to obtain statistical outcomes. The best, the worst, mean and standard deviations were used to compared BMO with other algorithms quantitatively. Tables 1, 2 and 3 show the results for three classifications of 23 benchmark functions respectively.

From the results, it can be seen that BMO has emerged as the best algorithm compared to PSO, DE and GA except for F-6 where BMO's performance next to PSO and DE, as depicted in Table 1. As been known, the characteristic of unimodal functions is consisting of one global optimum. Thus, the performance shown in this table stated that BMO is outperformed GA, DE and PSO in finding optimal solutions for unimodal test functions which exhibits the better exploitability of BMO.

Table 2 show that BMO able to achieve three best results (F-9, F-10 and F-11) which are at par with DE (F-8, F-12 and F-13). The characteristic of multi-modal functions is consisting several local optima solutions. As exhibited from the results incurred, BMO is performed well and efficiently to avoid from stuck into local optima during optimization process as good as DE and slightly better compared to GA and PSO.



	Variable #1	...	...	Variable #N
Barnacle #1	0.9	...	...	12
Barnacle #2	0.95	...	...	2
⋮	...	...	...	...
⋮	...	...	...	...
Barnacle #7	1.01	...	...	-10
Barnacle #8	1.00	...	...	25
Barnacle #9	0.99	...	...	10
Barnacle #10	0.97	...	...	8

Barnacle #1 mate with Barnacle #7

Fig. 2. Selection for BMO mating

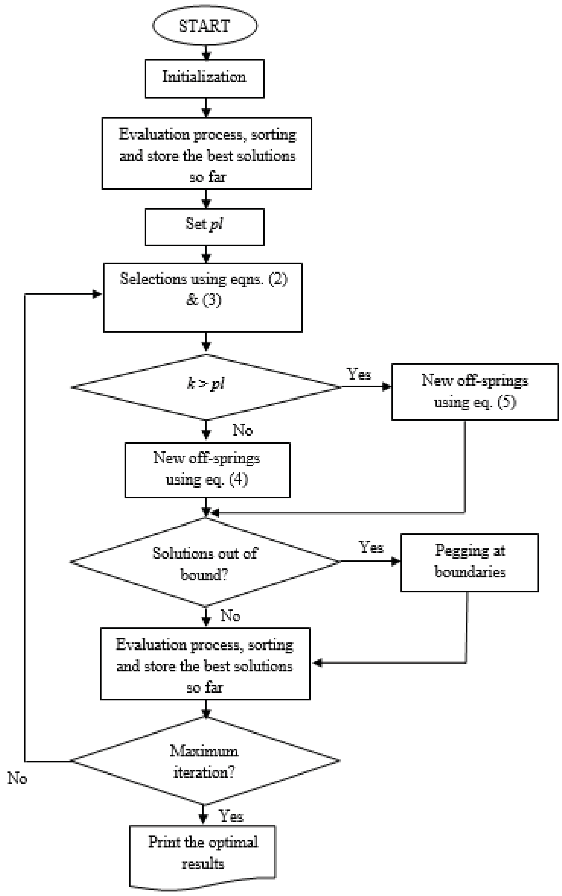


Fig. 3. Flow of the proposes BMO for solving optimization problems

Table 3 shows the results of the composite test functions. It can be seen that BMO gave close results to PSO, GA and DE. A proper strategy for balancing between exploration and exploitation to obtain global optima are designed for composite test functions. From this table, the proposed BMO exhibits a proper balance in processing the exploration and exploitation to solve near global optima solutions. Convergence curve for selected functions for all algorithms are depicted in Figs. 4-6.

TABLE I. UNIMODAL BENCHMARK FUNCTIONS RESULTS

Functions		BMO	GA	DE	PSO
F-1	The Best	**4.7E-149**	2.04E-01	3.40E-04	3.63E-14
	The Worst	**1.4E-124**	1.64E+00	1.06E-03	1.09E-05
	Mean	**5.4E-126**	7.05E-01	5.63E-04	7.85E-07
	Std dev	**2.6E-125**	3.65E-01	1.62E-04	2.68E-06
F-2	The Best	**5.18E-79**	7.50E-02	2.00E-03	2.59E-03
	The Worst	**5.53E-67**	3.13E-01	4.02E-03	4.16E-01
	Mean	**2.17E-68**	1.81E-01	2.77E-03	7.03E-02
	Std dev	**1.01E-67**	6.57E-02	3.81E-04	9.65E-02
F-3	The Best	**5.5E-140**	3.48E+03	1.90E+04	1.31E+01
	The Worst	**3.8E-110**	1.88E+04	3.80E+04	8.22E+02
	Mean	**2.9E-111**	1.19E+04	3.18E+04	1.14E+02
	Std dev	**9.6E-111**	3.68E+03	3.96E+03	1.52E+02
F-4	The Best	**3.65E-79**	8.23E+00	1.12E+01	6.72E-01
	The Worst	**5.33E-59**	3.12E+01	1.61E+01	4.56E+00
	Mean	**2.32E-60**	1.93E+01	1.31E+01	2.36E+00
	Std dev	**9.85E-60**	5.90E+00	1.32E+00	1.09E+00
F-5	The Best	2.68E+01	8.92E+01	9.24E+01	**1.13E+01**
	The Worst	2.80E+01	3.28E+03	2.63E+02	**1.73E+02**
	Mean	**2.72E+01**	5.22E+02	1.66E+02	5.63E+01
	Std dev	**3.12E-01**	8.82E+02	4.28E+01	4.38E+01
F-6	The Best	9.14E-03	9.48E-02	2.68E-04	**1.59E-12**
	The Worst	5.81E-01	2.11E+00	1.18E-03	**1.01E-04**
	Mean	8.29E-02	8.55E-01	5.50E-04	**3.94E-06**
	Std dev	1.28E-01	4.87E-01	1.93E-04	**1.85E-05**
F-7	The Best	**1.59E-04**	4.00E-02	2.36E-02	1.04E-02
	The Worst	**2.80E-03**	1.74E-01	8.51E-02	5.85E-02
	Mean	**9.00E-04**	8.80E-02	5.31E-02	2.76E-02
	Std dev	**6.61E-04**	3.29E-02	1.39E-02	1.23E-02

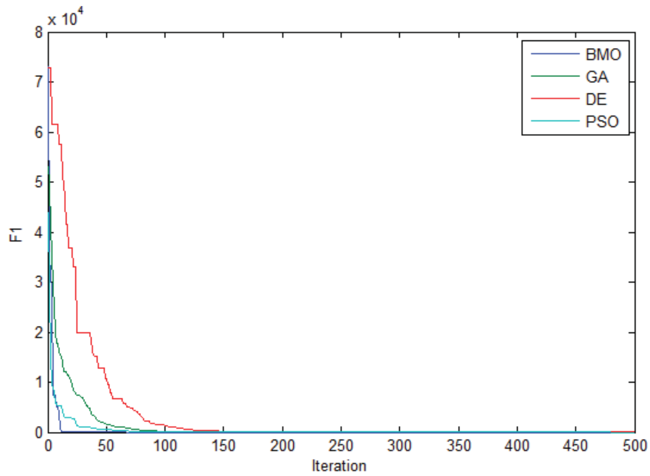


Fig. 4. Convergence curve for F1 using BMO, GA, DE and PSO for F1

TABLE II. MULTI-MODAL BENCHMARK FUNCTIONS RESULTS

Functions		BMO	GA	DE	PSO
F-8	The Best	-1.74E+03	-1.76E+03	**-1.86E+03**	-1.56E+03
	The Worst	-1.64E+03	-1.32E+03	**-1.78E+03**	-1.04E+03
	Mean	-1.68E+03	-1.53E+03	**-1.82E+03**	-1.28E+03
	Std dev	2.67E+01	1.11E+02	**1.87E+01**	1.26E+02
F-9	The Best	**0.00E+00**	5.85E-01	7.31E+01	4.18E+01
	The Worst	**0.00E+00**	8.91E+00	1.02E+02	1.33E+02
	Mean	**0.00E+00**	3.68E+00	9.13E+01	7.53E+01
	Std dev	**0.00E+00**	1.86E+00	7.50E+00	2.43E+01
F-10	The Best	**8.88E-16**	2.00E+01	2.00E+01	3.79E+00
	The Worst	**8.88E-16**	2.00E+01	2.00E+01	2.03E+01
	Mean	**8.88E-16**	2.00E+01	2.00E+01	1.94E+01
	Std dev	**4.01E-31**	7.76E-04	0.00E+00	2.96E+00
F-11	The Best	**0.00E+00**	1.83E-02	2.29E-05	9.29E-13
	The Worst	**0.00E+00**	1.38E-01	4.04E-03	4.43E-02
	Mean	**0.00E+00**	4.94E-02	3.47E-04	1.39E-02
	Std dev	**0.00E+00**	2.32E-02	7.60E-04	1.43E-02
F-12	The Best	5.72E-03	1.67E-05	2.21E-07	**4.84E-17**
	The Worst	2.13E-01	1.04E-01	**9.63E-07**	4.15E-01
	Mean	4.17E-02	3.61E-03	**4.78E-07**	2.42E-02
	Std dev	4.02E-02	1.89E-02	**1.89E-07**	8.02E-02
F-13	The Best	4.64E-02	8.52E-02	**8.51E-04**	1.10E-02
	The Worst	2.97E+00	8.96E-01	**3.70E-03**	3.61E+00
	Mean	5.49E-01	2.58E-01	**2.10E-03**	9.06E-01
	Std dev	6.74E-01	1.94E-01	**7.34E-04**	1.11E+00

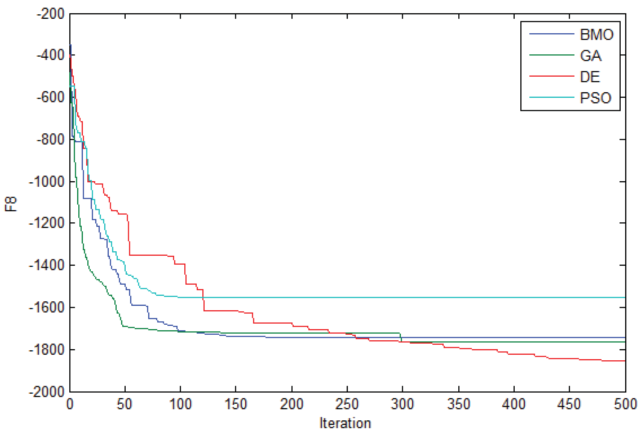


Fig. 5. Convergence curve for F8 using BMO, GA, DE and PSO for F8

TABLE III. COMPOSITE BENCHMARK FUNCTIONS RESULTS

Functions		BMO	GA	DE	PSO
F-14	The Best	9.98E-01	**9.98E-01**	**9.98E-01**	9.98E-01
	The Worst	7.87E+00	**9.98E-01**	**9.98E-01**	1.64E+01
	Mean	2.45E+00	**9.98E-01**	**9.98E-01**	5.22E+00
	Std dev	1.95E+00	**4.52E-16**	**4.52E-16**	3.74E+00
F-15	The Best	3.08E-04	3.94E-04	5.05E-04	**3.07E-04**
	The Worst	2.04E-02	2.07E-02	**8.04E-04**	2.04E-02
	Mean	1.97E-03	5.76E-03	**7.00E-04**	2.45E-03
	Std dev	5.00E-03	7.25E-03	**6.85E-05**	6.08E-03
F-16	The Best	**-1E+0**	**-1E+0**	**-1E+0**	**-1E+0**
	The Worst	**-1E+0**	**-1E+0**	**-1E+0**	**-1E+0**
	Mean	**-1E+0**	**-1E+0**	**-1E+0**	**-1E+0**
	Std dev	**6.7E-16**	**6.7E-16**	**6.7E-16**	**6.7E-16**
F-17	The Best	**3.98E-1**	**3.98E-1**	**3.98E-1**	**3.98E-1**
	The Worst	**3.98E-1**	**3.98E-1**	**3.98E-1**	**3.98E-1**
	Mean	**3.98E-1**	**3.98E-1**	**3.98E-1**	**3.98E-1**
	Std dev	**1.69E-16**	**3.65E-06**	**1.69E-16**	**1.69E-16**
F-18	The Best	**3.0E+00**	3.0E+00	**3.0E+00**	**3.0E+00**
	The Worst	**3.0E+00**	3.0E+01	**3.0E+00**	**3.0E+00**
	Mean	**3.0E+00**	3.9E+00	**3.0E+00**	**3.0E+00**
	Std dev	**0E+00**	4.93E+0	**0E+0**	**0E+0**
F-19	The Best	**-3E-01**	**-3E-01**	**-3E-01**	**-3E-01**
	The Worst	**-3E-01**	**-3E-01**	**-3E-01**	**-3E-01**
	Mean	**-3E-01**	**-3E-01**	**-3E-01**	**-3E-01**
	Std dev	**1.13E-16**	**1.130E-16**	**1.130E-16**	**1.130E-16**
F-20	The Best	-3.3E+0	-3.3E+0	**-3.3E+0**	-3.3E+0
	The Worst	-3.1E+0	-3.2E+0	**-3.2E+0**	-3.2E+0
	Mean	-3.3E+0	-3.3E+0	**-3.3E+0**	-3.3E+0
	Std dev	8.17E-02	5.83E-02	**2.59E-02**	6.03E-02
F-21	The Best	-1.0E+01	-1.0E+01	**-1.0E+01**	-1.0E+01
	The Worst	-2.6E+0	-2.6E+0	**-5.7E+00**	-2.6E+00
	Mean	-5.8E+0	-5.0E+0	**-9.9E+00**	-5.9E+00
	Std dev	2.3E+0	3.5E+0	**9.1E-01**	3.6E+00
F-22	The Best	-1.0E+01	-1.0E+01	**-1.0E+01**	-1.0E+01
	The Worst	-2.8E+0	-2.8E+0	**-1.0E+01**	-1.8E+00
	Mean	-7.1E+0	-6.6E+0	**-1.0E+01**	-6.7E+00
	Std dev	3.010E+00	3.660+0	**0.000E+00**	3.780E+00

	The Best	-1.1E+01	-1.1E+01	**-1.1E+01**	-1.1E+01
	The Worst	-2.4E+00	-2.4E+0	**-1.1E+01**	-2.4E+0
	Mean	-7.1E+00	-6.10E+0	**-1.1E+01**	-7.5E+0
F-23	Std dev	3.410E+0	3.750E+0	**0.000E+0**	3.610E+0

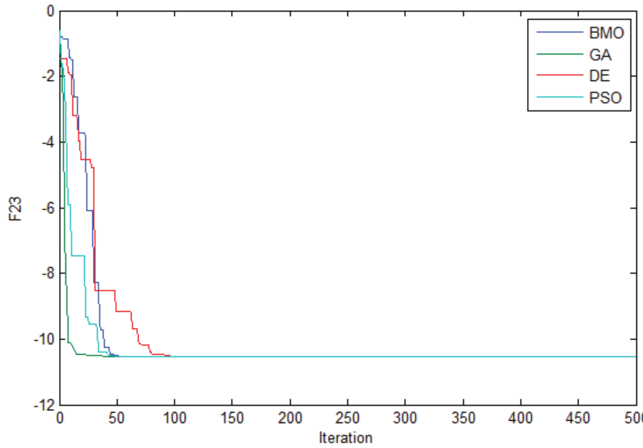


Fig. 6. Convergence curve for F23 using BMO, GA, DE and PSO for F23

V. Conclusion

A new evolutionary algorithm has been successfully proposed in this paper which is inspired by barnacle's mating behavior namely Barnacle Mating Optimizer. The performance of BMO is evaluated using the well-known 23 test benchmark functions and the results show that BMO outperformed other selected algorithms: GA DE and PSO especially for unimodal functions. The application of BMO in real practical engineering problems will be proposed in the near future.

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