Instituto Politécnico Nacional

Escuela Superior de Física y Matemáticas

Licenciatura en Matemática Algorítmica

Formulario

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1. Álgebra

1.1. Álgebra básica

- $(a \pm b)^2 = a^2 \pm 2ab + b^2 = (a \mp b)^2 \pm 4ab$
- $(a+b)(a+c) = a^2 + a(b+c) + bc$
- $(a+b)(a-b) = a^2 - b^2$
- $2(a^2+b^2) = (a+b)^2 + (a-b)^2$
- $4ab = (a+b)^2 - (a-b)^2$
- $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$
- $\log_a(1) = 0$
- $\log_a(a) = 1$
- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^n) = n \log_a(x)$
- $\log_a(b) = \frac{1}{\log_b(a)}$
- $\log_{b^n}(a^n) = \log_b(a)$
- $\log_{a^n}(a^m) = \frac{m}{n}$
- $\log_b(a) = \frac{\log_x(a)}{\log_x(b)}$

1.2. Sumas y series

- $\sum_{k=1}^{n} cf(k) = c \sum_{k=1}^{n} f(k)$
- $\sum_{k=1}^{n} (f(k) + g(k)) = \sum_{k=1}^{n} f(k) + \sum_{k=1} g(k)$
- $\sum_{k=1}^{n} f(k) = \sum_{k=1}^{m} f(k) + \sum_{k=m+1}^{n} f(k)$
- $\sum_{k=1}^{n} c = nc$
- $\sum_{k=1} k = \frac{n(n+1)}{2}$
- $\sum_{k=1} k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1} k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}; r \neq 1$
- $\sum_{k=a}^{n} (f(k) - f(k+1)) = f(a) - f(n+1)$
- $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
- $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

1.3. Números complejos

- $i^n = \begin{cases} 1, & n = 0 \mod 4 \\ i, & n = 1 \mod 4 \\ -1, & n = 2 \mod 4 \\ -i, & n = 3 \mod 4 \end{cases} ; n \geq 0$
- $\overline{z+w} = \overline{z} + \overline{w}$
- $z + \overline{z} = 2\Re(z)$
- $z - \overline{z} = 2\Im(z)\, i$
- $\overline{zw} = \overline{z} \cdot \overline{w}$
- $|z| = \sqrt{\Re(z)^2 + \Im(z)^2}$
- $z\overline{z} = |z|^2$
- $z^{-1} = \frac{\overline{z}}{|z|^2}$
- $\arg(z) = \arctan\left(\frac{\Im(z)}{\Re(z)}\right)$
- $|z| = 0 \iff z = 0$
- $|z+w| \leq |z| + |w|$
- $||z| - |w|| \leq |z - w|$
- $|zw| = |z|\,|w|$
- $z = |z|\, e^{i \arg(z)} = |z| (\cos(\arg(z)) + i \sin(\arg(z)))$
- Raíces de la unidad
  Sea $n \in \mathbb{N}$, entonces para cada $0 \leq k < n$, la k-ésima ráiz de 1 dado n es:

$$\omega_k = \exp\left(\frac{2\pi k}{n} i\right)$$

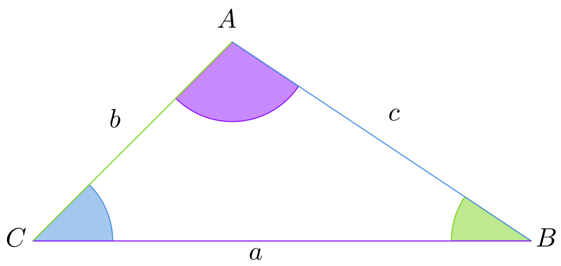
- $\sqrt[n]{z} = \bigcup_{k=0}^{n-1} \{|z|\, \omega_k\}; n \in \mathbb{N}$

2. Trigonometría

- Ángulos notables

	0	30°	45°	60°	90°	180°	270°
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan(x)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–
$\cot(x)$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0
$\sec(x)$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	–	-1	–
$\csc(x)$	–	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	–	1

- Leyes de senos, cosenos, tangentes y proyecciones



  - Ley de senos

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

  - Ley de cosenos

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

  - Ley de tangentes

$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$$
$$\frac{a+c}{a-c} = \frac{\tan\left(\frac{A+C}{2}\right)}{\tan\left(\frac{A-C}{2}\right)}$$
$$\frac{b+c}{b-c} = \frac{\tan\left(\frac{B+C}{2}\right)}{\tan\left(\frac{B-C}{2}\right)}$$

  - Ley de proyecciones

$$a\cos(B) + b\cos(A) = c$$
$$a\cos(C) + c\cos(A) = b$$
$$b\cos(C) + c\cos(B) = a$$

- $\sin(-x) = -\sin(x)$
- $\cos(-x) = \cos(x)$
- $\sin(x)\csc(x) = 1$
- $\cos(x)\sec(x) = 1$
- $\tan(x)\cot(x) = 1$
- $\tan(x) = \dfrac{\sin(x)}{\cos(x)} = \dfrac{1}{\cot(x)}$
- $\cot(x) = \dfrac{\csc(x)}{\sec(x)} = \dfrac{1}{\tan(x)}$
- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $\cot^2(x) + 1 = \csc^2(x)$
- $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
- $\tan(x \pm y) = \dfrac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$
- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(2x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$
- $\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
- $\sin\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1-\cos(x)}{2}}$
- $\cos\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1+\cos(x)}{2}}$
- $\tan\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1-\cos(x)}{1+\cos(x)}}$
- $\tan\left(\dfrac{x}{2}\right) = \dfrac{1-\cos(x)}{\sin(x)} = \dfrac{\sin(x)}{1+\cos(x)}$
- $\sin(x) \pm \sin(y) = 2\sin\left(\dfrac{x \pm y}{2}\right)\cos\left(\dfrac{x \mp y}{2}\right)$
- $\cos(x+y) = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$
- $\cos(x-y) = -2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)$
- $\sin(x)\sin(y) = \dfrac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$
- $\cos(x)\cos(y) = \dfrac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$
- $\sin(x)\cos(y) = \dfrac{1}{2}\left(\sin(x+y) + \sin(x-y)\right)$

3. Límites

- $\lim_{x\to a} k = k$
- $\lim_{x\to a} kf(x) = k \lim_{x\to a} f(x)$
- $\lim_{x\to a} (f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
- $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$
- $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
- Sean $F, f : \mathbb{R} \to \mathbb{R}$ con F continua, entonces:

$$\lim_{x\to a} F(f(x)) = F\left(\lim_{x\to a} f(x)\right)$$

- $\lim_{x\to 0} \dfrac{\sin(kx)}{kx} = \lim_{x\to 0} \dfrac{kx}{\sin(kx)} = 1$
- $\lim_{x\to 0} \dfrac{1-\cos(x)}{x} = 0$
- $\lim_{x\to 0} \dfrac{1-\cos(x)}{x^2} = \dfrac{1}{2}$
- $\lim_{x\to 0} \dfrac{\tan(kx)}{kx} = \lim_{x\to 0} \dfrac{kx}{\tan(kx)} = 1$
- $\lim_{n\to\infty} \left(1+\dfrac{x}{n}\right)^n = e^x$

4. Derivadas

- $\dfrac{d}{dx}(f(x)+g(x)) = \dfrac{d}{dx}f(x) + \dfrac{d}{dx}g(x)$
- $\dfrac{d}{dx}kf(x) = k\dfrac{d}{dx}f(x)$
- $\dfrac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
- $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\dfrac{d}{dx}f(u(x)) = f'(u(x))u'(x)$
- $\dfrac{d}{dx}x = 1$
- $\dfrac{d}{dx}x^n = nx^{n-1}$
- $\dfrac{d}{dx}\sqrt[n]{x} = \dfrac{1}{n\sqrt[n]{x^{n-1}}}$
- $\dfrac{d}{dx}\sqrt[n]{x^m} = \dfrac{m}{n}\sqrt[n]{x^{m-n}}$
- $\dfrac{d}{dx}\ln(x) = \dfrac{1}{x}$
- $\dfrac{d}{dx}\log_a(x) = \dfrac{1}{x\ln(a)}$
- $\dfrac{d}{dx}e^x = e^x$
- $\dfrac{d}{dx}a^x = a^x\ln(a)$
- $\dfrac{d}{dx}\sin(x) = \cos(x)$
- $\dfrac{d}{dx}\cos(x) = -\sin(x)$
- $\dfrac{d}{dx}\tan(x) = \sec^2(x)$
- $\dfrac{d}{dx}\cot(x) = -\csc^2(x)$
- $\dfrac{d}{dx}\sec(x) = \sec(x)\tan(x)$
- $\dfrac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
- $\dfrac{d}{dx}\arcsin(x) = \dfrac{1}{\sqrt{1-x^2}}$
- $\dfrac{d}{dx}\arccos(x) = -\dfrac{1}{1-x^2}$
- $\dfrac{d}{dx}\arctan(x) = \dfrac{1}{1+x^2}$
- $\dfrac{d}{dx}\operatorname{arccot}(x) = -\dfrac{1}{1+x^2}$
- $\dfrac{d}{dx}\operatorname{arcsec}(x) = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\dfrac{d}{dx}\operatorname{arccsc}(x) = -\dfrac{1}{|x|\sqrt{x^2-1}}$

5. Integrales indefinidas

- $\displaystyle\int (f(x)+g(x))\,dx = \int f(x)\ dx + \int g(x)\ dx$
- $\displaystyle\int kf(x)\ dx = k\int f(x)\ dx$
- $\displaystyle\int f(u(x))u'(x)\ dx = \int f(u)\ du$
- $\displaystyle\int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)\ dx$
- $\displaystyle\int k\ dx = kx + C$
- $\displaystyle\int x^n\ dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$
- $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$

- $\int e^x \ dx = e^x + C$
- $\int a^x \ dx = \frac{a^x}{\ln(a)} + C$
- $\int \sin(x) \ dx = -\cos(x) + C$
- $\int \cos(x) \ dx = \sin(x) + C$
- $\int \tan(x) \ dx = \ln|\sec(x)| + C$
- $\int \cot(x) \ dx = \ln|\sin(x)| + C$
- $\int \sec(x) \ dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x) \ dx = \ln|\csc(x) - \cot(x)| + C$
- $\int \sec^2(x) \ dx = \tan(x) + C$
- $\int \csc^2(x) \ dx = \cot(x) + C$
- $\int \sec(x)\tan(x) \ dx = \sec(x) + C$
- $\int \csc(x)\cot(x) \ dx = -\csc(x) + C$
- $\int \frac{1}{\sqrt{a^2 - x^2}} \ dx = \arcsin\left(\frac{x}{a}\right) + C$
- $\int \frac{1}{x^2 + a^2} \ dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$
- $\int \frac{1}{x\sqrt{x^2 - a^2}} \ dx = \frac{1}{a}\operatorname{arcsec}\left(\frac{x}{a}\right) + C$
- $\int \frac{1}{x^2 - a^2} \ dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$
- $\int \sin^2(x) \ dx = \frac{1}{2}(x - \sin(x)\cos(x)) + C$
- $\int \cos^2(x) \ dx = \frac{1}{2}(x + \sin(x)\cos(x)) + C$
- $\int \tan^2(x) \ dx = -x + \tan(x) + C$
- $\int \cot^2(x) \ dx = -x - \cot(x) + C$
- Sea $F(n) = \int \sin^n(x) \ dx$, entonces:

$$F(n) = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}F(n-2)$$

- Sea $F(n) = \int \cos^n(x) \ dx$, entonces:

$$F(n) = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}F(n-2)$$

- Sea $F(n) = \int \tan^n(x) \ dx$, entonces:

$$F(n) = \frac{1}{n-1}\tan^{n-1}(x) - F(n-2)$$

- Sea $F(n) = \int \cot^n(x) \ dx$, entonces:

$$F(n) = -\frac{1}{n-1}\cot^{n-1}(x) - F(n-2)$$

- Sea $F(n) = \int \sec^n(x) \ dx$, entonces:

$$F(n) = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}F(n-2)$$

- Sea $F(n) = \int \csc^n(x) \ dx$, entonces:

$$F(n) = -\frac{1}{n-1}\csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}F(n-2)$$

- $\int x^n e^x \ dx = x^n e^x - n\int x^{n-1}e^x \ dx$
- $\int \frac{1}{1+e^x} \ dx = x - \ln(1+e^x) + C$
- $\int e^{ax}\sin(bx) \ dx = \frac{e^{ax}(a\sin(bx) - b\cos(bx))}{a^2+b^2}$
- $\int e^{ax}\cos(bx) \ dx = \frac{e^{ax}(a\cos(bx) + b\sin(bx))}{a^2+b^2}$
- $\int \ln(x) \ dx = x(\ln(x) - 1) + C$

- $\int x \ln(x)\ dx = \frac{x^2}{4}\left(2\ln(x) - 1\right) + C$

- $\int x^n \ln(x)\ dx = \frac{x^{n+1}}{(n+1)^2}\left((n+1)\ln(x) - 1\right) + C$

- $\int (\ln(x))^2\ dx = x\Big((\ln(x))^2 - 2\ln(x) + 2\Big) + C$

- $\int (\ln(x))^n\ dx = x(\ln(x))^n - n\int (\ln(x))^{n-1}\ dx$

- Sustitución trigonométrica

Expresión	Sustitución	Raíz
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$	$a\cos(\theta)$
$\sqrt{x^2 + a^2}$	$x = a\tan(\theta)$	$a\sec(\theta)$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$	$a\tan(\theta)$

6. Integrales definidas

- Teorema fundamental del cálculo (2)
  Sea $f : [a,b] \to \mathbb{R}$ integrable y $F : \mathbb{R} \to \mathbb{R}$ una de sus antiderivadas, entonces:

$$\int_a^b f(x)\ dx = [F(x)]_a^b = F(b) - F(a)$$

- $\int_a^b (f(x) + g(x))\, dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$

- $\int_a^b kf(x)\ dx = k\int_a^b f(x)\ dx$

- $\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$

- $\int_a^a f(x)\ dx = 0$

- $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$

- Si $f : [-a,a] \to \mathbb{R}$ es par:

$$\int_{-a}^a f(x)\ dx = 2\int_0^a f(x)\ dx$$

- Si $f : [-a,a] \to \mathbb{R}$ es impar:

$$\int_{-a}^a f(x)\ dx = 0$$

- $\int_a^b f(u(x))u'(x)\ dx = \int_{u(a)}^{u(b)} f(x)\ dx$

- $\int_a^b f(x)g'(x)\ dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)\ dx$