A Mixture of Gaussians Model for Fading Channels

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Outline

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Physical Channel Modeling Phenomena

- Path Loss
  - $Power \propto \frac{1}{distance^n}$
- Large Scale Fading
  - Lognormal, Gamma, Inverse-Gaussian
- Small Scale Fading
  - Rayleigh, Nakagami-m, Rician
  - α-μ, η-μ, κ-μ

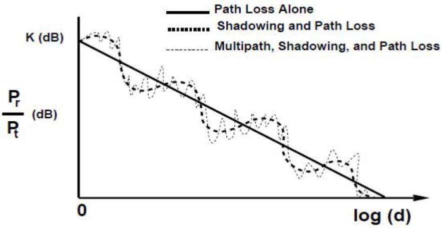

Fig.1.1 Path loss, shadowing and multipath versus distance

Essential Background

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  - $f_\alpha(\alpha|\sigma) = \frac{2m^m}{\sigma^m \Gamma(m)} \alpha^{2m-1} e^{-m\frac{\alpha^2}{\sigma}}$

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  - $f_\alpha(\alpha|\sigma) = \frac{2m^m}{\sigma^m\Gamma(m)} \alpha^{2m-1} e^{-m\frac{\alpha^2}{\sigma}}$
- This leads to an intractable integral
  - Symbol Error Rate (SER), Moment Generation Function (MGF), Amount of Fading (AF), ... are cumbersome.

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  - Contains the the modified Bessel function of second type.
- RIGD and G Distributions by Karmeshu *et al.* and Laourine *et al.*
  - Replace Lognormal by Inverse-Gaussian, and solve.
  - Again, contains the modified Bessel function of second type.

Some Previous Works, Cont.

- Mixture Gamma (MG) Distribution by Attapattue *et al.*
  - Used Gauss-Quadrature Methods (Moment Matching).
  - $f_\gamma(\gamma) = \sum_{i=1}^{C} \alpha_i (\frac{\gamma}{\gamma_0})^{\beta_i - 1} \exp\{-\zeta_i \frac{\gamma}{\gamma_0}\}$

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  - $f_\gamma(\gamma) = \sum_{i=1}^{C} \alpha_i (\frac{\gamma}{\gamma_0})^{\beta_i - 1} \exp\{-\zeta_i \frac{\gamma}{\gamma_0}\}$
  - Approximates:
    - Nakagami-m, η-μ, κ-μ, ...
    - Yet, only applicable to certain integral formulations.

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The MoG Model

We propose the use of Mixture of Gaussians distribution...

The MoG Model

$$f_\alpha(x) = \sum_{j=1}^{C} \frac{\omega_j}{\sqrt{2\pi}\eta_j} \exp(-\frac{(x-\mu_j)^2}{2\eta_j^2})$$

$$f_\gamma(\gamma) = \sum_{j=1}^{C} \frac{\omega_j}{\sqrt{8\pi\overline{\gamma}}\eta_j} \frac{1}{\sqrt{\gamma}} \exp(\frac{-(\sqrt{\frac{\gamma}{\overline{\gamma}}}-\mu_j)^2}{2\eta_j^2})$$

- We need to estimate:
  - ω_j and $\theta_{\mathbf{j}} = \{\mu_j, \zeta_j\}, j = 1, ..., C$
  - C is the number of components.

Expectation-Maxmization

- Utilize the **EM** algorithm to approximate fading channels.
  - Coined by Dempster *et al.* in 1977.
  - Iterative Solution to the Maximum Likelihood Estimation.
- Pros:
  - Automated (Unsupervised Algorithm)
  - Proven not to get worse, as it iterates by.
- Cons:
  - Might get stuck in a local maxima; works well in practice.

Expectation-Maxmization | Approximation Methodology

- Proplem Formulation:
  - Let the i^{th} entry of a random data vector $Y = (y_1, ..., y_2)$, which represents the **envelopes** of the composite models, be regarded as **incomplete data** and modeled as a finite mixture of Gaussians.

$$p(y_i|\theta) = \sum_{j=1}^{C} \omega_j \phi(y_i, \theta_j), \; i = 1, ..., n.$$

where $\phi(y_i, \theta_j) = \mathcal{N}(\mu_j, \zeta_j)$.

Expectation-Maxmization | Approximation Methodology

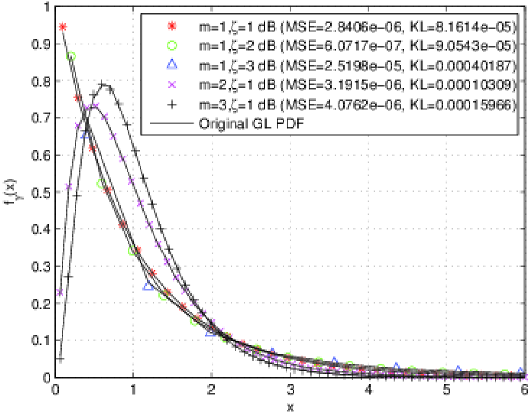
- Solution consists of two iterative steps:
  - $\mathbb{E}-\text{step} \rightarrow \; \rho_{ij}^{(m)} = \frac{\omega_j^{(m)}\phi\left(y_i|\mu_j^{(m)},\eta_j^{(m)}\right)}{\sum_{l=1}^{C}\omega_l^{(m)}\phi\left(y_i|\mu_l^{(m)},\eta_l^{(m)}\right)}.$
  - $\mathbb{M}-\text{step} \rightarrow$
    - $\omega_j^{(m+1)} = \frac{1}{n}\sum_{i=1}^{n}\rho_{ij}^{(m)}, \; j = 1, ..., C,$
    - $\mu_j^{(m+1)} = \frac{1}{n_j^{(m)}}\sum_{i=1}^{n}\rho_{ij}^{(m)} y_i, \; j = 1, ..., C,$
    - $\eta_j^{(m+1)} = \frac{1}{n_j^{(m)}}\sum_{i=1}^{n}\rho_{ij}^{(m)}\left(y_i - \mu_j^{(m+1)}\right)^2, \; j = 1, .., C.$
- Keep iterating between $\mathbb{E}$ and $\mathbb{M}$ until convergence, when $\left|L^{(m+1)} - L^{(m)}\right| < \delta$
  - $L^{(m)} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\sum_{j=1}^{C}\omega_j^{(m)}\phi\left(y_i|\mu_j^{(m)},\eta_j^{(m)}\right)\right), \; i = 1, ..., n,$

Outline

1. Background and Literature Review
   - Main Problem
   - Some Previous Works
2. Our Contribution
   - Proposed Approach
   - Approximation Results
   - Performance Analysis Metrics
3. Summary and Conclusions

Nakagami

As an example, we approximate Nakagami-Lognormal Scenarios. . .

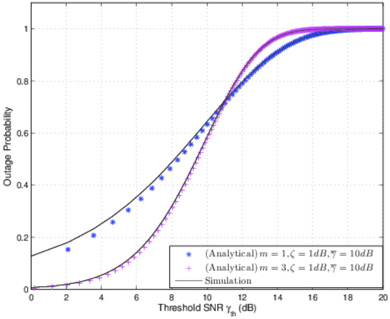


Outline

Outage Probability

- Standard performance criterion for fading channels.

$$F\left(\gamma_{th}\right) = \sum_{i=1}^{C} \omega_i \, Q\left(-\frac{\sqrt{\frac{\gamma_{th}}{\overline{\gamma}}} - \mu_i}{\eta_i}\right).$$



Moment Generation Function

- Critical for raw moments and error analysis.

Moment Generation Function

$$M_\gamma(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{\beta_i}} \exp(\frac{\mu_i^2 s}{\beta_i}) Q(\frac{-\mu_i}{\eta_i \sqrt{\beta_i}}).$$

Raw Moments

- $\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \omega_i \overline{\gamma}^n \frac{d^{(2n)} M_X(s)}{dx^{(2n)}} |_{s=0}$
    - $X_i \sim \mathcal{N}(\mu_i, \zeta_i)$

Raw moments

$$\mathbb{E}[\gamma^n] = \sum_{i=1}^{C} \omega_i \overline{\gamma}^n \eta_i^{2n} 2^n \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}} {}_1F_1[-n, \frac{1}{2}, -\frac{-\mu_i^2}{2\eta_i^2}]$$

- e.g. to evaluate Amount of Fading

$$\mathrm{AF} = \frac{\mathbb{E}[\gamma^2] - (\mathbb{E}[\gamma])^2}{(\mathbb{E}[\gamma])^2}.$$

Average Ergodic Capacity

- When only the receiver has knowledge about the channel state information, then

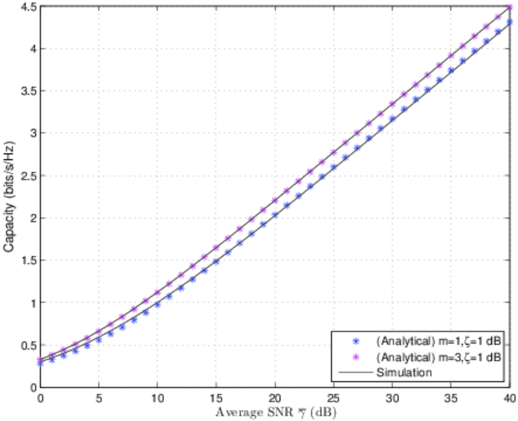
$$C_{erg} = \frac{B}{\ln 2} \int_0^{\infty} \ln(1+\gamma)\, f_{\gamma}(\gamma)\, \mathrm{d}\gamma,$$

$$\ln(1+\gamma) = \ln(1+\mathbb{E}[\gamma]) + \sum_{w=1}^{\infty} \frac{(-1)^{w-1}}{w} \frac{(x-\mathbb{E}[\gamma])^w}{(1+\mathbb{E}[\gamma])^w}.$$

Average Ergodic Capacity

$$C_{erg} \approx \frac{B}{\ln 2}[\ln(1+\mathbb{E}[\gamma]) - \frac{\mathbb{E}[\gamma^2]-\mathbb{E}^2[\gamma]}{2(1+\mathbb{E}[\gamma])^2}],$$

Average Ergodic Capacity, Cont.



Symbol Error Analysis

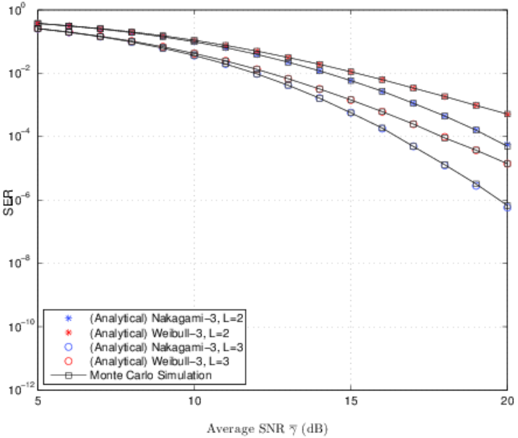
- We consider L-branch MRC diversity receiver

$$\gamma_{MRC} = \sum_{k=1}^{L} \gamma_k .$$

$$p_s(E) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \prod_{k=1}^{L} M_{\gamma_k}\left(\frac{-\sin^2(\frac{\pi}{M})}{\sin^2(\theta)}\right) d\theta,$$

$$\begin{aligned} P_s(E) &= \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\left[\int_0^{\frac{\pi}{2}} \prod_{k=1}^{L} M_{\gamma_k}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right) d\theta \right. \\ &\quad \left. - \int_0^{\frac{\pi}{4}} \prod_{k=1}^{L} M_{\gamma_k}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right) d\theta\right], \qquad (1) \end{aligned}$$

Symbol Error Analysis, Cont.



Summary

- Represented non-composite and composite fading channels by MoG distribution, using EM algorithm.
- Resulted in tractable analytical tools for performance analysis over generalized fading channels.
- Watch out for the Journal edition (arXiv:1503.00877):
  - Details of Approximation Methodology.
  - Determination of number of components.
  - Additional application to cognitive radio networks.
- Framework is easily extendable to other scenarios: *diversity systems, cooperative communications and others* ...