Robust Online Composition, Routing and NF Placement for NFV-enabled Services

Omar Alhussein, *Student Member, IEEE,* and Weihua Zhuang, *Fellow, IEEE*

***Abstract*—Network function virtualization (NFV) fosters innovation in the networking field and reduces the complexity involved in managing modern-day conventional networks. Via NFV, the provisioning of a network service becomes more agile, whereby virtual network functions can be instantiated on commodity servers and data centers on demand. Network functions can be either mandatory or best-effort. The former type is strictly necessary for the correctness of a network service, whereas the latter is preferrable yet not necessary. In this paper, we study the online provisioning of NFV-enabled network services. We consider both unicast and multicast NFV-enabled services with multiple mandatory and best-effort NF instances. We propose a primal-dual based online approximation algorithm that allocates both processing and transmission resources to maximize a profit function, subject to resource constraints on physical links and NFV nodes. The online algorithm resembles a joint admission mechanism and an online composition, routing and NF placement framework. The online algorithm is derived from an offline formulation through a primal-dual based analysis. Such analysis offers direct insights and a fundamental understanding on the nature of the profit-maximization problem for NFV-enabled services with multiple resource types.**

***Index Terms*—NFV, online algorithms, primal-dual scheme, profit maximization, competitive analysis.**

I. INTRODUCTION

Network function virtualization (NFV) has established itself as a prominent concept for the provisioning of modern communication networks. Traditionally, network elements, such as routers and middlewares, were implemented in proprietary hardware boxes. With NFV, such network elements are virtualized as NF instances, and can be deployed in the data plane in commodity servers and cloud data centers. This gives rise to service-customized networks – a provisioning mechanism that can meet new demands and agile use cases. A service-customized network resembles a network application, whereby a data flow passes by virtual NF instances for processing before arriving at the destination(s). In such paradigm, a service provider submits a request to reserve network resources (e.g., transmission and processing) to orchestrate its own virtual network. The reserved resources should be guaranteed according to some agreed-upon quality of service. The infrastructure provider aims at maximizing some "profit" function, while minimizing the provisioning costs of the network services.

A considerable number of works are carried out for the orchestration and provisioning of NFV-enabled service requests [1]–[8]. Earlier research focuses on the orchestration of a single service request, where the focus is on minimizing the provisioning cost of a single service without taking other services into consideration [5], [9]–[11]. However, the admission and embedding of one service request affects the service provisioning of other requests, thereby the need for orchestrating multiple service requests jointly. To this end, the relevant literature can be classified based on their handling of service requests into offline and online. In the former, all service requests are known a priori, and all service requests are assumed to arrive in one batch. In practice, network services arrive in an online and random manner without knowledge of future requests [12], [13].

Due to the increased flexibility and agility brought-forth by NFV, future (over-the-top) service requests are envisaged to be hardly predictable [14]. Future service quality and data traffic patterns for new use cases are arguably not well understood, and an advanced knowledge of future patterns can be difficult to obtain or predict. Moreover, such traffic patterns can vary dramatically over short periods due to the inherent agility of NFV-based networks.

Some relevant studies deal with the online handling of service requests without statistical assumptions [12], [13], [15]–[18]. The NFV-enabled frameworks are based on the seminal work by Awerbuch et al. [19], where some new aspects are due to the inclusion of NF instances and the need for an admission mechanism for service requests with multiple resource types. To our knowledge, in the existing NFV-enabled works, service requests have either one resource type or one NF instance. Also, online (routing and NF placement) algorithms can be classified as either *all-or-nothing* or *all-or-something*. In the all-or-nothing scenario, service requests need to be fully served in the network substrate. In the all-or-something scenario, services can be partially (fractionally) served, e.g., admitting a service request while reducing the required data rate [14], [20]. Current works in the relevant NFV literature can be considered as all-or-nothing schemes.

This paper deals with two resource types simultaneously, namely the processing and transmission resources. The two resource types are often conflicting in their utilization. Therefore, there is a need to design a generalized admission mechanism and an online joint composition, routing, and NF placement (JCRP) algorithm that take the multiple resource types into account. We consider unicast and multicast service requests that can have multiple NF instances. Furthermore, we consider two NF types, namely best-effort and mandatory. A successful placement of a network service is contingent only on successfully placing the set of mandatory NFs. The functionality of a best-effort NF is not necessary for the correctness of a network service [21]. Therefore, the set of best-effort NFs can be removed from a service request when it is deemed "too prohibitive". In practice, best-effort NFs can improve either the performance, the quality of service, or the security of a network service, such as in the case of compression and intrusion detection. Consider for instance a video/image compression NF type for a voice over IP

Omar Alhussein and Weihua Zhuang are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada, N2L 3G1 (emails: {oalhusse, wzhuang}@uwaterloo.ca)

network service. Such NF type enhances the quality of service by compressing the incoming data flow. However, when the available processing resources (or the available subscription) for the NF type in the network substrate is scarce, a network service can take a rather unnecessarily long (i.e., expensive) route, which would be too costly and can conversely degrade the overall quality of service. Therefore, such NF type can be declared as best-effort, whereby including it should be contingent on whether a certain profit is achieved.

The objective of this work is to develop a robust admission mechanism and an online joint composition, routing and NF placement (JCRP) framework (online algorithm, in short) that aims to maximize a profit function, which is proportional to the so-called amortized throughput, while considering unicast and multicast NFV-enabled services with best-effort and mandatory NF instances, subject to resource constraints on physical links and NFV nodes. The amortized throughput is defined as the weighted total transmission and processing resources reserved for all the accepted service requests.

The online algorithm relies on two main components, i.e., (i) an admission mechanism that rejects or accepts a service request based on a profit function while taking best-effort NFs into consideration, and (ii) an online JCRP algorithm that provides unicast-enabled and multicast-enabled routing and NF placement configurations for the admitted service requests. The online algorithm is developed through a primal-dual analysis, which provides an approximately optimal result with provable competitive performance. A formal defintion of the competitive ratio (performance) is stated as follows. For a profit-maximization problem, let $A_{\text{OPT}}(\sigma)$ be the profit of the (optimal) offline solution for a sequence of requests (σ). An online algorithm is c-competitive if the produced solution is feasible and its profit is at least $A_{\text{OPT}}(\sigma)/c - e$, where e is an additive term that is independent of the service requests [20]. The primal-dual approach exists for solving offline optimization problems. Buchbinder and Naor extended the framework for the treatment of online algorithms [20]. This work offers the following new contributions:

- We propose a primal-dual based online algorithm to allocate both processing and transmission resources for network services with multiple NF instances. In addition, we consider heterogeneous NFV-enabled services with mandatory and best-effort NFs. We provide a natural generalization to relevant works that focus on the provisioning of services without a processing requirement or with only one NF instance. The online algorithm can be regarded as an all-or-nothing/all-or-something algorithm in the sense that the requested data rate and the required processing resource for each NF instance should be fully satisfied, yet best-effort NF instances need not necessarily be included in the accepted service request, thereby providing the flexibility to recompose the logical topology of a service request before admission;
- The primal-dual analysis offers an alternative analysis and generalized treatment to approaches adopted in recent relevant works [12], [13], [16]–[18]. For instance, the competitive performance in the aforementioned works is shown to associate not only with an optimal integer solution but also with an optimal fractional solution;
- We propose a “one-step” algorithm for the routing and NF placement of unicast and multicast services for an unconstrained scenario. The algorithm relies mainly on the construction of an auxiliary network transformation that has a one-to-one mapping from the NF placement and routing problem to an equivalent routing problem.

The rest of the paper is organized as follows. Section II gives an overview of related works. Section III describes the system model under consideration, followed by the problem description. Section IV presents the problem formulation, which includes the design of a profit function, and the primal-dual based problem formulation for the offline routing and NF placement framework. In Section V, we develop the primal-dual based admission mechanism, followed by an analysis of the competitive performance of the proposed admission mechanism. Section VI presents the routing and NF placement algorithm for the proposed admission mechanism. Finally, Section VII presents some discussions on the proposed framework, followed by simulation results to investigate and corroborate the competitive performance of the proposed work.

II. Related Works

A. *Routing and NF Placement*

A growing body of literature has been evolving for the composition, routing and NF placement for both unicast and multicast NFV-enabled services. Most of the earlier works focus on the orchestration of unicast services, cf. [7], [8]. In its most basic form, the orchestration of an NFV-enabled service poses two correlated and conflicting subproblems, i.e., how to place (or select) the NF instances, and how to route the traffic to traverse the NF instances. Placing a minimal feasible number of NF instances can lead to a large link provisioning cost; conversely, deploying more NF instances can reduce the link provisioning cost at the expense of an increased function provisioning cost. This tradeoff becomes more conspicuous when considering a multicast service in which traffic is routed to more than one destination. To that end, several works have been proposed for the NF placement and routing of a multicast service [9], [22]–[29]. The aforementioned works consider designing heuristic (or approximation) algorithms to orchestrate one service request without taking future service requests into consideration. In this paper, we consider an online setting, for which more relevant literature is addressed in Subsection II-C.

B. *Competitive Online Routing (Predating NFV)*

Prior to enabling NFV, in traditional circuit switching, call requests (respectively, service requests) resembled a routing request from the source to the destinations with a data rate requirement that need to be routed in a capacitated network substrate [19]. With the emergence of NFV, service requests subsumed call requests with the additional requirement that virtual NFs need to be instantiated on commodity servers or data centers (NFV nodes) along the route.

Related works can be classified according to the parameter that measures the performance of the intended design, typically the throughput or the congestion. In throughput-maximization frameworks, we measure the transmission resources of all admitted service requests. In congestion-minimization, we measure the maximum link congestion, i.e., the maximum ratio of the allocated transmission resources on a link to its total transmission capacity. This work can be considered as a generalized case of the throughput-maximization framework.

In 1993, Aspnes et al. developed a competitive strategy for congestion-minimization that achieves a competitive ratio of $O(\log n)$ for service requests of infinite holding time, where n is the total number of nodes in the substrate network [30]. Assuming service requests have finite holding time (which is revealed only upon the arrival for each service request), the authors extended their result to achieve an $O(\log nT)$ competitive ratio, where T is the maximum holding time of all service requests. For the throughput-maximization model, Awerbuch et al. achieved a competitive ratio of $O(\log n)$ [19].

C. Competitive Online Routing and NF Placement

One main challenge in devising a competitive online routing and NF placement algorithm is pertaining to the inclusion of processing resources along with transmission resources. To this end, Lukovszki et al. consider that all unicast service requests contain the same set of requested NFs and identical transmission rates, where service requests vary with regard to the source and destination [13]. They propose an $O(\log K)$-competitive admission mechanism, where K is the number of NFs of a service request. Interestingly, the competitive ratio is logarithmic with the number of NFs, which is small in practice. In [16], [31], the authors consider both unicast and multicast requests without NFs, where routing a request utilizes transmission resources from the physical links and routing rules from the forwarding table of the traversed switches. Notably, the achieved competitive ratio can be shown to be $O(\max\{\log 2L, \log 2E\})$, where L and E are the maximum number of physical links and switches for a service request, respectively. The competitive ratio for the two resources is balanced since $L = E - 1$.

Xu et al. consider a multicast request with one NF, and develop an $O(\log L)$-competitive algorithm [17]. Ma et al. consider the dynamic admission of delay-aware requests for services in a distributed cloud with the objective of maximizing a designed profit function [12]. They first provide a heuristic algorithm for the delay-aware scenario, followed by an online algorithm with an $O(\log L)$-competitive ratio for the special case where the end-to-end delay is negligible.

In this paper, we propose a primal-dual based online algorithm that accommodates both unicast and multicast service requests with multiple NF instances that can be deployed at different NFV nodes. We consider heterogeneous services with best-effort and mandatory NFs. In doing so, we propose an all-or-nothing/all-or-something admission mechanism that recomposes the logical topology of the service request before admitting it (depending on whether or not best-effort NF instances can be included). Moreover, based on a primal-dual framework, we offer new alternative, generalized description, and analysis to the aforementioned NFV-enabled literature.

III. System Model and Problem Description

In this section, we present the system model under consideration, followed by the problem description.

A. Network Functions

With NFV, traditional applications and functionalities (which used to be implemented in the control plane or at the end-users) are now deployable in the data plane in NFV nodes. Examples of NFs include firewall, intrusion detection, Web cache, proxy, and service gateway. From the perspective of quality of service, we consider two types of NFs, namely mandatory and best-effort. As discussed, a successful placement of a network service is contingent on successfully placing only the set of mandatory NFs, whereas best-effort NFs are not necessary for the correctness of a network service.

B. Online Service Requests

We consider an ongoing input sequence of unicast and multicast service requests $\sigma = (S^1, S^2, \dots)$ that arrive in an online fashion. The rth service request is expressed as

$$S^r = (s^r, \mathcal{D}^r, \mathcal{V}^r, d^r), \qquad S^r \in \sigma \tag{1}$$

where the source and destination nodes are s^r and $\mathcal{D}^r$, respectively; parameter d^r denotes the required transmission rate in packet per second (packet/s); $\mathcal{V}^r = \{f_1^r, f_2^r, \dots, f_{|\mathcal{V}^r|}^r\}$ represents the set of NFs that need to be traversed in an ascending order for the source-destination pair. For simplicity, for each service request, each NF requires an equal amount of processing resources of $C(f^r)$ in packet/s. The online algorithm to be developed thereafter can be generalized for NFs with arbitrary processing requirements. The sets of mandatory and best-effort NFs are denoted by $\mathcal{V}_m^r$ and $\mathcal{V}_b^r$, respectively.

C. Network Substrate

We are given a capacitated network substrate $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ and $\mathcal{L}$ are the sets of nodes and links, respectively. Each physical link l $(\in \mathcal{L})$ has a residual transmission resource, $B(l)$, in packet/s. Each node n $(\in \mathcal{N})$ has a residual processing resource, $C(n)$, in packet/s. Nodes can be either (i) switches that are capable of forwarding traffic only (with $C(n) = 0$), or NFV nodes (e.g., commodity servers) that are capable of both forwarding traffic and operating a set of NF instances. An NFV node is capable of provisioning a number of NF instances simultaneously as long as the available processing resources satisfy the deployed NF processing requirements. Denote the set of NFV nodes that can host an NF f_i by $\mathcal{F}_i$, where $\mathcal{F}_i \subseteq \mathcal{N}$.

D. Problem Description

We are given a sequence of service requests σ that is revealed over time, i.e., the service requests arrive one by one without knowledge of future arrivals. We need to define a profit function, whose main goal is to maximize the amortized processing and transmission throughput. Recall the amortized throughput is the weighted total transmission and processing resources reserved for all the accepted service requests. However, the profit function (and the online algorithm) should capture both the mode of communication (i.e., unicast and multicast) and the heterogeneity of the NF types (i.e., best-effort and mandatory). That is, maximizing the amortized throughput alone would unfairly favor unicast to multicast services due to the larger number of destinations in the latter. Yet, the multicast mode of communication is more efficient as it is shown to reduce the bandwidth consumption in backbone networks by over 50% in contrast to the unicast mode [32]. Moreover, although best-effort NFs are optional, their use should be incentivized.

In the following, we first define a profit function that accurately captures the system model and operational design requirements. Then, we develop a path-based formulation for an offline profit-maximization problem. The offline formulation is omniscient, where it has complete a priori knowledge of the entire sequence of service requests. Moreover, it yields the optimal combination of service requests and their routing and NF placement configurations, such that the profit function is maximized. Given the offline formulation, through a primal-dual analysis, we develop an all-or-nothing/all-or-something online algorithm to deal with each service request in a dynamic manner, while providing competitive guarantees against the optimal offline adversary.

IV. Problem Formulation

A. The Objective (Profit) Function

We consider two profit functions, ϱ^r and ρ^r, that correspond to the transmission and processing resource types, respectively. We know that the number of physical links required for a service in multicast mode is always less than or equal to the overall number of links needed for the equivalent services in unicast mode. Therefore, for the rth service, an upper bound on the ratio of the number of links in a multicast topology to the number of links in an equivalent unicast service is given by $|\mathcal{D}^r|$. Notably, it has been experimentally shown that the respective ratio is $|\mathcal{D}^r|^k$, where $k = 0.8$ for many real and generated network topologies [33]. Therefore, for the transmission resources, to provide a non-discriminatory treatment between multicast and unicast services, we define ϱ^r to be proportional to both (i) the required data rate of the service request and (ii) the kth power of the number of included destinations,

$$\varrho^r = d^r |\mathcal{D}^r|^k, \qquad S^r \in \sigma \tag{2}$$

where $k = 0.8$.

For the processing resources, let the amount of the incentive (and disincentive) for including (and excluding) the set of best-effort NFs for the rth service request be given by η^r_b (and η^r_m), respectively, where $\eta^r_b \geq \eta^r_m \geq 1$. We let ρ^r be proportional to (i) the processing throughput accrued from placing the NFs, and (ii) the incentive from including (or excluding) the set of best-effort NFs,

$$\rho^r = \eta^r C(f^r), \qquad S^r \in \sigma \tag{3}$$

where $\eta^r \in \{\eta^r_m, \eta^r_b\}$ is a decision variable, with $\eta^r = \eta^r_b$ indicating that the set of best-effort NFs from the rth service is included, and $\eta^r = \eta^r_m$ indicating otherwise. One method is to set η^r to the number of included NFs in the rth service (e.g., $\eta^r_b = |\mathcal{V}^r|$ when all best-effort NFs are accepted, and $\eta^r_m = |\mathcal{V}^r_m|$ otherwise). In contrast to existing relevant literature, ρ^r varies with the logical topology of the composed service request. Solutions that exclude the set of best-effort NFs can have lower provisioning costs since they require less number of NF instances, and therefore can be accepted if otherwise not feasible by the admission mechanism. Since we have two resource types, the overall profit from accepting the rth service request is given by $\alpha\varrho^r + \beta\rho^r$, where α and β are two coefficients to indicate the relative importance (or scalarization) of each profit function, with $\alpha, \beta \geq 1$.

B. Primal-Dual Schema

First, we develop a path-based formulation for the offline multi-resource profit-maximization problem. There are two possible routing and NF placement models for the offline formulation, namely unsplittable (or fixed) and splittable. In the unsplittable model, a service request is restricted to an integral solution, where only one path is used for a unicast service (or only one tree is used for a multicast service).

Formally, let all the possible paths/trees for unicast/multicast service request S^r be given by set $\mathcal{P}(r)$. Let P ($\in \mathcal{P}(r)$) be a path/tree on the network substrate that is selected to host service request S^r. Here, P comprises the physical links and NFV nodes that host the virtual edges and NF instances, respectively. Hereafter, the term "path" is used liberally; throughout the paper, P can be regarded as a *tree* when provisioning a multicast service. Define y^r_P as the fraction of flow allocated for service S^r along path P ($\in \mathcal{P}(r)$). In the unsplittable model, $y^r_P \in \{0, 1\}$ and $\sum_{P\in\mathcal{P}(r)} y^r_P = 1$. In the splittable model, a service can be fractionally routed on several paths, where multipath routing is enabled and NF instances can be split, i.e., $y^r_P \in [0, 1]$ and $\sum_{P\in\mathcal{P}(r)} y^r_P = 1$. Clearly, the optimal splittable model provides a larger profit compared to the unsplittable variant. Even stronger performance (in terms of maximizing the profit) can be achieved when the splittable model is linearly relaxed to allow for the sum of fractional allocations to be at most 1, i.e., $\sum_{P\in\mathcal{P}(r)} y^r_P \leq 1$. This is due to the fact that the linear relaxation provides an upper bound to the unsplittable (combinatorial) problem.

In data networks (such as in fifth-generation networks and the Internet), the scale of demands is large relative to the granularity at which it can be managed/routed [34], especially in software-defined networks. Therefore, it is desired to design and measure the (competitive) performance of a designed apparatus against a splittable offline model (i.e., with multipath routing and NF splitting) [35]. Therefore, although the online algorithm provides an unsplittable solution, its performance will be measured against the splittable offline model.

The path-based offline profit-maximization formulation for the linearly-relaxed splittable model is expressed in (4).

Dual – profit-maximization problem

$$\max \alpha \sum_{S^r\in\sigma} \sum_{P\in\mathcal{P}(r)} \varrho^r y^r_P + \beta \sum_{S^r\in\sigma} \sum_{P\in\mathcal{P}(r)} \rho^r y^r_P \tag{4a}$$

subject to :

$$\forall S^r \in \sigma : \sum_{P \in \mathcal{P}(r)} y_P^r \leq 1 \tag{4b}$$

$$\forall l \in \mathcal{L} : \sum_{S^r \in \sigma} \sum_{P \in \mathcal{P}(r) | l \in P} d^r y_P^r \leq B(l) \tag{4c}$$

$$\forall n \in \mathcal{N} : \sum_{S^r \in \sigma} \sum_{P \in \mathcal{P}(r) | n \in P} C(f^r) y_P^r \leq C(n) \tag{4d}$$

$$\forall S^r \in \sigma, P \in \mathcal{P}(r) : y_P^r \geq 0. \tag{4e}$$

If all service requests in σ are known a priori, solving the formulation in (4) yields the optimal splittable all-or-something/all-or-something packing configuration for all the accepted services from σ. In (4a), we maximize the overall profit function accrued by the accepted service requests.

The first set of constraints in (4b) requires that the sum of the fractional allocations for each service request along all possible paths is bounded above by unity. Constraints (4c) and (4d) represent the transmission and processing resource constraints on the physical links and the NFV nodes, respectively.

In the context of the online version of the problem, service requests in σ are revealed over time (in discrete steps). The idea is to develop an online solution that maintains a feasible set whenever a new service request arrives *in a controlled manner* to guarantee certain competitive performance. This is achieved by first deriving the primal of (4). Second, we need to ensure that the online algorithm produces solutions such that the objective function of the primal and dual are bounded, which will be explained in Subsection V-A.

Next, we present the corresponding primal formulation in (5). Given (4b), we assign variable z^r for each request S^r, where $z^r \in [0, \max\{\varrho^r, \rho^r\}]$. Given (4c) and (4d), we assign variables $\bar{x}(l)$ and $\tilde{x}(n)$ for each physical link l ($\in \mathcal{L}$) and NFV node n ($\in \mathcal{N}$), respectively, where $\bar{x}(l) \in [0, |\mathcal{D}|_{\max}]$, $\tilde{x}(n) \in [0, \frac{\eta_{\max}}{\eta_{\min}}]$, $\eta_{\max} = \max_{S^r \in \sigma} \eta^r$, $\eta_{\min} = \min_{S^r \in \sigma} \eta^r$, and $|\mathcal{D}|_{\max} = \max_{S^r \in \sigma} |\mathcal{D}^r|$. Through the tableau method, the primal formulation is expressed in (5).

Primal

$$\min \sum_{l \in \mathcal{L}} B(l)\, \bar{x}(l) + \sum_{n \in \mathcal{N}} C(n)\, \tilde{x}(n) + \sum_{S^r \in \sigma} z^r \tag{5a}$$

subject to :

$$\forall S^r \in \sigma, P \in \mathcal{P}(r) : \sum_{l \in P \cap \mathcal{L}} d^r \bar{x}(l) + z^r \geq \alpha \varrho^r \tag{5b}$$

$$\forall S^r \in \sigma, P \in \mathcal{P}(r) : \sum_{n \in P \cap \mathcal{N}} \tilde{x}(n) + z^r \geq \beta \rho^r \tag{5c}$$

$$\forall S^r \in \sigma, l \in \mathcal{L}, n \in \mathcal{N} : z^r, \bar{x}(l), \tilde{x}(n) \geq 0. \tag{5d}$$

In what follows, we develop an admission mechanism that is based on the primal-dual formulation in (4) and (5).

V. Primal-Dual based Admission Mechanism

A. The Approach

In this subsection, we provide a systematic approach to deriving the operational cost model of the physical links and NFV nodes as well as the admission mechanism. Let J and D be the value of the objective function of the primal and the dual solutions as a result of the online algorithm, respectively. Using the weak duality concept, for the aforementioned primal-dual formulation, we know that $D \leq J$. Therefore, in order to have a provable competitive ratio, we need to bound the objective functions of the primal and dual such that

$$J \leq 2\xi D, \tag{6}$$

while maintaining the constraints of the primal and dual formulations satisfied, where 2ξ will be the competitive ratio. However, service requests arrive in an online fashion. Therefore, instead, it is sufficient for the online algorithm to bound the change between the primal and dual objectives whenever a new service request arrives such that [20]

$$\frac{\partial J}{\partial y_P^r} \leq 2\xi \frac{\partial D}{\partial y_P^r}, \quad S^r \in \sigma. \tag{7}$$

Due to the multi-resource form of the objective functions in (4a) and (5a), we can re-express inequality (7) as

$$\sum_{l \in \mathcal{L}} B(l) \frac{\partial \bar{x}(l)}{\partial y_P^r} + \sum_{n \in \mathcal{N}} C(n) \frac{\partial \tilde{x}(n)}{\partial y_P^r} + \frac{\partial z^r}{\partial y_P^r} \leq 2\varphi\alpha\varrho^r + 2\phi\beta\rho^r,\ S^r \in \sigma \tag{8}$$

where $\xi = \max\{\varphi, \phi\}$, and φ and ϕ are some other constants. The right-hand side of inequality (8) has two profit functions, each of which corresponds to a resource type. Therefore, to satisfy inequality (8), it is sufficient to find some functions, $\bar{x}(l)$, $\tilde{x}(n)$ and z^r, such that

$$\sum_{l \in \mathcal{L}} B(l) \frac{\partial \bar{x}(l)}{\partial y_P^r} \leq 2\varphi\alpha\varrho^r, \quad S^r \in \sigma \tag{9a}$$

$$\sum_{n \in \mathcal{N}} C(n) \frac{\partial \tilde{x}(n)}{\partial y_P^r} \leq 2\phi\beta\rho^r, \quad S^r \in \sigma \tag{9b}$$

$$\frac{\partial z^r}{\partial y_P^r} \leq 0, \quad S^r \in \sigma \tag{9c}$$

while maintaining the constraints of the primal and dual formulations satisfied.

Starting with the transmission resource type, we need to satisfy (9a) while maintaining feasibility in constraints (4b) and (5b). To do so, let the solution of the first partial derivative in (9a) follow the following form,

$$\sum_{l \in \mathcal{L}} B(l) \frac{\partial \bar{x}(l)}{\partial y_P^r} = \varphi \sum_{l \in \mathcal{L}} \left(d^r \bar{x}(l) + \frac{\alpha \varrho^r}{L}\right), \quad S^r \in \sigma \tag{10}$$

where L is the maximum number of hops in a path for a unicast service (or maximum number of physical links in a tree for a multicast service), i.e., $\sum_{l \in \mathcal{L}} 1 \leq L$. After substituting (10) in (9a), we impose a requirement that $\sum_{l \in \mathcal{L}} d^r \bar{x}(l) \leq \alpha\varrho^r$ for (9a) to hold. Having satisfied (9a), we need to derive cost function $\bar{x}(l)$ by solving the differential equation in (10). Rearranging the terms in (10), we have a differential equation of the following form,

$$\sum_{l \in \mathcal{L}} \frac{\partial \bar{x}(l)}{\partial y_P^r} + \sum_{l \in \mathcal{L}} \frac{-\varphi d^r}{B(l)} \bar{x}(l) = \sum_{l \in \mathcal{L}} \frac{\varphi \alpha \varrho^r}{B(l) L}. \tag{11}$$

Define the integrating factor

$$I = \exp\Big(\sum_{S^r \in \sigma | l \in P \in \mathcal{P}(r)} \int \frac{-\varphi d^r}{B(l)} \partial y_P^r\Big)$$
$$= C \exp\Big(\frac{-\varphi}{B(l)} \sum_{S^r \in \sigma | l \in P \in \mathcal{P}(r)} d^r y_P^r\Big) \qquad (12)$$

where C is an arbitrary constant, and multiply both sides of (11) by I, we get

$$I\Big(\frac{\partial \bar{x}(l)}{\partial y_P^r} + \frac{-\varphi d^r}{B(l)} \bar{x}(l)\Big) = \frac{\varphi \alpha \varrho^r}{B(l) L} I. \qquad (13)$$

Through the following identity by the product rule,

$$\frac{\partial}{\partial y_P^r}(Ix) = I \frac{\partial x}{\partial y_P^r} + \frac{-\varphi d^r}{B(l)} I x, \qquad (14)$$

we can express $\bar{x}(l)$ as

$$\bar{x}(l) = I^{-1} \frac{\varphi d^r}{B(l) L} \int I \partial y_P^r,$$
$$= \frac{1}{L}\big(-C e^{\frac{\varphi}{B(l)} \sum_{S^r \in \sigma | l \in P \in \mathcal{P}(r)} d^r y_P^r} - 1\big), \quad l \in \mathcal{L}.$$

Initially, before the arrival of any service request, we require that $\bar{x}(l) = 0$, which occurs when $\frac{1}{B(l)} \sum_{S^r \in \sigma | l \in P \in \mathcal{P}(r)} d^r y_P^r = 0$. Thus, we set $C = -1$. We also require that $\bar{x}(l) \geq \alpha |\mathcal{D}^r|^k$ when physical link l $(\in \mathcal{L})$ is saturated, i.e. when $\frac{1}{B(l)} \sum_{S^r \in \sigma | l \in P \in \mathcal{P}(r)} d^r y_P^r = 1$. Hence, we need $\varphi \geq \ln(\alpha L |\mathcal{D}|_{\max}^k + 1)$. Therefore, $\bar{x}(l)$ can be expressed as

$$\bar{x}(l) = \frac{1}{L}\big(e^{\varphi \frac{\sum_{S^r \in \sigma | l \in P \in \mathcal{P}(r)} d^r y_P^r}{B(l)}} - 1\big), \quad l \in \mathcal{L}. \qquad (15)$$

Note that constraint (4c) is maintained feasible over the whole range of $\bar{x}(l)$. In the admission mechanism, we need to ensure that there is a sufficient protection to the resources before accepting a service to avoid the scenario of accidentally violating the resources (which occurs when $\bar{x}(l) \geq \alpha |\mathcal{D}^r|$). It turns out that, due to Lemma 1 (to be derived later), if the required data rate is bounded above by $d^r \leq \frac{\min_{l \in \mathcal{L}} B(l)}{\varphi}$, we need $\varphi \geq \ln(2\alpha L |\mathcal{D}|_{\max}^k + 2)$. Note that the edge costs include variables from future requests (i.e., y_P^r, $\forall S^r \in \sigma$). However, since this is an online algorithm, future variables can be initialized to zero until the respective service requests are parsed through the admission mechanism. We can express the edge costs in a multiplicative recursive manner as

$$\bar{x}^r(l) = \bar{x}^{r-1}(l) e^{\varphi \frac{d^r}{B(l)}} + \frac{1}{L}\big(e^{\varphi \frac{d^r}{B(l)}} - 1\big), \quad l \in \mathcal{L}, S^r \in \sigma \quad (16)$$

where $\bar{x}^r(\cdot)$ is the edge cost after embedding the rth service request, and $\bar{x}^0(\cdot)$ is set to zero. Now, we need to ensure that constraint (5b) is maintained feasible. Since we set $\sum_{l \in \mathcal{L}} d^r \bar{x}(l) \leq \alpha \varrho^r$, we require that $z^r \geq \alpha \varrho^r - \sum_{l \in \mathcal{L}} d^r \bar{x}(l)$ for (5b) to be maintained feasible, and for (9c) to hold.

By following a similar procedure, for the processing resource type, we need to satisfy (9b) while maintaining feasibility in constraints (4d) and (5c). To do so, let the solution of the partial derivative in (9b) follow the following form,

$$\sum_{n \in \mathcal{N}} C(n) \frac{\partial \tilde{x}(n)}{\partial y_P^r} = \phi \sum_{n \in \mathcal{N}} \big(C(f^r) \tilde{x}(n) + \frac{\beta \rho^r}{K}\big), \quad S^r \in \sigma \quad (17)$$

where K is the maximum number of NF instances in a service request (including both best-effort and mandatory NF instances), i.e., $\sum_{n \in \mathcal{N}} 1 \leq K$. Similarly, we can satisfy (9b) by imposing a condition that $\sum_{n \in \mathcal{N}} C(f^r) \tilde{x}(n) \leq \beta \rho^r$. Now, we need to solve differential equation (17). A similar procedure yields a cost function that is expressed as

$$\tilde{x}(n) = \frac{1}{K}\big(-C e^{\phi \frac{\sum_{S^r \in \sigma | n \in P \in \mathcal{P}(r)} C(f^r) y_P^r}{C(n)}} - 1\big), \quad n \in \mathcal{N}.$$

Initially, before the arrival of any service request, we require that $\tilde{x}(n) = 0$, which occurs when $\frac{1}{C(n)} \sum_{S^r \in \sigma} C(f^r) y_P^r = 0$. Therefore, we have $C = -1$. We also require that $\tilde{x}(n) \geq \beta \eta^r$ when NFV node n $(\in \mathcal{N})$ is saturated, i.e. when $\frac{1}{C(n)} \sum_{S^r \in \sigma} C(f^r) y_P^r = 1$. Therefore, we need $\phi \geq \ln(\beta K \frac{\eta_{\max}}{\eta_{\min}} + 1)$. Hence, $\tilde{x}(n)$ can be expressed as

$$\tilde{x}(n) = \frac{1}{K}\big(e^{\phi \frac{\sum_{S^r \in \sigma | n \in P \in \mathcal{P}(r)} C(f^r) y_P^r}{C(n)}} - 1\big), \quad n \in \mathcal{N}. \qquad (18)$$

Note that constraint (4d) is maintained feasible over the whole range of $\tilde{x}(n)$. Similar to the transmission resource, due to Lemma 2 (to be given in Subsection V-C), if the required processing rate is bounded above by $C(f^r) \leq \frac{\min_{n \in \mathcal{N}} C(n)}{\phi}$, we need $\phi \geq \ln(2\beta K \frac{\eta_{\max}}{\eta_{\min}} + 2)$ to ensure a sufficient protection against violating the processing resources. Since we set $\sum_{n \in \mathcal{N}} C(f^r) \tilde{x}(n) \leq \beta \rho^r$, we require that $z^r \geq \max\{\alpha \varrho^r - \sum_{l \in \mathcal{L}} d^r \bar{x}(l), \beta \rho^r - \sum_{n \in \mathcal{N}} C(f^r) \tilde{x}(n)\}$ for (5b) and (5c) to be maintained feasible, and for (9c) to hold. The cost function can be updated multiplicatively each time after embedding a service request as follows,

$$\tilde{x}^r(n) = \tilde{x}^{r-1}(n) e^{\phi \frac{C(f^r)}{C(n)}} + \frac{1}{K}\big(e^{\phi \frac{C(f^r)}{C(n)}} - 1\big), \quad n \in \mathcal{N}, S^r \in \sigma$$

where $\tilde{x}^r(n)$ is the cost of the NFV node n $(\in \mathcal{N})$ after embedding the rth service request, and $\tilde{x}^0(n)$ is set to zero. Now, we are ready to state the all-or-nothing/all-or-something admission mechanism.

B. Admission Mechanism

The procedure commences with the arrival of an rth service request, which resembles an augmentation of a new decision variable (y_P^r) to the dual formulation. Correspondingly, in the primal formulation, the arrival is equivalent to the augmentation of two new constraints, namely (5b) and (5c). The rth service request is accepted if there exists a path, P[1], such that the following two conditions hold:

$$\sum_{l \in P \cap \mathcal{L}} d^r \bar{x}^{r-1}(l) \leq \alpha \varrho^r \qquad (19)$$

and

$$\sum_{n \in P \cap \mathcal{N}} C(f^r) \tilde{x}^{r-1}(n) \leq \beta \rho^r. \qquad (20)$$

If the two conditions are satisfied, accept the request and route it on P, and set $y_P^r = 1$. To maintain feasibility in (5b) and (5c), set

$$z^r = \max\big(\alpha \varrho^r - \sum_{l \in P \cap \mathcal{L}} d^r \bar{x}(l), \beta \rho^r - \sum_{n \in P \cap \mathcal{N}} C(f^r) \tilde{x}(n)\big). \quad (21)$$

[1]How to find path P for the routing and NF placement problem is addressed in Subsection VI.

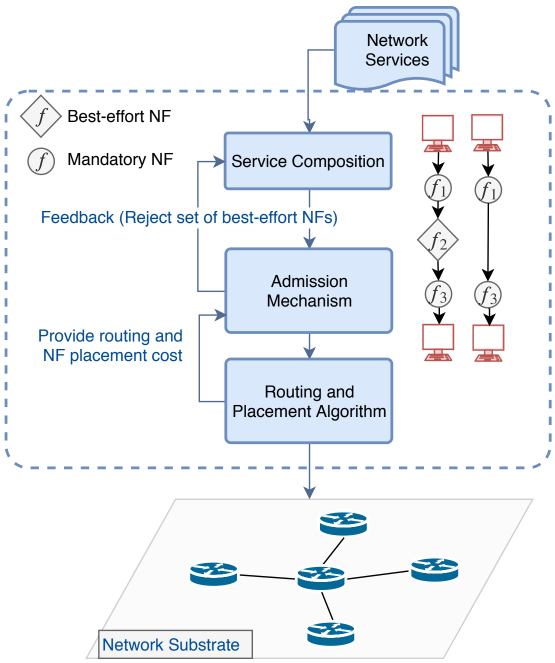

Figure 1: The joint all-or-nothing/all-or-something admission mechanism and online JCRP framework.

Finally, update the costs of the edge variables $\bar{x}(l)$ and NFV nodes $\tilde{x}(n)$ in a multiplicative manner as follows,

$$\bar{x}^r(l) = \bar{x}^{r-1}(l)e^{\varphi\frac{d^r}{B(l)}} + \frac{1}{L}(e^{\varphi\frac{d^r}{B(l)}} - 1), \quad l \in P \cap \mathcal{L} \tag{22}$$

$$\tilde{x}^r(n) = \tilde{x}^{r-1}(n)e^{\phi\frac{C(f^r)}{C(n)}} + \frac{1}{K}(e^{\phi\frac{C(f^r)}{C(n)}} - 1), \quad n \in P \cap \mathcal{N} \tag{23}$$

where $\varphi = \ln(2\alpha L|\mathcal{D}|^k_{max} + 2)$ and $\phi = \ln(2\beta K\frac{\eta_{\max}}{\eta_{\min}} + 2)$.

Treatment of best-effort NFs – We first find a routing and NF placement configuration that includes the set of best-effort NFs since it provides a larger (incentivized) profit with $\rho^r = C(f^r)\eta^r_b$. If such configuration is rejected by the admission mechanism, we find another configuration that excludes the set of best-effort NFs, and check against the admission mechanism with the nominal profit function $\rho^r = C(f^r)\eta^r_m$. If both configurations (with and without the set of best-effort NFs) do not satisfy (19) and (20), the service request is rejected. Fig. 1 provides a representation of the all-or-nothing/all-or-something admission mechanism. Algorithm 1 summarizes the online admission framework. In the algorithm's pseudocode, an assignment is denoted by "←".

Algorithm 1: Admission control and online JCRP framework

<u>Procedure</u> OnlineJPR($\mathcal{G}, S^r$);
Input : $\mathcal{G}(\mathcal{N}, \mathcal{L})$, S^r
Output: Embed or reject S^r
$\bar{x}(l) \leftarrow 0$ (*only once*); $\tilde{x}(n) \leftarrow 0$ (*only once*);
services $\leftarrow (S^r, S^r - \mathcal{V}^r_b)$;
for $i = 1 : 2$ **do**
  $P \leftarrow$ JRP($\mathcal{G}$, services[i]); ▷ JRP is in Algorithm 2 in Section VI.
  if $\sum_{l\in P\cap\mathcal{L}} d^r \bar{x}(l) \le \alpha\varrho^r$ *and* $\sum_{n\in P\cap\mathcal{N}} C(f^r)\tilde{x}(n) \le \beta\rho^r$ **then**
    Accept request ($y^r_P \leftarrow 1$);
    $z^r \leftarrow \max\big(\alpha\varrho^r - \sum_{l\in P\cap\mathcal{L}} d^r \bar{x}(l), \beta\rho^r - \sum_{n\in P\cap\mathcal{N}} C(f^r)\tilde{x}(n)\big)$;
    $\bar{x}(l) \leftarrow \bar{x}(l)e^{\varphi\frac{d^r}{B(l)}} + \frac{1}{L}(e^{\varphi\frac{d^r}{B(l)}} - 1), \quad \forall l \in P \cap \mathcal{L}$;
    $\tilde{x}(n) \leftarrow \tilde{x}(n)e^{\phi\frac{C(f^r)}{C(n)}} + \frac{1}{K}(e^{\phi\frac{C(f^r)}{C(n)}} - 1), \quad \forall n \in P \cap \mathcal{N}$;
    return;
  end
end
return (Reject request);

C. Performance Analysis

In what follows, we analyze the performance of the admission mechanism. We show that the proposed mechanism does not violate the transmission and processing resources of physical links and NFV nodes. Then, we prove the competitive ratio for the all-or-nothing/all-or-something admission mechanism.

Lemma 1. *The transmission resource constraint on the physical links cannot be violated, i.e.,* $\sum_{S^r\in\sigma|l\in P\in\mathcal{P}(r)} d^r \le B(l)$, $l \in \mathcal{L}$, *if* $\varphi \ge \ln(2\alpha L|\mathcal{D}|^k_{\max} + 2)$ *and* $d^r \le \frac{\min_{l\in\mathcal{L}} B(l)}{\varphi}$, $\forall S^r \in \sigma$.

Lemma 2. *The processing resource constraint on the NFV nodes cannot be violated, i.e.* $\sum_{S^r\in\sigma|n\in P\in\mathcal{P}(r)} C(f^r) \le C(n)$, $n \in \mathcal{N}$, *if* $\phi \ge \ln(2\beta K\frac{\eta_{\max}}{\eta_{\min}} + 2)$ *and* $C(f^r) \le \frac{\min_{n\in\mathcal{N}} C(n)}{\phi}$, $\forall S^r \in \sigma$.

Theorem 1. *The competitive ratio of the admission mechanism is* $\mathcal{O}\big(\max(\varphi, \phi)\big)$, *where* $d^r \le \frac{\min_{l\in\mathcal{L}} B(l)}{\varphi}$, $C(f^r) \le \frac{\min_{n\in\mathcal{N}} C(n)}{\phi}$, $\varphi = \ln(2\alpha L|\mathcal{D}|^k_{\max} + 2)$, *and* $\phi = \ln(2\beta K\frac{\eta_{\max}}{\eta_{\min}} + 2)$.

The proof of Lemmata 1 and 2 and Theorem 1 is given in Appendix.

Now, we discuss the method to find path P for a service request. Recall the admission conditions in (19) and (20). For each service request, the admission mechanism requires checking *all* possible paths for the performance guarantees to hold. If any of such paths satisfies the admission mechanism, the service request should be accepted. Notably, in general, it is not necessary to route the service on the minimum-cost path, but rather a secondary routing objective can be invoked. However, there exists an exponential number of possible paths for a service request. It is more convenient (and sufficient) to check against the minimum-cost path only. If it was rejected, all other paths would be rejected. Next, we propose an algorithm to find the minimum-cost routing and NF placement solution for a unicast and multicast service request.

VI. Routing and NF Placement Approximation Algorithm

Due to Lemmata 1 and 2, the proposed admission mechanism guarantees that a violation in the processing and transmission resources is always avoided if the exponential cost functions are used for the physical links and NFV nodes with $\varphi \ge \ln(2\alpha L|\mathcal{D}|^k_{\max} + 2)$ and $\phi \ge \ln(2\beta K\frac{\eta_{\max}}{\eta_{\min}} + 2)$. Therefore, it is sufficient to design a routing and NF placement algorithm for the unconstrained (or uncapacitated) scenario. Here, we propose a *one-step* algorithm for the routing and NF placement of unicast and multicast services for the unconstrained scenario. The algorithm relies mainly on the construction of an auxiliary multilayer network transformation that has a one-to-one mapping from the NF placement and routing problem to an equivalent routing problem. This facilitates the use of existing (approximation) algorithms, such as the Dijkstra shortest path for the unicast scenario and MST-based Steiner tree for the multicast scenario.

A. Auxiliary Network Transformation and Routing and NF Placement Algorithm

To jointly consider the provisioning costs of both NF (processing) and virtual links (transmission), for each service request, we construct an auxiliary multilayer graph from the network substrate, in which the constructed edges represent either (i) the hosting of a virtual link or (ii) the processing of some NF type. For the sake of exposition, since the joint routing and NF placement algorithm treats each service request separately, we drop superscript r (which alludes to the rth service request) in this subsection.

The auxiliary multilayer graph is modeled as a directed graph $\mathcal{G}_M = (\mathcal{N}_M, \mathcal{L}_M)$, where $\mathcal{N}_M \subseteq \mathcal{N} \times \mathcal{X}$ is the set that contains all nodes, in which node n $(\in \mathcal{N})$ is present in a corresponding layer a $(\in \mathcal{X})$; denote such a node by n^{α}. Correspondingly, $\mathcal{L}_M \subseteq \mathcal{N}_M \times \mathcal{N}_M$ is the set of all inter- and intra-layer edges. Intra-layer edges, $\mathcal{L}_A = \{(u^a, v^b) \in \mathcal{L}_M | a = b \in \mathcal{X}\}$, represent the routing connections between the network elements (nodes) in each layer. Inter-layer edges, $\mathcal{L}_I = \{(u^a, u^b) \in \mathcal{L}_M | a \neq b \in \mathcal{X}\}$, are used to encode the placement decisions in which a traversal of an edge from one layer a $(\in \mathcal{X})$ to another layer b $(\in \mathcal{X})$ maps to the processing of the ath NF instance in a service request. Hence, for service request S^r, the number of layers is equivalent to the number of NFs plus one (i.e., $|\mathcal{X}| = |\mathcal{V}| + 1$).

Upon the arrival of the rth service request, we construct a multilayer graph $\mathcal{G}_M$ as follows:

1) Create $|\mathcal{X}|$ $(= |\mathcal{V}|+1)$ copies of the network substrate $\mathcal{G}$. Each copy $(\mathcal{G}^i(\mathcal{N}^i, \mathcal{L}^i))$ represents one layer, where $i = \{0, \ldots, |\mathcal{V}|\}$. The transmission and processing resources for each NFV node $(n \in \mathcal{N})$ and physical link $(l \in \mathcal{L})$ are equal in each copy;
2) Assign the source node to its corresponding node at layer 0 $(= s^0,\ s^0 \in \mathcal{N}^0)$;
3) Assign the destination nodes to their corresponding nodes at the last layer $(= t^{|\mathcal{V}|},\ t^{|\mathcal{V}|} \in \mathcal{N}^{|\mathcal{V}|})$;
4) For the first $|\mathcal{V}|$ layers, construct inter-layer edges from layer i to layer $i+1$ $(l \leftarrow (n^i, n^{i+1}))$ for each NFV node that can host f_i (i.e., if $n^i \in \mathcal{F}_i$). The processing resources of each inter-layer edge, $l \leftarrow (n^i, n^{i+1})$, is that of the corresponding NFV node n^i.

We provide an illustrative example of the construction of the auxiliary graph transformation in Figs. 2 and 3. Fig. 2 illustrates a unicast service request of two NFs (f_1 and f_2), where the source is n_1 and the destination is n_4. We also have a network substrate of 4 NFV nodes that can host either NF type or both. Fig. 3 illustrates the construction of the auxiliary graph

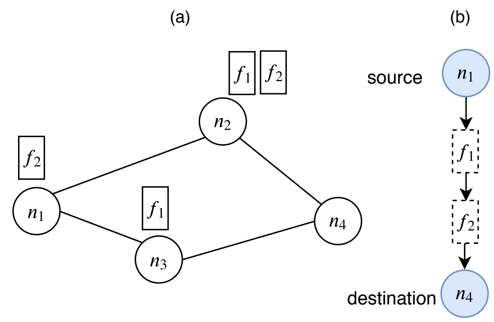

Figure 2: A problem input: (a) A network substrate along with the permissible NFs on each network element, and (b) the logical topology of a service request.

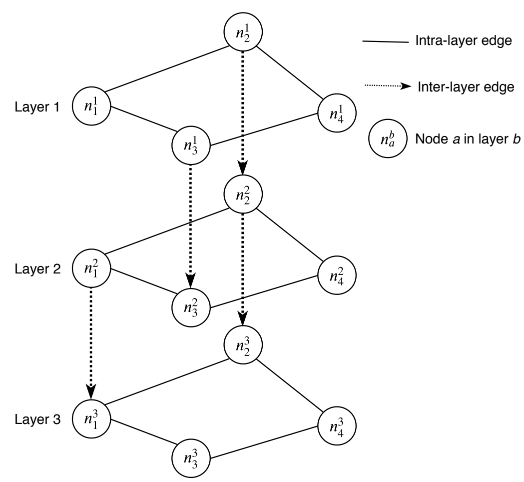

Figure 3: The auxiliary network transformation for the problem input in Fig. 2.

transformation. Since we have two NFs, the transformation has three layers. NFV nodes n_1 and n_3 can host f_1. Therefore, we construct the inter-layer edges, $n_1^1 \rightarrow n_1^2$ and $n_3^1 \rightarrow n_3^2$. Similarly, n_1 and n_2 can host f_2. Therefore, we construct the inter-layer edges, $n_1^2 \rightarrow n_1^3$ and $n_2^2 \rightarrow n_2^3$.

With the new auxiliary graph transformation, a path traversal (while considering only the edge costs) from node s^0 to node $t^{|\mathcal{V}|}$ represents a routing and NF placement solution in the original network substrate graph. Algorithm 2 summarizes the online JRP framework, which comprises the construction of the auxiliary graph transformation with the minimum-cost routing algorithm for a unicast or multicast service. For multicast services, the MST-based Steiner tree algorithm is a 2-approximation algorithm. Therefore, the competitive-ratio of the admission mechanism for the multicast services is $O(2\max\{\varphi, \phi\}) = O(\max\{\varphi, \phi\})$.

Theorem 2. *For unicast services, the overall time complexity of the online routing and NF placement framework is* $O(h \log h)$*, where* $h = |\mathcal{N}|(K+1)$*. For multicast services, the overall time complexity is* $O\big(|\mathcal{D}|^2 h\big)$.

Algorithm 2: JRP algorithm for a single service request

Procedure JRP($\mathcal{G}, S^r$);
Input : $\mathcal{G}_M$, $S^r = S = (s, \mathcal{D}, f_1, f_2, \ldots, f_{|\mathcal{V}|}, d)$
Output: P
$\{\mathcal{G}^k(N^k, L^k)\}_{k=0}^{|\mathcal{V}|+1} \leftarrow \mathcal{G}(N, L)$;
$s^0 \leftarrow s$ (source node at layer 0 as source s);
$\mathcal{D}^{|\mathcal{V}|} \leftarrow \mathcal{D}$;
$\mathcal{L}_I \leftarrow \{\}$;
for $k = 0 : (|\mathcal{V}| - 1)$ **do**
  if $n^k \in \mathcal{F}_k$ **then**
    Add $l \leftarrow (n^k, n^{k+1})$ to $\mathcal{L}_I$;
    $C_l(r) = C(n^k)$;
  end
end
For multicast services, find an MST-based Steiner tree from s^0 to $\mathcal{D}^{|\mathcal{V}|}$, while utilizing the cost functions in (22) and (23), and save on P;
For unicast services, find a Dijkstra shortest path from s^0 to $t^{|\mathcal{V}|}$, while utilizing the cost functions in (22) and (23), and save on P;
return P;

Proof. For an efficient run-time, the full construction of the auxiliary network transformation can be performed once in the beginning. With the arrival of a new service request, depending on the requested NFs, the relevant layers can be connected with each other, while temporarily deactivating irrelevant layers (by disconnecting them). Therefore, the major component of the runtime is due to the Dijkstra shortest path for unicast services and MST-based Steiner tree for multicast services that is run over the network transformation which has $|\mathcal{N}|(K+1)$ nodes. □

VII. Discussions and Simulation Results

A. Discussions

On the obtained competitive ratio – As seen from the derivations in Subsection V-A, in order to bound the performance of the primal and the dual, exponential cost functions are used for the physical links and NFV nodes. In doing so, if we can guarantee that the residual resources for the physical links and NFV nodes are not violated, the admission mechanism yields a competitive ratio of $O\big(\max\{\varphi, \phi\}\big) = O\big(\max\{\ln \alpha L|\mathcal{D}|^k_{\max}, \ln \beta K \frac{\eta_{\max}}{\eta_{\min}}\}\big)$. However, the routing and NF placement problem for the constrained scenario is NP-hard. Therefore, to protect against a possible violation in the resources without relying on a constrained routing and NF placement algorithm, Lemmata 1 and 2 require that $\varphi = \ln(2\alpha L|\mathcal{D}|^k_{\max} + 2)$ and $\phi = \ln(2\beta K \frac{\eta_{\max}}{\eta_{\min}} + 2)$ at least, for which the competitive ratio is increased to $O\big(\max\{\ln 2\alpha L|\mathcal{D}|^k_{\max}, \ln 2\beta K \frac{\eta_{\max}}{\eta_{\min}}\}\big)$. A consequential drawback is that the utilization of physical links and NFV nodes will not exceed $1 - \frac{1}{\varphi}$ and $1 - \frac{1}{\phi}$, respectively. Therefore, when φ or ϕ are relatively small, a considerable amount of processing and transmission resources will be wasted. Hence, the online algorithm is expected to not perform well for networks of a small size relative to the intended competitive performance.

On the design of profit function – The proposed framework works for both unicast and multicast services. Moreover, it includes both best-effort and mandatory NF types. The profit functions allows a variation that depends on the maximum number of included destinations and the maximum incentive for including the set of best-effort NFs. To this effect, we can observe that the optimality is penalized due to the large variation of the profit functions, where maximizing the amortized throughput only (i.e., with $\varrho^r = d^r$ and $\rho^r = C(f^r)$) improves the competitive ratio by a logarithmic factor in $|\mathcal{D}|^k_{\max}$ and $\frac{\eta_{\max}}{\eta_{\min}}$, respectively.

Recall that the derived competitive ratio is $O\big(\max\{\ln 2\alpha L|\mathcal{D}|_{\max}, \ln 2\beta K \frac{\eta_{\max}}{\eta_{\min}}\}\big)$. In practice, $L|\mathcal{D}|^k_{\max}$ is larger than the maximum number of NFs (K). Therefore, the use of an incentive for including the best-effort NFs can help to scale up the second term without necessarily degrading the competitive performance, which offers an appropriate generalization.

B. Numerical Analysis

In this subsection, we analyze three online algorithms. The first algorithm is the proposed approximation algorithm with $\varphi = \ln(2\alpha L|\mathcal{D}|^k_{\max} + 2)$ and $\phi = \ln(2\beta K \frac{\eta_{\max}}{\eta_{\min}} + 2)$ (as in Algorithm 1). As shown, a resource violation is always avoided, and the competitive performance is guaranteed. The second online algorithm is similar to the first one but with $\varphi = \ln(\alpha L|\mathcal{D}|^k_{\max} + 1)$ and $\phi = \ln(\beta K \frac{\eta_{\max}}{\eta_{\min}} + 1)$. Here, resources are not necessarily protected from future violations. As a heuristic algorithm, if the routing and NF placement solution violates any processing or transmission constraint, it is removed. The third online algorithm is a greedy algorithm that attempts to accept all services as long as there are sufficient resources. The greedy algorithm basically resembles the heuristic algorithm but without invoking the admission conditions in (19) and (20). That is, it is basically the output of Algorithm 2 (without Algorithm 1), and with an extra step of checking if the service request violates any processing or transmission constraint.

In the experiments, we analyze the performance of the three algorithms on linear, random, and real network substrate topologies. Throughout the experiments, we set the scalarization coefficients (α, β) to unity (since the processing and transmission resources are appropriately scaled). Each NFV node can host 2/3 of the possible NFs in random. The transmission and processing resources are randomly distributed between 1000 and 5000 packet/s. The required data rate for service requests is uniformly distributed between 1 and 20 packet/s. The processing rate requirement of NF instances are linearly proportional to the incoming data rate $C(f^r) = d^r$ [10], [36]. In all trials, we terminate an algorithm when it no longer can accept any request, i.e., when the network substrate instance reaches the maximum possible utilization.

In the first experiment, we generate a directed network substrate with a linear topology for unicast service requests. Each request has a random pair of source and destination nodes, and an overall number of required NFs ($|\mathcal{V}^r|$) of 3, with the number of best-effort NFs ($|\mathcal{V}^r_b|$) uniformly distributed between 0 and 3. The incentive for including the best-effort NFs (η^r) is set to unity. Fig. 4 shows the normalized aggregate

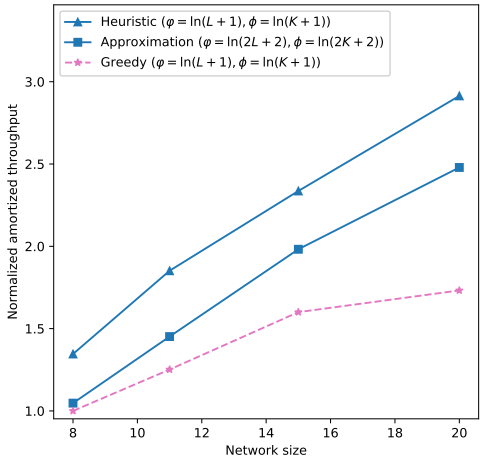

Figure 4: Normalized aggregate throughput ($\varrho^r + \rho^r$) for the three algorithms for a linear topology, with $L = K = 4$.

profit for the three algorithms as the size of the linear network substrate ($|\mathcal{N}|$) grows, with $L = K = 4$. The aggregate profit increases almost linearly for all the algorithms. The heuristic algorithm (with $\varphi = \ln(L + 1)$ and $\phi = \ln(K + 1)$) outperforms the approximation algorithm (with $\varphi = \ln(2L+2)$ and $\phi = \ln(2K + 2)$) by a constant gap of approximately 30%. Interestingly, this is equal to the wasted utilization by the latter algorithm, which is given by $1 - \frac{1}{\varphi}$ and $1 - \frac{1}{\phi}$ for the transmission and processing resources, respectively. The heuristic algorithm outperforms the greedy algorithm by almost 40%. When the network size is small, with $|\mathcal{N}| = 8$, the performance of the approximation and greedy algorithms is very close. This is expected since φ and ϕ are not small compared to the size of the network. Moreover, the approximation algorithm wastes a potential utilization of 30%.

In the second experiment, our aim is to observe the effect of the incentive for including the set of best-effort NFs (η^r) on the competitive performance. The experiment is performed over a linear topology with 20 nodes. We generate unicast service requests, each with an overall number of required NFs ($|\mathcal{V}^r|$) of 2, where either one or none of the NFs is set as best-effort in random. Here, we set η^r to the number of included NFs, i.e., $\eta_b = 2$ and $\eta_m = 1$. Fig. 5 shows the normalized aggregate profit for the three algorithms. As expected, when an incentive is used for including the best-effort NFs, the aggregate profit is scaled up for all the algorithms, which implies service requests with best-effort NFs are encouraged to maximize the aggregate profit. However, this comes at the expense of an increased competitive ratio compared to the greedy algorithm. With an incentivized profit function, the percent increase of the approximation algorithm to the greedy algorithm is 11%, whereas the percent increase without using an incentive is 39%.

Next, we evaluate the three algorithms on two real topologies from the Topology Zoo dataset [37]. The first topology, namely Bell Canada, is a commercial topology with 48 nodes and 64 links. The second topology, namely CESNET, is a research and education network (REN) with 52 nodes and 63

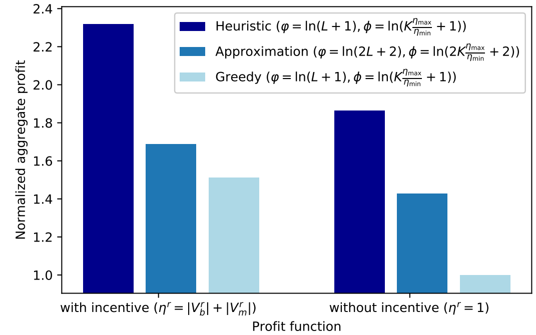

Figure 5: Normalized aggregate profit ($\varrho^r + \rho^r$) for three algorithms for a linear topology with and without incentivizing the use of best-effort NFs, with $L = 4$ and $K = 3$.

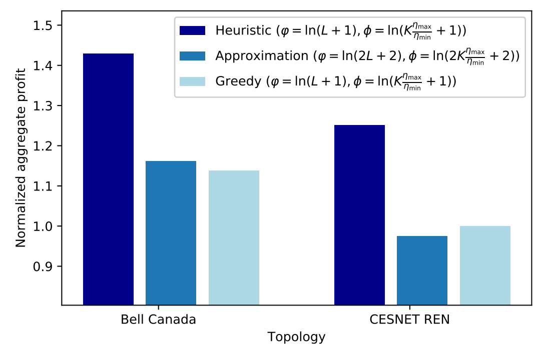

Figure 6: Normalized aggregate profit ($\varrho^r + \rho^r$) for the three algorithms for two real topologies, namely Bell Canada and CESNET REN, with $K = 5$ and $L = 13$ and 6, respectively.

links. We generate unicast service requests with 5 required NFs ($|\mathcal{V}^r|$). For each service request, the number of best-effort NFs ($|\mathcal{V}_b|$) is uniformly distributed between 1 and $|\mathcal{V}^r|$. In this experiment, the design goal is to penalize requests which would take unnecessarily long routes due to deploying NF instances that are far-away from the shortest path between the source and destination. Therefore, for the two topologies, we set L to the maximum shortest path between any pair of nodes, which corresponds to 13 and 6, respectively for each topology. Moreover, we set $K = 5$ and $\eta^r = 1$. Fig. 6 shows the normalized aggregate profit for the two topologies. The approximation and greedy algorithms have close performance, whereas the heuristic algorithm yields a 25% and 23% improvement for the two topologies.

The next experiment is to test the performance of the online algorithms on both multicast and unicast service requests over a random topology with 25 nodes ($|\mathcal{N}| = 25$). The random topology is generated using the Barabási–Albert preferential attachment model, which provides scale-free network topologies [38]. For each service request, the number of destinations varies randomly between 1 and 4, and the number of required NFs in each service request is uniformly random between 1 and 3. Recall that, to provide a non-discriminatory treatment between unicast and multicast service requests, $\varrho^r \propto |\mathcal{D}|^k$, where k is recommended to be 0.8. Fig. 7 shows the normalized aggregate profit for the online heuristic and greedy algorithms as $|\mathcal{D}|_{\max}$ grows for different values of k. The

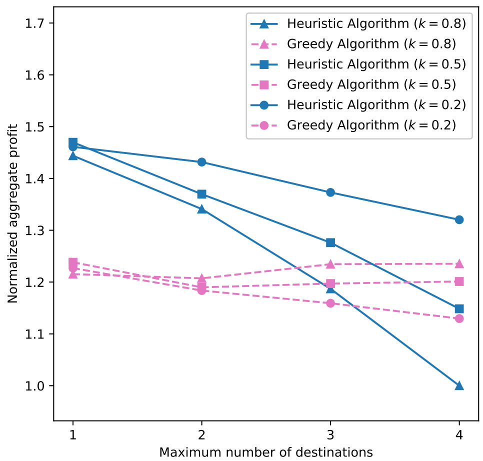

Figure 7: Normalized aggregate profit ($\varrho^r+\rho^r$) for the heuristic and greedy algorithms over random topology with $|\mathcal{N}| = 25$, $K = 4$, $\eta^r = 1$, and L is set to the maximum hop distance between any pair of nodes.

performance of the greedy algorithm remains almost constant as $|\mathcal{D}|^k_{\max}$ increases. However, the heuristic algorithm shows a downtrend as $|\mathcal{D}|^k_{\max}$ increases, especially for large k (e.g, $k = 0.8$). The experiment demonstrates that the competitive ratio of the online heuristic algorithm is increased due to the allowed variation in the profit function (as $|\mathcal{D}|^k_{\max}$ increases).

VIII. Conclusions

In this paper, we have proposed a joint admission mechanism and an online composition, routing and NF placement algorithm for unicast and multicast NFV-enabled services. We considered services with multiple mandatory and best-effort NF instances, which is shown to offer a natural generalization to previous works. Through a primal-dual based analysis, it is shown that a provable competitive performance can be achieved, which can be tuned depending on the allowed variability of the profit function and the desired optimality. This work does not assume any statistical models on the arrival pattern of the service requests nor does it have any probabilistic assumptions on the sources, destinations, and network functions. Therefore, this paper provides a fundamental understanding on the nature of the profit-maximization problem for NFV-enabled services with multiple resource types. Indeed, the analyzed worst-case competitive performance can be improved by incorporating more contextual assumptions (e.g., adding probabilistic/stochastic assumptions). In doing so, the performance of the online algorithm is expected to be improved, while retaining the robustness of the competitive analysis (at least in a probabilistic/stochastic sense).

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Appendix

A. Proof for Lemma 1

Assume that the transmission resources on physical link l ($\in \mathcal{L}$) become exceeded when the rth service request arrives. Then, we have $B(l)-\sum_{j=1}^{r-1} d^j < d^r$. Therefore, after admitting the $(r-1)$th service, the value of $\bar{x}^{r-1}(l)$ can be expressed as

$$\begin{aligned}\bar{x}^{r-1}(l) &= \frac{1}{L}(e^{\varphi\frac{\sum_{j=1}^{r-1} d^j}{B(l)}} - 1)\\ &= \frac{1}{L}(e^{\varphi(1-\frac{B(l)-\sum_{j=1}^{r-1} d^j}{B(l)})} - 1)\\ &> \frac{1}{L}(e^{\varphi(1-\frac{d^r}{B(l)})} - 1).\end{aligned} \tag{24}$$

Assuming that the required data rate of a service request is "small enough", i.e. $d^r \le \frac{\min_{l\in\mathcal{L}} B(l)}{\varphi}$, $\forall S^r \in \sigma$. Then, inequality (24) becomes

$$\begin{aligned}\bar{x}^{r-1}(l) &\ge \frac{1}{L}(e^{\varphi(1-\frac{1}{\varphi})} - 1)\\ &= \frac{1}{L}(\frac{e^{\varphi}}{e} - 1).\end{aligned} \tag{25}$$

Therefore, for the rth service request to be rejected, we need $d^r\bar{x}^{r-1}(l) \ge \alpha\varrho^r$, $\forall S^r \in \sigma$, which translates to $\bar{x}^{r-1}(l) \ge \alpha|\mathcal{D}|^k_{\max}$. Therefore, we need $\frac{1}{L}(\frac{e^{\varphi}}{e} - 1) \ge \alpha|\mathcal{D}|_{\max}$, which entails that $\varphi \ge \ln(2\alpha L|\mathcal{D}|^k_{\max} + 2)$. That is, φ is set such that the admission mechanism rejects any service request that would violate the transmission resources of a physical link. ∎

B. Proof for Lemma 2

Assuming that the processing resources on the NFV node n ($\in \mathcal{N}$) become exceeded when request S^r is accepted, we have $C(n) - \sum_{j=1}^{r-1}\sum_{f\in\mathcal{V}^j} C(f^j) < C(n)$. Therefore, the value of $\tilde{x}^{r-1}(n)$ can be expressed as

$$\begin{aligned}\tilde{x}^{r-1}(n) &= \frac{1}{K}(e^{\phi\frac{\sum_{j=1}^{r-1}\sum_{f\in\mathcal{V}^j} C(f^j)}{C(n)}} - 1)\\ &= \frac{1}{K}(e^{\phi(1-\frac{C(n)-\sum_{j=1}^{r-1}\sum_{f\in\mathcal{V}^j} C(f^j)}{C(n)})} - 1)\\ &> \frac{1}{K}(e^{\phi(1-\frac{\sum_{f\in\mathcal{V}^r} C(f^r)}{C(n)})} - 1).\end{aligned} \tag{26}$$

Under the assumption that the processing requirements of a service request is "small enough", i.e., $C(f^r) \le \frac{\min_{n\in\mathcal{N}} C(n)}{\phi}$, $S^r \in \sigma$, inequality (26) becomes

$$\begin{aligned}\tilde{x}^{r-1}(l) &\ge \frac{1}{K}(e^{\phi(1-\frac{1}{\phi})} - 1)\\ &= \frac{1}{K}(\frac{e^{\phi}}{e} - 1).\end{aligned} \tag{27}$$

Therefore, for the rth service to be rejected, we need $\sum_{n\in\mathcal{N}} C(f^r)\tilde{x}(n) \ge \beta\rho^r$, $\forall S^r \in \sigma$, which translates to $\frac{1}{K}(e^{\phi}/e - 1) \ge \beta\frac{\eta_{\max}}{\eta_{\min}}$, i.e., $\phi \ge \ln(2\beta K\frac{\eta_{\max}}{\eta_{\min}} + 2)$. That is, ϕ is set such that the admission mechanism rejects any request that would violate the processing resources of an NFV node. ∎

C. Proof for Theorem 1

Let ΔJ and ΔD be the change in the primal and dual cost in each iteration, respectively. Starting with $J = D = 0$, when the rth service request is accepted, the objective function of the dual formulation is increased by $\Delta D = \alpha\varrho^r + \beta\rho^r$. The objective function of the primal is increased by

$$\Delta J = \sum_{l\in\mathcal{L}} B(l)\big(\bar{x}^r(l) - \bar{x}^{r-1}(l)\big) + \sum_{n\in\mathcal{N}} C(n)\big(\tilde{x}^r(n) - \tilde{x}^{r-1}(n)\big) + z^r. \tag{28}$$

Substituting (22) and (23) in (28), we obtain

$$\begin{aligned}\Delta J =& \sum_{l\in\mathcal{L}} (e^{\varphi\frac{d^r}{B(l)}} - 1)(\bar{x}^{r-1}(l) + 1/L)B(l)\\ &+ \sum_{n\in\mathcal{N}} (e^{\phi\frac{C(f^r)}{C(n)}} - 1)(\tilde{x}^{r-1}(n) + 1/K)C(n) + z^r. \end{aligned}\tag{29}$$

Using inequality $e^x - 1 \le x$ for $0 \le x \le 1$, we get

$$\Delta J \le \varphi \sum_{l\in\mathcal{L}} d^r(\bar{x}^{r-1}(l) + 1/L) + \phi\sum_{n\in\mathcal{N}} C(f^r)(\tilde{x}^{r-1}(n) + 1/K) + z^r. \tag{30}$$

From the admission mechanism, substituting z^r from (21) in (30), we obtain

$$\begin{aligned}\Delta J =&\varphi \sum_{l\in\mathcal{L}} d^r(\bar{x}^{r-1}(l) + 1/L) + \phi\sum_{n\in\mathcal{N}} C(f^r)(\tilde{x}^{r-1}(n) + 1/K)\\ &+ \max\Big(\alpha\varrho^r - \sum_{l\in P\cap\mathcal{L}} d^r\bar{x}(l), \beta\rho^r - \sum_{n\in P\cap\mathcal{N}} C(f^r)\tilde{x}(n)\Big) \end{aligned}\tag{31}$$

$$\begin{aligned}&\le\varphi\sum_{l\in\mathcal{L}} d^r(\bar{x}^{r-1}(l) + \frac{\alpha\varrho^r}{L}) + \phi\sum_{n\in\mathcal{N}} C(f^r)(\tilde{x}^{r-1}(n) + \frac{\beta\rho^r}{K})\\ &+\alpha\varrho^r + \beta\rho^r - \sum_{l\in P\cap\mathcal{L}} d^r\bar{x}(l) - \sum_{n\in P\cap\mathcal{N}} C(f^r)\tilde{x}(n) \end{aligned}\tag{32}$$

$$\begin{aligned}&=(\varphi - 1)\sum_{l\in\mathcal{L}} d^r\bar{x}^{r-1}(l) + \alpha\varrho^r + \beta\rho^r + (\phi - 1)\\ &\times\sum_{n\in\mathcal{N}} C(f^r)\tilde{x}^{r-1}(n) + \alpha\varphi\varrho^r\sum_{l\in\mathcal{L}}\frac{d^r}{L} + \beta\phi\rho^r\frac{1}{K}\sum_{n\in\mathcal{N}} C(f^r) \end{aligned}\tag{33}$$

$$\begin{aligned}&\le\alpha\varrho^r + \beta\rho^r + (\varphi - 1)\sum_{l\in\mathcal{L}} d^r\bar{x}^{r-1}(l)\\ &+(\phi - 1)\sum_{n\in\mathcal{N}} C(f^r)\tilde{x}^{r-1}(n) + \alpha\varrho^r\varphi + \beta\rho^r\phi. \end{aligned}\tag{34}$$

Since the rth service request is accepted, i.e., $\sum_{l\in\mathcal{L}} d^r\bar{x}^{r-1}(l) \le \alpha\varrho^r$ and $\sum_{n\in\mathcal{N}} C(f^r)\tilde{x}^{r-1}(n) \le \beta\rho^r$, inequality (34) becomes

$$\Delta J \le 2\alpha\varrho^r\varphi + 2\beta\rho^r\phi \tag{35}$$

$$\le 2(\alpha\varrho^r + \beta\rho^r)\max\{\varphi, \phi\}. \tag{36}$$

It is shown in Lemmata 1 and 2 that the online algorithm ensures that the transmission and processing resource constraints are always satisfied, where variables $\bar{x}(l)$, $\tilde{x}(n)$, and z^r are designed such that a feasible primal solution is maintained. Therefore, using weak duality (i.e., $\Delta D \le \Delta J$) and from (36), a competitive performance of $O\big(\max\{\varphi, \phi\}\big)$ is concluded. ■

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