Análisis Factorial

Omar Sanchez Hernandez

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Introducción

El análisis factorial sirve para explicar un conjunto de variables observadas a través de un grupo de variables no observadas.Ayuda a la reducción de dimension. Se utiliza en la reducción de los datos para identificar un pequeño número de factores que explique la varianza observada en un número mayor de variables manifestadas.

1.- Lectura de la matriz de datos

Para el desarrollo de este ejemplo se va a trabajar con la matriz de datos ***state.x77*** que se encuentra pre cargada dentro del paquete *datasets* de R. La base de datos contiene datos sobre algunos indicadores como la cantidad de población o ingreso pormedio por estado en Estados Unidos.

```
library(datasets)
x<-as.data.frame(state.x77)
```

#2.- Quitar los espacios de los nombres

```
colnames(x)[4]="Life.Exp"
colnames(x)[6]= "HS.Grad"
```

#3.- Separa n (estados) y p (variables)

```
n<-dim(x)[1]
p<-dim(x)[2]
```

Caracteristicas de los datos

```
dim(x)
```

```
## [1] 50  8
```

```
str(x)
```

```
## 'data.frame':	50 obs. of  8 variables:
##  $ Population: num  3615 365 2212 2110 21198 ...
##  $ Income    : num  3624 6315 4530 3378 5114 ...
##  $ Illiteracy: num  2.1 1.5 1.8 1.9 1.1 0.7 1.1 0.9 1.3 2 ...
##  $ Life.Exp  : num  69 69.3 70.5 70.7 71.7 ...
##  $ Murder    : num  15.1 11.3 7.8 10.1 10.3 6.8 3.1 6.2 10.7 13.9 ...
##  $ HS.Grad   : num  41.3 66.7 58.1 39.9 62.6 63.9 56 54.6 52.6 40.6 ...
##  $ Frost     : num  20 152 15 65 20 166 139 103 11 60 ...
##  $ Area      : num  50708 566432 113417 51945 156361 ...
```

```
colnames(x)
```

```
## [1] "Population" "Income"     "Illiteracy" "Life.Exp"   "Murder"
## [6] "HS.Grad"    "Frost"      "Area"
```

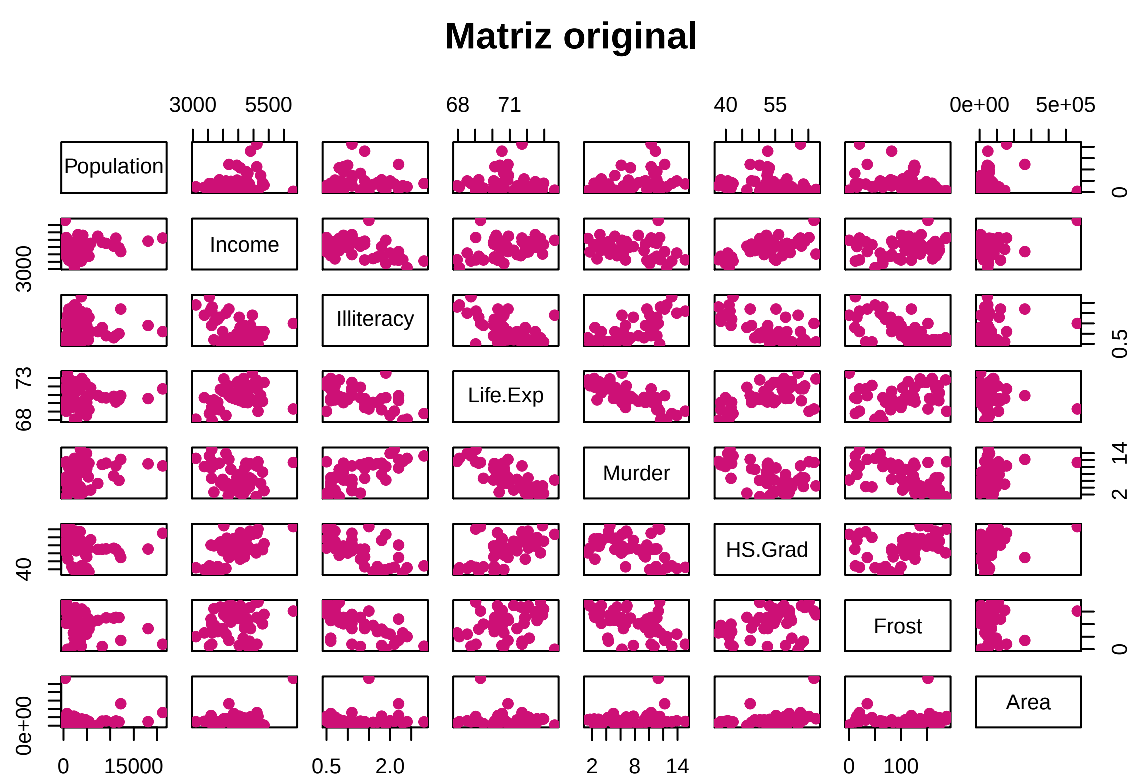
```
anyNA(x)
```

```
## [1] FALSE
```

La matriz de datos tiene 50 filas, 8 variables numericas y no posee datos faltantes.

#4.- Generacion de un scater plot para la visualización de variables originales.

```
pairs(x, col="#CD1076", pch=19, main="Matriz original")
```



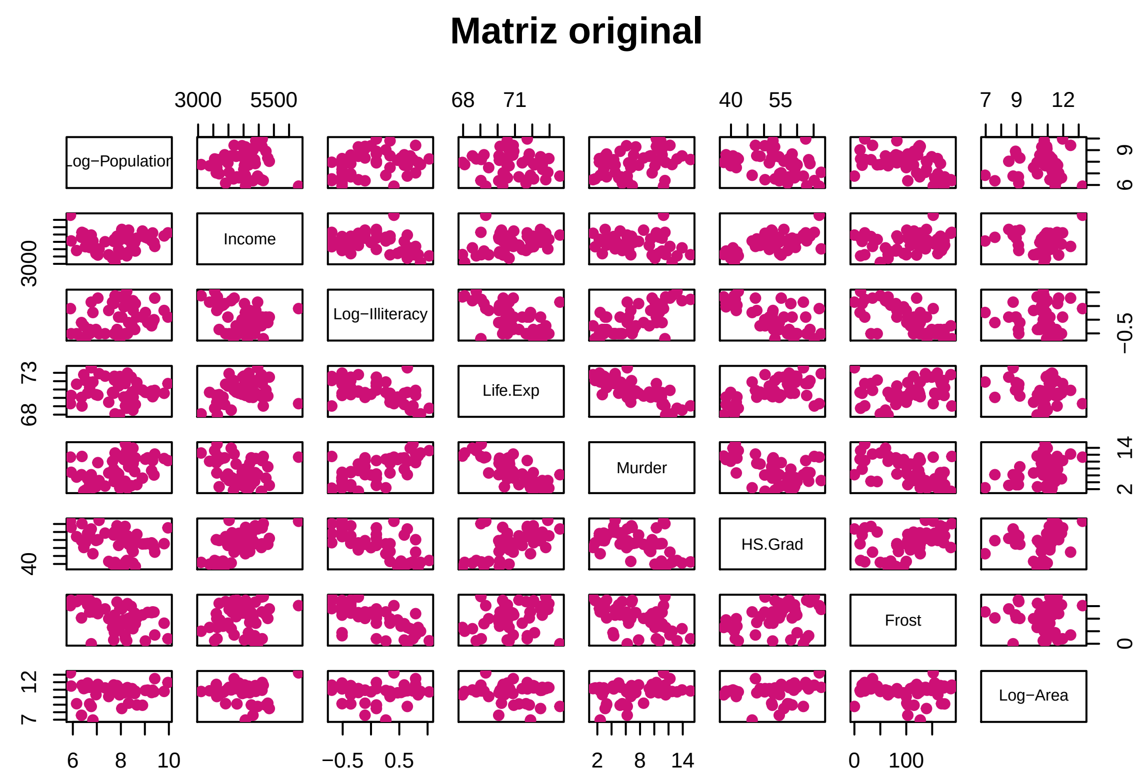
Transformación de alguna varibles

1.- Aplicamos logaritmo para las columnas 1,3 y 8

```
x[,1]<-log(x[,1])
colnames(x)[1]<-"Log-Population"
x[,3]<-log(x[,3])
colnames(x)[3]<-"Log-Illiteracy"
x[,8]<-log(x[,8])
colnames(x)[8]<-"Log-Area"
```

Gráfico scater para la visualizacion de la matriz original con 3 variables que se incluyeron

```
pairs(x,col="#CD1076", pch=19, main="Matriz original")
```



Nota

Como las variables tiene diferentes unidades de medida, se va a implementar la matriz de correlaciones para estimar la matriz de carga.

Reduccion de la dimensionalidad

Análsis Factorial de componentes principales (PCFA)

1.- Calcular la matriz de medias y de correlaciones

Matriz de medias

```r
mu<-colMeans(x)
mu
```

```
## Log-Population          Income Log-Illiteracy       Life.Exp         Murder
##   7.863443e+00   4.435800e+03   3.128251e-02   7.087860e+01   7.378000e+00
##        HS.Grad           Frost       Log-Area
##   5.310800e+01   1.044600e+02   1.066237e+01
```

Matriz de correlaciones

```r
R<-cor(x)
R
```

```
##                Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population     1.00000000  0.034963788     0.28371749 -0.1092630  0.3596542
## Income             0.03496379  1.000000000    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy     0.28371749 -0.351477726     1.00000000 -0.5699943  0.6947320
## Life.Exp          -0.10926301  0.340255339    -0.56999432  1.0000000 -0.7808458
## Murder             0.35965424 -0.230077610     0.69473198 -0.7808458  1.0000000
## HS.Grad           -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost             -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area           0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                   HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         1.0000000  0.36677970  0.196743429
## Frost           0.3667797  1.00000000 -0.021211992
## Log-Area        0.1967434 -0.02121199  1.000000000
```

2.- Reducción de la dimensionalidad mediante

Análisis factorial de componentes principales (PCFA).

1.- Calcular los valores y vectores propios.

```r
eR<-eigen(R)
```

2.- Valores propios

```r
eigen.val<-eR$values
eigen.val
```

```
## [1] 3.6796976 1.3201021 1.1357357 0.7517550 0.6168266 0.2578511 0.1366186
## [8] 0.1014132
```

3.- Vectores propios

```r
eigen.vec<-eR$vectors
eigen.vec
```

```
##             [,1]        [,2]        [,3]       [,4]        [,5]       [,6]
## [1,] -0.23393451 -0.41410075  0.50100922  0.2983839  0.58048485  0.0969034
## [2,]  0.27298977 -0.47608715  0.24689968 -0.6449631  0.09036625 -0.3002708
## [3,] -0.45555443  0.04116196  0.12258370 -0.1824471 -0.32684654 -0.6084112
## [4,]  0.39805075 -0.04655529  0.38842376  0.4191134 -0.26287696 -0.3565095
## [5,] -0.44229774 -0.27640285 -0.21639177 -0.2610739  0.02383706  0.1803894
## [6,]  0.41916283 -0.36311753 -0.06807465 -0.1363534 -0.34015424  0.3960855
## [7,]  0.36358674  0.21893783 -0.37542494 -0.1299519  0.59896253 -0.3507630
## [8,] -0.03545293 -0.58464797 -0.57421867  0.4270918 -0.06252285 -0.3012063
##            [,7]        [,8]
## [1,] -0.1777562 -0.23622413
## [2,]  0.3285840  0.12483849
## [3,] -0.3268997 -0.39825363
## [4,] -0.3013983  0.47519991
## [5,] -0.4562245  0.60970476
## [6,] -0.4808140 -0.40675672
## [7,] -0.4202943 -0.06001175
## [8,]  0.2162424 -0.05831177
```

4.- Calcular la proporcion de variabilidad

```
prop.var<-eigen.val/sum(eigen.val)
prop.var
```

```
## [1] 0.45996220 0.16501277 0.14196697 0.09396938 0.07710332 0.03223139 0.01707733
## [8] 0.01267665
```

5.- Calcular la proporcion de variabilidad acumulada

```
prop.var.acum<-cumsum(eigen.val)/sum(eigen.val)
prop.var.acum
```

```
## [1] 0.4599622 0.6249750 0.7669419 0.8609113 0.9380146 0.9702460 0.9873233
## [8] 1.0000000
```

Estimacion de la matriz de carga

Nota

se estima la matriz de carga usando los autovalores y autovectores.

Se aplica la rotación varimax

Primera estimación de Lamda mayuscula se calcula multiplicando la matriz de los 3 primeros autovectores por la matriz diagonal formada por la raiz cuadrada de los primeros 3 autovalores.

```
L.est.1<-eigen.vec[,1:3] %*% diag(sqrt(eigen.val[1:3]))
L.est.1
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.44874575 -0.47578394  0.53393005
## [2,]  0.52366367 -0.54700365  0.26312322
## [3,] -0.87386900  0.04729332  0.13063856
## [4,]  0.76356236 -0.05349003  0.41394671
## [5,] -0.84843932 -0.31757498 -0.23061066
## [6,]  0.80406070 -0.41720642 -0.07254777
## [7,]  0.69745163  0.25155014 -0.40009375
## [8,] -0.06800771 -0.67173536 -0.61195003
```

Rotación varimax

```
L.est.1.var<-varimax(L.est.1)
L.est.1.var
```

```
## $loadings
##
## Loadings:
##      [,1]   [,2]   [,3]
## [1,]                0.840
## [2,]  0.785 -0.106  0.121
## [3,] -0.665         0.583
## [4,]  0.763  0.384 -0.168
## [5,] -0.573 -0.528  0.517
## [6,]  0.825 -0.202 -0.323
## [7,]  0.281        -0.794
## [8,]        -0.906
##
##                 [,1]  [,2]  [,3]
## SS loadings    2.744 1.300 2.091
## Proportion Var 0.343 0.163 0.261
## Cumulative Var 0.343 0.506 0.767
##
## $rotmat
##            [,1]      [,2]       [,3]
## [1,]  0.7824398 0.1724744 -0.5983649
## [2,] -0.5274231 0.6944049 -0.4895169
## [3,]  0.3310784 0.6986089  0.6342970
```

Estimación de la matriz de los errores

1.- Estimación de la matriz de perturbaciones

```
Psi.est.1<-diag(diag(R-as.matrix(L.est.1.var$loadings)%*% t(as.matrix(L.est.1.var$loadings))))
Psi.est.1
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]      [,7]
## [1,] 0.2871756 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [2,] 0.0000000 0.3573295 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2170499 0.0000000 0.0000000 0.000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.2427595 0.0000000 0.000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1261156 0.000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.174162 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.2902087
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.1696637
```

2.- Se utiliza el método Análisis de factor principal (PFA) para estimación de autovalores y autovectores

```
RP<-R-Psi.est.1
RP
```

```
##                 Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population      0.71282441  0.034963788     0.28371749 -0.1092630  0.3596542
## Income              0.03496379  0.642670461    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy      0.28371749 -0.351477726     0.78295012 -0.5699943  0.6947320
## Life.Exp           -0.10926301  0.340255339    -0.56999432  0.7572405 -0.7808458
## Murder              0.35965424 -0.230077610     0.69473198 -0.7808458  0.8738844
## HS.Grad            -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost              -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area            0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                   HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         0.8258380  0.36677970  0.196743429
## Frost           0.3667797  0.70979126 -0.021211992
## Log-Area        0.1967434 -0.02121199  0.830336270
```

Calculo de la matriz de autovalores y autovectores

```
eRP<-eigen(RP)
```

Autovalores

```
eigen.val.RP<-eRP$values
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

Autovectores

```
eigen.vec.RP<-eRP$vectors
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

Proporcion de variabilidad

```
prop.var.RP<-eigen.val.RP/ sum(eigen.val.RP)
prop.var.RP
```

```
## [1]  0.564152306  0.180134556  0.143675179  0.079382934  0.057632455
## [6]  0.004585668 -0.011014811 -0.018548286
```

Proporcion de variabilidad acumulada

```
prop.var.RP.acum<-cumsum(eigen.val.RP)/ sum(eigen.val.RP)
prop.var.RP.acum
```

```
## [1] 0.5641523 0.7442869 0.8879620 0.9673450 1.0249774 1.0295631 1.0185483
## [8] 1.0000000
```

Estimación de la matriz de cargas

con rotación varimax

```
L.est.2<-eigen.vec.RP[,1:3] %*% diag(sqrt(eigen.val.RP[1:3]))
L.est.2
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.42621819 -0.27609775  0.56228420
## [2,]  0.48528446 -0.36092954  0.32467098
## [3,] -0.84791581  0.08163995  0.10816670
## [4,]  0.73812189  0.02688907  0.36866093
## [5,] -0.84699944 -0.34227865 -0.12211117
## [6,]  0.78817342 -0.40399024  0.04935203
## [7,]  0.66112453  0.12457105 -0.40191996
## [8,] -0.06868291 -0.77165602 -0.36531090
```

Rotacion varimax

```
L.est.2.var<-varimax(L.est.2)
```

Estimación de la matriz de covarianzas de los errores.

```r
Psi.est.2<-diag(diag(R-as.matrix(L.est.2.var$loadings)%*% t(as.matrix(L.est.2.var$loadings))))
Psi.est.2
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4259446 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [2,] 0.0000000 0.5288176 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2626737 0.0000000 0.0000000 0.0000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.3185422 0.0000000 0.0000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1505261 0.0000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.2131389 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.3858568
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.2663776
```

Obtencion de los scores de ambos métodos

PCFA

```r
FS.est.1<-scale(x)%*% as.matrix(L.est.1.var$loadings)
FS.est.1
```

```
##                       [,1]          [,2]       [,3]
## Alabama        -5.84072356 -1.3993671511  4.0008109
## Alaska          2.12443806 -3.6163397014 -1.3435941
## Arizona        -0.77245459 -1.1030150088  1.7864181
## Arkansas       -4.26961555 -0.1287634469  1.8680205
## California      1.57843978 -1.6386262821  3.0959757
## Colorado        3.35619481 -0.5747409714 -1.9955520
## Connecticut     2.96609993  2.5265114588 -1.0120520
## Delaware        0.15111765  2.2707877284 -1.3473631
## Florida        -0.91278118 -0.8518787165  3.2141818
## Georgia        -5.10406769 -1.5374188978  3.5972606
## Hawaii          1.68679592  2.0782245763  0.6972161
## Idaho           1.93931571  0.0374520725 -2.6403015
## Illinois        0.36572803 -0.9730363911  1.3246992
## Indiana         0.69870165  0.1740586327 -0.1660034
## Iowa            3.77325852  0.8634090197 -2.4308546
## Kansas          3.22079390  0.2206198504 -1.7333568
## Kentucky       -3.97957229 -0.1711842990  1.8581455
## Louisiana      -6.15095874 -1.1449716511  4.2193388
## Maine           0.38912287  0.9352663421 -2.8385772
```

```
## Maryland        0.54556931  0.6481615589  0.7313943
## Massachusetts   1.95531363  1.9508870989 -0.0699601
## Michigan        0.06109118 -0.8995742724  1.1610156
## Minnesota       3.83625590  0.7199310360 -2.2609012
## Mississippi    -6.73875213 -1.1336057288  3.0124928
## Missouri       -0.63621057 -0.5673516660  0.5606479
## Montana         1.70022911 -0.7530855537 -2.9827203
## Nebraska        3.31393569  0.5702899251 -2.6630094
## Nevada          1.83953234 -2.1624547546 -2.8632403
## New Hampshire   1.76672303  1.8835104424 -3.2522623
## New Jersey      1.23076573  1.5154423999  0.6483326
## New Mexico     -2.42369795 -1.2184859435  0.1095350
## New York       -0.55160991 -0.8431042602  2.9025469
## North Carolina -4.53932589 -0.7126552652  2.8168209
## North Dakota    3.26810535  1.0664889529 -3.5180166
## Ohio            0.67643704 -0.0394642439  0.5816740
## Oklahoma       -0.43628926  0.0293430043  0.2108486
## Oregon          2.64633236 -0.0126633017 -0.6563722
## Pennsylvania   -0.06313819  0.0425262164  0.8538298
## Rhode Island    0.25059508  4.0533333045 -1.3779994
## South Carolina -6.20030464 -0.7067780563  3.0142562
## South Dakota    2.51505516  0.8539599931 -3.9694575
## Tennessee      -3.75602365 -0.3764569265  2.4225536
## Texas          -2.74825842 -2.0176142597  4.0126966
## Utah            3.40911641  0.2638533973 -3.0642167
## Vermont         1.26368503  1.7670538099 -3.5748058
## Virginia       -1.45435214 -0.4332714574  1.8388594
## Washington      2.95298764  0.0002978623 -0.1436737
## West Virginia  -3.41599674  0.5649932020  0.5132111
## Wisconsin       2.58972274  0.8701285803 -1.5397225
## Wyoming         1.92267355 -0.8906222579 -3.6087703
```

PFA

```
FS.est.2<-scale(x)%*% as.matrix (L.est.2.var$loadings)
FS.est.2
```

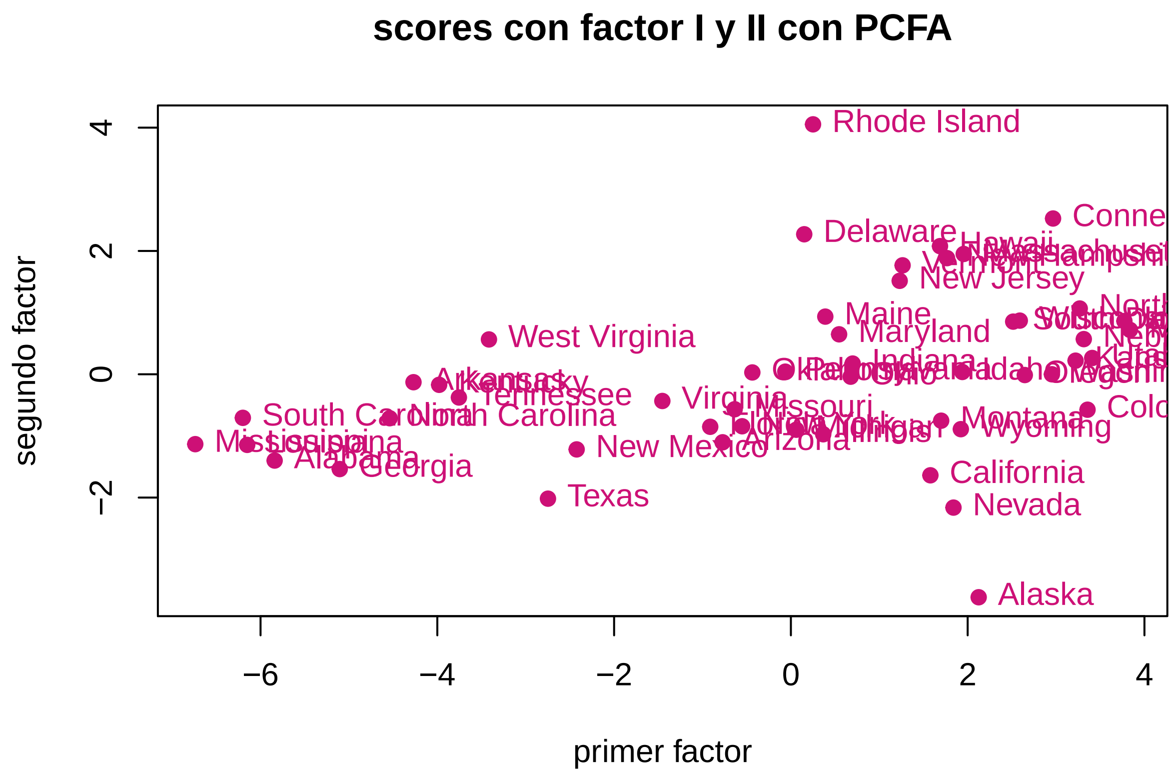
```
##                       [,1]         [,2]       [,3]
## Alabama        -5.69766092 -1.133005866  3.9030908
## Alaska          1.77921500 -3.310049553 -1.2425530
## Arizona        -0.80948635 -1.007423566  1.6833688
## Arkansas       -4.04451164 -0.036340306  1.8899610
## California      1.28900772 -1.589528660  2.7938220
## Colorado        3.21256763 -0.645092519 -1.9103448
## Connecticut     2.85639977  2.291700954 -1.1152442
## Delaware        0.22491218  2.168332191 -1.3109174
## Florida        -1.04778981 -0.760012075  2.9630979
## Georgia        -5.04193484 -1.243399542  3.4848855
## Hawaii          1.64548810  1.848120424  0.5487863
## Idaho           1.99602286 -0.067186945 -2.4442739
## Illinois        0.17329771 -0.870927790  1.1838509
```

```
## Indiana         0.66348403  0.140717116 -0.1900850
## Iowa            3.70915552  0.657976435 -2.3698485
## Kansas          3.13617617  0.071725764 -1.6894853
## Kentucky       -3.82119443 -0.051170443  1.8492550
## Louisiana      -5.97309240 -0.880509145  4.1021292
## Maine           0.58567717  0.845398887 -2.6098620
## Maryland        0.40855637  0.650876372  0.5867974
## Massachusetts   1.91021424  1.761365924 -0.1964750
## Michigan       -0.07208772 -0.823049544  1.0671998
## Minnesota       3.74953682  0.518054623 -2.2104937
## Mississippi    -6.45121865 -0.852611917  3.0320154
## Missouri       -0.64446964 -0.519762510  0.5472506
## Montana         1.72574501 -0.752576236 -2.7507980
## Nebraska        3.28773039  0.392513546 -2.5439122
## Nevada          1.69672312 -1.994626548 -2.6292009
## New Hampshire   1.87991014  1.704867403 -3.0632652
## New Jersey      1.10782292  1.425042094  0.4638907
## New Mexico     -2.26112419 -1.086582245  0.2653217
## New York       -0.72255151 -0.744949928  2.6624378
## North Carolina -4.42441540 -0.513264749  2.7372284
## North Dakota    3.22068093  0.897031063 -3.3556310
## Ohio            0.59453054 -0.051780182  0.4905274
## Oklahoma       -0.36512462  0.000708499  0.2244101
## Oregon          2.56050584 -0.129810062 -0.6934180
## Pennsylvania   -0.10451900  0.054229408  0.7553645
## Rhode Island    0.40356926  3.785456289 -1.3760426
## South Carolina -5.98815271 -0.435831413  2.9745853
## South Dakota    2.60764548  0.683975660 -3.7117087
## Tennessee      -3.63769564 -0.249263663  2.3593673
## Texas          -2.80670233 -1.827474308  3.8156526
## Utah            3.44131011  0.069209103 -2.8669774
## Vermont         1.44160727  1.580578146 -3.3086066
## Virginia       -1.50774364 -0.328200587  1.7151967
## Washington      2.81601549 -0.109025242 -0.2503494
## West Virginia  -3.18525955  0.632647668  0.5745805
## Wisconsin       2.55487697  0.699000994 -1.5141208
## Wyoming         1.92835024 -0.866073018 -3.3204601
```

Graficamos ambos scores

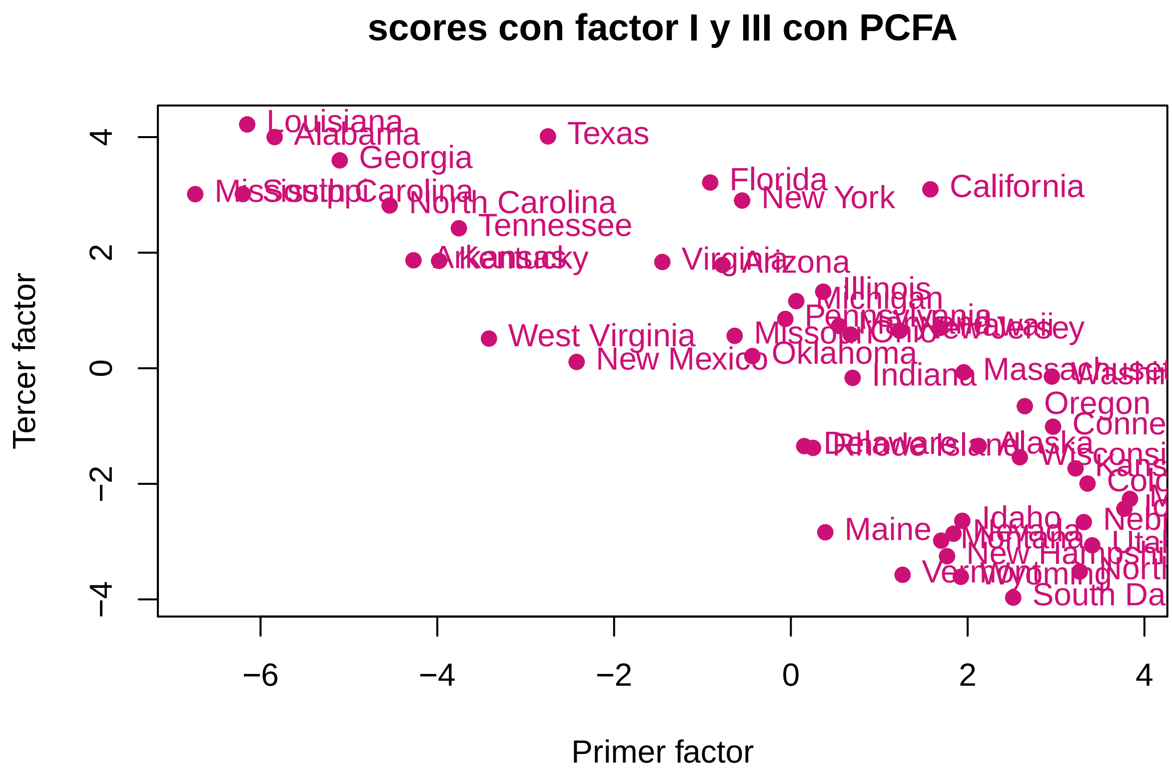
Factor I y II

```r
pl1<-plot(FS.est.1[,1], FS.est.1[,2], xlab="primer factor",
     ylab="segundo factor", main="scores con factor I y II con PCFA",
     pch=19, col="#CD1076")
text(FS.est.1[,1], FS.est.1[,2], labels = rownames(x), pos=4, col="#CD1076")
```



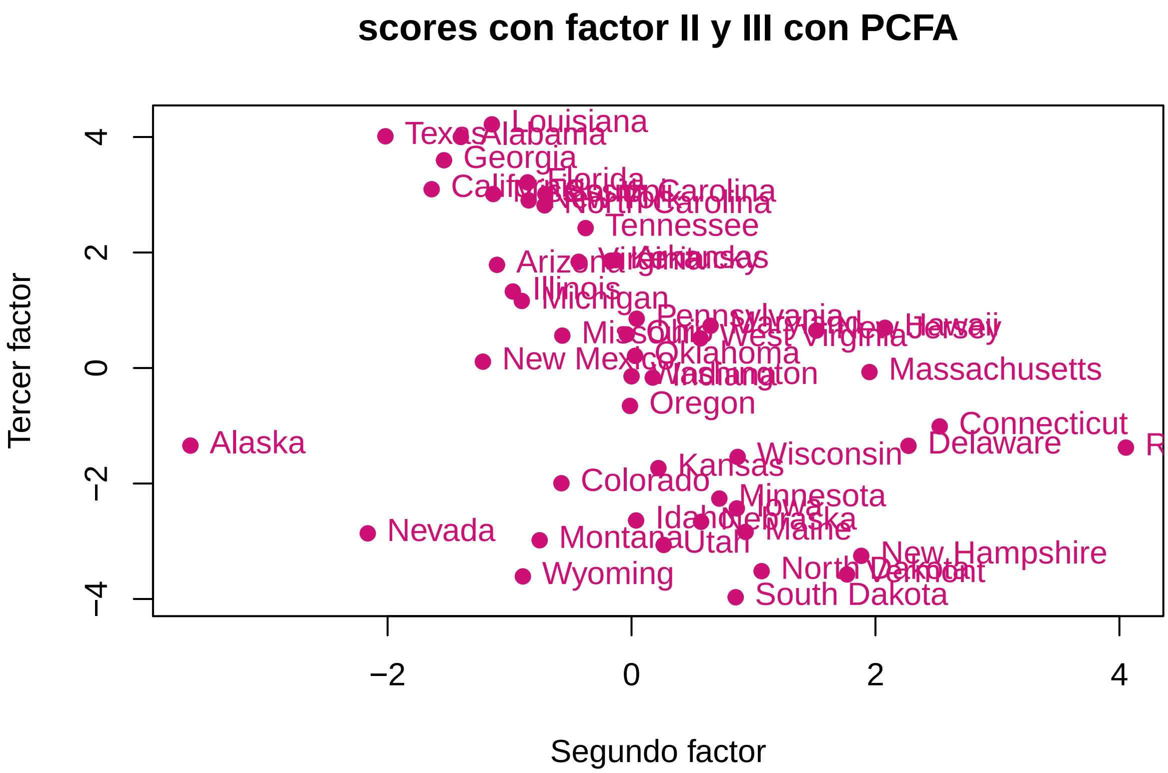
Factor I y III

```
pl2<-plot(FS.est.1[,1], FS.est.1[,3], xlab="Primer factor",
     ylab="Tercer factor", main="scores con factor I y III con PCFA",
     pch=19, col="#CD1076")
text(FS.est.1[,1], FS.est.1[,3], labels = rownames(x), pos=4, col="#CD1076")
```



Factor II y III

```r
pl3<-plot(FS.est.1[,2], FS.est.1[,3], xlab="Segundo factor",
      ylab="Tercer factor", main="scores con factor II y III con PCFA",
      pch=19, col="#CD1076")
text(FS.est.1[,2], FS.est.1[,3], labels = rownames(x), pos=4, col="#CD1076")
```


scores con factor II y III con PCFA
Tercer factor
Segundo factor
4
2
0
−2
−4
−2
0
2
4
Louisiana
Texas
Alabama
Georgia
Florida
North Carolina
Tennessee
Arizona
Illinois
Michigan
Pennsylvania
Missouri
West Virginia
Hawaii
New Mexico
Oklahoma
Washington
Massachusetts
Oregon
Connecticut
Alaska
Delaware
Wisconsin
Kansas
Colorado
Minnesota
Iowa
Nevada
Montana
Utah
Maine
New Hampshire
Wyoming
Vermont
South Dakota