II Exact method:-

given:-

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

دوال في x و y ←

لا يمكن فصلهم عن بعض

*الى قدام dx $M(x,y)$ - $N(x,y)$ = الى قدام dy

$\frac{\partial M}{\partial y}$ $\frac{\partial N}{\partial x}$ =

$$\boxed{\text{If } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \text{Exact}}$$

Sol.

$$\int M(x,y)\,dx + \int N(x,y)\,dy = 0$$

*① $xy\,dx + \frac{x^2}{2} + y + 3\,dy = 0$

$M(x,y) = yx$ $N(x,y) = \frac{x^2}{2} + y + 3$

$\frac{\partial M}{\partial y} = x$ $\frac{\partial N}{\partial x} = x$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ Exact DE

$\rightarrow \int xy\,dx + \int \frac{x^2}{2} + y + 3\,dy$

$$y\frac{x^2}{2} + \frac{x^2}{2}y + \frac{1}{2}y^2 + 3y = C$$

$\rightarrow \frac{x^2}{2}y + \frac{1}{2}y^2 + 3y = C$ المكرر ناخد واحد فقط

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$f(x,y)=x^2+y^2+xy+3y+3x+2.$

$f(x)=\frac{\partial f}{\partial x}=$ (x)التفاضل بالنسبة

$f(y)=\frac{\partial f}{\partial y}$ (y)التفاضل بالنسبة

$f(x)=2x+y+3$ $\qquad$ $f(y)=2y+x+3$

$\boxed{2}$ linear method:- application:- L.R.C

$e^{\alpha \ln x}=e^{\ln x^{\alpha}}=x^{\alpha}$

① $\frac{dy}{dx}+P(x)y=Q(x)$

الشروط ← $P(x), Q(x) \neq 0$

← $\frac{dy}{dx}$ ← المعامل $=1$

① $m(x)=e^{\int P(x)dx}$

2) Sol.

$m(x)\cdot y=\int m(x)\cdot Q(x)$

Ex:- $\frac{dy}{dx}+3y=e^{5x}$

1) $P(x)=3$ $\quad$ $,Q(x)=e^{5x}$

2) $m(x)=e^{\int 3dx}=e^{3x}$

3) $e^{3x}y=\int e^{3x}\cdot e^{5x}=\frac{1}{8}\int 8e^{8x}$

$\therefore e^{3x}y=\frac{1}{8}e^{8x}+c \Leftarrow \boxed{*e^{-3x}}$

$y=\frac{1}{8}e^{5x}+ce^{-3x}$

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$\boxed{*}$ $\cancel{\frac{dy}{dx} = 1}$

$\rightarrow \frac{dy}{dx} - \frac{1}{x} y = x^2$ $\quad P(x) = \frac{-1}{x}$. $\quad \phi(x) = x^2$

① $m(x) = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = x^{-1}$

$m(x) \cdot y = \int m(x) \cdot \phi(x) dx$

$\frac{1}{x} y = \int \frac{1}{x} \cdot x^2$

$= \int x \, dx$

$\frac{1}{x} y = \frac{1}{2} x^2 + c$ $\quad \boxed{\therefore y = (\frac{x^2}{2} + c) x}$

③ Bernaulli's method :-

$* \frac{dy}{dx} + P(x) y = \phi(x) \cdot y^n$ $\rightarrow$ nonlinear

$y^{-n} \frac{dy}{dx} + y^{-n+1} P(x) = \phi(x)$

Put :- $y^{-n+1} = u$

$(-n+1) y^{-n} \cdot \frac{dy}{dx} = \frac{du}{dx}$

$y^{-n} \cdot \frac{dy}{dx} = \frac{1}{(-n+1)} \cdot \frac{du}{dx}$

$\therefore \frac{1}{(1-n)} \cdot \frac{du}{dx} + P(x) u = \phi(x)$ $\quad * (-n+1)$

$\frac{du}{dx} + P(x) \cdot (-n+1) u = \phi(x)(-n+1)$

ex:- $\frac{dy}{dx} + \frac{1}{x} y = x y^2$

$y^{-2} \frac{dy}{dx} + \frac{1}{xy} = x \quad \longleftarrow \quad y^2 \frac{dy}{dx} = \frac{-du}{dx}$

let $y^{-1} = u$

$-y^{-2} \frac{dy}{dx} = \frac{du}{dx}$

$-\frac{du}{dx} + \frac{1}{x} u = x$

$\frac{du}{dx} - \frac{1}{x} u = -x$

$m(x) = e^{\int P(x) dx} = e^{\int -\frac{1}{x} dx}$

$= e^{-\ln(x)} = \frac{1}{x}$

$m(x) \cancel{y}^{u} = \int m(x) \phi(x) dx$

$\frac{\cancel{y}^{u}}{x} = \int \frac{-1}{x} * x \, dx$

$\frac{\cancel{y}^{u}}{x} = -x + C$

$\therefore \cancel{y}^{u} = (-x + c) x$