El-Manzala Higher Institute of Engineering and Technology

Fundamentals of Electrical Engineering

First YEAR

By

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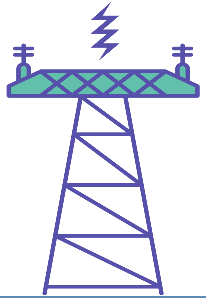
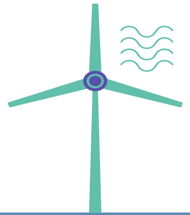
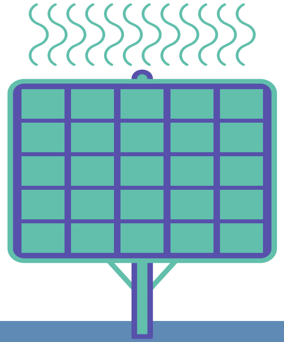






Sinusoidal Alternating Voltage and Current

Chapter 7

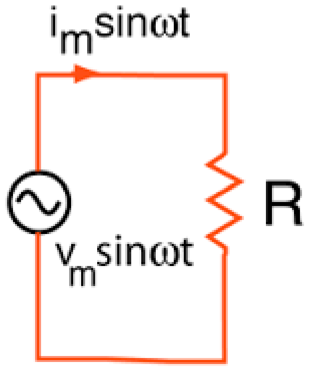
Chapter Content

CH7: Sinusoidal Alternating Voltage and Current

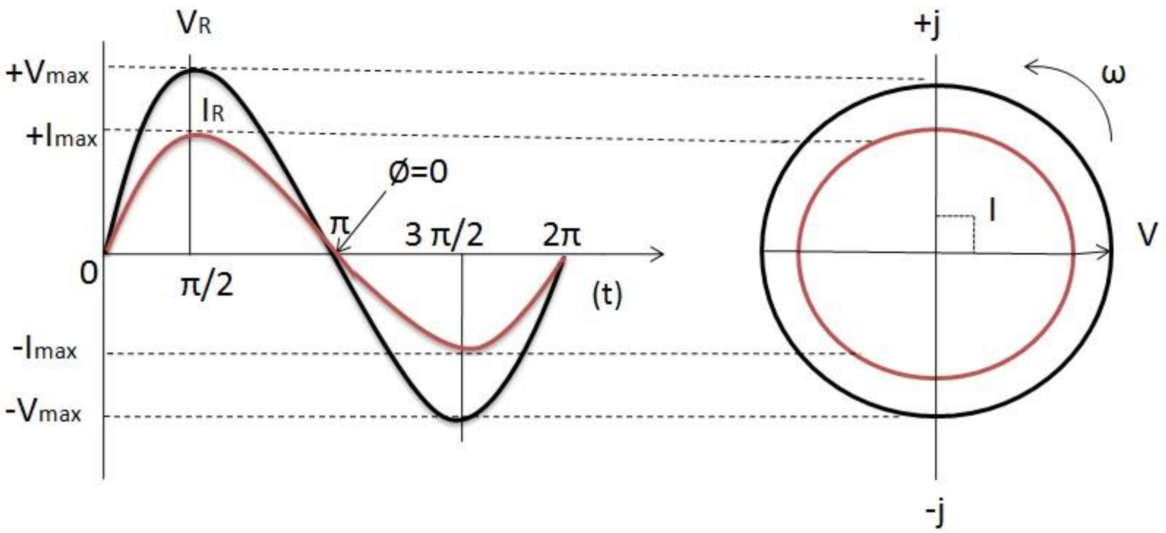
1. Resistor Sinusoidal Response
2. Effective Value
3. Inductor Sinusoidal Response
4. Capacitor Sinusoidal Response

2. Resistor Sinusoidal Response

- Ohm law $V = I R$ $\qquad$ $i = v/R$
- $v = Vm \sin(wt+\Theta)$
- $i = \frac{Vm}{R} \sin(wt+\Theta) \Rightarrow i = Im \sin(wt+\Theta)$
- The resistor current and voltage are in phase
- The current peak is lower than the voltage peak
- $P = IV = i*v = Vm\, Im \sin^2(wt+\Theta)$ $\qquad$ $\sin^2(wt+\Theta) = \frac{1-\cos 2X}{2}$
- $P = \frac{Vm*Im}{2} - \frac{Vm*Im}{2} \cos(2wt+2\Theta)$



2. Resistor Sinusoidal Response



2. Resistor Sinusoidal Response

- The <u>instantaneous</u> resistor power can never be negative. why?
- The power never being negative means that a resistor never delivers power to a circuit. Instead, resistor dissipates as heat all energy it receives
- So the term $-\frac{Vm * Im}{2}$ cos (2wt+ 2 ϴ)

$$P = \frac{Vm * Im}{2}$$

$$P = \frac{Vm\,^\wedge 2}{2\,R} = \frac{Im\,^\wedge 2}{2} R$$

3. Effective Value

- The effective value of a periodic voltage or current equals the value of a dc voltage or current that would produce the same average power loss in a resistor that the periodic voltage or current would.

- The effective value of a periodic voltage or current (V_{eff} or I_{eff}) is the positive dc voltage or current that produces the same average power loss in resistor.

$$\mathrm{P} = \frac{Veff\ ^\wedge 2}{R} = Im\ ^\wedge 2 * \mathrm{R} = \frac{Vm\ ^\wedge 2}{R}$$

$Veff\ ^\wedge 2 = (Vm\ ^\wedge 2)/(\ 2)$ ⟹ $Veff = \sqrt{}(Vm\ ^\wedge 2)/(\ 2)$

$$Veff = Vm / \sqrt{}\ 2$$

3. Effective Value

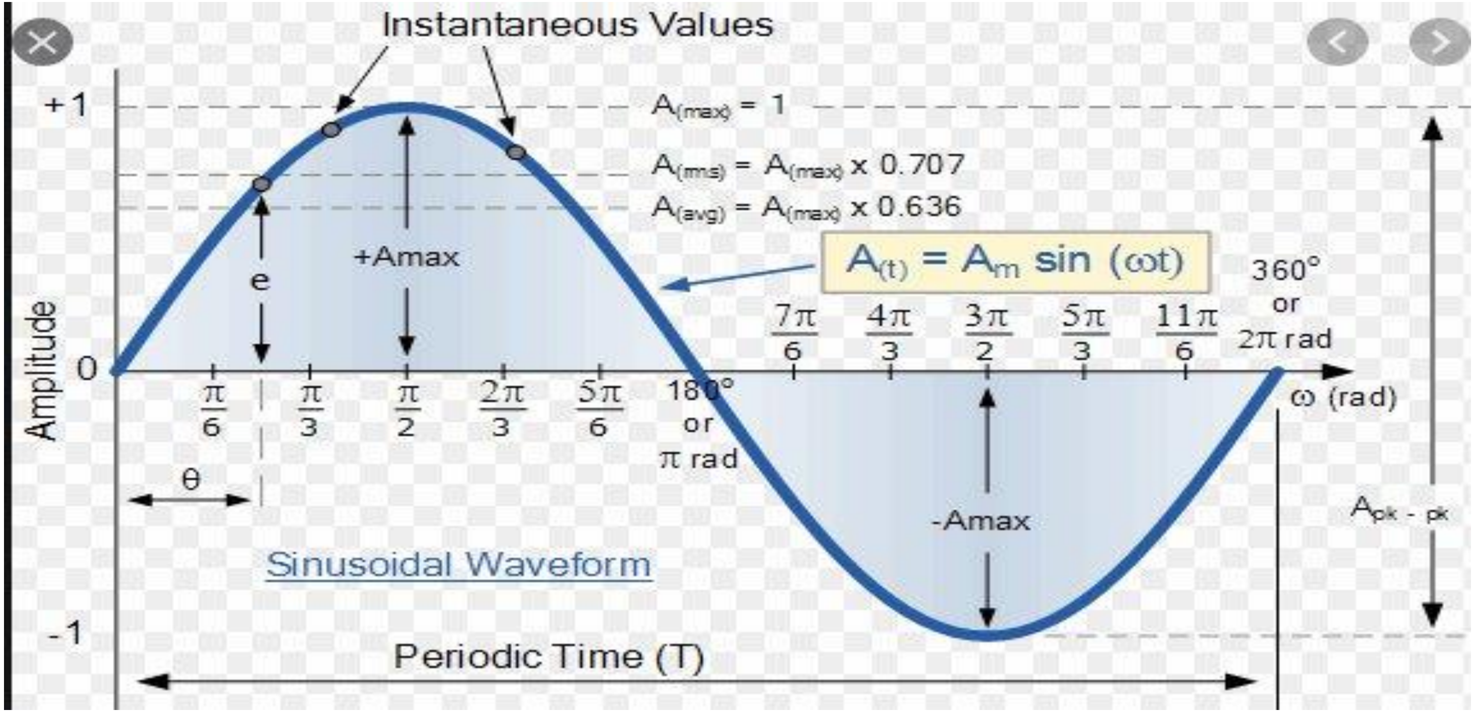
$$V_{eff} = V_m/\sqrt{2} = 0.707\, V_m$$
$$I_{eff} = I_m/\sqrt{2} = 0.707\, I_m$$

- Another name for effective value is root mean square (RMS)
- V_{rms}, I_{rms} are the same as V_{eff} and I_{eff}

- The name stems from a procedure for finding the effective or rms value, the procedure is to:

1. Square the periodic voltage or current
2. Find the average of this squared wave over one periode
3. Find the positive square root of this average

3. Effective Value



2. Effective Value

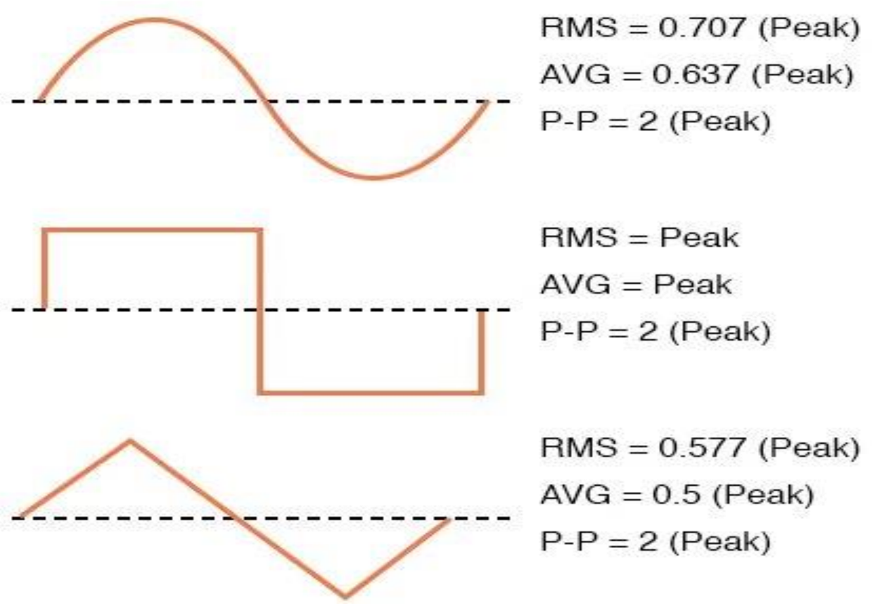
Solved problem

3. Effective Value

- Unfortunately, except for square type waves finding the area in step 2 require calculus.

- Incidentally , for sawtooth and a triangular wave, the result is the same effective value, as the peak value divided by √3

V_{eff} = Vrms = Vm /√ 3
(sawtooth and a triangular wave)



4. Inductor Sinusoidal Response

- If an inductor of L (henries) has a current $i = I\,m \sin(wt + \Theta)$ flowing through it , the voltage across the inductor is
- $v = L\frac{di}{dt} = L\frac{d}{dt}[\,Im \sin(wt+\Theta)] = L\,Im\,w\,\frac{d}{dt}[\sin(wt+\Theta)]$

$$v = L\,Im\,w \cos(wt+\Theta)$$

- The inductor voltage leads the inductor current by 90º $v = Vm \cos(wt + \Theta + 90)$
- The inductor current lags the inductor voltage by 90º

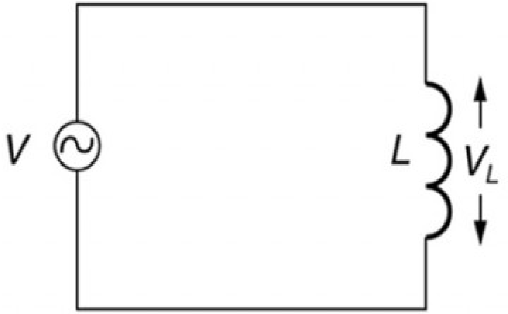
$$v = Vm \cos(wt+\Theta)$$

$$Vm = L\,Im\,w$$

$$X_L = Vm / Im = 2\,\pi\,f\,L$$

- The quantity wL is called <u>inductive reactance</u> of the inductor, with symbol (X_L)

4. Inductor Sinusoidal Response



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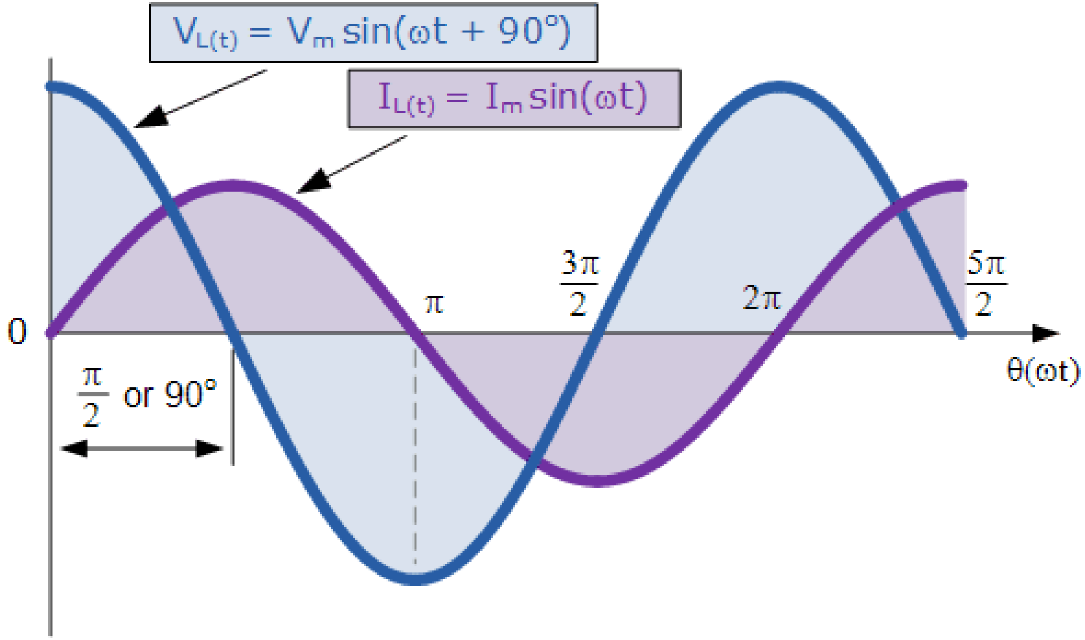


4. Inductor Sinusoidal Response

- Unlike resistance, inductive reactance depends on frequency.
- The greater the frequency, the greater inductive reactance value, the greater its current limiting action.

- With very low frequency, $X_L \approx$ Zero
- With very high frequency, $XL \approx \infty$, inductor is almost an open circuit

$$P = IV = i*v = Vm\ Im \cos(wt+\Theta) \sin(wt+\Theta) = \frac{Vm * Im}{2} \sin(2\,wt+2\,\Theta)$$

$$V_{eff} = V_m / \sqrt{2} \qquad I_{eff} = I_m / \sqrt{2}$$

$$P = V_{eff} * I_{eff} * \sin(2\,wt+2\,\Theta)$$

4. Inductor Sinusoidal Response

- The power is sinusoidal twice the voltage and current frequency.
- $P_{average} = 0$, a sinusoidal excited inductor absorbs zero average power
- P is positive, an inductor absorbs energy
- P is negative, an inductor returns energy to the circuit and acts as source
- Generally, over a period inductor deliver just as much energy as its receives

5. Capacitor Sinusoidal Response

- If a capacitor of C farads has a voltage v = Vm sin (wt + Ɵ) across it , the capacitor current is
- $i = C \frac{dv}{dt} = C \frac{d}{dt} [\ Vm \sin (wt+\Theta)] = C\ Vm\ w \frac{d}{dt} [\sin (wt+\Theta)]$

$$v = w\ C\ Vm \cos (wt+\Theta)$$

- The capacitor current leads the capacitor voltage by 90º
- The capacitor voltage lags the capacitor current by 90º

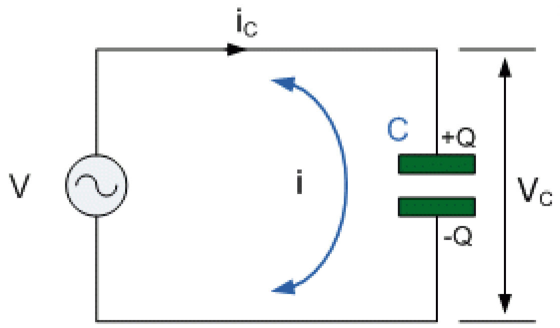
$$i = Im \cos (wt+\Theta)$$

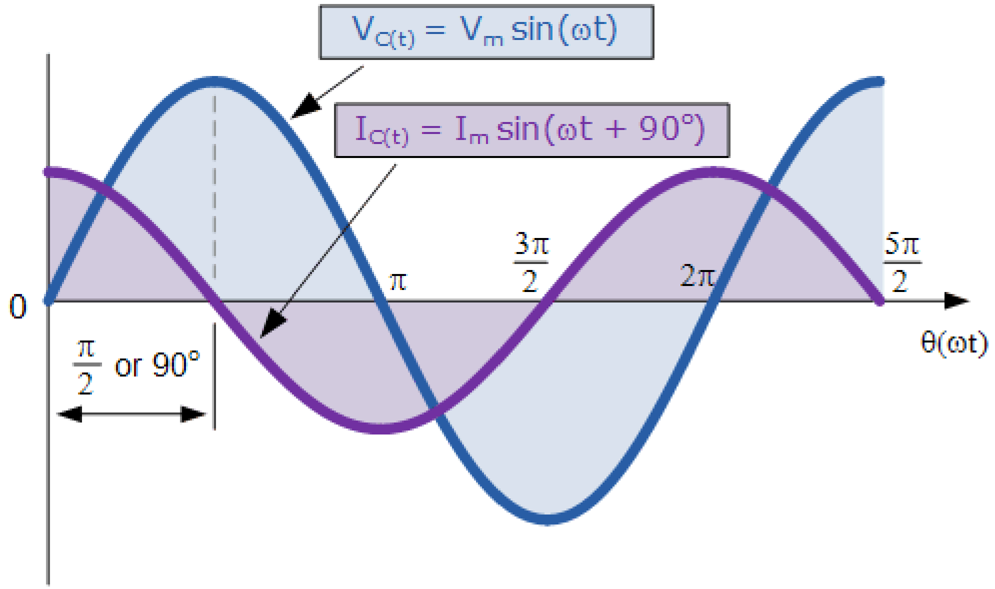
$$Im = w\ C\ Vm$$

$$XC = 1 / wC = \frac{1}{2 \pi f C}$$

- The quantity 1 / wC is called capacitive reactance of the capacitor, with symbol (X_c)

5. Capacitor Sinusoidal Response





5. Capacitor Sinusoidal Response

- Both X_L and X_C has the unit of ohm

  XC = −1 / wC The negative sign relates to phase shift

- Because 1 / wC is inversely proportional to frequency. The greater the frequency, the greater the current for the same voltage peak

- With very high frequency, $X_C \approx$ Zero , capacitor is almost a short circuit
- With very low frequency, XL $\approx \infty$, capacitor is almost an open circuit

5. Capacitor Sinusoidal Response

- The instantaneous power absorbed by a capacitor

- P = IV = i*v = Vm Im cos (wt+Ѳ) sin (wt+Ѳ) = $(Vm * Im)$/(2) sin (2 wt+2 Ѳ)
- $Veff$ =Vm /√ 2 I eff = Im /√ 2

P = $Veff$ * I eff * sin (2 wt+2 Ѳ)

- The instantaneous power absorbed is sinusoidal twice the voltage and current frequency and has zero a average value.
- A capacitor absorbs zero average power
- over a period , a capacitor delivers just as much energy as its absorbs.

End of Lecture 11