Sec 2

Solve:-

1. $\frac{dy}{dx} = \frac{y^2 + x y^2}{x^2 y - x^2}$ Separable

عامل مشترك

$$\frac{dy}{dx} = \frac{y^2(1+x)}{x^2(y-1)}$$

$$\int \frac{(y-1)}{y^2}\, dy = \int \frac{(1+x)}{x^2}\, dx$$

$$\int (y-1)\, y^{-2}\, dy = \int (1+x)\, x^{-2}\, dx$$

$$\int y^{-1} - y^{-2}\, dy = \int x^{-2} + x^{-1}\, dx$$

$$\ln y + \frac{1}{y} = -\frac{1}{x} + \ln x + c \quad \#$$

2. $y \tan x \frac{dy}{dx} = (4 + y^2) \sec^2 x$ Separable

مشتقتها مشتقتها

$$\frac{1}{2}\int \frac{2y}{4+y^2}\, dy = \int \frac{\sec^2 x}{\tan x}\, dx$$

$$\frac{1}{2} \ln(4+y^2) = \ln(\tan x) + c \qquad * 2$$

$$\ln(4+y^2) = 2\ln(\tan x) + 2c$$

$$\ln(4+y^2) = \ln \tan x + \ln \tan x + \ln A$$

$$(4+y^2) = (\tan^2 x) A$$

$$y^2 = (\tan^2 x) A - 4 \quad \#$$

put $2c = \ln A$
and $2\ln(\tan x) = \ln(\tan x) + \ln(\tan x)$
and $\ln(a) + \ln(b) = \ln(ab)$

3. $(2x^2)\frac{dy}{dx} = x^2 + y^2$ (Homogenous)

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$$\frac{dy}{dx} = \frac{x^2+y^2}{2x^2}$$

let $y = ux \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$

$$u + x\frac{du}{dx} = \frac{x^2 + u^2x^2}{2x^2} = \frac{\cancel{x^2}(1+u^2)}{2\cancel{x^2}}$$

$$u + x\frac{du}{dx} = \frac{1+u^2}{2}$$

$$x\frac{du}{dx} = \frac{1+u^2}{2} - 2u$$

$$x\frac{du}{dx} = \frac{(1+u^2-2u)}{2} = \frac{(u-1)^2}{2}$$

$$\int \frac{2}{(u-1)^2}\,du = \int \frac{1}{x}\,dx$$

$$-2\frac{1}{(u-1)} = \ln x + c \qquad \text{put } u = y/x$$

$$-2\frac{1}{(y/x - 1)} = \ln x + c$$

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4. $(x^3 + 3xy^2)\frac{dy}{dx} = y^3 + 3x^2y$ (Homogenous)

$$\frac{dy}{dx} = \frac{y^3+3x^2y}{x^3+3xy^2}$$

let $y = ux \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$

$$u + x\frac{du}{dx} = \frac{u^3x^3 + 3x^3u}{x^3 + 3x^3u^2} = \frac{(u^3+3u)}{(1+3u^2)} - u$$

$$x\frac{du}{dx} = \frac{u^3+3u-u-3u^3}{1+3u^2} = \frac{-2u^3 \cancel{+3u} - 2u}{1+3u^2}$$

$$\int -\frac{3u^2+1}{2u^3-2u}\,du = \int \frac{1}{x}\,dx \rightarrow \underline{\text{Sheet}}$$

$$\int -\frac{3u^2+1}{2u^3-2u}\,du = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}\int \frac{3u^2+1}{\cancel{2}(u-1)u(u+1)}\,du = \int \frac{1}{x}\,dx$$

$$= \ln x + C$$

$$\frac{3u^2+1}{(u-1)u(u+1)} = \frac{A}{(u-1)} + \frac{B}{u} + \frac{C}{(u+1)}$$

$u = 1 \Rightarrow A = \frac{4}{2} = 2$

$u = 0 \Rightarrow B = \frac{1}{-1} = -1$

$u = -1 \Rightarrow C = \frac{4}{2} = 2$

$$\int \frac{2}{u-1} + \frac{-1}{u} + \frac{2}{u+1}\,du$$

$$2\ln(u-1) - \ln(u) + 2\ln(u+1) = \ln x + C \quad \#$$

5. $x \sin y \, dx + y^2 + \frac{x^2}{2} \cos y \, dy = 0$

$M = x \sin y \qquad M_y = x \cos y$

$N = y^2 + \frac{x^2}{2} \cos y \qquad N_x = \frac{\cancel{2}x}{\cancel{2}} \cos y$

$M_y = N_x \qquad$ Exact.

$\int x \sin y \, dx + \int y^2 + \frac{x^2}{2} \cos y \, dy = 0$

$\frac{x^2}{2} \sin y + \frac{y^3}{3} + \frac{x^2}{2} \sin y + c = 0$

Solu. $\frac{x^2}{2} \sin y + \frac{y^3}{3} + c = 0$