(1) if $Z = f(x+ay) + F(x-ay)$, find $\frac{\partial^2 Z}{\partial x^2}$ and $\frac{\partial^2 Z}{\partial y^2}$ and hence prove that $\frac{\partial^2 Z}{\partial y^2} = a^2 \cdot \frac{\partial^2 Z}{\partial x^2}$ ✓

$$\frac{\partial Z}{\partial x} = f'(x+ay) + F'(x-ay)$$ المشتقة الأولى بالنسبة لـ x

$$\frac{\partial^2 Z}{\partial x^2} = f''(x+ay) + F''(x-ay)$$ المشتقة الثانية بالنسبة لـ x

$$\frac{\partial Z}{\partial y} = a\, f'(x+ay) - a\, F'(x-ay)$$

$$\frac{\partial^2 Z}{\partial y^2} = \underline{\underline{a^2}}\, f''(x+ay) + \underline{\underline{a^2}}\, F''(x-ay)$$

$$\frac{\partial^2 Z}{\partial y^2} = a^2 \underbrace{\left(f''(x+ay) + F''(x-ay) \right)}_{\frac{\partial^2 Z}{\partial x^2}}$$

$$\therefore \frac{\partial^2 Z}{\partial y^2} = a^2 \cdot \frac{\partial^2 Z}{\partial x^2}$$ ⋕

(2) Show that the equation $\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} = 0$, is satisfied by

$$Z = \ln \underbrace{\sqrt{x^2+y^2}}_{(x^2+y^2)^{\frac{1}{2}}} + \frac{1}{2} \underbrace{\tan^{-1}\left(\overset{u}{\frac{y}{x}}\right)} \rightarrow \frac{1}{u^2+1} \cdot u'$$

مشتق $\frac{y}{x} \Rightarrow \frac{\cancel{x \cdot 0} - y}{x^2}$

$$\frac{\partial Z}{\partial x} = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{\cancel{2}} \cdot (x^2+y^2)^{-\frac{1}{2}}\, \cancel{2}x + \frac{1}{2} \cdot \frac{1}{\frac{y^2}{x^2}+1} \cdot \frac{-y}{x^2}$$

$$= \frac{x}{x^2+y^2} + \frac{1}{2} \cdot \frac{-y}{y^2+x^2}$$

$$\frac{-y}{2(y^2+x^2)} \Rightarrow \frac{y\,2x}{2(y^2+x^2)^2}$$

$$\frac{\partial^2 Z}{\partial x^2} = \frac{\overset{-(x^2-y^2)}{-x^2+y^2}}{(x^2+y^2)^2} + \frac{yx}{(y^2+x^2)^2}$$

$$\frac{\partial Z}{\partial y} = \frac{y}{x^2+y^2} + \frac{1}{2}\, \frac{x}{y^2+x^2}$$

$$\frac{\partial^2 Z}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2} - \frac{xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} =$$

$$- \frac{(x^2-y^2)}{(x^2+y^2)^2} + \cancel{\frac{yx}{(y^2+x^2)^2}} + \frac{x^2-y^2}{(x^2+y^2)^2} - \cancel{\frac{xy}{(x^2+y^2)^2}}$$

$$= 0$$ ⋕

(3) if $Z = f(x-2y) + F(3x+y)$, where f and F are arbitrary functions, and if

$$\frac{\partial^2 Z}{\partial x^2} + a\frac{\partial^2 Z}{\partial x \partial y} + b\frac{\partial^2 Z}{\partial y^2} = 0$$ find the values of a & b

$$\frac{\partial Z}{\partial x} = f'(x-2y) + 3F'(3x+y)$$ ✓

$$\frac{\partial^2 Z}{\partial x^2} = f''(x-2y) + 9F''(3x+y)$$

$$\frac{\partial Z}{\partial y} = -2f'(x-2y) + F'(3x+y)$$

$$\frac{\partial^2 Z}{\partial y^2} = 4f''(x-2y) + F''(3x+y)$$

$$\frac{\partial^2 Z}{\partial x \partial y} = -2f''(x-2y) + 3F''(3x+y)$$

$$1\,f''(x-2y) + 9F''(3x+y) + a\left[-2f''(x-2y) + 3F''(3x+y)\right] +$$

$$b\left[4f''(x-2y) + F''(3x+y)\right] = 0$$

تساوي معاملات

$$1 - 2a + 4b = 0$$

$$9 + 3a + b = 0$$

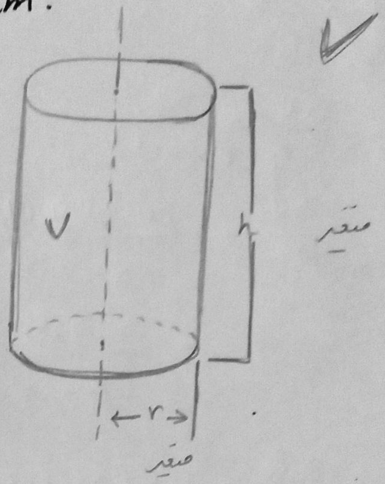
$$-2a + 4b = -1$$

$$3a + b = -9$$

$$a = \frac{-5}{2}$$

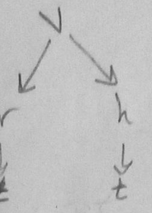
$$b = \frac{-3}{2}$$

① The radius of a cylinder increases at the rate of 0.2 cm/s while the height decreases at the rate of 0.5 cm/s. Find the rate at which the volume is changing at the instant when $r = 8$ cm and $h = 12$ cm.



$V = \pi r^2 h$ الحجم

يعتمد على متغيرين r, h



$$\frac{dV}{dt} = \frac{\partial V}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial V}{\partial h} \frac{dh}{\partial t}$$

$$\Rightarrow \frac{\partial V}{\partial r} = 2\pi r h \quad , \Rightarrow \frac{\partial V}{\partial h} = \pi r^2$$

$$\therefore \frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

لأن h decreasing

$$\frac{dr}{dt} = 0.2 \quad , \quad \frac{dh}{dt} = -0.5 \quad , r = 8 , \ h = 12$$

$$\frac{dv}{dt} = 2\pi(8)(12)(0.2) + \pi(8)^2(-0.5)$$

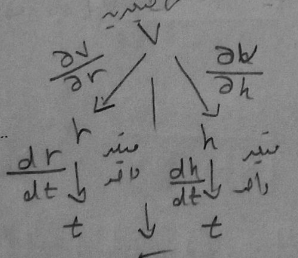
$$\frac{dv}{dt} = 20.1 \ cm^3/s$$

- التحديد معدل تغير حجم اسطوانة دائرية قائمة بمعلومية معدل تغير الطول ونصف القطر

① قانون الحجم ← $V = \pi r^2 h$ ← يعتمد على متغيرين r, h

والحجم يتغير في زمن t

نبني



$$\therefore \frac{dv}{dt} = \frac{\partial v}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial v}{\partial h} \cdot \frac{dh}{dt}$$

Show that $\phi = A e^{-kt/2} \sin(pt) \cos(qx)$, satisfies the equation

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2} \left\{ \frac{\partial^2 \phi}{\partial t^2} + k \frac{\partial \phi}{\partial t} \right\}$$

∴, provided that $p^2 = c^2 q^2 - \frac{k^2}{4}$

$$\frac{\partial \phi}{\partial x} = -A e^{-kt/2} \sin(pt) \sin(qx) (q)$$

$$\frac{\partial^2 \phi}{\partial x^2} = -A e^{-kt/2} \sin(pt) \cos(qx) (q^2)$$

$$\frac{\partial \phi}{\partial t} = \frac{-k}{2} A e^{-kt/2} \sin(pt) \cos(qx) + A e^{-kt/2} \cos(pt) \cos(qx) (p)$$

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{k^2}{4} A e^{-kt/2} \sin(pt) \cos(qx) - \frac{k}{2} A e^{-kt/2} \cos(pt) \cos(qx)\, p$$

$$\frac{-k}{2} A e^{-kt} \cos(pt) \cos(qx) (p) - A e^{-kt/2} \sin(pt) \cos(qx) (p^2)$$

$$= \frac{k^2}{4} A e^{-kt/2} \sin(pt) \cos(qx) - k A e^{-kt/2} \cos(pt) \cos(qx)\, p$$

$$- A e^{-kt/2} \sin(pt) \cos(qx) (p^2)$$

$$= A e^{-kt/2} \sin(pt) \cos(qx) \left(\frac{k^2}{4} - p^2 \right) - k A e^{-kt/2} \cos(pt) \cos(qx)\, p$$

$$\therefore \quad - A e^{-kt/2} \sin(pt) \cos(qx) (q^2) =$$

$$\frac{1}{c^2} \Big\{ A e^{-kt/2} \sin(pt) \cos(qx) \left(\frac{k^2}{4} - p^2 \right) - k A e^{-kt/2} \cancel{\cos(pt) \cos(qx)\, p}$$

$$- \frac{k^2}{2} A e^{-kt/2} \sin(pt) \cos(qx) + k A e^{-kt/2} \cancel{\cos(pt) \cos(qx) (p)}$$

$$- A e^{-kt/2} \cancel{\sin(pt) \cos(qx)} (q^2) = \frac{1}{c^2} \left(A e^{-kt/2} \cancel{\sin(pt) \cos(qx)} \left(\frac{k^2}{4} - p^2 - \frac{k^2}{2} \right) \right)$$

$$- q^2 = \frac{1}{c^2} \left(\frac{-k^2}{4} - p^2 \right)$$

$$- q^2 c^2 = \frac{-k^2}{4} - p^2$$

$$p^2 = q^2 c^2 - \frac{k^2}{4}$$