Lecture (5)

Light interference

❑ Research topic

Proof

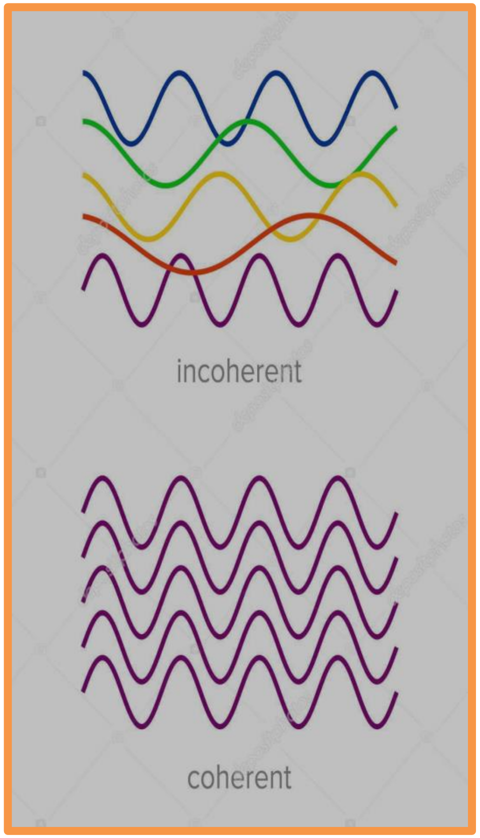
$$\boxed{I = 4I_0 \cos^2 \tfrac{1}{2}\phi,}$$

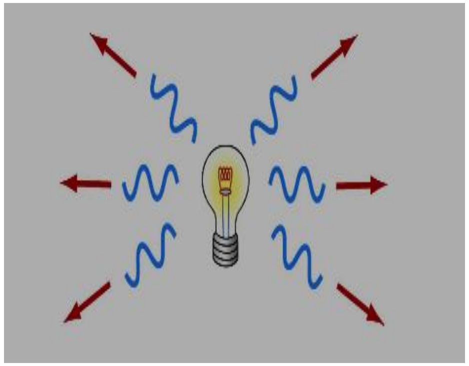
❑ Interference

In order to form an interference pattern, the incident light must satisfy two conditions:

(i) The light sources must be <u>coherent</u>. This means that the plane waves from the sources must maintain a constant phase relation.

(ii) The light must be <u>monochromatic</u>. This means that the light consists of just one wavelength .





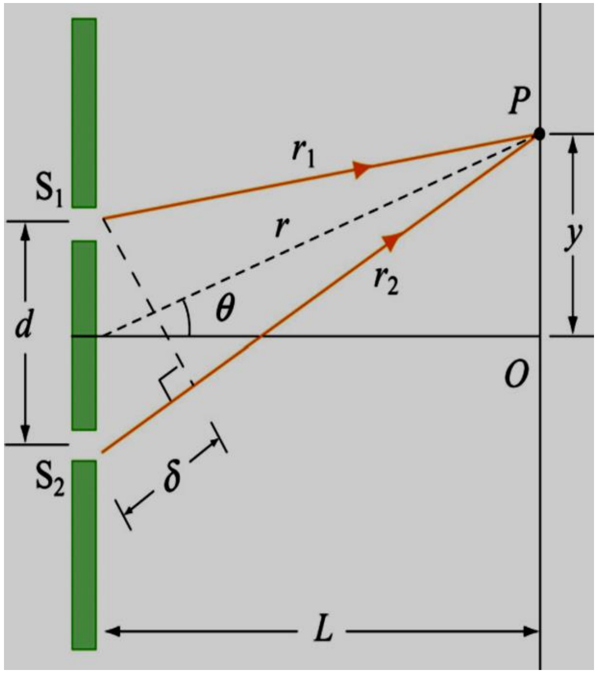
- The light waves emerging from the two slits interfere and form an interference pattern on the viewing screen. The bright bands (fringes) correspond to interference maxima, and the dark band interference minima.

- **Constructive interference** occurs when δ is zero or an integer multiple of the wavelength and led to **bright fringe** .

$$\delta = d\sin\theta = m\lambda, \quad m = 0, \pm1, \pm2, \pm3, \ldots \text{ (constructive interference)}$$

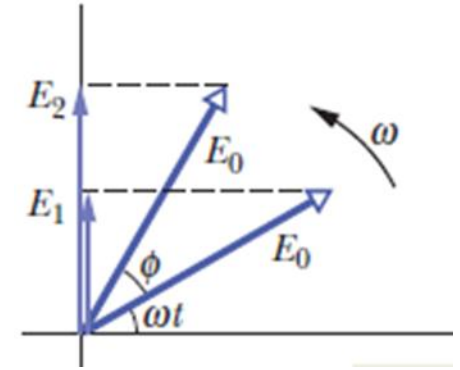
- **Destructive interference** occurs when δ is equal to an odd integer multiple of λ/2 and led to **dark fringe** .

$$\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm1, \pm2, \pm3, \ldots \text{ (destructive interference)}$$



❑ Intensity in Double-Slit Interference

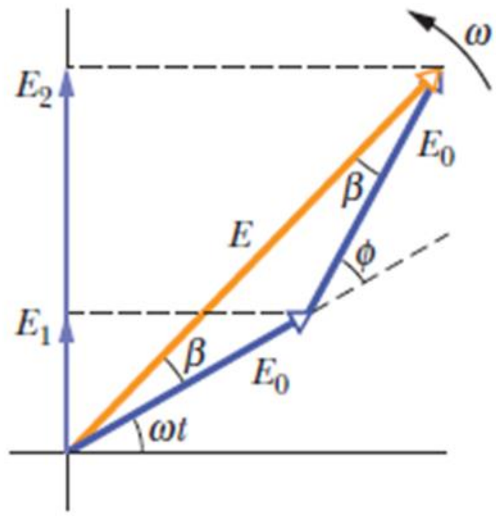
❖ Here we wish to derive an expression for the intensity I of the fringes as a function of θ.



❖ the electric field components of those waves at point P are not in phase and vary with time as

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin(\omega t + \phi),$$



❖ We shall show that these two waves will combine at P to produce an intensity I given by

$$\frac{I}{I_0} = \frac{E^2}{E_0^2}.$$

$$\boxed{I = 4I_0 \cos^2 \tfrac{1}{2}\phi,}$$

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \frac{2\pi}{\lambda}\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}.$$

$$\boxed{\phi = \frac{2\pi d}{\lambda}\sin\theta.}$$

Maxima. The intensity maxima will occur when

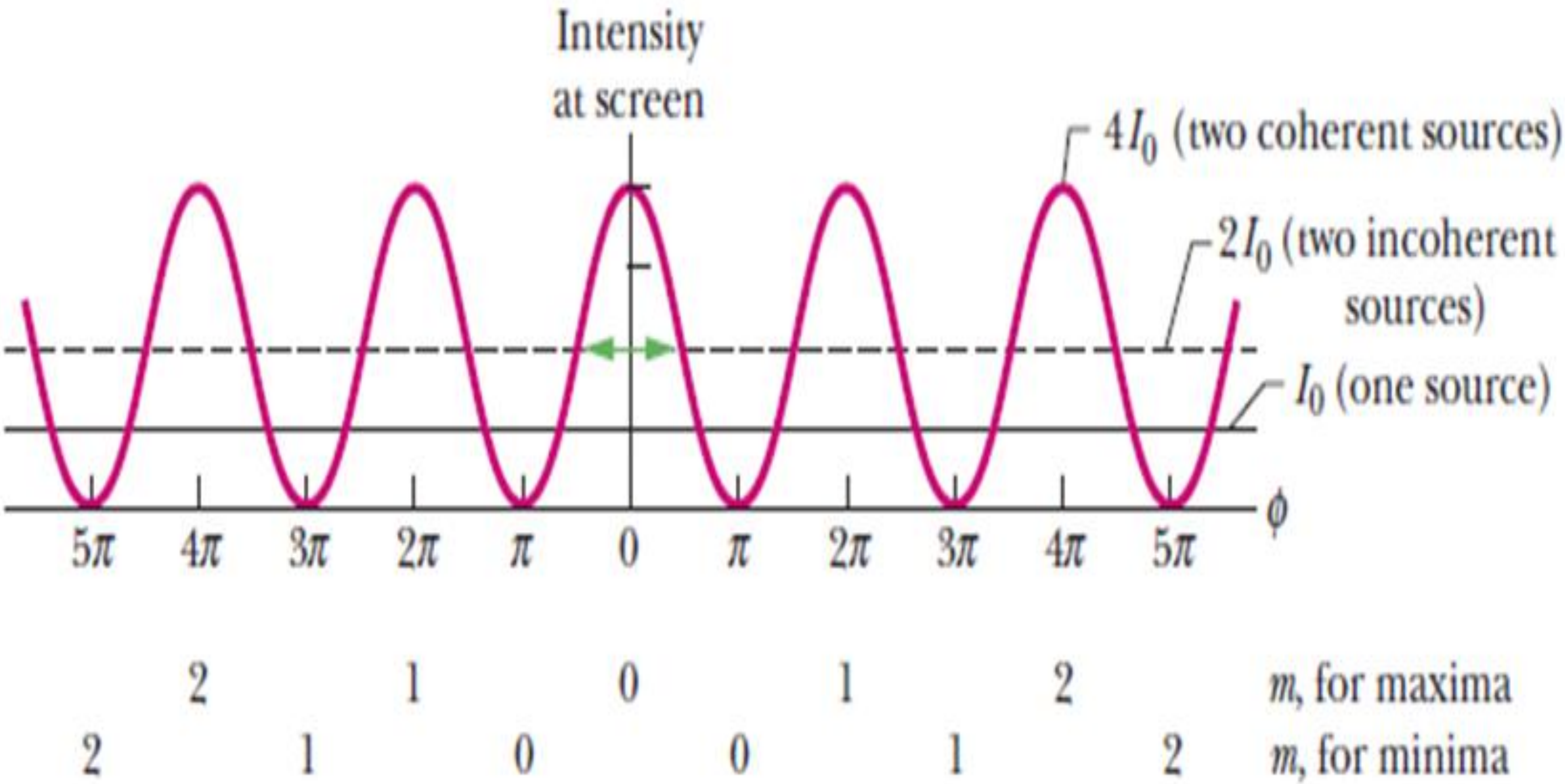
$$\tfrac{1}{2}\phi = m\pi, \qquad \text{for } m = 0, 1, 2, \ldots .$$

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima)},$$

Minima. The minima in the fringe pattern occur when

$$\tfrac{1}{2}\phi = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 0, 1, 2, \ldots .$$

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima)},$$



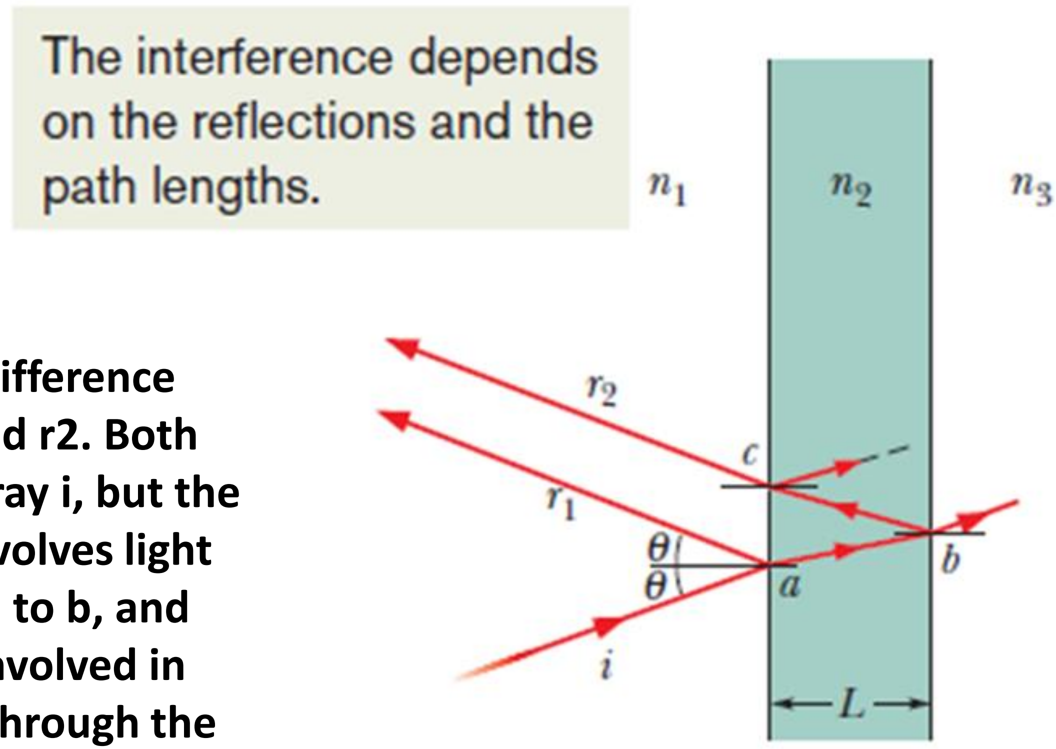
$$\frac{1}{2}\phi = m\pi, \qquad \text{for } m = 0, 1, 2, \ldots .$$

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$

$$\frac{1}{2}\phi = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 0, 1, 2, \ldots .$$

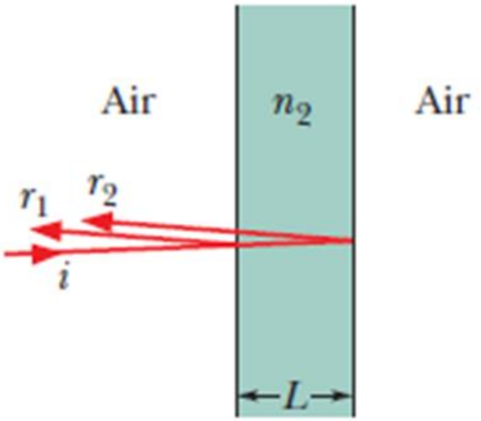
$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$

❑ Interference from Thin Films



The observer sees is the phase difference between the waves of rays r1 and r2. Both rays are derived from the same ray i, but the path involved in producing r2 involves light traveling twice across the film (a to b, and then b to c), whereas the path involved in producing r1 involves no travel through the film.

❑ **The path length difference** between the waves of r1 and r2 as 2L. However, to find the phase difference between the waves, we cannot just find the number of wavelengths ⍰ that is equivalent to a path length difference of 2L.



Thus, **for a bright film**, we must have

$$2L = \frac{\text{odd number}}{2} \times \text{wavelength}$$

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

❑ **The wavelength we need here is the wavelength λ_{n2} of the light in the medium containing path length 2L, ($\lambda_{n2} = \lambda/n_2$).**

For a dark film, we must have

$$2L = \text{integer} \times \text{wavelength}$$

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$

Example (1)

White light, with a uniform intensity across the visible wavelength range of 400 to 690 nm, is perpendicularly incident on a water film, of index of refraction n_2 =1.33 and thickness L = 320 nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

$$2L = \frac{\text{odd number}}{2} \times \text{wavelength}$$

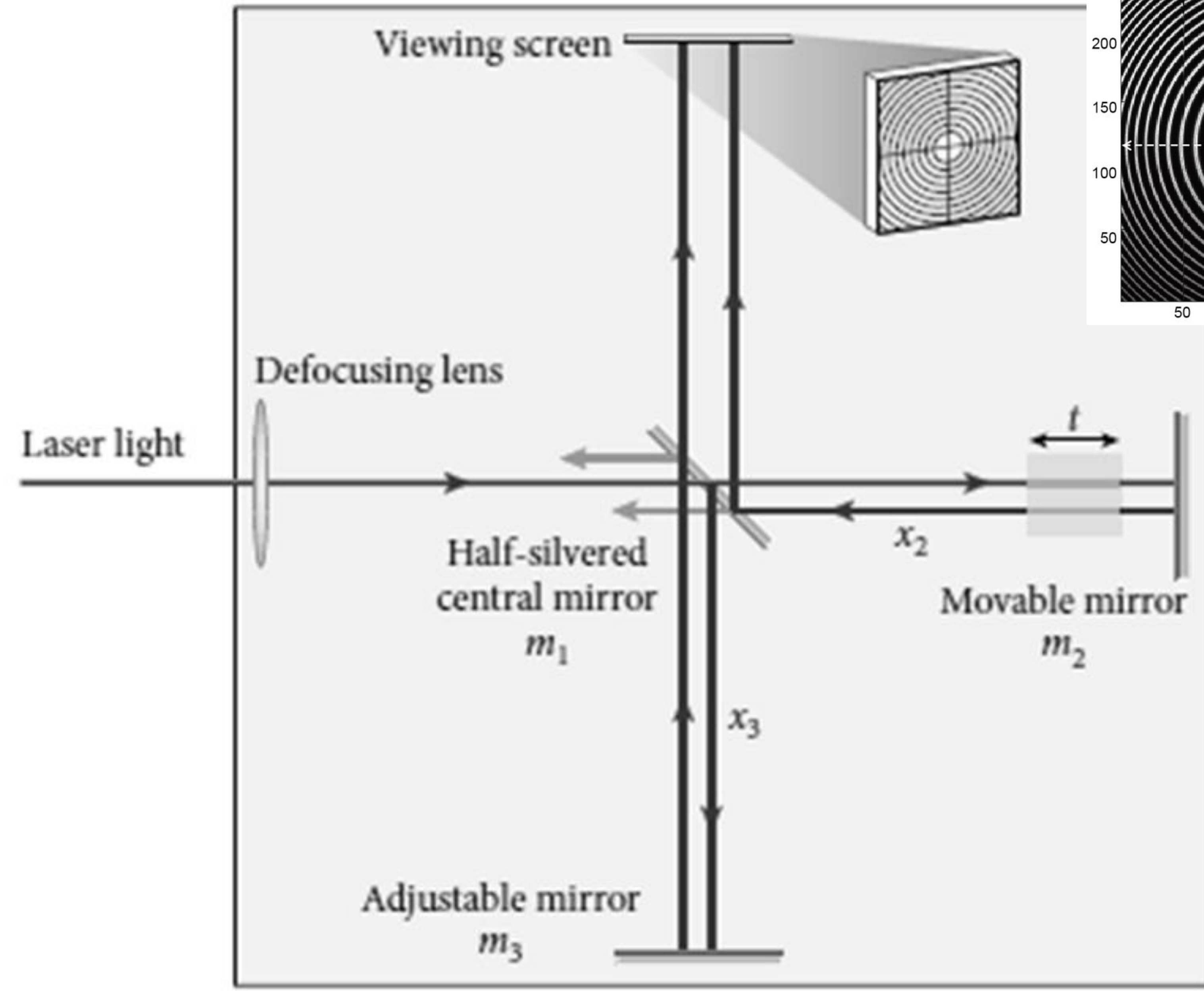
$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots$$

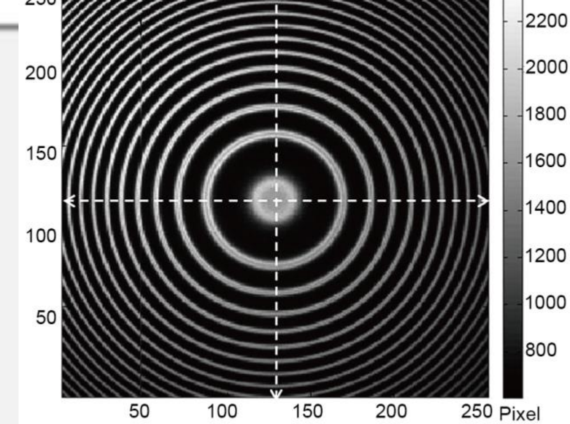
$$\lambda = \frac{2n_2L}{m + \frac{1}{2}} = \frac{(2)(1.33)(320\text{ nm})}{m + \frac{1}{2}} = \frac{851\text{ nm}}{m + \frac{1}{2}}.$$

For m = 0, this gives us λ= 1700 nm, which is in the infrared region. For m = 1, we find λ = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. Thus, the wavelength at which the light seen by the observer is brightest is λ = 567 nm.

A Michelson interferometer

- Interferometers generally are used to measure very small displacements by using the wave property of light. They measure changes of the interference pattern when waves with different phases overlap.


Viewing screen
Defocusing lens
Laser light
t
Half-silvered
central mirror
m_1
x_2
Movable mirror
m_2
x_3
Adjustable mirror
m_3


200
150
100
50
50
100
150
200
250
Pixel
2200
2000
1800
1600
1400
1200
1000
800

❑ Measurement of thickness

By placing a material with index of refraction n and thickness t in the path of the light traveling to the movable mirror m2, as depicted in Figure. The path length difference in terms of the number of wavelengths will change because the wavelength of light in the material λ_m is different from λ.

$$\boxed{\lambda_m = \frac{\lambda}{n}}$$

- ❖ The number of wavelengths in the material

$$\boxed{N_{\text{material}} = \frac{2t}{\lambda_n} = \frac{2tn}{\lambda}.}$$

- ❖ The number of wavelength that would have been there if the light traveled through only air

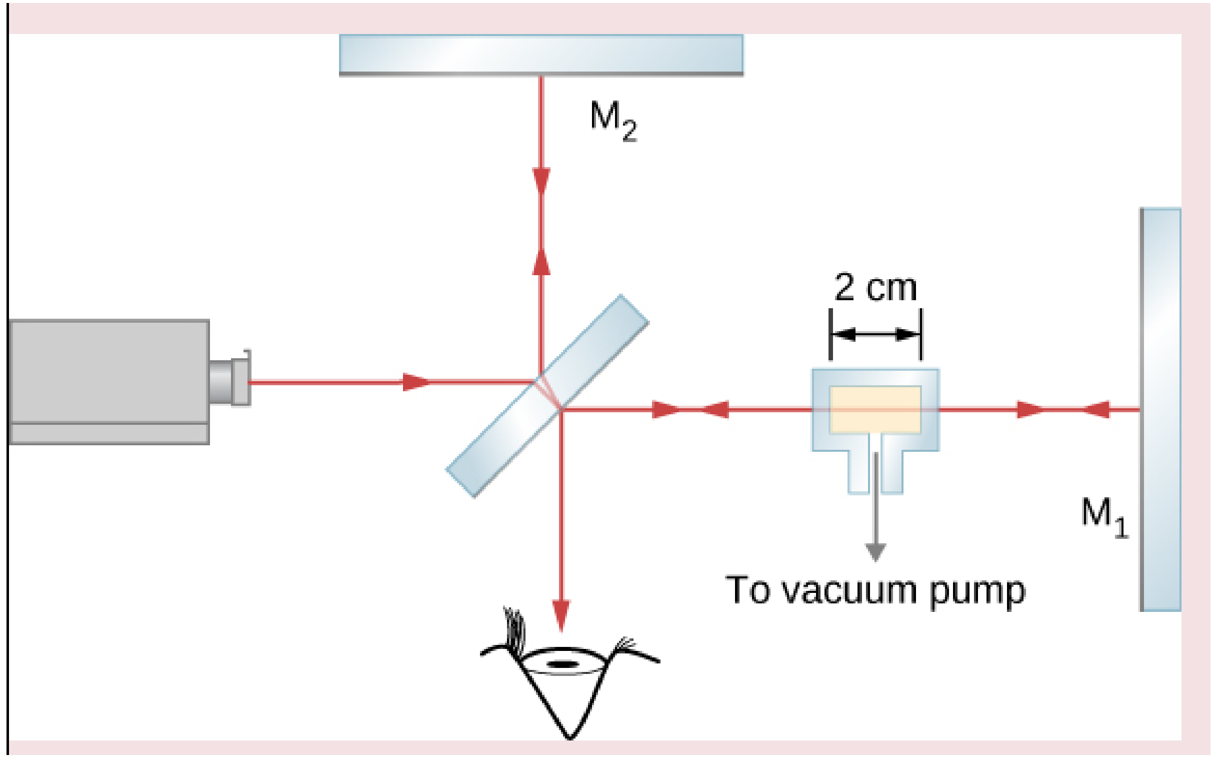
$$\boxed{N_{\text{air}} = \frac{2t}{\lambda}.}$$

- ❖ The difference in the number of wavelengths

$$\boxed{N_{\text{material}} - N_{\text{air}} = \frac{2tn}{\lambda} - \frac{2t}{\lambda} = \frac{2t}{\lambda}(n-1).}$$

Example

Measuring the Refractive Index of a Gas in one arm of a Michelson interferometer, a glass chamber is placed with attachments for evacuating the inside and putting gases in it. The space inside the container is 2 cm wide. Initially, the container is empty. As gas is slowly let into the chamber, you observe that dark fringes move past a reference line in the field of observation. By the time the chamber is filled to the desired pressure, you have counted 122 fringes move past the reference line. The wavelength of the light used is 632.8 nm. What is the refractive index of this gas?



$$N_{\text{material}} - N_{\text{air}} = \frac{2tn}{\lambda} - \frac{2t}{\lambda} = \frac{2t}{\lambda}(n-1).$$

$$\boldsymbol{n - 1 = 122 \times \frac{632.8 \times 10^{-9}}{2 \times 2 \times 10^{-2}} = 0.0019}$$

n = 1.0019

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