Sec 3

1- reducible to separable :- ✓ Sec 1 , Sec 2 , Sec 3

$$\frac{dy}{dx} = f(ax + by + c)$$

تفاضل

let $u = ax + by + c \Rightarrow \frac{du}{dx} = a + b\frac{dy}{dx}$

$$\rightarrow \frac{dy}{dx} = e^{x+y+1}$$

let $u = x + y + 1 \Longrightarrow \frac{du}{dx} = 1 + \frac{dy}{dx}$

$$\frac{du}{dx} = 1 + e^{u}$$

$$\int \frac{(1+e^u-e^u)}{1+e^u}\,du = \int dx$$

$$\int \frac{1+e^u}{1+e^u}\,du - \int \frac{e^u}{1+e^u}\,du = \int dx$$

$u - \ln(1+e^u) = x + c$ put $u = x + y + 1$

$$(x+y+1) - \ln(1 + e^{(x+y+1)}) = x + c$$

2- linear method :-

general form: $\frac{dy}{dx} + p(x)\,y = Q(x)$

- $p(x)$, $Q(x)$
- $IF = e^{\int p(x)\,dx}$
- $IF\,y = \int I.F\,Q(x)\,dx + c$

ملاحظ معامل $\frac{dy}{dx}$ يجب انه يكون (1)

$$x\frac{dy}{dx} - y = x^2 \qquad \div x$$

$$\frac{dy}{dx} - \frac{1}{x}y = x$$

$$P(x) = -\frac{1}{x}, \qquad Q(x) = x$$

$$IF = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$$

$$\frac{1}{x}y = \int \frac{1}{\cancel{x}}\cancel{x}\,dx + C$$

$$\frac{1}{x}y = x + C$$

$$y = x^2 + xC \quad \#$$

$$x\frac{dy}{dx} - 2y = x^3\cos x \qquad \div x$$

$$\frac{dy}{dx} - \frac{2}{x}y = x^2\cos x$$

$$P(x) = -2/x, \qquad Q(x) = x^2\cos x$$

$$IF = e^{\int -2/x\,dx} = e^{-2\ln x} = \cancel{e}^{\cancel{\ln} x^{-2}} = x^{-2} = \frac{1}{x^2}$$

$$\frac{1}{x^2}y = \int \frac{1}{\cancel{x^2}}\cancel{x^2}\cos x\,dx + C$$

$$\frac{1}{x^2}y = \sin x + C$$

$$y = x^2\cdot\sin x + x^2C \quad \#$$

Sheet $\qquad \frac{dy}{dx} + y\tanh x = 2\sinh x$

6- Bernolli's method:-

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n \qquad * \, y^{-n}$$

$$y^{-n}\frac{dy}{dx} + p(x)\,y^{1-n} = Q(x)$$

let $Z = y^{1-n} \Rightarrow \frac{dz}{dx} = (1-n)\,y^{-n}\frac{dy}{dx}$

$$\frac{1}{(1-n)}\frac{dZ}{dx} + p(x)\,Z = \cancel{\frac{1}{1-n}}\,Q(x)$$

$$IF = e^{\int p(x)\,dx}$$

$$Z.IF = \int IF\,Q(x)\,dx + c$$

$$\frac{dy}{dx} + y/x = y^3 \qquad * \, y^{-3}$$

$$y^{-3}\frac{dy}{dx} + \frac{1}{x}y^{-2} = 1$$

let $Z = y^{-2} \Rightarrow \frac{dZ}{dx} = -2\,y^{-3}\frac{dy}{dh}$

$$-\frac{1}{2}\frac{dZ}{dx} + \frac{1}{x}Z = 1$$

$$\frac{dZ}{dx} - 2/x\,Z = -2$$

$p(x) = -2/x \quad , \quad Q(x) = -2$

$$I.F. = e^{\int -2/x\,dx} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$$

$$\frac{1}{x^2}Z = \int \frac{1}{x^2}\,-2\,dx + c$$

$$\frac{1}{x^2}Z = -2 \cdot -\frac{1}{x} + c$$

$$\frac{1}{x^2}Z = \frac{2}{x} + c$$

$$Z = 2x + x^2c$$

$$y^{-2} = 2x + x^2c$$

$$y^2 = \frac{1}{2x + x^2c}$$

Sheet $\quad x\dfrac{dy}{dx} + 3y = x^2y^2$

Solve

$\Rightarrow$ Exact

$$xy^2 + 2 = (3 - x^2y)\,y'$$

$$xy^2 + 2 = (3 - x^2y)\frac{dy}{dx}$$

$$(xy^2+2)\,dx = (3 - x^2y)\,dy$$

$$\underbrace{(xy^2+2)}_{M}\,dx\ \underbrace{(-3+x^2y)}_{N}\,dy = 0$$

$M = xy^2 + 2 \qquad M_y = 2xy$

$N = x^2y - 3 \qquad N_x = 2xy$

$$M_y = N_x \quad \Leftarrow \quad \text{Exact}$$

$$\int xy^2 + 2\,dx - \int 3 + x^2y\,dy = 0$$

$$\frac{x^2}{2}y^2 + 2x - 3y + \frac{x^2y^2}{2} + C = 0$$

Solu. $\quad \dfrac{x^2}{2}y^2 + 2x - 3y + C = 0$ ⁂

Quiz [illegible]

Class [illegible]

$(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$

Solve : $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$ if $Z = x-2y$

$$\frac{dy}{dx} = \frac{x-2y+1}{(2)(x-2y)}$$

let $Z = x-2y$

$$\frac{dZ}{dx} = 1-2\frac{dy}{dx}$$

$$2\frac{dy}{dx} = 1-\frac{dZ}{dx}$$

$$2\frac{dy}{dx} = \frac{Z+1}{Z}$$

$$1-\frac{dZ}{dx} = \frac{Z+1}{Z}$$

$$\frac{dZ}{dx} = 1-\frac{Z+1}{Z}$$

$$\left(\frac{Z-(Z+1)}{Z}\right.$$

$$\frac{dZ}{dx} = \frac{\cancel{Z}+1-\cancel{Z}}{Z} = \frac{-1}{Z}$$

$$\int Z \, dZ = \int -dx$$

$$\frac{Z^2}{2} = -x + c$$

$$x^2y - x^3\frac{dy}{dx} = y^4 \cos x \quad * -$$

$$x^3\frac{dy}{dx} - x^2y = -y^4\cos x \quad \div x^3$$

$$\frac{dy}{dx} - \frac{1}{x}y = -\frac{y^4 \cos x}{x^3} \quad * y^{-4}$$

$$y^{-4}\frac{dy}{dx} - \frac{1}{x}y^{-3} = \frac{-\cos x}{x^3}$$

$$y^{-3} = Z \Rightarrow \frac{dZ}{dx} = -3y^{-4}\frac{dy}{dx}$$

$$\frac{1}{-3}\frac{dZ}{dx} - \frac{1}{x}Z = \frac{-\cos x}{x^3}$$

$$\frac{dZ}{dx} + \frac{3}{x}Z = \frac{3\cos x}{x^3}$$

$$P(x) = \frac{3}{x}, \quad Q(x) = \frac{3\cos x}{x^3}$$

$$I.F = e^{\int P(x)\,dx} = e^{3\ln x} = e^{\ln x^3} = x^3$$

$$Zx^3 = \int \cancel{x^3}\,\frac{3\cos x}{\cancel{x^3}}\,dx + c$$

$$Zx^3 = 3\sin x + c$$

$$Z = \frac{3\sin x + c}{x^3}$$

$$y^3 = \frac{x^3}{3\sin x + c}$$

$$x\frac{dy}{dx} + 3y = x^2y^2$$

$$\frac{dy}{dx} + \frac{3}{x}y = xy^2$$

$$y^{-2}\frac{dy}{dx} + \frac{3}{x}y^{-1} = x$$

let $z = y^{-1}$, $\frac{dz}{dx} = -1\, y^{-2}\frac{dy}{dx}$

$$-\frac{dz}{dx} + \frac{3}{x}z = x$$

$$\frac{dz}{dx} - \frac{3}{x}z = -x$$

$$IF = e^{-3\int\frac{1}{x}} = e^{-3\ln x} = e^{\ln x^{-3}} = x^{-3} = \frac{1}{x^3}$$

$$\frac{1}{x^3}z = \int \frac{1}{\cancel{x^3}}\cdot -\cancel{x}\; dx + c$$

$$\frac{1}{x^3}z = \frac{1}{x} + c$$

$$z = x^2 + \frac{c}{\cancel{x^3}}x^3$$

$$y^{1} = \cancel{\frac{1}{x^2} + \frac{x^3}{c}} \quad \frac{1}{x^2 + cx^3}$$