Solve:-

Sec 4

1. $2xyy' = y^2 - x^2$ ✓

$$2xy \frac{dy}{dx} = y^2 - x^2$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

let $y = ux$ $\Longrightarrow$ $\frac{dy}{dx} = u + x\frac{du}{dx}$

$u = y/x$

$$u + x\frac{du}{dx} = \frac{u^2\cancel{x^2} - \cancel{x^2}}{2\cancel{x^2}u}$$

$$u + x\frac{du}{dx} = \frac{u^2-1}{2u} - u$$

$$x\frac{du}{dx} = \frac{u^2 - 1 - 2u^2}{2u} = \frac{-u^2-1}{2u}$$

$$x\frac{du}{dx} = -\frac{(u^2+1)}{2u}$$

$$\int \frac{2u}{u^2+1}\,du = \int -\frac{1}{x}\,dx$$

$$\ln(u^2+1) = -\ln x + c$$

$$\ln\left(y^2/x^2 + 1\right) = -\ln x + c$$

$$y^2/x^2 + 1 = \frac{e^c}{x} \leftarrow \text{ثابت}$$

$$y^2/x^2 + 1 = c/x$$

$$y^2 = (c/x - 1)\,x^2$$

2. $x\frac{dy}{dx} = y + x e^{y/x}$

$$\frac{dy}{dx} = y/x + \frac{\cancel{x} e^{y/x}}{\cancel{x}}$$

$$\frac{dy}{dx} = y/x + e^{y/x}$$

let $u = y/x$ $\qquad \frac{dy}{dx} = u + x\frac{du}{dx}$

$$\cancel{u} + x\frac{du}{dx} = \cancel{u} + e^{u}$$

$$x\frac{du}{dx} = e^{u}$$

$$\int \frac{1}{e^{u}} du = \int \frac{dx}{x}$$

$$-\frac{1}{e^{u}} = \ln x + c$$

$$\frac{1}{e^{u}} = -c - \ln x$$

$$e^{-(\frac{y}{x})} = -(\ln x + c)$$

$$-y/x = \ln(-\ln x + c)$$

$$y = -x \ln(-\ln x + c)$$ #

3. $2xy \frac{dy}{dx} = x^2 - y^2$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

let $y = ux \Rightarrow \frac{dy}{dx} = u + x \frac{du}{dx}$

$$u + x\frac{du}{dx} = \frac{\cancel{x^2} - u^2 \cancel{x^2}}{2\cancel{x^2} u}$$

$$\underline{u +} x \frac{du}{dx} = \frac{1 - u^2}{2u} - u$$

$$x \frac{du}{dx} = \frac{1 - u^2 - 2u^2}{2u} = \frac{1 - 3u^2}{2u} = \frac{-(3u^2+1)}{2u}$$

$$x \frac{du}{dx} = -\frac{(3u^2+1)}{2u}$$

$$\frac{1}{6}\int \frac{6u}{3u^2+1} du = \int -\frac{1}{2} \frac{dx}{x}$$

$$\frac{1}{6} \ln(3u^2+1) = -\frac{1}{2} \ln x + C$$

4. Determine the general solution of the equation:-

$$\frac{dy}{dx} + \{\underbrace{\frac{1}{x} - \frac{2x}{1-x^2}}_{P(x)}\} \cdot y = \underbrace{\frac{1}{1-x^2}}_{Q(x)}$$

$$P(x) = \frac{1}{x} - \frac{2x}{1-x^2}, \quad Q(x) = \frac{1}{1-x^2}$$

$$m(x) = e^{\int p(x)\, dx}$$

$$\therefore \int p(x)\, dx = \int \frac{1}{x} - \frac{-2x}{1-x^2} dx = \ln x + \ln(1-x^2)$$
$$= \ln(x(1-x^2))$$

$$e^{\ln(x(1-x^2))} = (x - x^3)$$

$$y(x - x^3) = \int \frac{x - x^3}{1-x^2} dx$$
$$= \int \frac{x(\cancel{1-x^2})}{\cancel{1-x^2}} dx$$

$$y(x - x^3) = \frac{x^2}{2} + C$$

$$y = \frac{x}{2(x-1)(x+1)} + \frac{C}{x(1-x^2)}$$