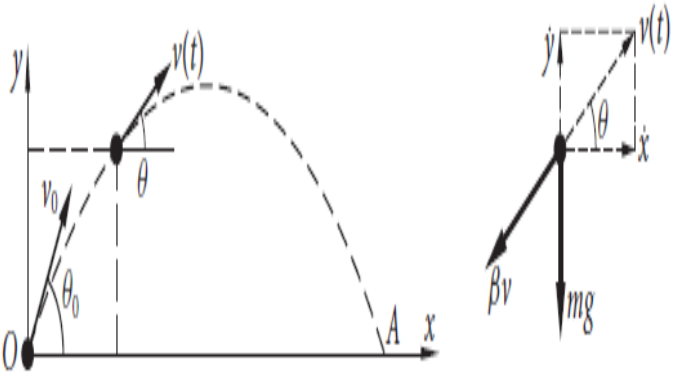
application of differential equation

(a) Modeling is the process of writing a differential equation to describe a physical situation. The following **four** models represent a number of engineering problems which can be modeled using differential equations.

1) Consider the projectile of mass *m* launched with initial velocity v_0 at angle θ_0 at time *t = 0*. The atmosphere exerts a resistance force R on the mass, which is proportional to the instantaneous velocity, $R = \beta v$, β is a constant, the governing equation take the form



$$m\ddot{y}(t) + \beta \dot{y}(t) + mg = 0,$$

$$m\ddot{x}(t) + \beta \dot{x}(t) = 0$$

Solve the modeling equations for

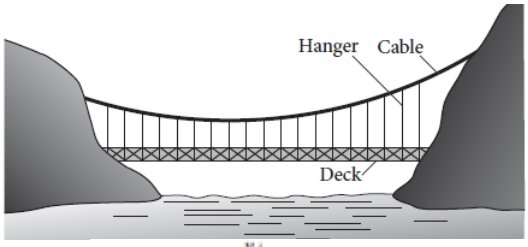
$m = 1Kg$, $\beta = 1$, $g = 10 m/s^2$

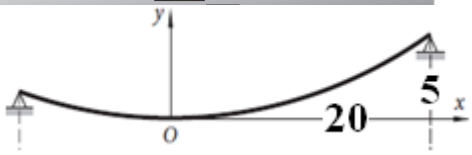
application of differential equation

2) Consider the suspension bridge as shown, which consists of the main cable, the hangers, and the deck. The equation governing the shape of the cable is given by

$$\frac{d^2y}{dx^2} = \frac{w(x)}{H},$$

where $w(x)$ **is a distributed load and** H **is the tension in the cable at the lowest point O.**



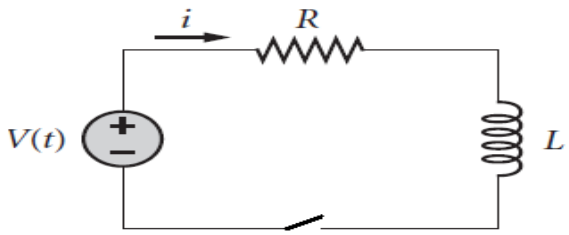


- **Find the shape of cable** $y(x)$ **for**

  $w(x) = x^2$ *KN* ***and*** $H = 1KN$

application of differential equation

3) A circuit consisting of a resistor *R*, an inductor L, and a voltage source V(t) connected in series. The governing equation is



$$v(t) = R\,i(t) + L\frac{d}{dt}i(t)$$

- **Find the current $i(t)$ for**

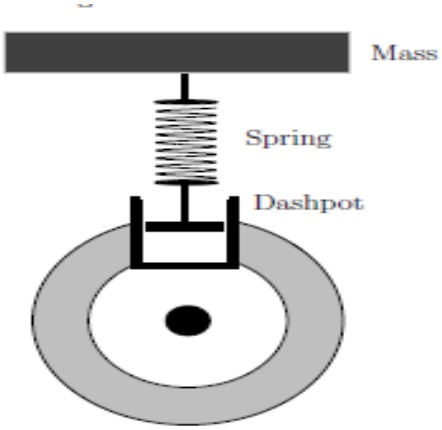
  $v(t) = 5V$, $R = 1\,\Omega$, $L = 1\,H$

application of differential equation

4) The equation of motion of a shock absorber can be described in terms of the vertical displacement *y* of the mass *m* as follow.

$$m\frac{d^2y}{dt^2}+\mu\frac{dy}{dt}+k\,y=0,\;\; y(0)=1,\; y'(0)=0$$

where *k* is the spring constant, μ is damping constant.



- **Find the vertical displacement *y***

  for $m=100\,kg,\ \mu=0,\ k=100\,N/m$

application of differential equation

5- A homicide victim is found at 6:00PM in an office building that is maintained at 72˚F.When the victim was found, his body temperature was at 85 ˚F. Three hours later at 9:00PM, his body temperature was recorded at 78˚F. Assume the temperature of the body at the time of death is your typical normal temperature of 98.6˚F.

The estimated time of death most nearly is:

(A) 2:11 PM

(B) 3:13 PM

(C) 4:34 PM

(D) 5:12 PM