Applications

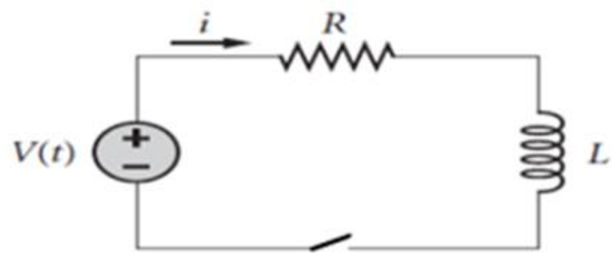
Chapter1:

(الكترونيات و مدني)

A circuit consisting of a resistor R, an inductor L, and a voltage source V(t) connected in series. The governing equation is

$$v(t) = R\,i(t) + L\frac{d}{dt}i(t)$$

- **Find the current $i(t)$ for**
  $v(t) = 5V$, $R = 1\,\Omega$, $L = 1\,H$



$$5 = I + \frac{dI}{dt}\ , \quad p(t) = 1, q(t) = 5$$

$$m(t) = e^{\int p(t)dx} = e^{\int dt} = e^{t}$$

$$I.m(t) = \int m(t).Q(t)dt$$

$$e^{t}.I = \int 5\,e^{t}\,dt$$

$$e^{t}.I = 5\,e^{t} + \mathrm{c}$$

$$I(0) = 0\ ,\ c = -5$$

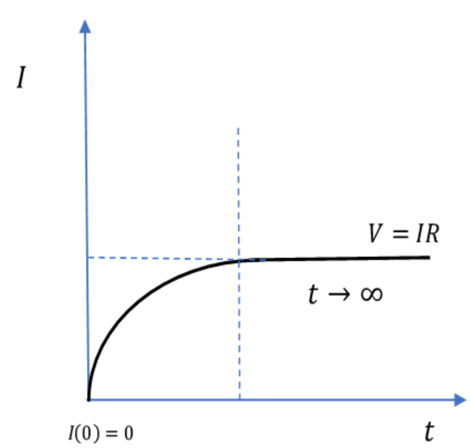
$$I = 5 - 5\,e^{-t}$$

$$\boldsymbol{I = 5(1 - e^{-t})}$$

And steady state for I

$$I_{(t)} = 5(1 - e^{-t})$$

$$\boldsymbol{I_{(\infty)} = 5(1 - e^{-\infty}) = 5}$$



(الكترونيات و مدني)

A homicide victim is found at 6:00PM in an office building that is maintained at 72˚F.When the victim was found, his body temperature was at 85 ˚F. Three hours later at 9:00PM, his body temperature was recorded at 78˚F. Assume the temperature of the body at the time of death is your typical normal temperature of 98.6˚F.

The estimated time of death most nearly is:

(A) 2:11 PM
(B) 3:13 PM
(C) 4:34 PM
(D) 5:12 PM

Solution:

$$\theta_{(6)} = 85°\text{F} \quad \theta_{(9)} = 78°\text{F} \quad \theta_{(body)} = 98.6°\text{F}$$

The governing equation for the temperature θ of the body is

$$\frac{d\theta}{dt} \propto k(\theta - \theta_a)$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_a)$$

$$\frac{d\theta}{dt} = -k(\theta - 72°)$$

$$\int \frac{1}{(\theta - 72°)} d\theta = -k\, dt$$

$$\ln(\theta - 72°) = -kt + c$$

$$\ln(\theta - 72°) = -kt + \ln A$$

$$\ln(\theta - 72°) - \ln A = -kt$$

$$\ln \frac{(\theta - 72°)}{A} = -kt$$

$$\frac{(\theta - 72°)}{A} = e^{-kt}$$

$$(\theta - 72°) = A\, e^{-kt}$$

$$\boxed{(\boldsymbol{\theta}) = \boldsymbol{A\, e^{-kt} + 72°}}$$

At $t = 9\, pm$ $\quad 78 = A\, e^{-9k} + 72°$ $\quad \boxed{A\, e^{-9k} = 6}$ $\quad$ 1

At $t = 6\, pm$ $\quad 85 = A\, e^{-6k} + 72°$ $\quad \boxed{A\, e^{-6k} = 13}$ $\quad$ 2

Sub. 2 , 1

$$e^{3k} = \frac{13}{6}$$

$$3k = \ln\frac{13}{6}$$

$$k = \frac{1}{3}\ln\frac{13}{6} = 0.25773$$

$$A\, e^{-9(0.25773)} = 6$$

$$A = 61.028$$

$$(\theta) = 61.028\, e^{-(0.25773)t} + 72°$$

$$98.6 = 61.028\, e^{-(0.25773)t} + 72°$$

$$t = 3.221$$

$$0.221 * 60 = 13.3$$

$$\boxed{\boldsymbol{t = 3 : 13\; pm}}$$

(الكترونيات و مدني)

The rate of decay of a radioactive substance is proportional to the amount A remaining at any instant. If $A = A_0$ at $t = 0$, prove that, if the time taken for the amount of the substance to become $\frac{1}{2}A_0$ is T ,then $A_0 e^{-t\,ln2/T}$,Prove also that the time taken for the amount remaining to be reduced to $\frac{1}{20}A_0$ is 4.32 T

Solution:

1- At t=0 , A = A_0
2- At t=T $A = \frac{1}{2}A_0$
3- t=?? when $A = \frac{1}{20}A_0$

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = KA$$ Separable equation

$$\int \frac{dA}{A} = \int K\ dt$$

$$\ln A = Kt + c$$

$$\boxed{A = e^{Kt+c}} \leftarrow 1$$

We have to find c & k

First find c:

Put A = A_0 , at t=0

$$A_0 = e^{K(0)+c}$$

$$A_0 = e^{c}$$

$$\ln A_0 = c$$

Sub. Into 1

$$A = e^{Kt+\ln A_0}$$

$$\boxed{A = A_0 e^{Kt}} \leftarrow 2$$

second find k:

Put A = $\frac{1}{2}A_0$, at t=T

Sub into 2

$$\frac{1}{2}A_0 = A_0 e^{KT}$$

$$\frac{1}{2} = e^{KT}$$

$$\ln\frac{1}{2} = KT$$

$$\ln 1 - \ln 2 = KT$$

Ln 1=0

$$-\ln 2 = KT$$

$$K = \frac{-\ln 2}{T}$$

Sub into 2

$$\boldsymbol{A = A_0 e^{\frac{-t\ \ln 2}{T}}}$$

Third:

Put A = $\frac{1}{20}A_0$ in $A = A_0 e^{\frac{-\ln 2}{T}t}$

$$\frac{1}{20}A_0 = A_0 e^{\frac{-t\ \ln 2}{T}}$$

$$\ln\frac{1}{20} = \frac{-t\ln 2}{T}$$

$$-\ln 20 = \frac{-t\ln 2}{T}$$

$$-t\ln 2 = -\ln 20\,T$$

$$t = \frac{\ln 20}{\ln 2}\,T$$

Ln 20=2.9957322, ln 2= 0.6931471

$$\boldsymbol{t = 4.32\,T}$$

(مدني)

$$\boldsymbol{a = 4t + 3 \quad, V(0) = 0, \quad S(0) = 3\ m}$$

1- **Find the velocity when $\boldsymbol{t = 5s}$**
2- **Find the position when $\boldsymbol{t = 4\ s}$**

Solution:

$$a = \frac{dV}{dt} = 4t + 3$$

$$\int dV = \int 4t + 3\ dt$$

$$V(t) = \frac{4t^2}{2} + 3t + c$$

$$V(0) = 0 + 0 + c$$

$$c = 0$$

$$V(t) = 2t^2 + 3t$$

$$\boldsymbol{V(5) = 2(5)^2 + 3(5) = 65\ m/s}$$

$$V = \frac{dS}{dt} = 2t^2 + 3t$$

$$\int dS = \int 2t^2 + 3t\ dt$$

$$S(t) = \frac{2t^3}{3} + \frac{3t^2}{2} + c$$

$$S(0) = 0 + 0 + c = 3$$

$$c = 3$$

$$S(t) = \frac{2t^3}{3} + \frac{3t^2}{2} + 3$$

$$\boldsymbol{S(4) = \frac{2(4)^3}{3} + \frac{3(4)^2}{2} + 3 = 69.67\ m}$$

Chapter 2:

(الكترونيات و مدني)

For a horizontal cantilever of length $\boldsymbol{l}$, with load $\boldsymbol{w}$ per unit length, the equation of bending is

$$\boldsymbol{EI\ \frac{d^2y}{dx^2} = \frac{w}{2}(l-x)^2}$$

Where $\boldsymbol{E, I, w\ and\ l}$ are constants. If $\boldsymbol{y = 0\ and\ \frac{dy}{dx} = 0\ at\ x = 0}$, find $\boldsymbol{y}$ in terms of $\boldsymbol{x}$. Hence find the value of $\boldsymbol{y}$ when $\boldsymbol{x = l}$.

Solution

$$\frac{d^2y}{dx^2} = \frac{w}{2\ EI}(l-x)^2$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{w}{2\ EI}(l-x)^2$$

$$\frac{dy}{dx} = \int \frac{w}{2\ EI}(l-x)^2\ dx$$

$$\frac{dy}{dx} = \frac{-w}{2\ EI}\ \frac{(l-x)^3}{3} + c_1$$

$$0 = \frac{-w}{2\ EI}\ \frac{(l)^3}{3} + c_1 \quad \rightarrow \quad c_1 = \frac{wl^3}{6IE}$$

$$\frac{dy}{dx} = \frac{-w}{2\ EI}\ \frac{(l-x)^3}{3} + \frac{wl^3}{6IE}$$

$$\frac{dy}{dx} = \frac{w}{6IE}\ (-(l-x)^3 + l^3)$$

$$\int dy = \frac{w}{6IE}\int(-(l-x)^3 + l^3)\ dx$$

$$y = \frac{w}{6IE}\left[\frac{(l-x)^4}{4} + l^3\ x\right] + c_2$$

$$0 = \frac{w}{6IE}\left[\frac{(l)^4}{4} + 0\right] + c_2 \quad \rightarrow \quad c_2 = \frac{-wl^4}{24IE}$$

$$y = \frac{w}{6IE}\left[\frac{(l-x)^4}{4} + l^3\,x\right] - \frac{wl^4}{24IE}$$

$$at\ x = l$$

$$y = \frac{w}{6IE}\,[0 + l^3\,] - \frac{wl^4}{24IE}$$

$$y = \frac{wl^4}{6IE}\left[1 - \frac{1}{4}\right]$$

$$y = \frac{wl^4}{6IE}\left[\frac{3}{4}\right]$$