Fundamentals of Electrical Engineering

First YEAR

By

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DC Circuit Analysis

Chapter 4

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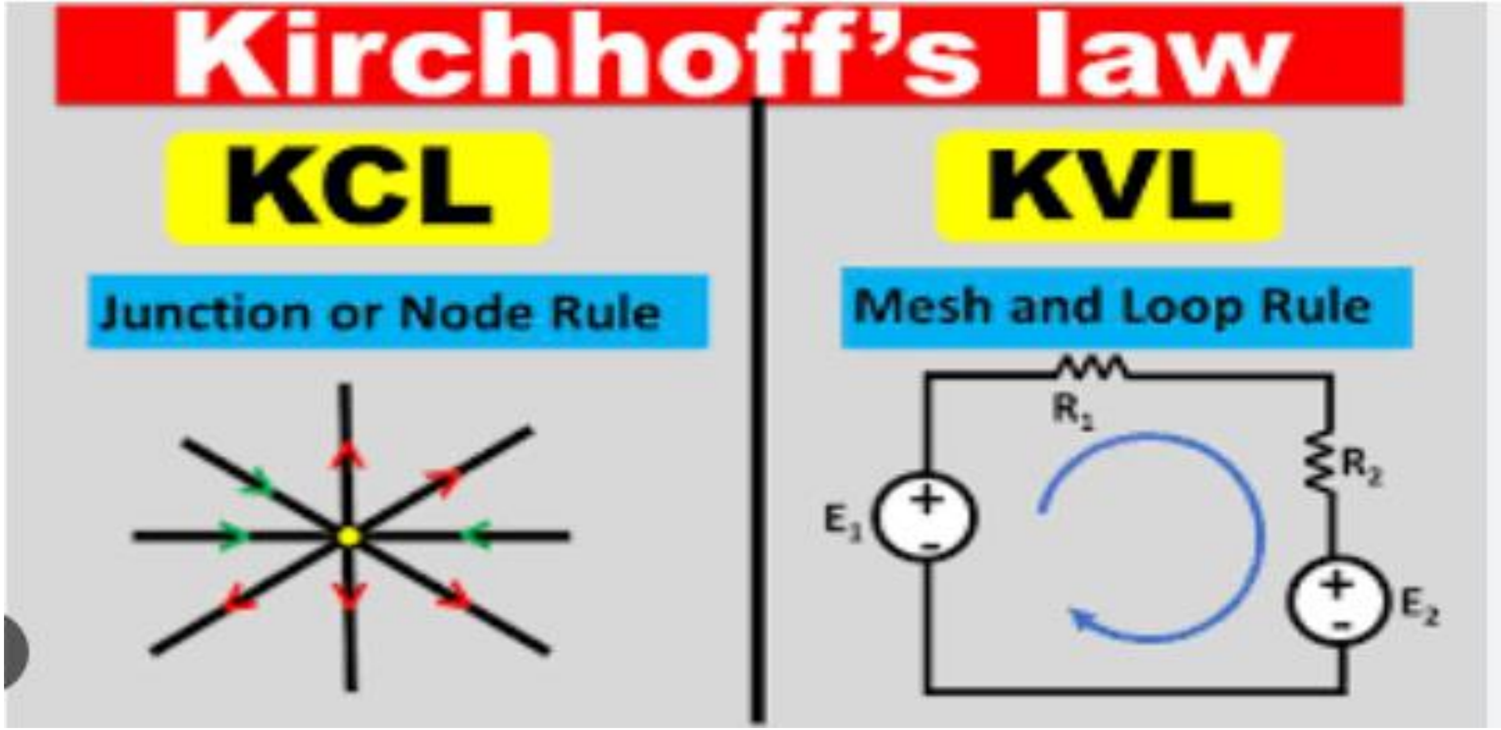
Chapter Content

CH4: DC Circuit Analysis

- KIRCHHOFF'S CURRENT LAW
- CURRENT DIVISION
- SOURCE TRANSFORMATIONS
- MESH ANALYSIS
- NODAL ANALYSIS

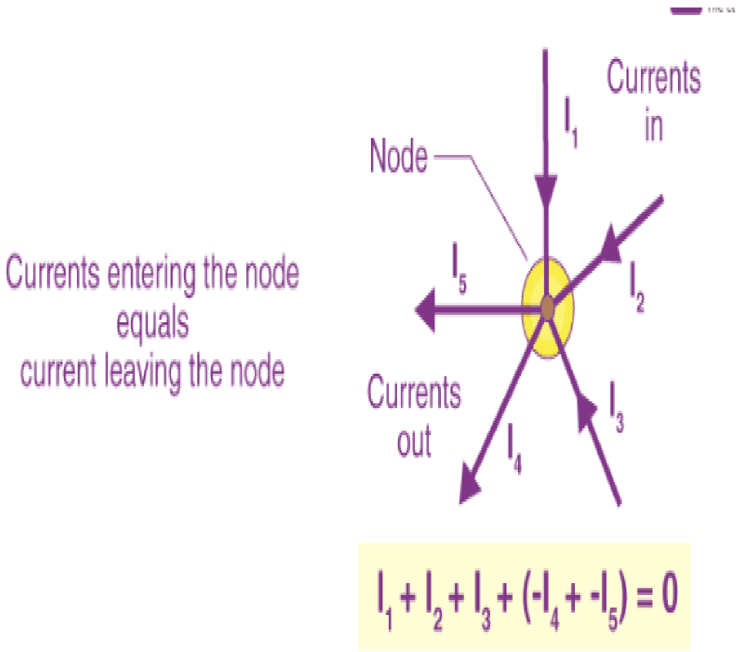

Kirchhoff's law
KCL
Junction or Node Rule
KVL
Mesh and Loop Rule
R_1
R_2
E_1
E_2

1. KIRCHHOFF'S CURRENT LAW

- Kirchhoff's Current Law, abbreviated KCL, has three equivalent versions:

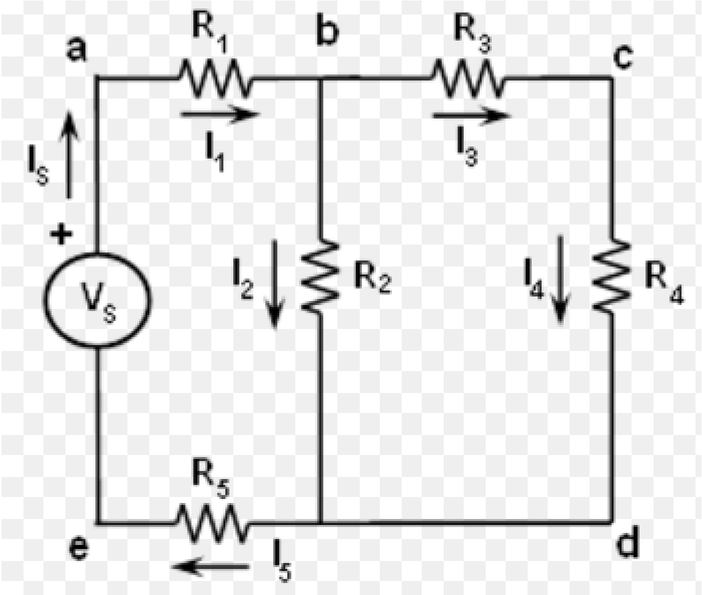
At any instant in a circuit,

- ✓ The algebraic sum of the currents leaving a closed surface is zero.
- ✓ The algebraic sum of the currents entering a closed surface is zero.
- ✓ The algebraic sum of the currents entering a closed surface equals the algebraic sum of those leaving.



1. KIRCHHOFF'S CURRENT LAW

- Remember that: a current entering is a negative current leaving, and that a current leaving is a negative current entering.
- for a node that has no voltage sources connected the most convenient KCL version is often the third one, restricted such that <u>the currents entering are from current sources and the currents leaving are through resistors.</u>



EX1

Find current i_3 at the node shown below.

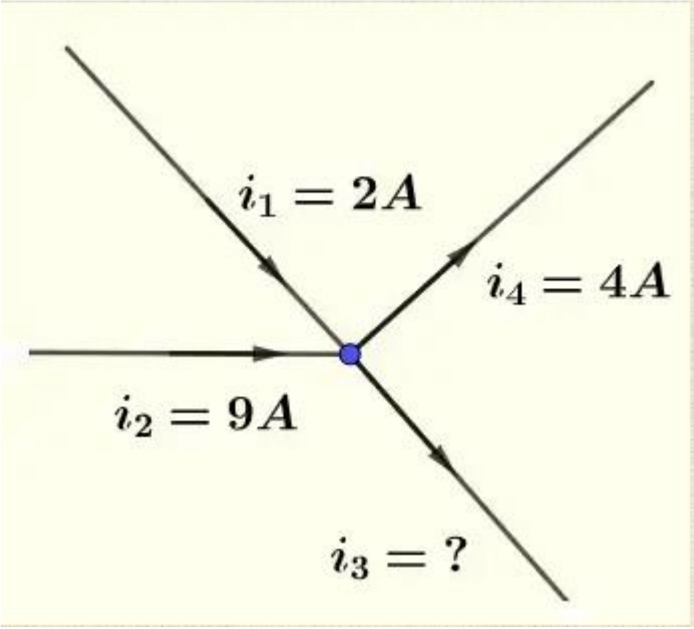
SOL: Currents i_1 and i_2 are flowing into the node and currents i_3 and i_4 are flowing out of the node. Apply KCL at the given node.

$$i_1+i_2 = i_3+i_4$$

Substitute the known quantities

$2+9 = i_3+4$

$$i_3=7 \text{ A}$$

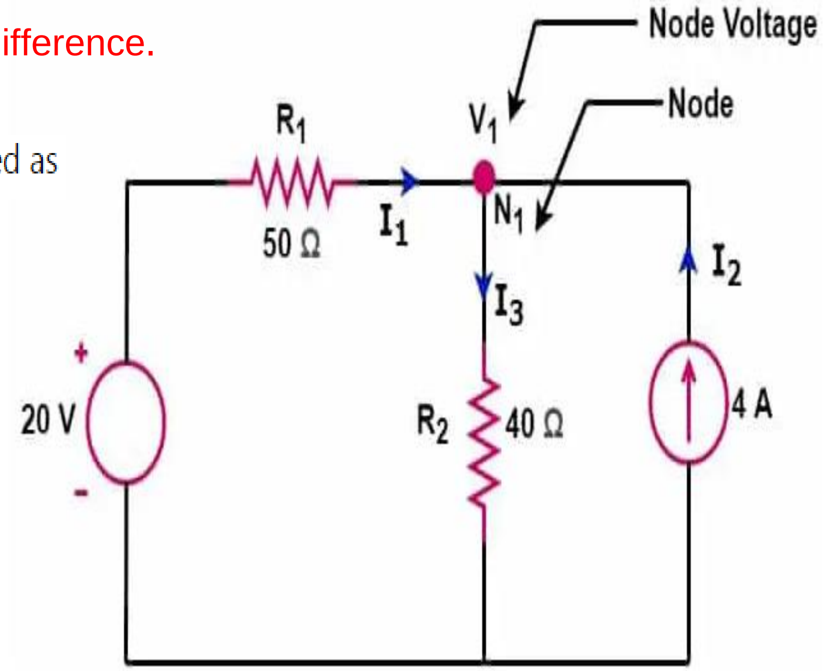


EX2

Determine the current across each resistor and potential difference.

KCL at node N1 $I_1 + I_2 - I_3 = 0$ ———————(1)

The voltage at node N_1 is V_1, the magnitude of I_1 and I_2 can be determined as



$I_1 = (20-V_1)/50$ ———————-(2)

$I_2 = 4$ A ————————————-(3)

$I_3 = V_1/40$ ———————————(4)

Putting the value of I_1, I_2 and I_3 in equation(1), we get

$I_1 + I_2 - I_3 = 0$

$(20-V_1)/50 + 4 - V_1/40 = 0$

$4(20-V_1) + 800 - 5V_1 = 0$

$80 - 4V_1 + 800 - 5V_1 = 0$

EX2

$9V_1 = 880$

$V_1 = 97.78$ Volts ————————(5)

Current through Resistance R_1

$I_1 = (20 - V_1)/50$

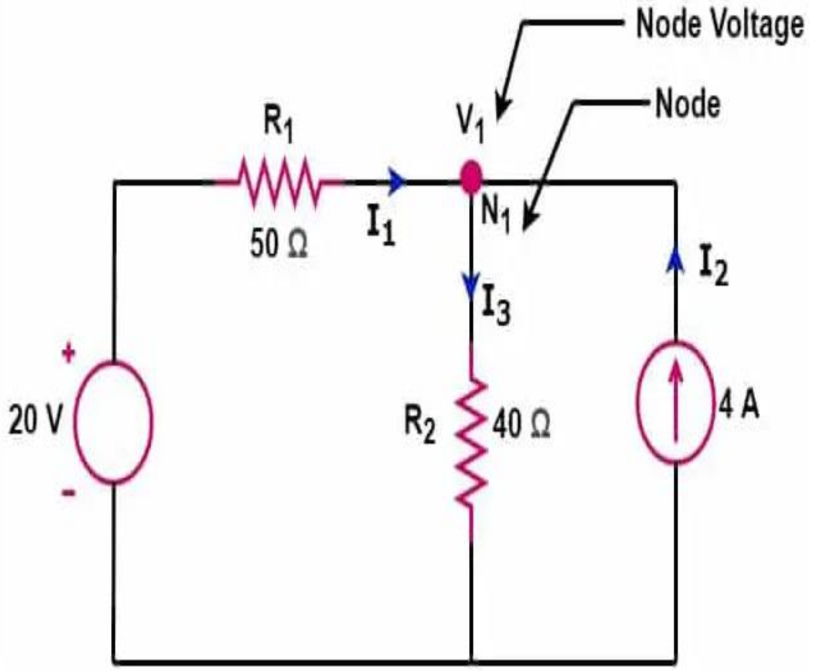
$I_1 = (20 - 97.78)/50$

$= -77.78/50$

$\mathbf{I_1 = -1.5556\ A}$

Current through Resistance R_2

$I_3 = V_1/40 = 97.78/40 =$ **2.4445 A**



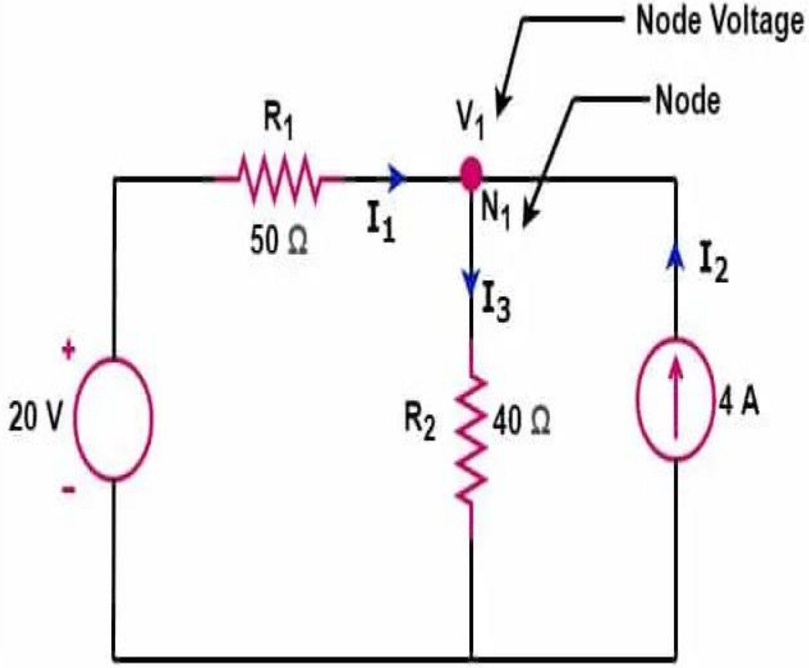
EX2

$I_1 + I_2 = I_3$ **(KCL Equation for this circuit)**

$-1.5556 + 4 = 2.4445$

$4 = 2.4445 + 1.5556$

$4 = 4$ **[KCL proved]**



EX3

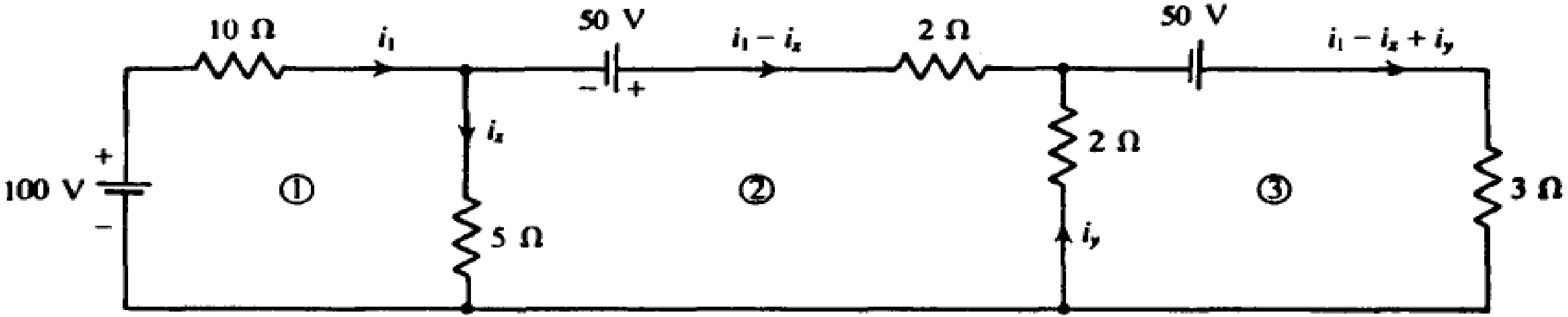


Fig.

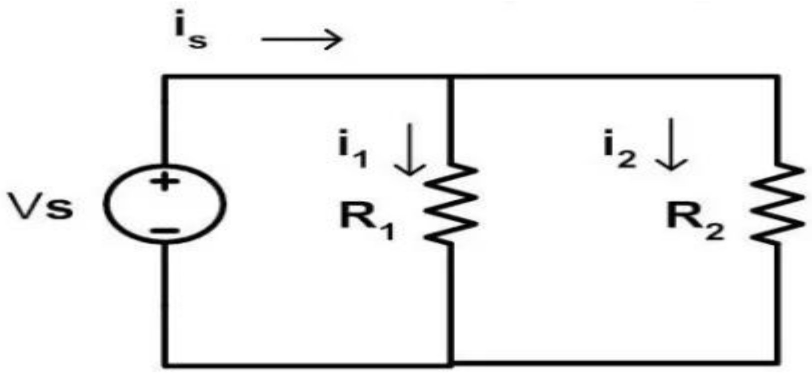
Determine the currents i_x and i_y in the network shown in Fig.

On the basis of KCL, the currents in the remaining branches are also marked in Fig. By KVL for meshes 1, 2, and 3,

$$100 = 10i_1 + 5i_x \qquad 50 = 2(i_1 - i_x) - 2i_y - 5i_x \qquad 50 = 3(i_1 - i_x + i_y) + 2i_y$$

2. CURRENT DIVISION

- The current division or current divider rule applies to resistors in parallel.
- It gives the current through any resistor into the parallel combination



$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$

2. CURRENT DIVISION

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} \dots\dots\dots\dots(2)$$

$$I = \frac{V\,(R_1 + R_2)}{R_1 R_2} \dots\dots\dots(4)$$

$$V = I_1 R_1 = I_2 R_2$$

$$I = \frac{I_1 R_1 (R_1 + R_2)}{R_1 R_2} = \frac{I_1}{R_2}(R_1 + R_2)$$

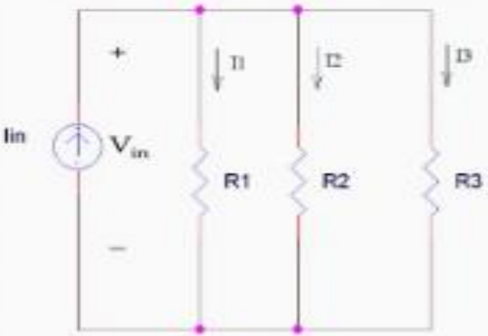
$$I = \frac{I_2 R_2 (R_1 + R_2)}{R_1 R_2} = \frac{I_2}{R_1}(R_1 + R_2)$$

$$I_1 = I\,\frac{R_2}{R_1 + R_2} \quad \text{and} \quad I_2 = I\,\frac{R_1}{R_1 + R_2}$$

2. CURRENT DIVISION

Current Division

All resistors in parallel share the same voltage



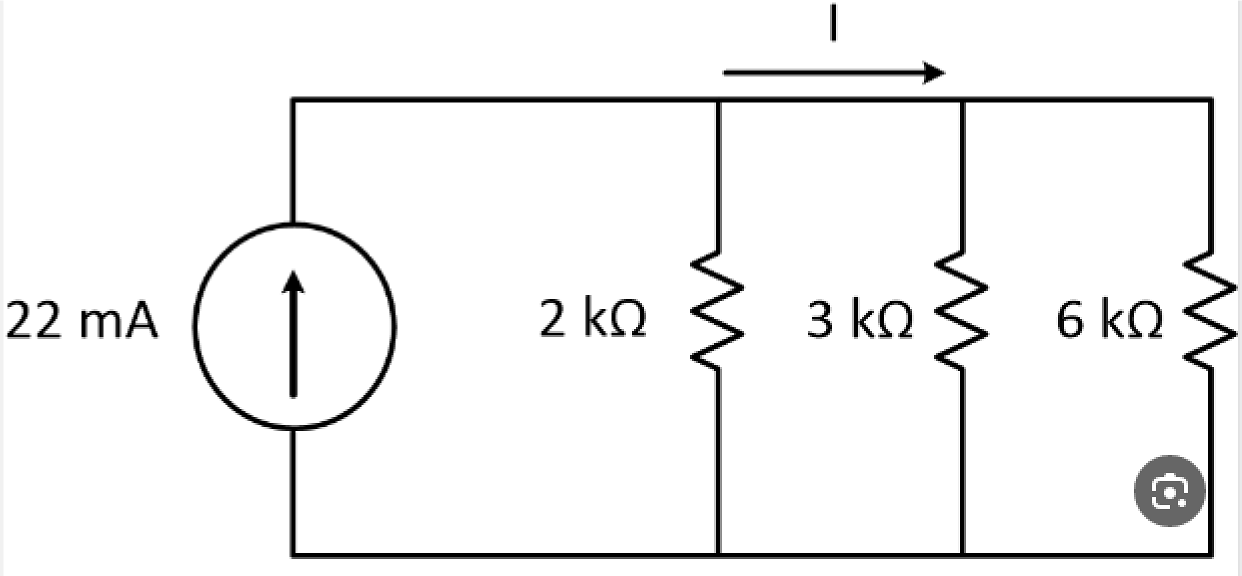
$$I_1 = \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} I_{in}$$

$$I_2 = \frac{R_1 \| R_3}{R_2 + R_1 \| R_3} I_{in}$$

$$I_3 = \frac{R_1 \| R_2}{R_3 + R_1 \| R_2} I_{in}$$

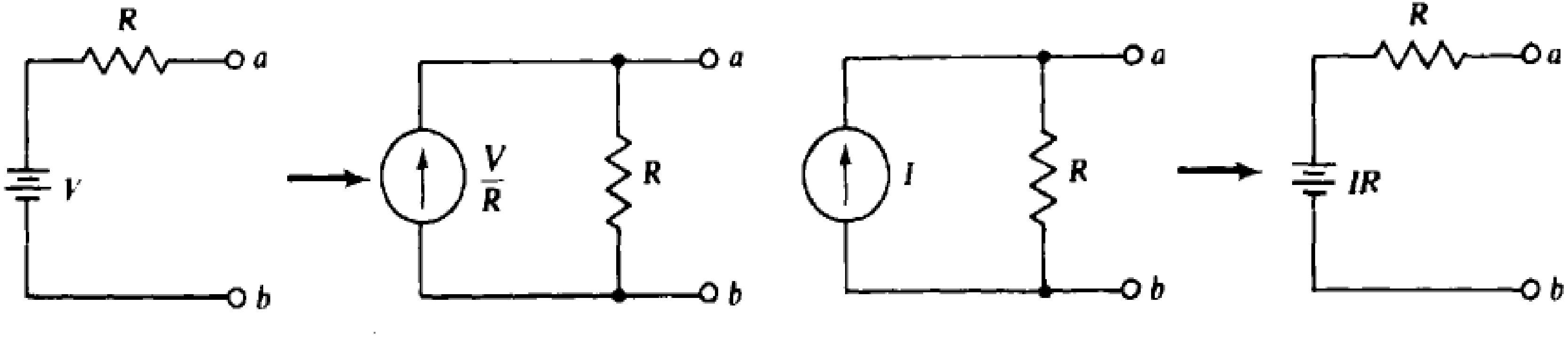
Assignment

Determine the current across each resistor and potential difference.



3.SOURCE TRANSFORMATIONS

Figure (a) shows the transformation from a voltage source to an equivalent current source.
Figure (b) the transformation from a current source to an equivalent voltage source.

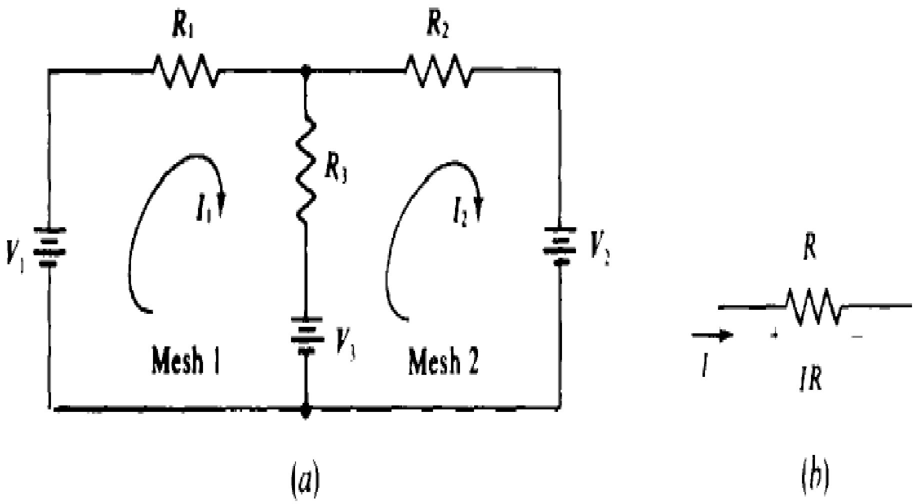


4. MESH ANALYSIS

<u>Mesh analysis</u> is defined as
The method in which the current flowing through a planar circuit is calculated.

$$I_1R_1 + (I_1 - I_2)R_3 = V_1 - V_3$$

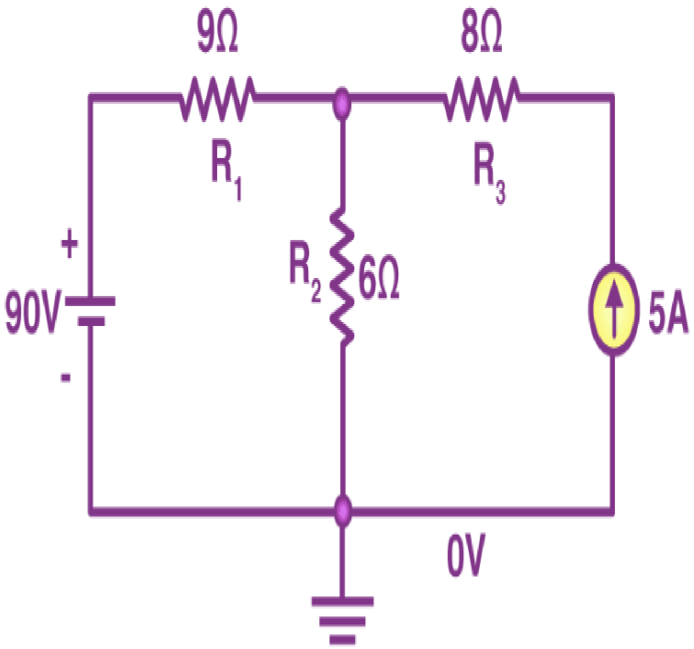
$$-R_3I_1 + (R_2 + R_3)I_2 = V_3 - V_2$$



EX

Example :

In the given circuit 90v is the battery value, 5A is the current source and the three resistors are 9 ohms, 6 ohms, and 8 ohms. Using mesh analysis, determine the current across each resistor and potential difference.



$$90 - I_1R_1 - R_2(I_1 - I_2) = 0$$

$$90 - 9I_1 - 6(I_1 - I_2) = 0$$

$$-15I_1 + 6I_2 = -90$$

$5I_1 - 2I_2 = 30$ (this is obtained by dividing the equation with -3)

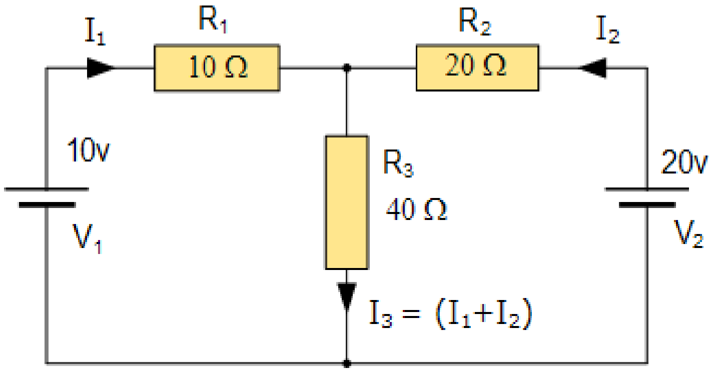
$$I_1 = 4A$$

So, through R_1, 4A current is flowing and through R_3, 5A current is flowing.

EX

Example :

determine the current across each resistor and potential difference.



$i_1 = I_1$, $i_2 = -I_2$ and $I_3 = I_1 - I_2$

$$10 = 50I_1 - 40I_2$$

$$-20 = -40I_1 + 60I_2$$

$$I_1 = -0.143A$$

$$I_2 = -0.429A$$

5. NODAL ANALYSIS

<u>Nodal analysis</u> is used for solving any electrical network, and it is defined as
The mathematical method for calculating the voltage distribution between the circuit nodes.

This method is also known as the node-voltage method since the node voltages are with respect to the ground. The following are the three laws that define the equation related to the voltage that is measured between each circuit node:

1. Ohm's law
2. Kirchhoff's voltage law
3. Kirchhoff's current law

5. NODAL ANALYSIS

Example 2: Determine the voltage at each node of the given circuit using nodal analysis.

Let node 2 be the reference node, and this node's voltage will be zero.

Using Kirchhoff's current law at each node, we get

$$\frac{V_1}{30} + \frac{V_1 - 100}{5} + \frac{V_1 - V_3}{10} = 0$$

(eq.1)

This is a result of KCL at node 1

$$\frac{V_3 - V_1}{10} + \frac{V_3}{10} + \frac{V_3}{20} = 0$$

(eq.2)

This is a result of KCL at node 3

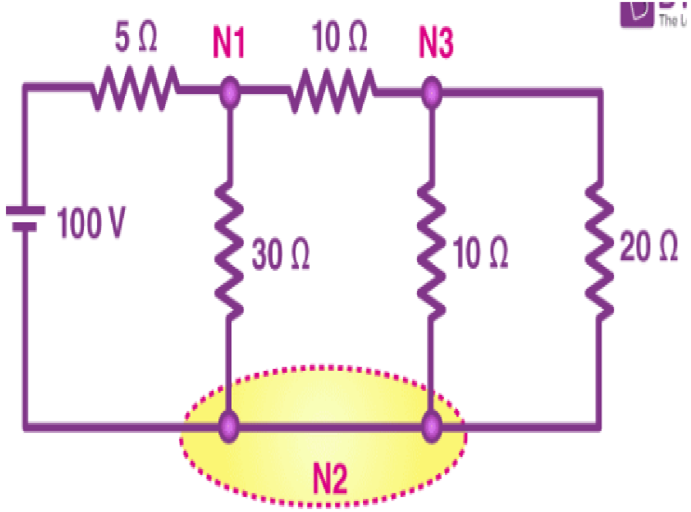
$$\left(\frac{1}{30} + \frac{1}{5} + \frac{1}{10}\right)V_1 - \left(\frac{1}{10}\right)V_3 = \frac{100}{5}$$

$$-\left(\frac{1}{10}\right)V_1 + \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right)V_3 = 0$$

Solving the above equations we get

V_1 = 68.2v

V_3 = 27.3v



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