CHAPTER 1: First Order Differential Equation

1-Classification:

1) $y''' + 2xy'' + \sin x \ y' + x^2 y = 0$

2) $y''' + 2xy'' + \sin x \ y' + x^2 y - x = 0$

3) $\frac{d^2 y}{dx^2} + 2x^2 \frac{dy}{dx} + \sin x \ y = 0$

4) $y''' + (y''')^2 + 2 \ x \ y''' + x^2 y'' = 0$

5) $y'' + 2xy' + xy = 0$

6) $\frac{\partial y}{\partial x} + xy = \sin x \ y$

7) $y^{(4)} + y \ y'' + xy^2 = x$

8) $(y''')^2 + (y')^3 + 3x^2 y = 0$

9) $\frac{\partial^2 y}{\partial x^2} + \frac{\partial y}{\partial x} + xy = \cos x \ y'$

10) $\frac{d^3 y}{dx^3} + x \ \frac{d^2 y}{dx^2} + y \frac{dy}{dx} = x$

11) $\frac{\partial^2 x}{\partial y \partial z} + \left(\frac{\partial x}{\partial y}\right)^5 + \frac{\partial^2 x}{\partial z^2} = 0$

12) $y^{(4)} + 4xy''' + \sin x \ y'' + x^2 \ y' + xy = 0$

2-solution of differential equation:

1) $\frac{dy}{dx} = (x^2 + 3x + e^x)$

2) $\frac{dy}{dx} = (x^2 + 1)(y^2 + 1)$

3) $\frac{dy}{dx} = \frac{y+1}{x^2+1}$

4) $x(y^2 + 1)dy + y(x^2 + 1)dx = 0$

5) $\frac{dy}{dx} = (xy + y)(x + 1)$

6) $\frac{dy}{dx} = \tan^2(x + y + 3)$

7) $\frac{dy}{dx} = 1 + e^{x+y+3}$

8) $\frac{dy}{dx} = \sin^2(x + y + 3)$

9) $\frac{dy}{dx} = \sin(x + y + 3)$

10) $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$

11) $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$

12) $xy\frac{dy}{dx} = x^2 + y^2$

13) $xy\,dy - (x^2 + y^2)dx = 0$

14) $\frac{dy}{dx} = x^2y + y + x^2 + 1$

15) $(x^2 + 1)y\,dx + x(y^2 + y)dy = 0$

16) $\frac{dy}{dx} = \frac{(y^2+1)}{x+1}$

17) $\frac{dy}{dx} = \frac{x^2+4}{y^2+5}$

18) $\frac{dy}{dx} = x^3 + 3x^2$

19) $x\sin y\,dx + y^2 + \frac{x^2}{2}\cos y\,dy = 0$

20) $xy\,dx + \left(\frac{x^2}{2} + y + 3\right)dy = 0$

21) $\frac{dy}{dx} + 3y = e^{5x}$

22) $\frac{dy}{dx} - \frac{1}{x}y = x^2$

23) $\frac{dy}{dx} - \frac{1}{x}y = x\,y^2$

24) $\frac{dy}{dx} = \frac{1+y}{2+x}$

25) $\frac{dy}{dx} = \frac{y^2+xy^2}{x^2y-x^2}$

26) $\frac{dy}{dx} = \frac{y^2-1}{x}$

27) $y\tan x\,\frac{dy}{dx} = (4 + y^2)\sec^2 x$

28) $x^2 \, y - x^3 \, \dfrac{dy}{dx} = y^4 \, \cos(x)$

29) $y - 2\,x\,\dfrac{dy}{dx} = x \, (x+1) \, y^3$

30) $2\,x^2\,\dfrac{dy}{dx} = x^2 + y^2$

31) $\left(x^3 + 3\,x\,y^2\right)\dfrac{dy}{dx} = y^3 + 3\,x^2\,y$

32) $\dfrac{dy}{dx} = e^{x+y+1}$

33) $x\,\dfrac{dy}{dx} - y = x^2$

34) $x\,\dfrac{dy}{dx} - 2\,y = x^3 \, \cos(x)$

35) $\dfrac{dy}{dx} + y \, \tanh(x) = 2 \, \sinh(x)$

36) $\dfrac{dy}{dx} + \dfrac{y}{x} = y^3$

37) $x\,\dfrac{dy}{dx} + 3\,y = x^2\,y^2$

38) $x\,y^2 + 2 = \left(3 - x^2\,y\right)\,y'$

39) $\dfrac{dy}{dx} = \dfrac{x - 2\,y + 1}{2\,x - 4\,y}$ if z = x − 2 y

40) $r \, \dfrac{\tan(\Theta)}{a^2 - r^2} \, \dfrac{dr}{d\Theta} = 1$

41) $\dfrac{dy}{dx} = \dfrac{x + y}{x - y}$

42) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{x^2 - y^2}$

43) $2\,x\,y\,y' = y^2 - x^2$

44) $x\,\dfrac{dy}{dx} = y + x\,e^{\frac{y}{x}}$

45) $\dfrac{dy}{dx} + \dfrac{1}{x} - \dfrac{2\,x}{1 - x^2} = \dfrac{1}{1 - x^2}$

46) $\left(x^2\,y + 2\,x^2\right)\,\dfrac{dy}{dx} = x\,y + 2\,y + x + 2$

47) $\dfrac{dx}{dy} = \dfrac{1}{y + x^2}$

48) $x\,\dfrac{dy}{dx} = \dfrac{y}{\ln(y) - \ln(x)}$

49) $\dfrac{dy}{dx} = \dfrac{2\,x\,y\,e^{\left(\frac{x}{y}\right)^2}}{y^2 + y^2\,e^{\left(\frac{x}{y}\right)^2} + 2\,x^2\,e^{\left(\frac{x}{y}\right)^2}}$

50) $x\,\dfrac{dy}{dx} = 2\,x^2 + 2\,x + 3$