Ministry of Higher Education		
Higher Institute for Engineering and Technology at Manzala		
First Semester: 2023/2024		Date: 12/11/2023
Midterm Exam		Level: 1
Department: Basic Science		Time allowed: 60 min
Total Marks: 40		Code: BS 110
Course title: Mathematics 3	Examiner: Dr. Hamouda Abueldahab	
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Q.1. A circuit consisting of a resistor R, an inductor L, and a voltage source V(t) connected in series. Deduce the mathematical model for the R-L circuit, then solve it, $\boldsymbol{i(0) = 0}$

[4 Mark]

$$V = RI + l\frac{dI}{dt}$$

$$\frac{dI}{dt} + \frac{R}{l}I = \frac{V}{l} \qquad p(t) = \frac{R}{l}, \; Q(t) = \frac{V}{l}$$

$$m(t) = e^{\int \frac{R}{l}dt} = e^{\frac{R}{l}t}$$

$$m(t).I = \int Q(t).m(t)\,dt$$

$$e^{\frac{R}{l}t}.I = \int \frac{V}{l}.e^{\frac{R}{l}t}\,dt$$

$$e^{\frac{R}{l}t}.I = \frac{V}{l}.e^{\frac{R}{l}t} + c$$

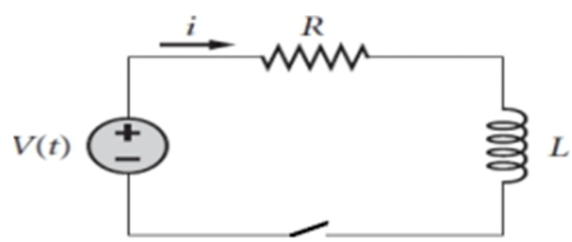
$$e^{\frac{R}{l}t}.I = \int \frac{V}{l}.e^{\frac{R}{l}t}\,dt$$

$$e^{\frac{R}{l}t}.I = \frac{V}{l}.e^{\frac{R}{l}t} + c$$

$$c = -\frac{V}{l}$$

$$e^{\frac{R}{l}t}.I = \frac{V}{l}.e^{\frac{R}{l}t} - \frac{V}{l}$$

$$\boldsymbol{I = \frac{V}{l}\left(1 - e^{-\frac{R}{l}t}\right)}$$



Q.2. Solve the following differential equations: **[16Mark]**

1- $\frac{ds}{dt} = 3t^2 + 2t + 3 \qquad , s(0) = 2$

2- $\frac{dy}{dx} = \frac{2xy\,e^{\left(\frac{x}{y}\right)^2}}{y^2 + y^2 e^{\left(\frac{x}{y}\right)^2} + 2x^2 e^{\left(\frac{x}{y}\right)^2}}$

3- $\frac{dy}{dx} = \sin(x + y + 3)$

4- $xy\frac{dy}{dx} = x^2 + y^2$

5- $\frac{dx}{dy} = \frac{1}{y + x^2}$

6- $\frac{d^2y}{dx^2} + 4y = 0$

7- $\frac{d^2y}{dx^2} - 9y = 0$

8- $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

Q .3. Find the D.E and the general solution of the D.E. whose characteristic equation is

$(r - 1)(r + 2)(r^2 + 4) = 0$ **[3 Mark]**

Q .4. Choose the correct answer from, a, b, c, or d. **[10 Mark]**

1- if $D = \frac{d}{dx}$ then the general type of the equation: $(D + 3)\,2\,y = 2x\,2 - 1\,is$:

(a) Ordinary differential equation (ODE) (b) Partial differential equation (PDE)

(c) Algebraic equation (d) Transcendental equation

2- The type of the first order ordinary differential equation: $y' = y + x$ is:

(a) Separable (b) Homogenous

(c) Linear (d) Bernoulli

3- If c is a constant, then the general solution of the ordinary differential equation (ODE) $y\,y' = x$ is:

(a) $y = cx$ **(b) $y^2 - x^2 = c$**

(c) $y^2 + x^2 = c$ (d) $y = c\,e^{x}$

4- If c_1 and c_2 are constants, then the general solution of the ODE $y'' - y' = 0$ is:

(a) $c_1 e + c_2\, e^{-x}$ **(b) $c_1 + c_2 e^{x}$**

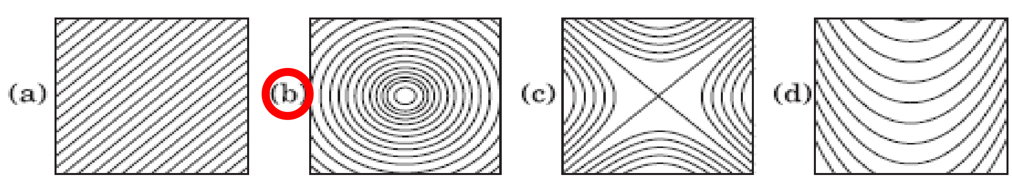
(c) $y = c_1\, e^{x}$ (d) $c_1\, e^{2x} + c_2 e^{x}$

If $y = e^{2x}$ is a solution of $\frac{d^2y}{dx^2} - 4ky = 0$ then k =

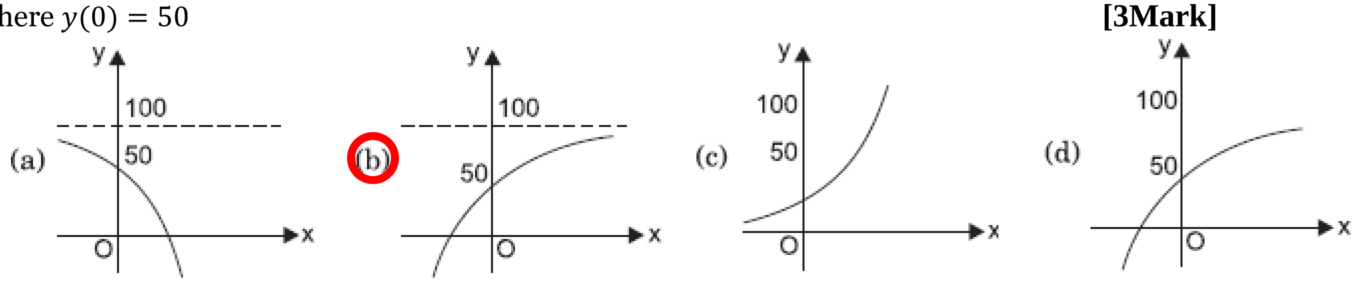
(a) -4 **(b) 1**

(c) -1 (d) 4

Q (5) The general solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ is a family of curves which looks most like which of the following? **[4 Mark]**



Q (6) Which one of the following curves represents the solution of the initial value problem $\frac{dy}{dx} = 100 - y$ where $y(0) = 50$ **[3Mark]**



Model answer for Q2:

1- $\int ds = \int 3t^2 + 2t + 3\ dt$

$s = t^3 + t^2 + 3t + c \qquad\qquad 2 = c$

$s = t^3 + t^2 + 3t + 2$

2-

$$\frac{dx}{dy} = \frac{y^2 + y^2 e^{(\frac{x}{y})^2} + 2x^2 e^{(\frac{x}{y})^2}}{2xye^{(\frac{x}{y})^2}}$$

Let $u = \frac{x}{y}$, $x = uy$, $\frac{dx}{dy} = u + y\frac{dx}{dy}$

$$u + y\frac{du}{dy} = \frac{y}{2x}e^{-(\frac{x}{y})^2} + \frac{y}{2x} + \frac{x}{y}$$

$$u + y\frac{du}{dy} = \frac{1}{2u}e^{-u^2} + \frac{e^{u^2}}{e^{u^2}2u}$$

$$y\frac{du}{dy} = \frac{1}{2u}e^{-u^2} + \frac{e^{u^2}}{e^{u^2}2u} - u$$

$$y\frac{du}{dy} = \frac{e^{u^2} + 1}{e^{u^2}2u}$$

$$\int \frac{e^{u^2}2u}{e^{u^2} + 1}du = \int \frac{dy}{y}$$

$$ln(e^{u^2} + 1) = lny + c$$

$$ln(e^{(\frac{x}{y})^2} + 1) = lny + c$$

3-

Let $u = x + y + 3$

$$\frac{du}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{du}{dx} = 1 + \sin u$$

$$\int \frac{du}{1+\sin u} = \int dx$$

$$\int \frac{du}{1+\sin u} * \frac{1-\sin u}{1-\sin u} = \int dx$$

$$\int \frac{1-\sin u}{1-sin^2 u}\,du = \int dx$$

$$\int \frac{1-\sin u}{cos^2 u}\,du = \int dx$$

$$\int sec^2 u - \tan u \sec u \;\; du = \int dx$$

$$\tan u - \sec u = x + c$$

$$\tan(x+y+3) - \sec(x+y+3) = x + c$$

4- $\dfrac{dy}{dx} = \dfrac{x^2+y^2}{xy}$

$$let\ y = ux, \qquad \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{x^2+u^2x^2}{x^2u}$$

$$x\frac{du}{dx} = \frac{1+u^2}{u} - u$$

$$x\frac{du}{dx} = \frac{1}{u}$$

$$\int u\,du = \int \frac{dx}{x}$$

$$\frac{u^2}{2} = \ln x + c$$

$$\frac{(\frac{y}{x})^2}{2} = \ln x + c$$

5-

$$\frac{dy}{dx} = y + x^2$$

$$\frac{dy}{dx} - y = x^2 \qquad ,p(x) = -1, Q(x) = x^2$$

$$m(x) = e^{\int -\mathbf{1}\,dx} = \boldsymbol{e^{-x}}$$

$$e^{-x}.y = \int e^{-x}.x^2 dx$$

$$e^{-x}.y = -x^2e^{-x} - 2xe^{-x} - 2e^{-x}$$

$$y = -x^2 - 2x - 2 + e^x c$$

6-

$$m^2 + 4 = 0$$

$$m_{1,2} = 2i$$

$$y = c_1 \cos 2x + c_2 \sin 2x$$

7-

$$m^2 - 9 = 0$$

$$m_{1,2} = \pm 3$$

$$y = c_1 e^{3x} + c_2 e^{-3x}$$

8-

$$m^2 + 2m + 1 = 0$$

$$m_{1,2} = -1$$

$$y = (c_1 + c_2 x)e^{-x}$$

Model answer for Q.3:

$$(r^2 + r - 2)(r^2 + 4) = 0$$

$$r^4 + r^3 + 2r^2 + 4r - 8 = 0$$

D.E. $\quad y^{(4)} + y''' + 2y'' + 4y' - 8y = 0$

$$y = c_1 e^{x} + c_2 e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$$