Lecture (10)

- Schrödinger equation
- Einstein's Principle of Relativity

❑ Schrödinger equation

The Schrödinger equation, sometimes called the Schrödinger wave equation, is a partial differential equation. It uses the concept of energy conservation (Kinetic Energy + Potential Energy = Total Energy) to obtain information about the behavior of an electron bound to a nucleus. Solving the Schrödinger equation gives us Ψ and Ψ^2 . With these we get the quantum numbers and the shapes and orientations of orbitals that characterize electrons in an atom or molecule .

❑ There is a time-dependent Schrödinger equation and a time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m}\nabla^2\Psi(\mathrm{r}) + V(r)\Psi(\mathrm{r}) = E\Psi(\mathrm{r})$$

$$\mathit{Kinetic\ Energy} + \mathit{Potential\ Energy} = \mathit{Total\ Energy}$$

$\hbar$ is the reduced Planck constant,

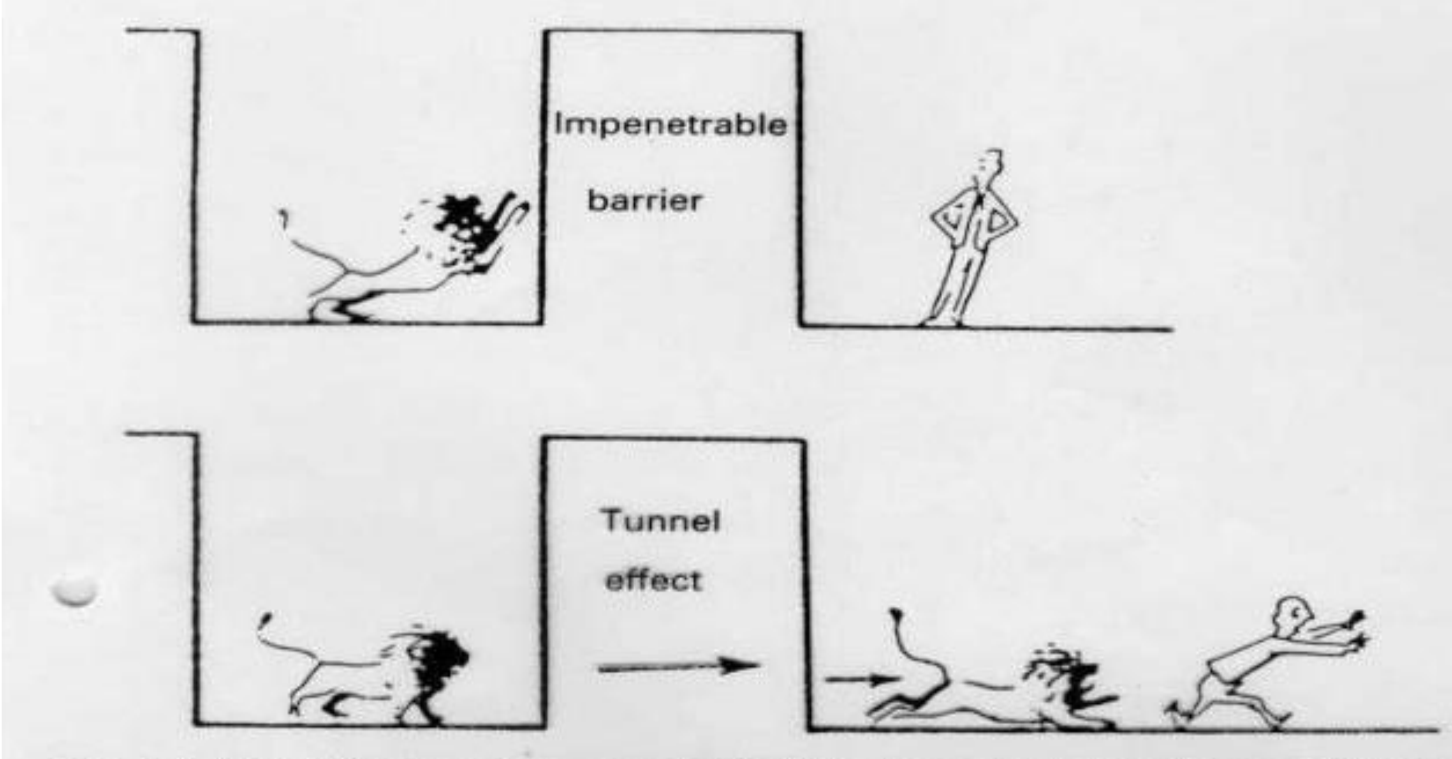
m **is the electron mass,**

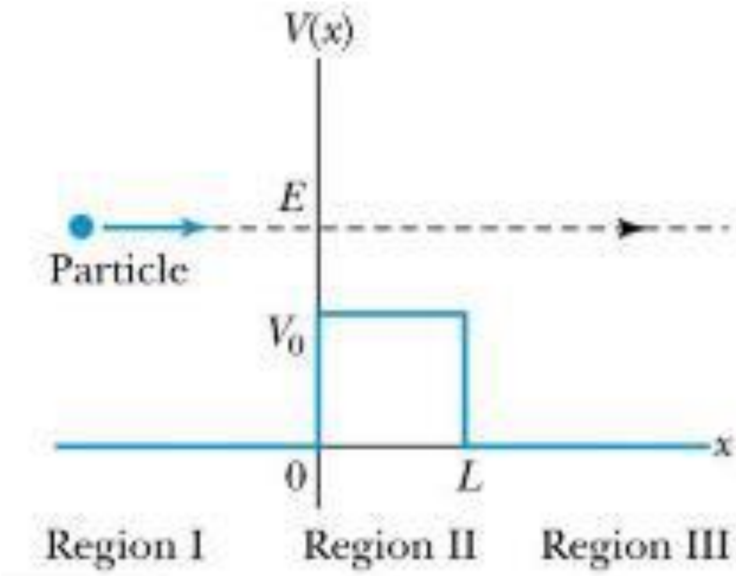
∇ is the Laplacian operator,

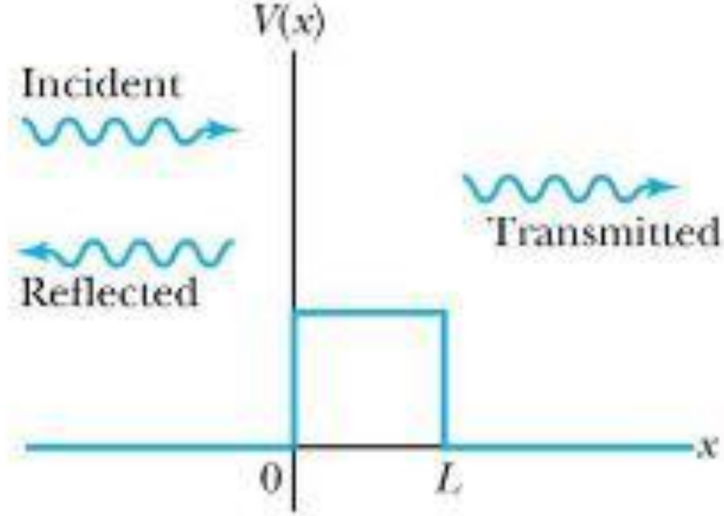
Ψ is the wave function,

V **is the potential energy,**

E **is the energy eigenvalue,**


Impenetrable
barrier
Tunnel
effect


V(x)
E
Particle
V0
0
L
x
Region I
Region II
Region III


V(x)
Incident
Reflected
Transmitted
0
L
x

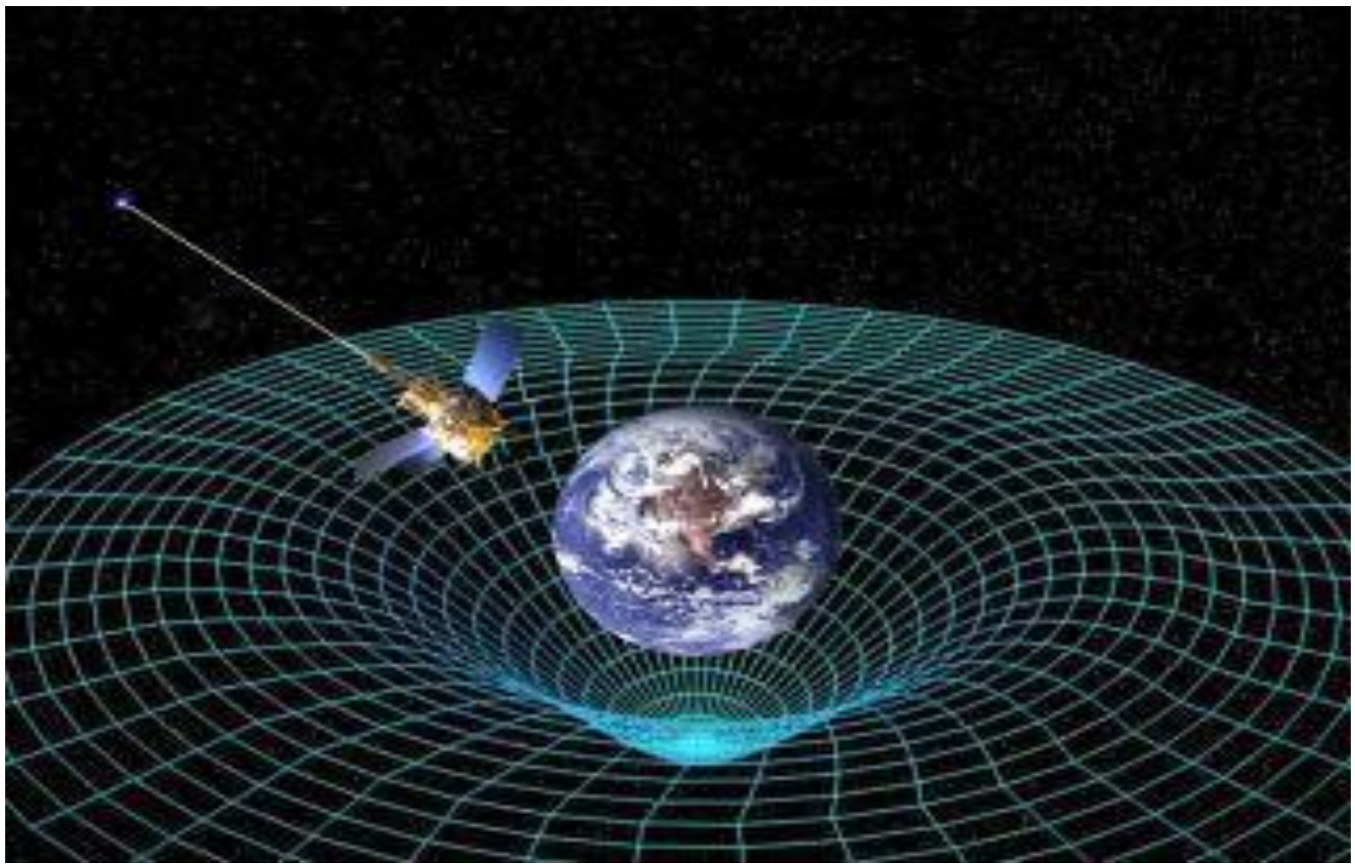
Einstein's Principle of Relativity

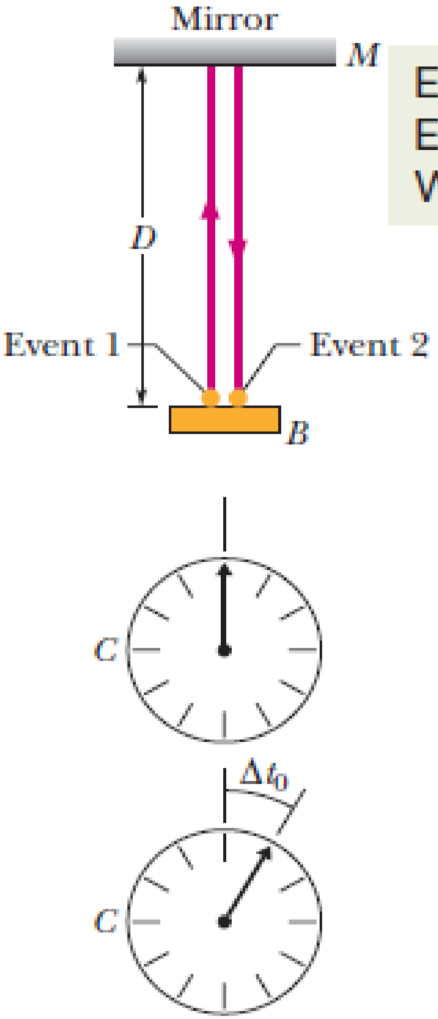
- **The first postulate** asserts that all the laws of physics, those dealing with mechanics, electricity and magnetism, optics, thermodynamics, and so on ,are the same in all reference frames moving with constant velocity relative to one another.

- **The second postulate** assets that the speed of light in free space has the same value c in all inertial frames of reference.

❖ The Relativity of Time

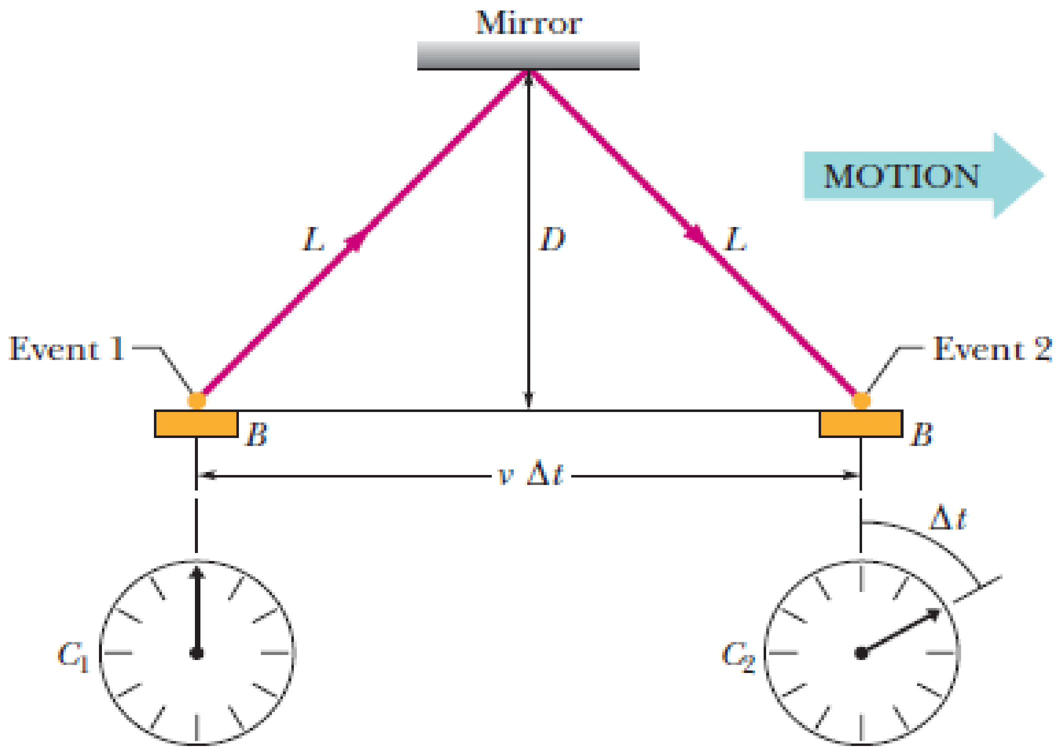
If observers who move relative to each other measure the time interval between two events, they generally will find different results. Why? Because the spatial separation of the events can affect the time intervals measured by the observers.





Event 1 is the emission of light.
Event 2 is the return of the light.
We want the time between them.

The measure of that time interval on Sally's clock differs from that on Sam's clock due to the relative motion.



$$\Delta t_0 = \frac{2D}{c}$$

$$\Delta t = \frac{2L}{c}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}.$$

Example

Your starship passes Earth with a relative speed of 0.9990c. After traveling 10.0 y (your time), you stop at lookout post LP13, turn, and then travel back to Earth with the same relative speed. The trip back takes another 10.0 y (your time). How long does the round trip take according to measurements made on Earth? (Neglect any effects due to the accelerations involved with stopping, turning, and getting back up to speed.)

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}$$

$$= \frac{10.0\text{ y}}{\sqrt{1-(0.9990c/c)^2}} = (22.37)(10.0\text{ y}) = 224\text{ y}.$$

$$\boxed{\Delta t_{total} = (2)(224\text{ y}) = 448\text{ y}}$$

❑ Length Contraction

Let L_0 be the length of a rod that you measure when the rod is stationary (meaning you and it are in the same reference frame, the rod's rest frame). If, instead, there is relative motion at speed v between you and the rod along the length of the rod, then with simultaneous measurements you obtain a length L given by

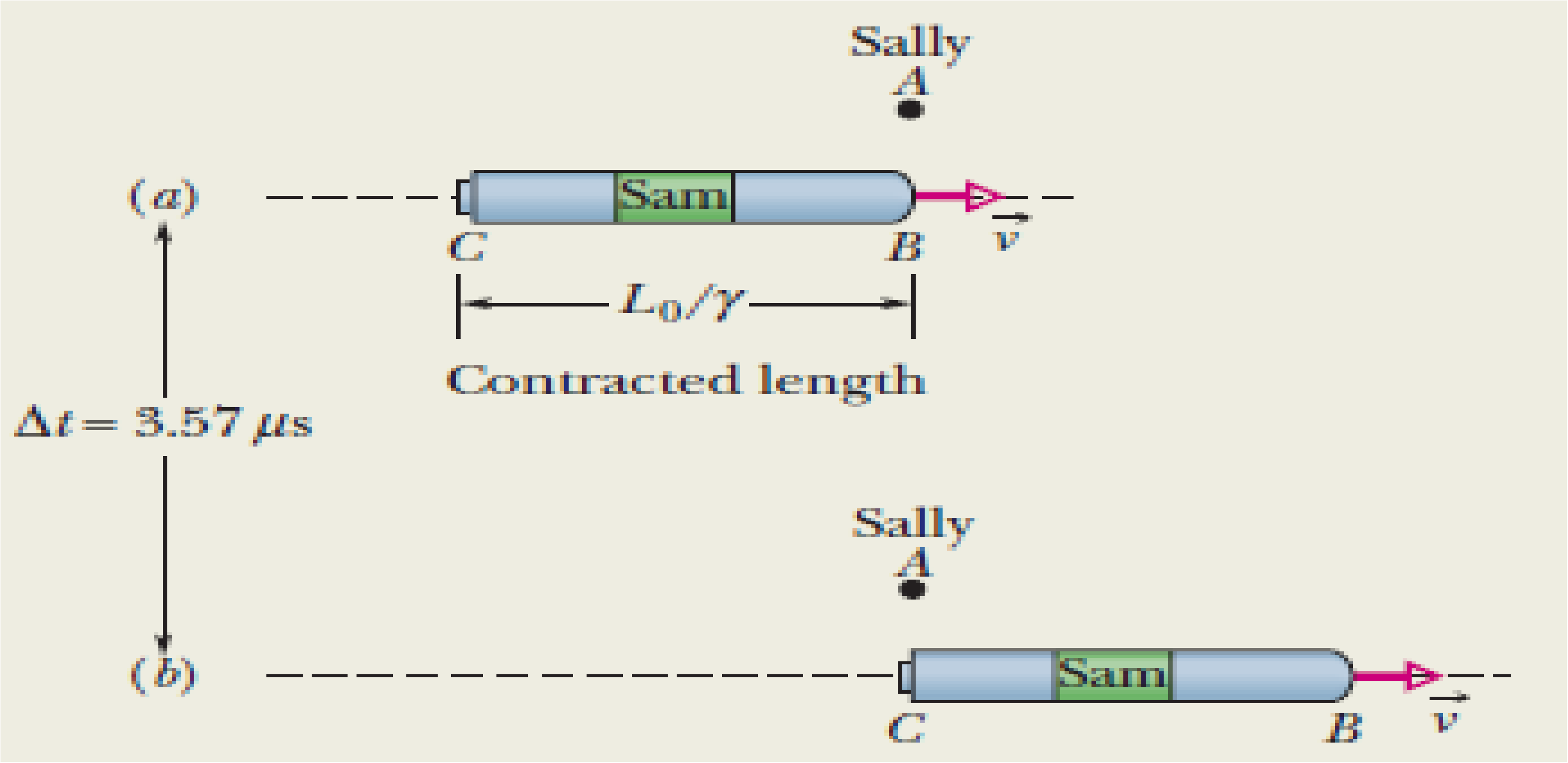
$$\boxed{L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma} \quad \text{(length contraction).}}$$

Where $\beta = v/c$ is the speed parameter and

The Lorentz factor γ , $\gamma = 1/\sqrt{1-\beta^2}$

Example

In Figure, Sally (at point A) and Sam's spaceship (of proper length L_0=230 m) pass each other with constant relative speed v. Sally measures a time interval of 3.57 ms for the ship to pass her (from the passage of point B in Fig.a to the passage of point C in Fig.b). In terms of c, what is the relative speed v between Sally and the ship?



Answer

$$v = \frac{L_0/\gamma}{\Delta t} = \frac{L_0\sqrt{(1 - (v/c)^2}}{\Delta t}.$$

Solving this equation for v (notice that it is on the left and also buried in the Lorentz factor) leads us to

$$v = \frac{L_0 c}{\sqrt{(c\,\Delta t)^2 + L_0^2}}$$

$$= \frac{(230 \text{ m})c}{\sqrt{(299\,792\,458 \text{ m/s})^2(3.57 \times 10^{-6} \text{ s})^2 + (230 \text{ m})^2}}$$

$$= 0.210c. \qquad \text{(Answer)}$$

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