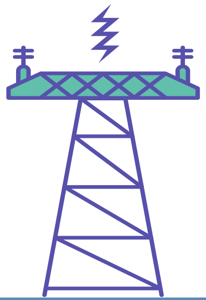
El-Manzala Higher Institute of Engineering and Technology

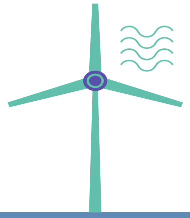
Fundamentals of Electrical Engineering

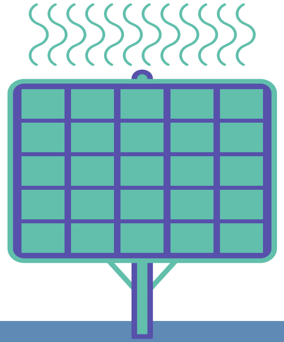
First YEAR

By

Dr. Eman Ahmed Awad Megahed







Sinusoidal Alternating Voltage and Current

Chapter 8

Chapter Content

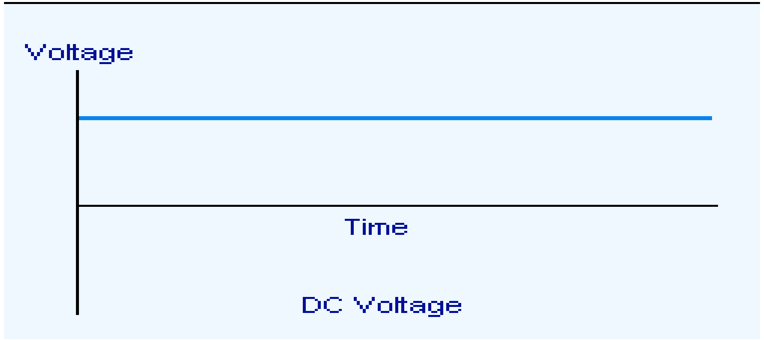
CH8: Sinusoidal Alternating Voltage and Current

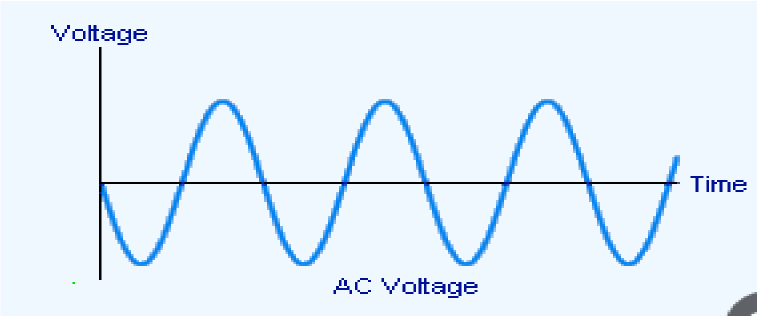
1 INTRODUCTION
2 SINE AND COSINE WAVES
3 PHASE RELATIONS

E. A. MEGAHED

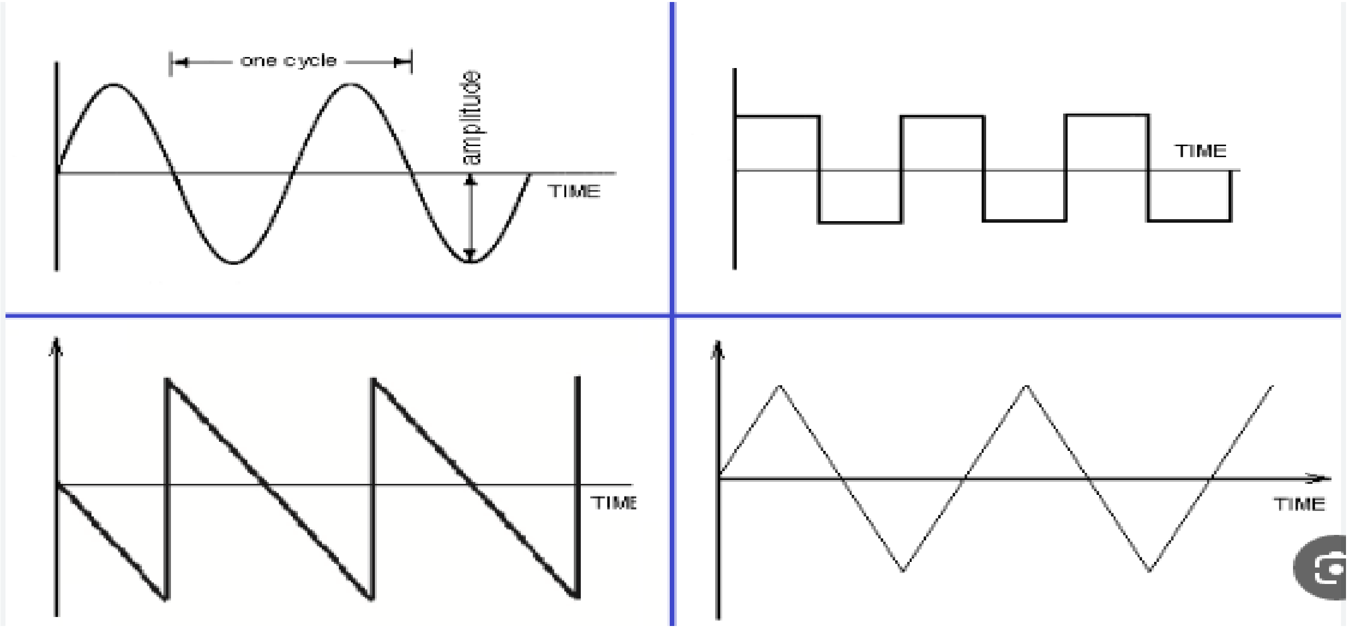
1. INTRODUCTION

- In the circuits considered so far, the independent sources have all been DC. From this point on, the circuits have Alternating Current (AC) sources.
- An AC voltage (or AC current) varies sinusoidally with time.
- This is a periodic voltage since it varies with time such that it continually repeats.





1. INTRODUCTION



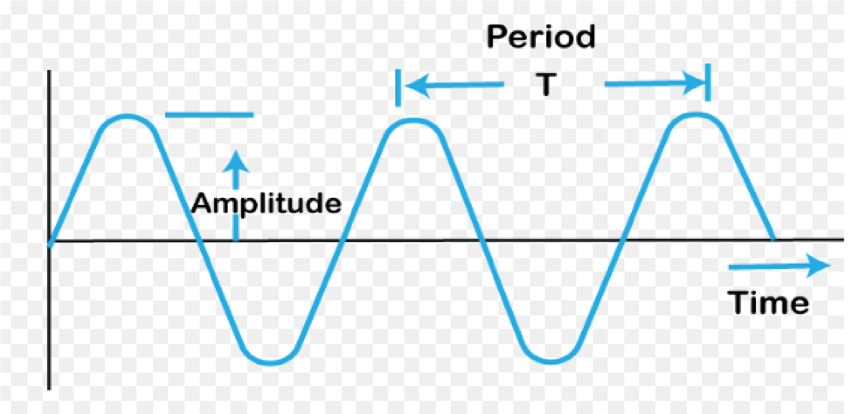
1. INTRODUCTION

- The smallest non repeatable portion of a periodic waveform is a **cycle**, and the duration of a cycle is the **period** (**T**) of the wave.

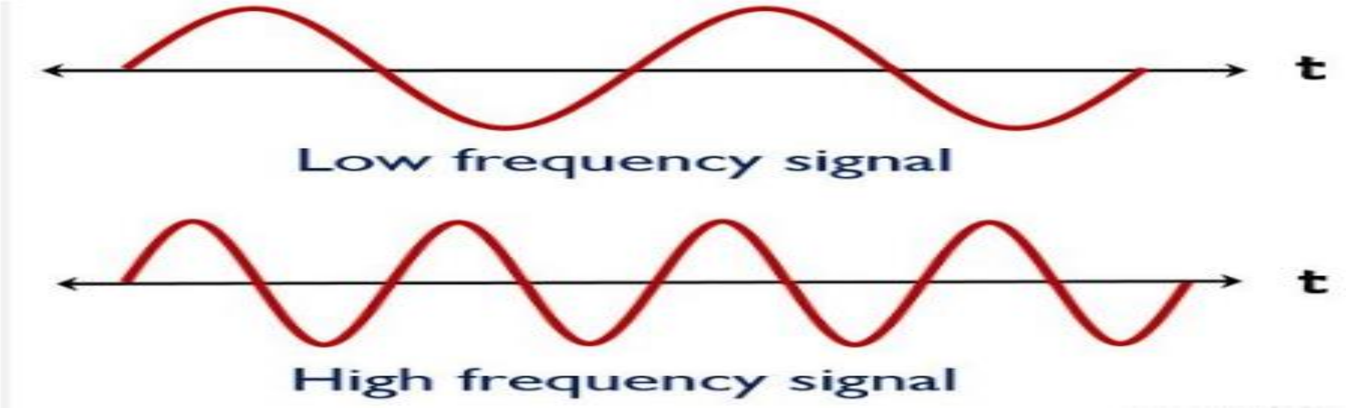
- The reciprocal of the period, and the number of cycles in a period, is the **frequency** (f).

$$T = 1/f$$

- The SI unit of frequency is the **hertz**, with unit symbol **Hz**.



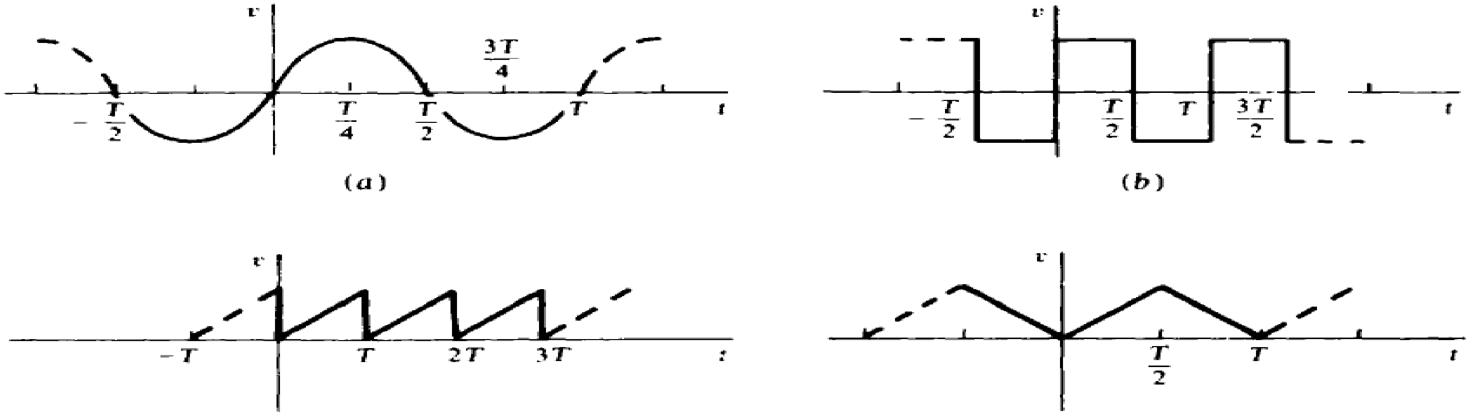
1. INTRODUCTION



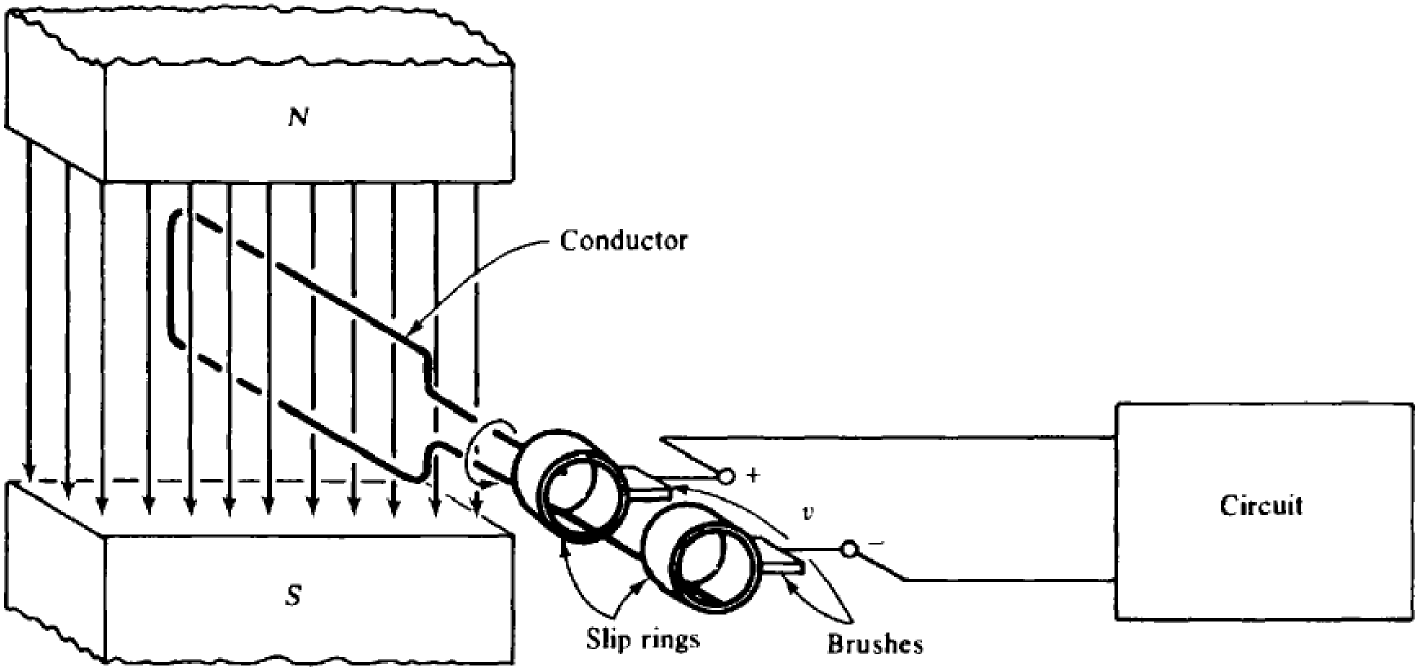
- In these definitions, notice the terms Wave and Waveform. They do not refer to the same thing. A **wave** is a varying voltage or current, but a **waveform** is a graph of such a voltage or current. Often, however, these terms are used interchangeably.

1. INTRODUCTION

- Although the **sine wave** is by far the most common periodic wave, there are other common periodic ones: such as a **square** wave, a **saw tooth** wave, and a **triangular** wave.



2. SINE AND COSINE WAVES



2. SINE AND COSINE WAVE

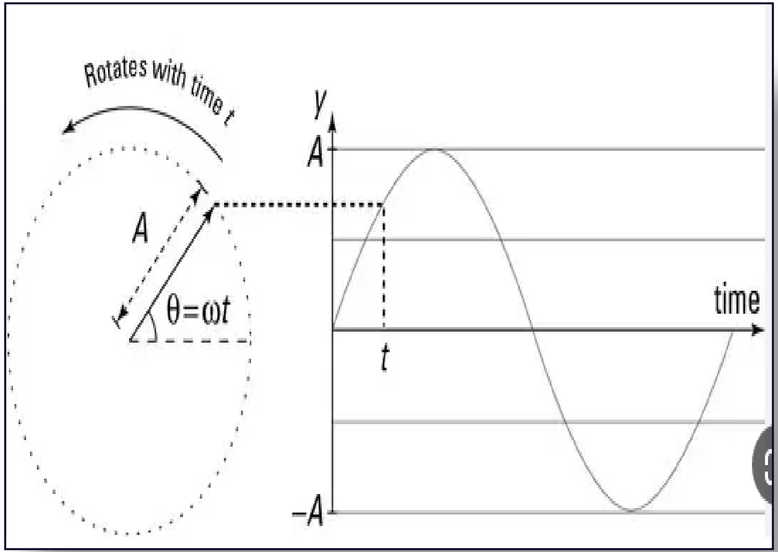
- The Figure shows the basics of an AC generator or alternator for generating a sinusoidal voltage.
- The conductor, which in practice is a coil of wire, is rotated by a steam turbine or by some other source of mechanical energy.
- This rotation causes a continuous change of magnetic flux linking the conductor,
- Thereby inducing a sine wave voltage in the conductor.
- This change of flux, and so the induced voltage, varies from zero when the conductor is horizontal to a maximum when the conductor is vertical.

2. SINE AND COSINE WAVES

- If t=0 s corresponds to a time when the conductor is horizontal and the induced voltage is increasing.
- the induced voltage is $v = V_m \sin\omega t$,
- where ,$\mathbf{V_m}$ is the peak value or amplitude,

sin is the operation designator for a sine wave,

ωt is the argument, and ω is the quantity symbol for the **radian frequency** of the voltage. Also called **angular velocity** or "**angular frequency**"



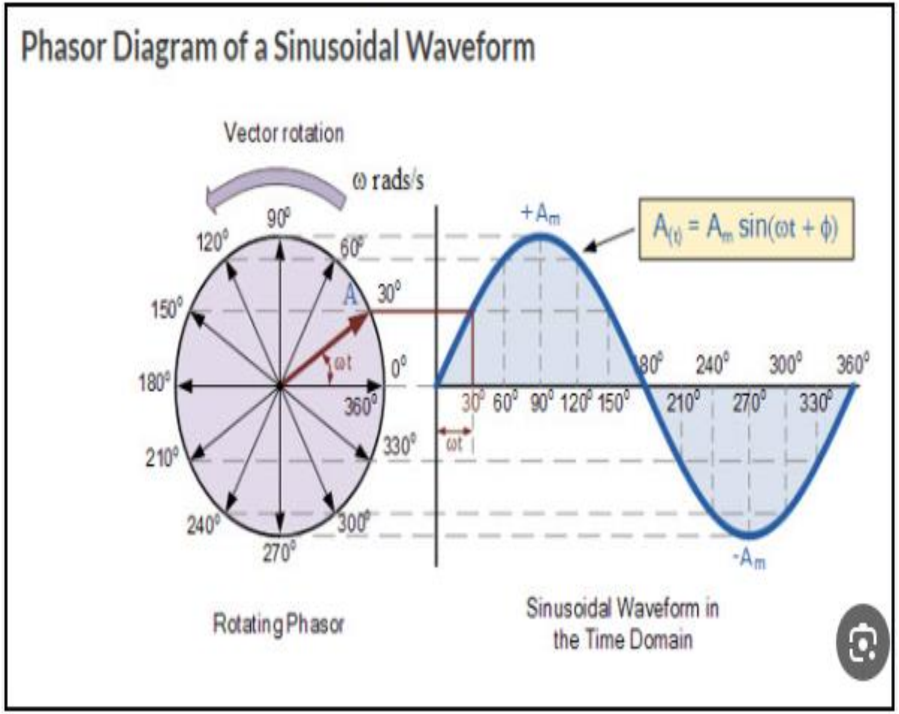
2. SINE AND COSINE WAVES

- The SI unit of radian frequency is radian per second, and the unit symbol is **rad/s**.
- Frequency f and the radian frequency ω are related by

$$\omega = 2\pi f$$

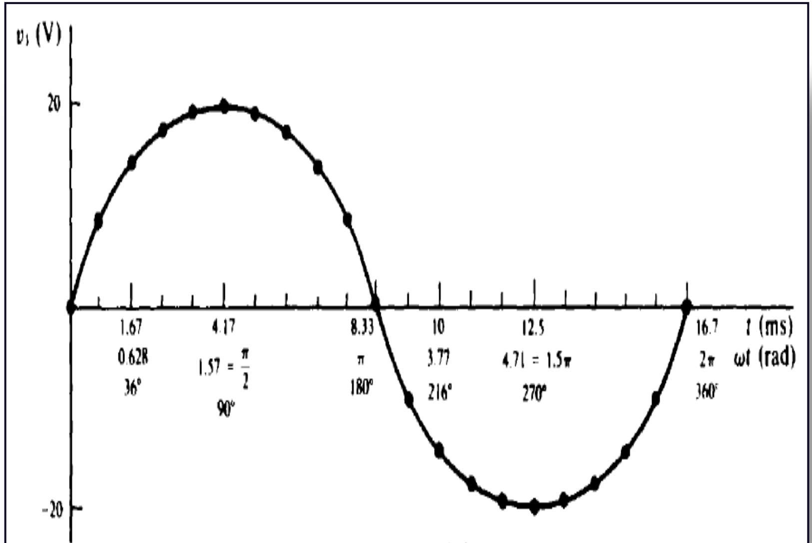
- sin ωt can be evaluated with a calculator operated in the radians mode.
- Alternatively, the argument can be converted to degrees and the calculator operated in the more popular decimal degrees mode.

For example, $\sin(\pi/6) = \sin 30^{\circ} = 0.5$



2. SINE AND COSINE WAVES

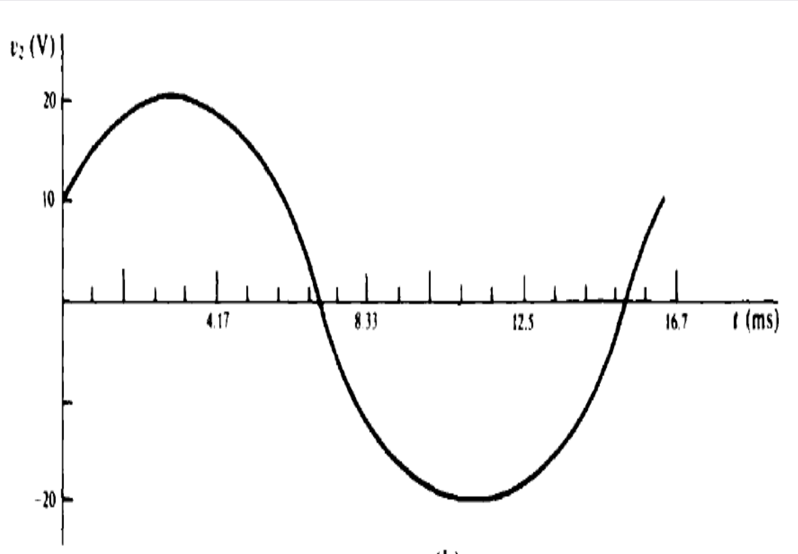
- Consider the graphing of one cycle of a specific AC voltage: $v_1 = 20 \sin 377t$ V.
- The peak value or amplitude is 20 V because sin 377t has a maximum value of 1.
- The radian frequency is $\omega = 377$ rad/s,
- which corresponds $f = \omega/2\pi = 60$ Hz, the frequency of the electrical power systems in the United States.
- The period is T= 1/60 = 16.7 ms.



(a)

2. SINE AND COSINE WAVES

- one cycle of $v_2 = 20 \sin(377t + 30\,)$ V.
- Notice that the argument is the sum of two terms, the first of which is in radians and the second of which is in degrees.
- Either the first term must be converted to degrees or the second term must be converted to radians.
- The 30° in the argument is called the phase angle.



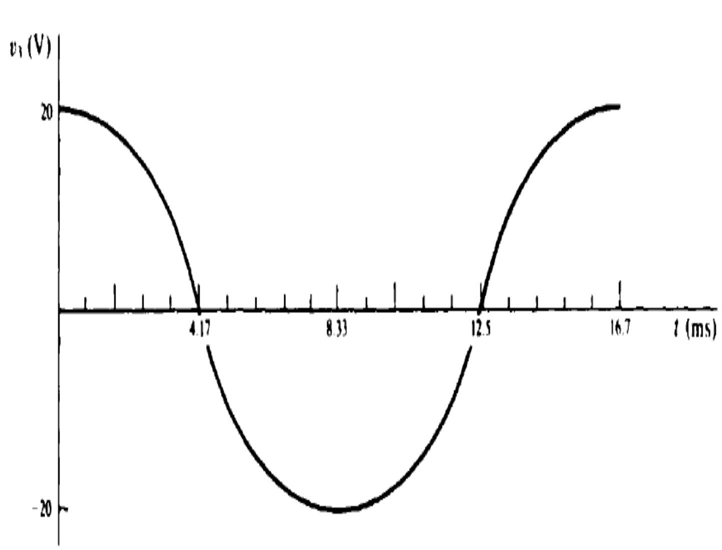
(b)

2. SINE AND COSINE WAVES

- The cosine wave, with designator cos, is as important as the sine wave.
- Its waveform has the same shape as the sine waveform, but is shifted 90º a fourth of a period ahead of it .

$$v_3 = 20 \sin (377t + 90^{\circ}) = 20 \cos 377t \text{ V.}$$

The values of the cosine wave v_3 occur one-fourth period earlier than corresponding ones for the sine wave v_1



(c)

2. SINE AND COSINE WAVES

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \sin(x + 90^\circ) = \cos x$$

$$\sin(x - 90^\circ) = -\cos x \qquad \cos(x + 90^\circ) = -\sin x \qquad \cos(x - 90^\circ) = \sin x$$

$$\sin(x \pm 180^\circ) = -\sin x \qquad \cos(x \pm 180^\circ) = -\cos x \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin(x + y) = \sin x \cos y + \sin y \cos x$$

$$\sin(x - y) = \sin x \cos y - \sin y \cos x$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$\sin x = \sin(x \pm N \times 360^\circ)$ and $\cos x = \cos(x \pm N \times 360^\circ)$ for any integer N

EX.

EX1: What is (a) the maximum value and (b) the period of a waveform given by v = 100 sin 377t V?

SOL:

$$(a)\quad V_m = 100\ \text{V} \qquad (b)\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{377} = 16.7\ \text{ms}$$

EX2: Find the frequency in hertz and the angular frequency in rad/s of a waveform whose period is 5ms.

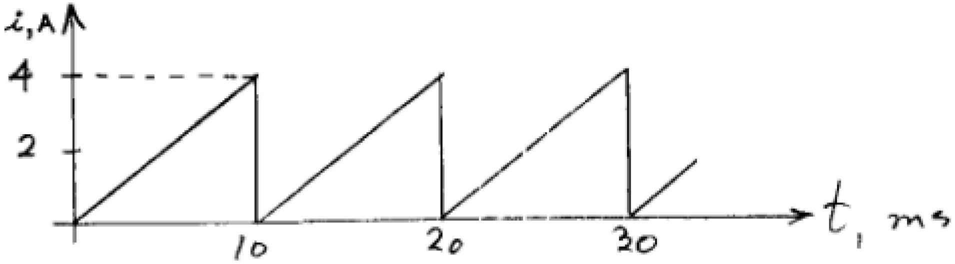
SOL:

$$f = \frac{1}{T} = \frac{1}{5\times 10^{-3}} = 200\ \text{Hz} \qquad \omega = 2\pi f = 2\pi(200) = 1256.64\ \text{rad/s}$$

EX.

EX3: Find the period, frequency, and the amplitude of the current waveform shown in Fig.

SOL:

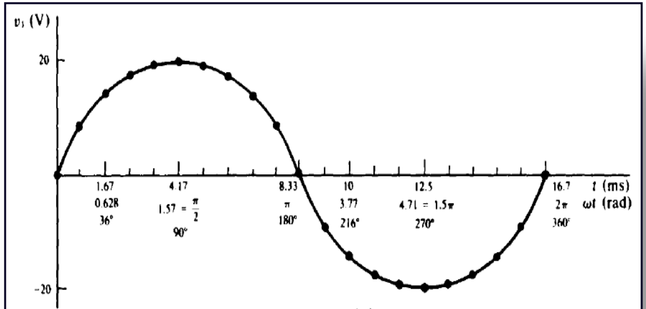


$$T = 10\ \text{ms} \qquad f = \frac{1}{T} = \frac{1}{10 \times 10^{-3}} = 100\ \text{Hz} \qquad I_m = 4\ \text{A}$$

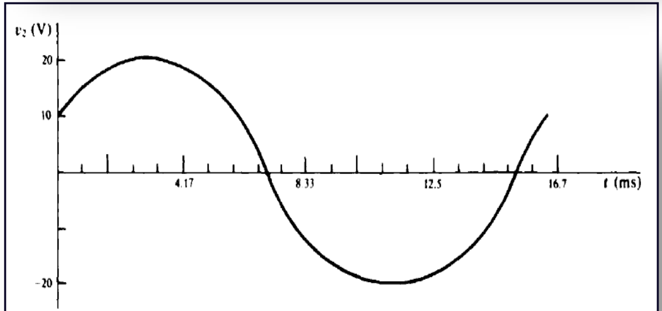
3. PHASE RELATIONS

- Sinusoids of the same frequency have phase relations that have to do with the angular difference of the sinusoidal arguments.
- For example, because of the added 30º in its argument, $v = 20 \sin(377t + 30°)$ V of the last section **leads** <u>v = 20 sin 377t by 30°</u>

- Alternatively. v_1 lags v_2 by 30°.
- This means that the peaks, zeros, and other values of v_2 occur earlier than those of v_1 by a time corresponding to 30°



(a)



(b)

EX

EX4: What are the phase relationships between the following waveforms?

(*a*) $v = 100 \sin(\omega t + 30^\circ)$ $\quad i = 10 \sin(\omega t + 60^\circ)$

(*b*) $v = 100 \sin(\omega t + 30^\circ)$ $\quad i = 10 \sin(\omega t - 30^\circ)$

(*c*) $v = 100 \sin(\omega t - 60^\circ)$ $\quad i = 10 \sin(\omega t - 90^\circ)$

Sol:

(*a*) i leads v by $(60^\circ - 30^\circ) = 30^\circ$.

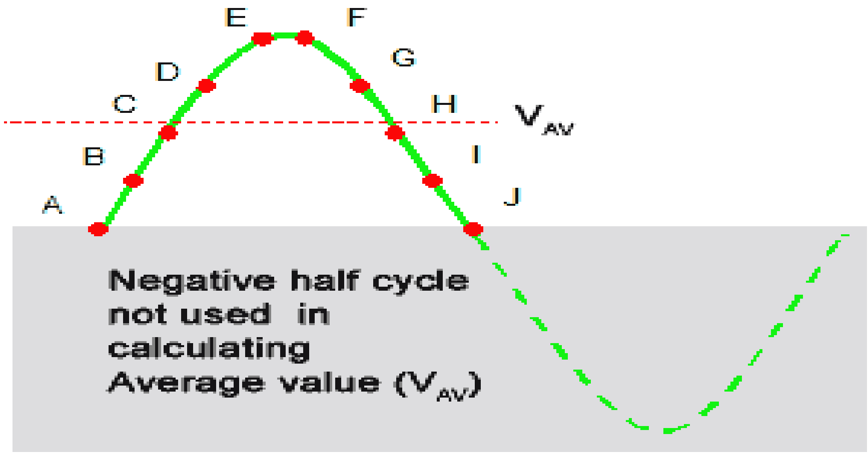
(*b*) i lags v by $[30^\circ - (-30^\circ)] = 60^\circ$.

(*c*) i lags v by $[-60^\circ - (-90^\circ)] = 30^\circ$.

1. Sinusoidal Average Value

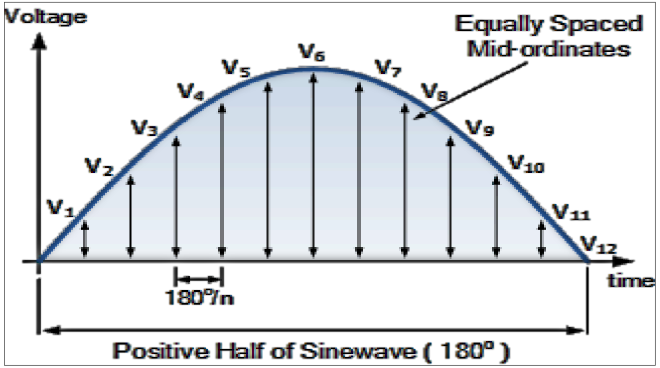
- The average value of aperiodic wave is a quotient of area and time
- حاصل قسمة مساحة المنطقة بين المنحني و المحاور الرئيسية والزمن
- With the area being between the wave and the abscissa axis over one period and with the time being this period.
- The area above the abscissa axis is a positive and that below is negative.
- The average value of any sinusoid is zero.
- By definition, this average is the average over a positive half cycle.
- This is the area under this half cycle divided by a half period.

1. Sinusoidal Average Value

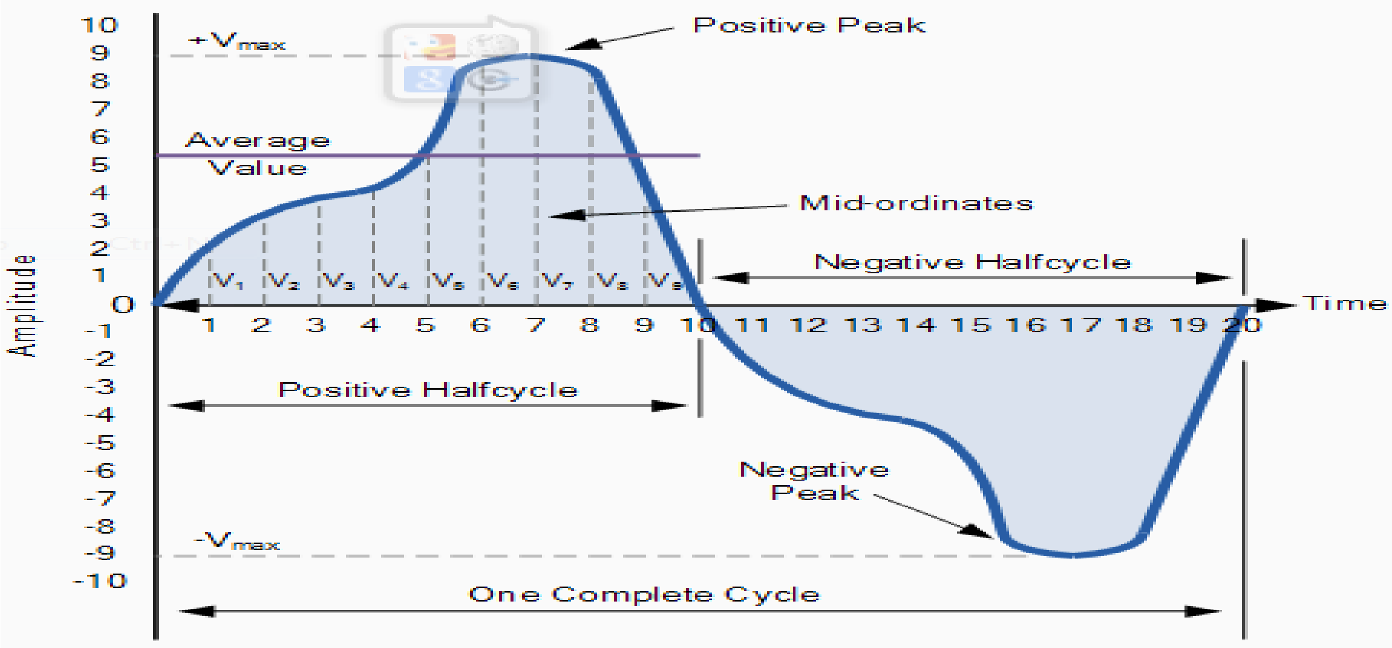


V_{AV} (for the positive half cycle) = $\dfrac{A+B+C+D+E+F+G+H+I+J}{10}$

For a pure sine wave V_{AV} will always be the PEAK value (V_{PK}) x 0.637



1. Sinusoidal Average Value



1. Sinusoidal Average Value

- Sine wavepeak value is Vm and period T, the area of a positive half cycle (Vm/ π) * T

The average value = Area/ Time

$$= \frac{Vm * T}{\pi} / \frac{T}{2} = \frac{2\,Vm}{\pi}$$

V average = $\frac{2\,Vm}{\pi}$ = 0.637 Vm

I average = $\frac{2\,Im}{\pi}$ = 0.637 Im

The average value of sinusoid turns out to be 0.637 times the peak value.

Notices that the average value of sinusoid does not depend on the period or on the phase angle.

End of Lecture 10