method of Soluation of DE :-

1) direct method (direct integration)

* given $\frac{dy}{dx} = f(x)$

* Req $y(x) = \boxed{?}$ → العلاقة بين المتغيرات والمشتقة المستقل

* method of Solution

Ex:- $\frac{dy}{dx} = (x^2 + 3x + e^x$

Sol:-

$$\int dy = \int (x^2 + 3x + e^x)\, dx$$

$$y(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + e^x$$

2) Separable method :- فصل المتغيرات

1) $\frac{dy}{dx} = f(x) \cdot g(y)$

* $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{f(y)}{g(x)}$

* $f_1(x) \cdot g_1(y)dx + f_2(x)\, g_2(y)\, dy = 0$

* مجموع الدوال من x ودوال y يسهل اخذ عامل مشترك بينهم لتصبح حاصل ضرب دالتين

Ex:- $\frac{dy}{dx} = (x^2+1)(y^2+1)$

$$\int \frac{dy}{y^2+1} = \int (x^2+1)\, dx$$

$$\tan^{-1}(y) = \frac{x^3}{3} + x + C$$

$$y(x) = \tan\left(\frac{1}{3}x^3 + x + C\right)$$

ex:- $\dfrac{dy}{dx} = \dfrac{y+1}{x^2+1}$

$$\int \frac{dy}{y+1} = \frac{dx}{x^2+1}$$

$$\ln(y+1) = \tan^{-1}(x) + c$$

$$y+1 = e^{\tan^{-1}(x)+c}$$

$$y(x) = e^{\tan^{-1}(x)+c} - 1$$

ex:- $x(y^2+1)\,dy + y(x^2+1)\,dx = 0$

نقسم على حاصل ضرب المختلفة امام dy, dx

$$\frac{(y^2+1)}{y}\,dy + \frac{(x^2+1)}{x}\,dx$$

$$\left(y + \frac{1}{y}\right)dy + \left(x + \frac{1}{x}\right)dx$$

$$\frac{y^2}{2} + \ln(y) + \frac{x^2}{2} + \ln(x) \pm c$$

[illegible] * ex $\dfrac{dy}{dx} = xy + y + x + 1$

$$\frac{dy}{dx} = y(x+1) + x + 1$$

$$(x+1)(y+1)$$

$$\int \frac{dy}{y+1} = \int (x+1)\,dx$$

$$\ln(y+1) = \frac{x^2}{2} + x + c$$

$$y(x) + 1 = e^{\frac{1}{2}x^2 + x + c}$$

$$y(x) = e^{\frac{1}{2}x^2 + x + c} - 1$$

$\boxed{3}$ reducible to separables:-

* given $\frac{dy}{dx} = f(ax+by+c)$
* Req $y(x) = \boxed{?}$
* method of Solution:- let $ax+by+c = u$

$\frac{dy}{dx} = f(u)$ diff $a + b\frac{dy}{dx} = \frac{du}{dx}$

$\frac{1}{b}\left(\frac{du}{dx} - a\right) = f(u)$

Ex: $\frac{dy}{dx} = \tan^2(x+y+3)$

let. $x+y+3 = u$

$1 + \frac{dy}{dx} + \cancel{3} = \frac{du}{dx}$

$1+\tan^2(u) = \frac{du}{dx}$

$\sec^2(u) = \frac{du}{dx}$

$\int \frac{du}{\sec^2(u)} = \int \cancel{4}\, d(x)$

$\int \cos^2(u) = \int dx$

$\frac{1}{2}\int (1+\cos(2u))\,du = \int dx$

$\frac{1}{2}\left[\cancel{x}\, u + \frac{1}{2}\sin(2u)\right] = x + c$

$\frac{1}{2}\left[x+y+3 + \frac{1}{2}\sin(2u)\right] = x + c$

Ex:- $\frac{dy}{dx} = 1 + e^{x+y+3}$

let

$x + y + 3 = u$

$1 + \frac{dy}{dx} = \frac{du}{dx}$

$1 + 1 + e^{(u)} = \frac{d(u)}{d(x)}$

$2 + e^{(u)} = \frac{d(u)}{d(x)}$

$\int d(x) = \int \frac{d(u)}{2 + e^{(u)}}$

$\frac{1}{2} \int \frac{2 + e^{u} - e^{u}}{2 + e^{(u)}} = \int dx$

$\frac{1}{2} \int 1 - \frac{e^{u}}{2 + e^{(u)}} = \int dx$

$\frac{1}{2} \left[u - \ln(2 + e^{(u)}) \right] = x + c$

H.W $\Rightarrow$ ① $\frac{dy}{dx} = \sin^2(x + y + 3)$

② $\frac{dy}{dx} = \sin(x + y + 3)$

③ $\frac{dy}{dx} = \frac{2xy\, e^{(x/y)^2}}{y^2 + y^2 e^{(x/y)^2} + 2x^2 e^{(x/y)^2}}$

4 Homogenous DE:

لو درجة البسط قد المقام *

① $\frac{dy}{dx} = f(y/x)$ or $f(x/y)$

$\ln(y/x)$, $e^{y/x}$, $\cos(y/x)$ * $\sin(x/y)$

2) method of Solution:- let $y = ux$

Ex:- $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$

2) $= \frac{x}{y} + \frac{y}{x}$

3) $xy\frac{dy}{dx} = x^2+y^2$

4) $yx\,dy - (x^2+y^2)dx = 0$

let $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$u + x\frac{du}{dx} = \frac{\cancel{x^2}+u^2\cancel{x^2}}{\cancel{x^2}u}$

$u + x\frac{du}{dx} = \frac{1+u^2}{u}$

$x\frac{du}{dx} = \frac{1+u^2}{u} - u$

$x\frac{du}{dx} = \frac{1}{u} + u - u$

$x\frac{du}{dx} = \frac{1}{u}$

$\int u\,du = \int \frac{dx}{x}$

$\therefore \frac{1}{2}u^2 = \ln(x) + c$

1] $\frac{dy}{dx} = \frac{1+y}{2+x}$

$$\int \frac{dy}{1+y} = \int \frac{dx}{2+x}$$

$$\ln(1+y) = \ln(2+x) + \ln A$$
$$= \ln(A(2+x))$$

$\cancel{1+y = c}$

$* \; 1+y = (2+x)A$

$y = A(2+x) - 1$

2] $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2 y - x^2} = \frac{y^2(1+x)}{x^2(y-1)}$

$$\frac{dy(y-1)}{y^2} = \frac{dx(x+1)}{x^2}$$

$$\int \frac{y}{y^2} - \frac{1}{y^2}\, dy = \int \frac{x}{x^2} + \frac{1}{x^2}\, dx$$

$$\int \frac{1}{y} - y^{-2}\, dy = \int \frac{1}{x} + x^{-2}\, dx$$

$$\ln(y) + \frac{1}{y} = \ln x - \frac{1}{x} + c$$

3] $y \operatorname{Tan}(x) \frac{dy}{dx} = (y^2+4)\sec^2 x$

$$\frac{1}{2}\int \frac{2y\,dy}{(y^2+4)} = \int \frac{\sec^2 x}{\operatorname{Tan}(x)}\, dx$$

$$\frac{1}{2}\ln(y^2+4) = \ln \operatorname{Tan}(x) + c$$

$$\cancel{\ln(y^2+4) = \ln \operatorname{Tan}(x) \cdot \ln \operatorname{Tan}(x) + \ln A}$$

$$\cancel{y^2 + 4 = \operatorname{Tan}^2(x) + A}$$

$\cancel{\therefore y = \sqrt{\operatorname{Tan}^2(x) - 4 + A}}$

$$y^2 = \operatorname{Tan}^2(x) - 4 + A$$

PAGE

DATE

4) $\frac{dy}{dx} = \frac{x+y}{x-y}$ let $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$u + x\frac{du}{dx} = \frac{x+ux}{x-ux} = \frac{1+u}{1-u}$

$x\frac{du}{dx} = \frac{1+u}{1-u} - \frac{u}{1}$

$= \frac{1+u-(1-u)u}{1-u}$

$x\frac{du}{dx} = \frac{1\cancel{+u}\cancel{-u}+u^2}{1-u} = \frac{1+u^2}{1-u}$

$\int du \frac{1-u}{1+u^2} = \int \frac{1}{x} dx$

$\frac{1}{1+u^2} - \frac{u}{1+u^2} = \ln(x) + C$

$\tan^{-1}(u) - \frac{1}{2}\ln(1+u^2) = \ln(x) + C$

$\boxed{5}$ If $y = Ax^2 + Bx$ $y' = 2Ax + B$

$y' = 2\left(\frac{y''}{2}\right)x^2 + Bx$ $y'' = 2A$ $A = \frac{y''}{2}$

$y' = y''x^2 + B\cancel{x}$ $B = y' - y''\cancel{x}$

$y = \frac{y''}{2}x^2 + (y' - y''\cancel{x})x$

$y = \frac{y''}{2}x^2 + y'x - y''x^2$

$y = y'x - \frac{y''}{2}x^2$

$\frac{y''}{2}x^2 - y'x + y = 0$