reducible to separable تؤدل للفصل

$$\frac{dy}{dx} = f(ax + by + c)$$

لا يمكن فصلها

let $u = ax + by + c$

نفاضل بالنسبة لـ x $\quad \frac{du}{dx} = a + b\frac{dy}{dx}$

نزلها $\quad b\frac{dy}{dx} = \frac{du}{dx}\frac{1}{b} - \frac{a}{b}$

(ثم نعوض في المعادله الاصليه)

⑨ Exact

$$\overset{M}{f_1(x)\,g(y)}\,dx + \overset{N}{f_2(x)\,g(x)}\,dy = 0$$

ولا يمكن فصلها او حلها

① Test لنا exact

$M_y = N_x \quad \therefore$ at

ثم نكامل كل طرف لوحده بالنسبة لـ له اللي جمبه

⑥ Bernollis

$$\frac{dy}{dx} + p(x)\,y = Q(x)\,y^n$$

بنخليها linear

① $* \; y^{-n}$

$$y^{-n}\frac{dy}{dx} + p(x)\boxed{y^{1-n}} = Q(x)$$

let $y^{1-n} = u$, $\frac{du}{dx} = (1-n)\,y^{-n}\frac{dy}{dx}$

ثم نعوض

⑤ linear

general form:-

$$\frac{dy}{dx} + p(x)\,y = Q(x)$$

① $\frac{dy}{dx}$ معامل $= 1$

② $p(x)$, $Q(x)$

③ $e^{\int p(x)\,dx} = m(x)$

④ $m(x) \cdot y = \int m(x)\,Q(x)\,dx$

1) $x y\, dx + (1+x^2)\, dy = 0$

$x y\, dx = -(1+x^2)\, dy$

$\frac{1}{2}\int \frac{2x}{1+x^2}\, dx = \int -\frac{1}{y}\, dy$

$\frac{1}{2}\ln(1+x^2) + c = -\ln y$

$\ln y = -\frac{1}{2}\ln(1+x^2) - \underset{\ln c}{c}$

$y = \frac{c}{\sqrt{1+x^2}}$

$\ln c = c$

[illegible]

($\ln$ ثابت)

$\alpha \ln a = \ln(a)^{\alpha}$

$\ln a - \ln b = \ln \frac{a}{b}$

$e^{\ln a} = a$

$\ln a + \ln b = \ln ab$

② $y - x y' = y^2 + y'$

$y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$

$y - x\frac{dy}{dx} - y^2 - \frac{dy}{dx} = 0$

عامل مشترك

$(y - y^2) - \frac{dy}{dx}(x+1) = 0$

$(y - y^2) = (x+1)\frac{dy}{dx}$

$\int \frac{dy}{y-y^2} = \int \frac{dx}{x+1}$

عامل y مشترك
واعملها كسور جزئية

$\int \frac{1}{y(1-y)}\, dy = \ln(x+1) + c$

$\frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{(1-y)}$

$A = 1 \quad , B = 1$

$\int \frac{1}{y} + \frac{1}{1-y}\, dy = \ln(x+1) + c$

$\ln y - \ln(1-y) = \ln(x+1) + \underset{\ln c}{c}$

$\ln\left(\frac{y}{1-y}\right) = \ln((x+1)c)$

$\frac{y}{1-y} = (x+1)c$

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③ $\frac{dy}{dx} = (x+y+5)^2$ ✓

let $u = x+y+5 \rightarrow \frac{du}{dx} = 1 + \frac{dy}{dx}$

$$\frac{du}{dx} = 1 + u^2$$

$$\int \frac{du}{1+u^2} = \int dx$$

$$\tan^{-1} u = x + c$$

$$u = \tan(x+c)$$

$$x+y+5 = \tan(x+c) \quad \#$$

$x \frac{dy}{dx} = y + x e^{y/x}$

④ $x^2 \frac{dy}{dx} = y^2 - xy \frac{dy}{dx}$ ✓

$$0 = y^2 - xy \frac{dy}{dx} - x^2 \frac{dy}{dx}$$

$$0 = y^2 - \frac{dy}{dx}(x.y + x^2)$$

$$\frac{dy}{dx} = \frac{y^2}{xy + x^2}$$

← درجة ثانية
← ولاعليه نصادم

let $y = ux \rightarrow \frac{dy}{dx} = u + x \frac{du}{dx}$

$$u + x \frac{du}{dx} = \frac{u^2 \cancel{x^2}}{\cancel{x^2} u + \cancel{x^2}} = \frac{u^2}{u+1}$$

$$x \frac{du}{dx} = \frac{u^2}{u+1} - u = \frac{\cancel{u^2} - \cancel{u^2} - u}{u+1} = \frac{u}{u+1}$$

$$\int \frac{u+1}{u} du = \int \frac{dx}{x}$$

$$\int 1 + \frac{1}{u} \, du = \ln x + c$$

$$u + \ln u = \ln x + c$$

$$y/x + \ln y/x = \ln x + c \quad \#$$

"2 $\frac{dy}{dx} = e^{3x - 2y}$

$e^{\alpha + \beta} = e^{\alpha} \cdot e^{\beta}$

$e^{\alpha - \beta} = \frac{e^{\alpha}}{e^{\beta}}$

$\frac{dy}{dx} = \frac{e^{3x}}{e^{2y}}$

$\frac{1}{2}\int 2e^{2y}\, dy = \frac{1}{3}\int 3e^{3x}\, dx$

$\frac{1}{2} e^{2y} = \frac{1}{3} e^{3x} + c$

6- $[y(1 + \frac{1}{x}) + \cos y]\, dx + [x + \ln x - x \sin y]\, dy = 0$

$M = y(1 + \frac{1}{x}) + \cos y$, $M_y = (1 + \frac{1}{x}) - \sin y$

$N = x + \ln x + x \sin y$, $N_x = 1 + \frac{1}{x} - \sin y$

$M_y = N_x$ Exact

$\int y + y/x + \cos y\, dx + \int x + \ln x - x \sin y\, dy = 0$

$yx + y\ln x + x\cos y + xy + y\ln x + x\cos y = 0$

$yx + y\ln x + x\cos y + c = 0$

7- $(1 - x^3)\frac{dy}{dx} + x^2 y = x^2(1 - x^3)$

$\frac{dy}{dx} + \frac{x^2}{1 - x^3} y = x^2$

$p(x) = \frac{x^2}{1 - x^3}$, $Q(x) = x^2$

$m(x) = e^{\int p(x)dx} = e^{\frac{1}{-3}\int \frac{-3x^2}{1-x^3} dx} = e^{-\frac{1}{3}\ln(1 - x^3)} = (1 - x^3)^{-\frac{1}{3}} = \frac{1}{(1 - x^3)^{\frac{1}{3}}}$

$\frac{1}{(1 - x^3)^{\frac{1}{3}}} y = \int \frac{x^2}{(1 - x^3)^{\frac{1}{3}}} dx$

$\frac{1}{(1 - x^3)^{\frac{1}{3}}} y = \frac{1}{-3}\int -3x^2 (1 - x^3)^{-\frac{1}{3}} dx$

$\frac{1}{(1 - x^3)^{\frac{1}{3}}} y = -\frac{1}{2}(1 - x^3)^{\frac{2}{3}} + c$

$y = -\frac{1}{2}(1 - x^3) + c(1 - x^3)^{\frac{1}{3}}$

(8) $\frac{dy}{dx} + y \tan x = y^3 \sec^4 x \quad * y^{-3}$

$y^{-3} \frac{dy}{dx} + \tan x \boxed{y^{-2}} = \sec^4 x$

let $u = y^{-2} \rightarrow \frac{du}{dx} = -2 y^{-3} \frac{dy}{dx}$

$-\frac{1}{2} \frac{du}{dx} + \tan x \, u = \sec^4 x$

$\frac{du}{dx} - 2 \tan x \, u = -2 \sec^4 x$

$P(x) = -2 \tan x \quad , \quad Q(x) = -2 \sec^4 x$

$m(x) = e^{-2\int \tan x \, dx} = e^{-2 \ln \sec x} = \sec^{-2} x = \frac{1}{\sec^2 x}$

$\frac{1}{\sec^2 x} u = -2 \int \frac{\sec^{\cancel{4}} x}{\sec^{\cancel{2}} x} dx$

$\frac{1}{\sec^2 x} u = -2 \int \sec^2 x \, dx$

$\frac{1}{\sec^2 x} u = -2 \tan x + C$

$u = -2 \tan x \sec^2 x + C \sec^2 x$

$\tan = \frac{\sin}{\cos} \quad \frac{1}{\cos^2}$

$\sec^2 = \frac{1}{\cos^2}$

$u = -2 \frac{\sin x}{\cos^3 x} + \frac{C}{\cos^2 x}$

$y^2 = \frac{1}{-2 \frac{\sin x}{\cos^3 x} + \frac{C}{\cos^2 x}}$ //