

## Example 8

EX1: what is the total resistance of  $4\Omega$ ,  $6\Omega$ ,  $7\Omega$ ,  $10\Omega$  and  $3\Omega$  resistors in series?

Sol ↓

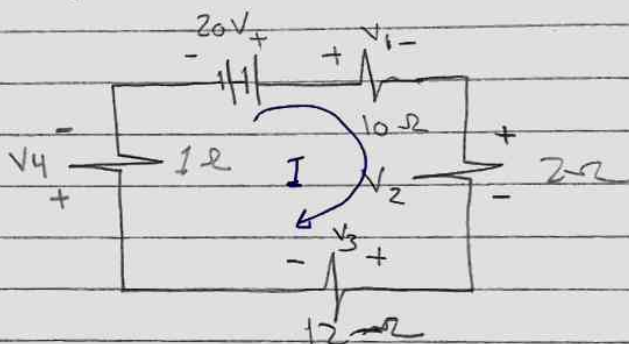
$$R_T = R_1 + R_2 + R_3 + R_4 + R_5 \\ = 4 + 6 + 7 + 10 + 3 = 30\Omega$$

EX2: In the circuit in the figure, find the current and all the unknown voltages.

$$R_T = R_1 + R_2 + R_3 + R_4 \\ = 10 + 2 + 12 + 1 \\ R_T = 25\Omega$$

$$I = V_s / R_T = 20 / 25$$

$$I = 0,8 \text{ A}$$



$$V_1 = I R_1 = 0,8 \times 10 = 8 \text{ V}$$

$$V_2 = I R_2 = 0,8 \times 2 = 1,6 \text{ V}$$

$$V_3 = I R_3 = 0,8 \times 12 = 9,6 \text{ V}$$

$$V_4 = I R_4 = 0,8 \times 1 = 0,8 \text{ V}$$

$$V_s = +8 + 1,6 + 9,6 + 0,8 = 20 \text{ V}$$

$R_T \leftarrow \text{pl ①}$

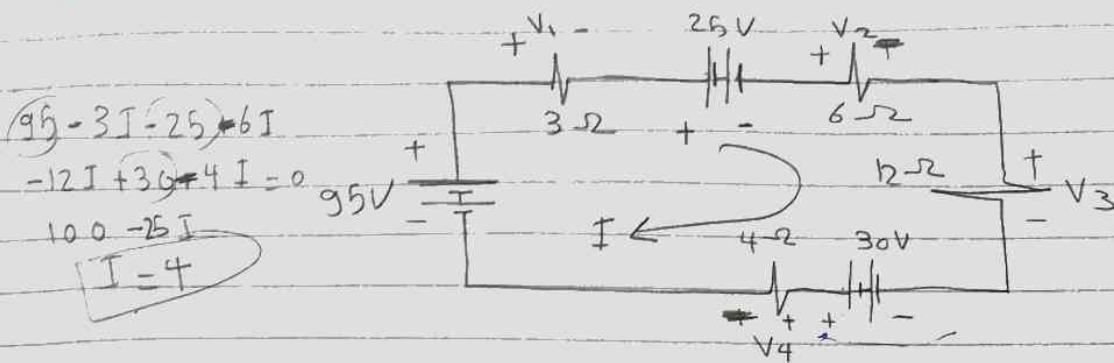
$V_s \leftarrow \text{pl ②}$

$$-1I + 20 - 10I - 2I - 12I$$

$$+ 20 - 25I = 0$$

$$I = \frac{20}{25} = 0,8 \text{ A}$$

Find the current and the unknown voltages in the circuit



$$R_T = R_1 + R_2 + R_3 + R_4$$

$$= 3 + 6 + 12 + 4 = 25 \Omega$$

$$V_T = 95 - 25 + 30 = 100V$$

$$I = V_T / R_T = \frac{100}{25} = 4A$$

$$V_1 = IR_1 = 4 \times 3 = +12V$$

$$V_2 = IR_2 = 4 \times 6 = -24V$$

$$V_3 = IR_3 = 4 \times 12 = +48V$$

$$V_4 = IR_4 = 4 \times 4 = -16V$$

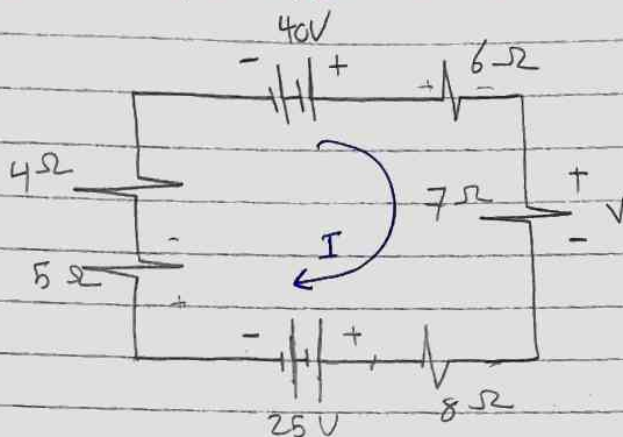
$$V_T = 12 - (-24) + 48 - (-16) = 100V$$

أولاً :- امسك  $\sum R_T$   
ثانياً :- امسك  $\sum V$   
التيار + لوانه يمر في البطارية  
منه - الى +  
والتيار - لوانه يمر في البطارية  
منه + الى -

ثالثاً :-  
امسك  $I = \frac{V_T}{R_T}$   
التيار  
التيار + لوانه يمر في البطارية  
منه - الى +  
والتيار - لوانه يمر في البطارية  
منه + الى -

36  
95  
12  
12

EX: 4 → In the circuit in the Figure find the current and the voltage <sup>at</sup> ~~of~~ ~~all~~ the resistance ~~7~~  $\Omega$  (V)



$$R_T = \overset{13}{6} + \overset{21}{7} + \overset{26}{8} + 5 + 4 = 30 \Omega$$

$$V_T = 40 - 25 = 15 \text{ V}$$

$$I = \frac{15}{30} = \frac{V_T}{R_T} = 0,5 \text{ A}$$

$$V = I R = + 0,5 * 7 = 3,5 \text{ V}$$

$$-25(-5I - 4I) + 40(-6I - 7I - 8I) = 0$$

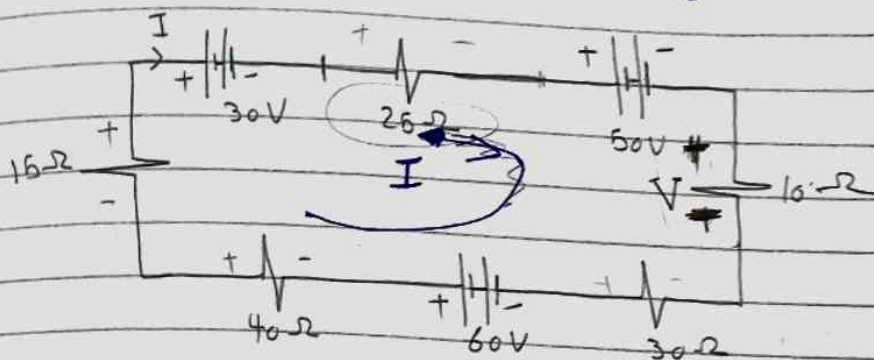
$$-30 I = -15$$

$$I = 0,5 \text{ A}$$

$$V = 0,5 * 7 = 3,5 \text{ V}$$



Repeat EX4 for the circuit in the Figure



$$R_T = 25 + 10 + 30 + 40 + 15 = 120 \Omega$$

$$V_T = -30 - 50 + 60 = -20 \text{ V} \checkmark$$

$$I = V_T / R_T = -20 / 120 = -0,167 \text{ A}$$

$$V = I R = -(-0,167 \times 10) = 1,67 \text{ V}$$

$$\begin{array}{c} -5I \quad \quad -25I \\ (30) - 15I - 40I - (60) - 30I + 10I + (50) = 0 \\ -120I = -20 \end{array}$$

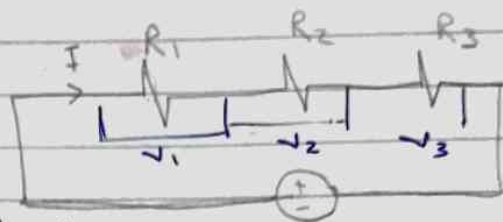
$$\begin{array}{c} (-50) - 10I - 30I + (60) - 40I - 15I - (30) - 25I = 0 \\ -20 - 120I = 0 \end{array}$$

$$I = -\frac{20}{120} = -0,167$$

## L.3: Series and parallel DC Circuits

### 1. Resistors in Series

The figure shows an electric circuit in which three resistors having resistances  $R_1, R_2, R_3$  respectively.



The resistances are joined end to end.

Here the resistors are said to be connected in series.

The potential difference  $V$  is equal to the sum of potential difference  $V_1, V_2, V_3$ .

$$V = V_1 + V_2 + V_3$$

Applying ohm's law  $V = IR$

$$IR = IR_1 + IR_2 + IR_3$$

$$R_T = R_s = R_1 + R_2 + R_3$$

EX: How much current will flow through a  $2\ \Omega$  resistor connected in series with a  $4\ \Omega$  resistor, and the combination connected across a  $12\text{ V}$  source? what is the voltage across each resistor?

sol ↓

$$R_s = R_1 + R_2 = 2 + 4 = 6$$

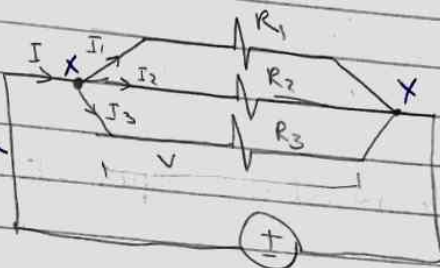
$$I = \frac{V_s}{R_s} = \frac{12}{6} = 2A$$

$$V_1 = IR_1 = 2 \times 2 = 4V$$

$$V_2 = IR_2 = 2 \times 4 = 8V$$

## 2. Resistors in parallel

The Figure shows a combination of resistors in which three resistors are connected together between points X and Y



Here, the resistors are said to be connected in parallel.

The total current \$I\$, is equal to the sum of the separate currents through each branch of the combination

$$I = I_1 + I_2 + I_3$$

Applying ohm's law  $I = V/R_p$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$R_{Ts} \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

for 2 resistors in parallel



sol ↓

$$R_s = R_1 + R_2 = 2 + 4 = 6$$

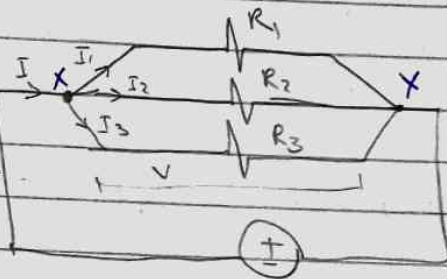
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## 2. Resistors in parallel

The Figure shows a combination of resistors in which three resistors are connected together between points X and Y



Here, the resistors are said to be connected in parallel.

The total current  $I$ , is equal to the sum of the separate currents through each branch of the combination

$$I = I_1 + I_2 + I_3$$

Applying ohm's law  $I = V/R_p$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$R_T = \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

resistors in parallel

EX2: what is the total resistance of the combination of  $2\Omega$  and  $4\Omega$  resistance in parallel?

Calculate the current supplied by a  $12V$  source connected across the combination.

Sol ↓

$$\frac{1}{R_p} = \frac{1}{2} + \frac{1}{4} = \frac{6}{8} = \frac{3}{4}$$

$$\frac{1}{R_p} = R_T = \frac{4}{3} \Omega$$

$$I = \frac{V_s}{R_T} = \frac{12 \times 3}{4} = 9A$$

Find the resistance equ. of the circuit shown below

$$R_1 \Rightarrow 12\Omega // 6\Omega // 4\Omega$$

$$R_1 = \frac{1}{R_p} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

$$R_p = 0.5\Omega$$

$$R_1 = 2\Omega$$

$$R_2 = 2\Omega \text{ series } 3\Omega$$

$$R_2 = 5\Omega$$

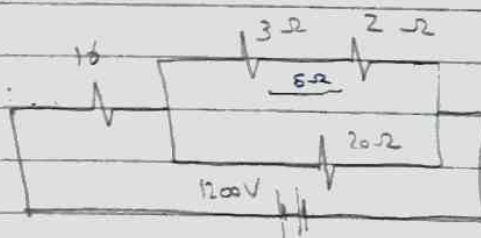
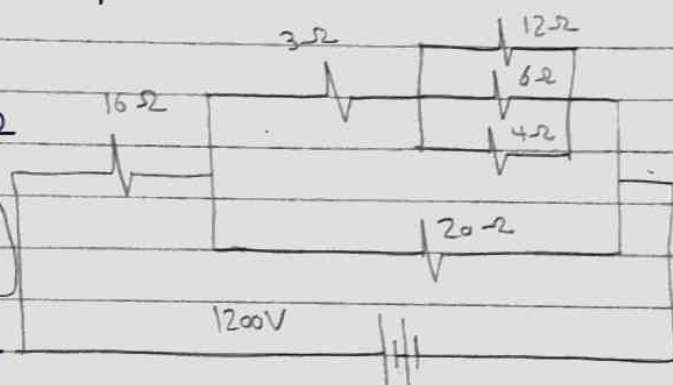
$$R_3 = \frac{1}{R_{p2}} = \frac{5 \times 20}{5 + 20} = 4\Omega$$

$$R_4 = 4\Omega \text{ series } 16\Omega$$

$$R_4 = 20\Omega$$

$$R_{eq} = 20\Omega$$

$$I = \frac{V_s}{R_{eq}} = \frac{1200}{20} = 60A$$





### 3. Branches, Nodes, loops, Meshes.

Branch: a group of components that carry the same current

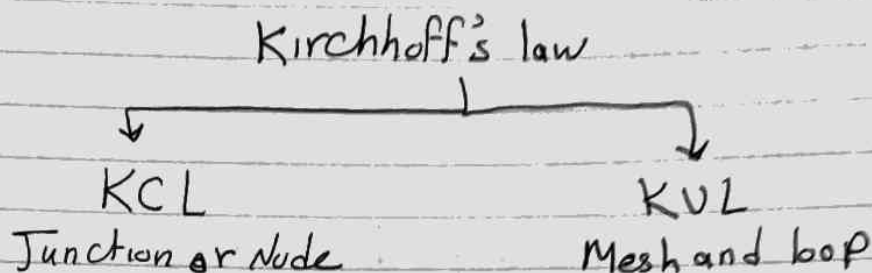
Node: is a connection point between two or more branches.

loop: is any simple closed path in a circuit

mesh: is a loop that does not have a closed path in its interior

No components are inside mesh.

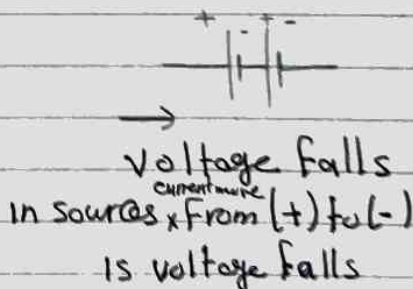
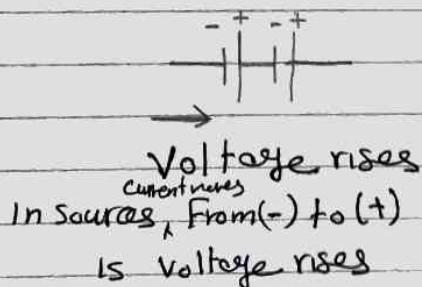
### 3. Kirchhoff's law



#### 1. Kirchhoff's voltage law. (KVL)

At any instant around a loop, in either a clock wise or counter clock wise direction.

- ✓ The algebraic sum of the voltage drops is Zero
- ✓ The algebraic sum of the voltage rises is Zero
- ✓ The algebraic sum of the voltage drops equals the algebraic sum of the voltage rises.





Voltage full

التيار يمر في المقاومة من + الى -

لوانه التيار المفروض له loop لنفسه الاتجاه  
اذنه ذ الجهد بالسالب لانه يتناقص



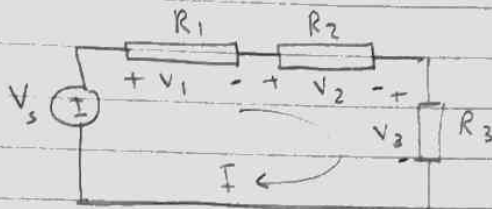
Voltage rises

التيار يمر في المقاومة من + الى -

لوانه التيار المفروض له loop في عكسه  
لهذا الاتجاه فانه الجهد يزداد ويكون موجب

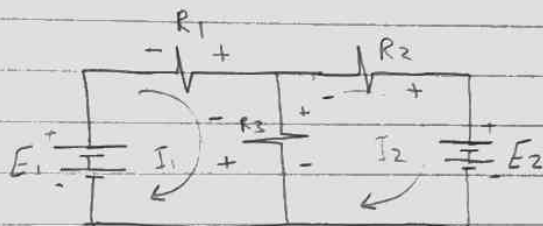
$$+V_s - V_1 - V_2 - V_3 = 0$$

$$V_s = V_1 + V_2 + V_3$$



$$+E_1 + I_1 R_1 + I_1 R_3 = 0$$

$$I_2 R_3 + I_2 R_2 = E_2$$



Find  $V_3$  and its polarity IF the current  $I$  in the circuit =  $0.4A$

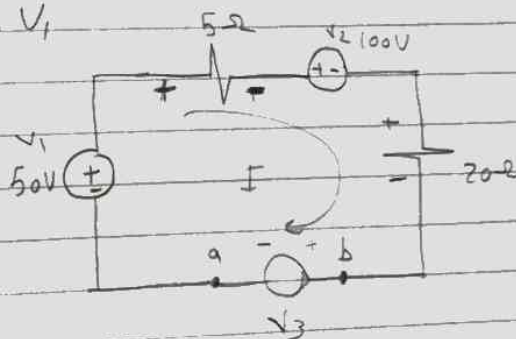
Assume  $V_3$  has the same polarity as  $V_1$

$$V_1 - I(5) - V_2 - I(20) + V_3 = 0$$

$$50 - 5I - 100 - 20I + V_3 = 0$$

$$V_3 = -30V$$

b is positive and a is negative





## voltage divider

The voltage division or voltage divider rule applies to resistors in series.

$$V_s = I(R_1 + R_2 + R_3)$$

$$V_1 = IR_1$$

$$\frac{V_1}{V_s} = \frac{I R_1}{I(R_1 + R_2 + R_3)}$$

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V_s$$

$$V_x = \frac{R_x}{R_T} V_s \quad \text{in general!}$$

EX:  $R_1 = 2\Omega$   $R_2 = 4\Omega$  in series  $V_s = 12$

The voltage across each resistor??

$$V_1 = \frac{R_1}{R_1 + R_2} V_s = \frac{2}{2 + 4} \times 12 = \frac{2}{6} \times 12 = 4V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_s = \frac{4}{2 + 4} \times 12 = \frac{4}{6} \times 12 = 8V$$

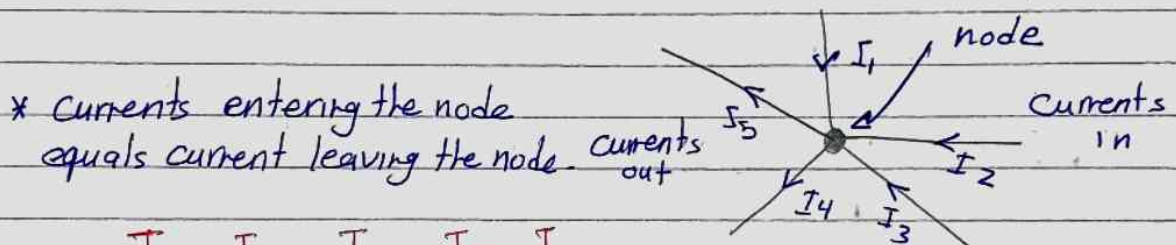
## Kirchhoff's Current Law

Kirchhoff's Current law abbreviated KCL, has three equivalent versions:

At any instant in a circuit.

The algebraic sum of the currents leaving a closed surface is zero  
The algebraic sum of the currents entering a closed surface is zero

The algebraic sum of the currents entering a closed surface equals  
~~the~~ algebraic sum of those leaving

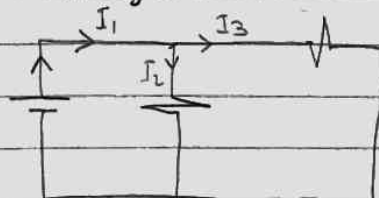


$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

→ a current entering is a negative current leaving, and that a current leaving is a negative current entering.

→ For a node that has no voltage source KCL is often the third one.

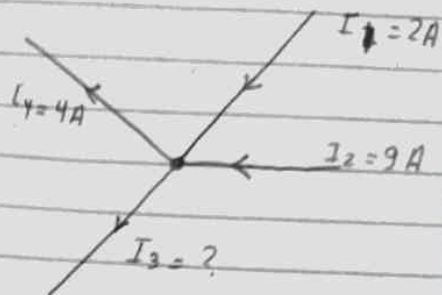
The currents entering are from current source and the current leaving are through resistors



Find current  $i_3$  at the node shown below.

Currents  $i_1$  and  $i_2$  are flowing into the node

Currents  $i_3$  and  $i_4$  are flowing out of the node.



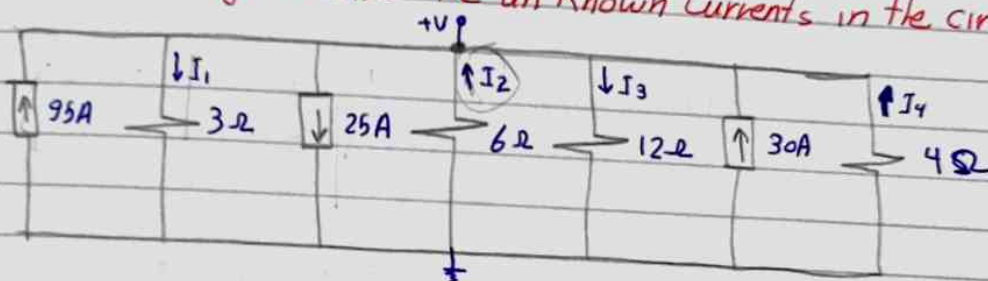
Apply Kcl at the given node

$$i_1 + i_2 = i_3 + i_4$$

$$2 + 9 = i_3 + 4$$

$$i_3 = 7A$$

Find the voltage  $V$  and the unknown currents in the circuit



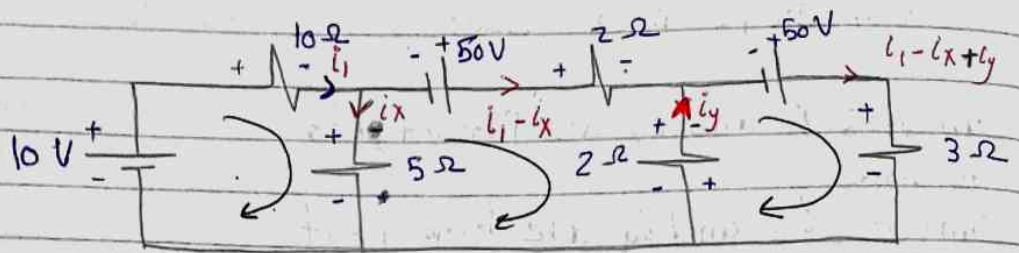
Sol ↓

$$95 - 3I_1 - 25 + 6I_2 - 12I_3 + 30 + 4I_4 = 0$$



95  
3  
11





sol ↓

$$10 = 10i_1 - 5i_x = 0 \quad 10 = 10i_1 + 5i_x \rightarrow (1)$$

$$+5i_x + 50 - 2(i_1 - i_x) + 2i_y = 0$$

$$50 = 2i_1 - 2i_x - 2i_y - 5i_x$$

$$50 = 2i_1 - 7i_x - 2i_y \rightarrow (2)$$

$$-2i_y + 50 - 3(i_1 - i_x + i_y) = 0$$

$$50 = 3(i_1 - i_x + i_y) + 2i_y$$

$$50 = 3i_1 - 3i_x + 3i_y + 2i_y$$

$$50 = 3i_1 - 3i_x + 5i_y \rightarrow (3)$$

$$10 = 10i_1 + 5i_x \rightarrow (1)$$

$$2 = 2i_1 + i_x \quad (i_x = 2 - 2i_1) \rightarrow (4)$$

$$50 = 2i_1 - 7(2 - 2i_1) - 2i_y$$

$$50 = 2i_1 - 14 + 14i_1 - 2i_y$$

$$64 = 18i_1 - 2i_y \rightarrow (5)$$

$$50 = 3i_1 - 3(2 - 2i_1) + 5i_y$$

$$50 = 3i_1 - 6 + 6i_1 + 5i_y$$

$$56 = 9i_1 + 5i_y \rightarrow (6)$$

solving 5, 6

32

8

$$64 = 16i_1 - 2i_y \Rightarrow i_y = 8i_1 - 32$$

$$56 = 9i_1 + 5i_y$$

$$56 = 9i_1 + 5(8i_1 - 32)$$

$$56 = 49i_1 - 160$$

$$216 = 49i_1$$

$$i_1 = 4,41 \text{ A}$$

$$i_x = 2 - 2i_1 = -6,82 \text{ A}$$

$$i_y = 3,28 \text{ A}$$

EX.

$$\text{Kcl} \rightarrow I_1 + I_2 - I_3 = 0$$

$$I_1 = \frac{20 - V_1}{50}$$

$$I_2 = 4 \text{ A}$$

$$I_3 = \frac{V_1 - 0}{40}$$

$$\frac{20 - V_1}{50} + 4 - \left( \frac{V_1}{40} \right) = 0$$

$$\frac{4(20 - V_1) + 800 - 5V_1}{200} = 0$$

$$-4V_1 + 80 + 800 - 5V_1 = 0$$

$$\therefore V_1 = 880 \quad V_1 = 97.78$$

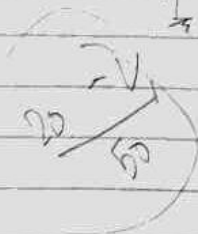
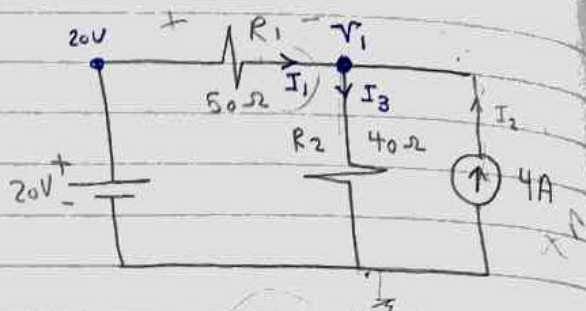
$$I_1 = \frac{20 - 97.78}{50} = -1.556 \text{ A}$$

$$I_3 = \frac{V_1}{40} = \frac{97.78}{40} = 2.445 \text{ A}$$

$$\text{Kcl} \Rightarrow -1.554 + 4 = 2.445$$

$$I_1 + I_2 = I_3$$

✓



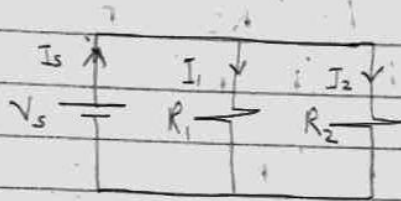


## Current Division

→ The current Division or current Divider rule applies to resistors in parallel.

It gives the current through any terms resistors into the parallel combination

$$I_1 = \frac{R_2}{R_1 + R_2} I_s$$



$$I_2 = \frac{R_1}{R_1 + R_2} I_s$$

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$I_1 = \frac{V_1}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$R = \frac{R_1 * R_2}{R_1 + R_2}$$

$$I = \frac{V(R_1 + R_2)}{R_1 * R_2}$$

$$V = I_1 R_1 = I_2 R_2$$

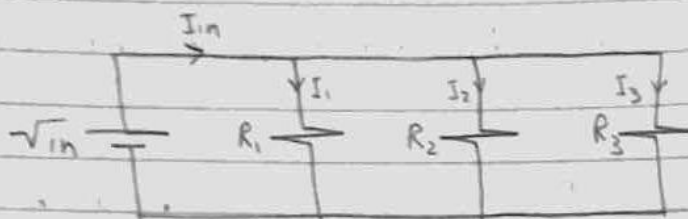
$$I = \frac{I_1 R_1 (R_1 + R_2)}{R_1 R_2} = \frac{I_1}{R_2} (R_1 + R_2)$$

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I$$

$$I = \frac{I_2 R_2 (R_1 + R_2)}{R_1 R_2} = \frac{I_2}{R_1} (R_1 + R_2)$$

All resistors in parallel share the same voltage

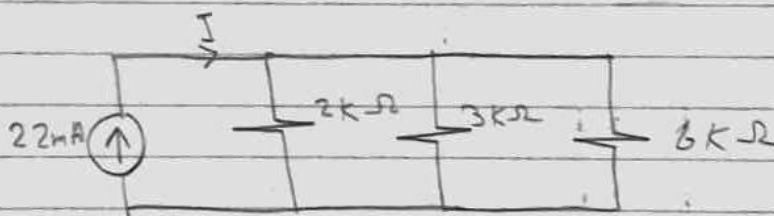


$$I_1 = \frac{R_2 \parallel R_3}{R_1 + (R_2 \parallel R_3)} I_{in}$$

$$I_2 = \frac{R_1 \parallel R_3}{R_2 + (R_1 \parallel R_3)} I_{in}$$

$$I_3 = \frac{R_1 \parallel R_2}{R_3 + (R_1 \parallel R_2)} I_{in}$$

EX:



$$I_1 = \frac{2k\Omega \times 22mA}{2 + 2} = 0,5 \times 22 \times 10^{-3} = 11mA$$

$$I_2 = \frac{1,5k\Omega \times 22mA}{3 + 1,5} = 0,33 \times 22 \times 10^{-3} = 7,26mA$$

$$I_3 = \frac{1,2k\Omega \times 22mA}{6 + 1,2} = 0,1667 \times 22 \times 10^{-3} = 3,67mA$$

22 mA

## Source transformation

Figure (a) shows the transformation from a voltage source to an equivalent current source

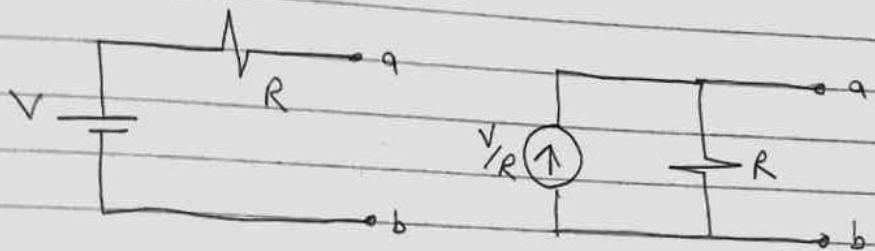
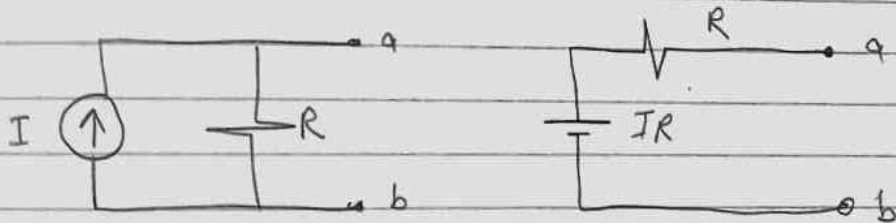


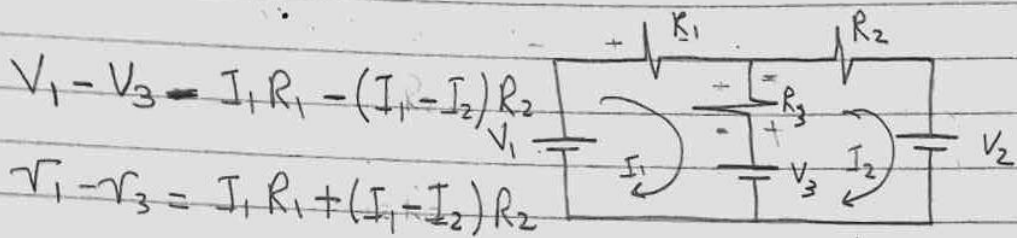
Figure (b) shows the transformation from a current source to an equivalent voltage source





## Mesh Analysis

Mesh Analysis : is defined as the method in which the current flowing through a planar circuit is calculated.



$$V_1 - V_3 = I_1 R_1 - (I_1 - I_2) R_2$$

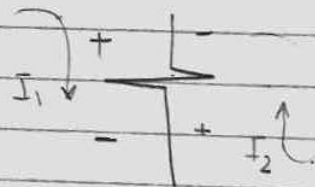
$$V_1 - V_3 = I_1 R_1 + (I_1 - I_2) R_2$$

$$V_3 - V_2 - (I_2 - I_1) R_3 - R_2 I_2 = 0$$

$$V_3 - V_2 = I_2 R_3 - I_1 R_3 + I_2 R_2$$

mesh 1

$$- (I_1 - I_2) R_1$$



mesh 2

$$- (I_2 - I_1) R_1$$

EX  
Using mesh analysis, Determine the current across each resistor and potential difference

sol ↓

$$90 - 9I_1 - 6(I_1 - I_2) = 0$$

$$90 = 15I_1 - 6I_2 \rightarrow \textcircled{1}$$

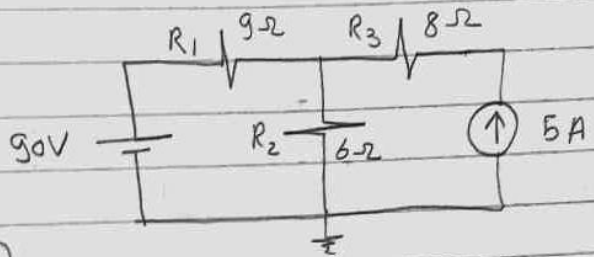
$$I_2 = 5 \text{ A} \quad \text{current source}$$

$$I_1 = 4 \text{ A}$$

$$V_1 = 4 \times 9 = 36 \text{ V}$$

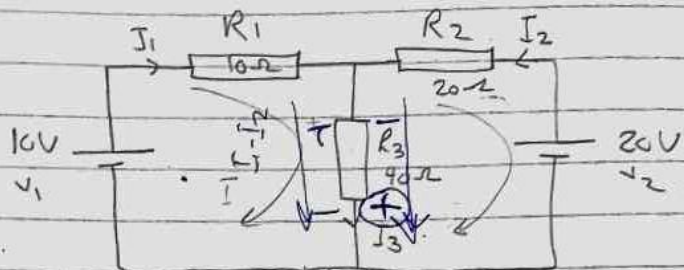
$$V_2 = 6 \text{ V}$$

$$V_3 = 40 \text{ V}$$



Ex:  
Determine the current across each resistor and potential difference

Sol ↓



$$10 - 10I_1 - 40(I_1 - I_2) = 0$$

$$10 = 50I_1 - 40I_2 \rightarrow (1)$$

$$+ (I_2 - I_1)$$

$$-20 - 40(I_2 - I_1) - I_2 \cdot 20 = 0$$

$$-20 = 60I_2 - 40I_1 \rightarrow (2)$$

$$10 = 50I_1 - 40I_2$$

$$-20 = -40I_1 + 60I_2$$

$$I_2 = -0,25 + 1,25 I_1$$

$$I_2 = 1,25 I_1 - 0,25$$

$$-2 = -4I_1 + 7,5I_1 - 1,5$$

$$-0,5 = 3,5I_1 \quad I_1 = -0,143A$$

$$I_2 = -0,429A$$



## Nodal analysis

is used for solving any electrical network, and it is defined as the mathematical method for calculating the voltage distribution between the circuit nodes.

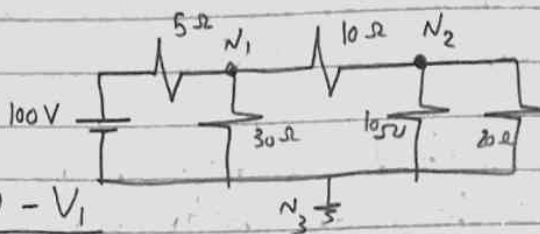
This method is also known as the node-voltage method since the node voltages are with respect to the ground.

The following are the three laws that define the equation related to the voltage that is measured between each circuit node:

- 1- ohm's law
- 2- kirchoff's voltage law
- 3- kirchoff's current law

Ex: using the Node analysis  
Determine the voltage at each node of the given circuit  
sol:

at Node 1



$$\frac{V_1}{630} + \frac{V_1 - V_2}{210} = \frac{100 - V_1}{5}$$

$$2V_1 + 6(V_1 - V_2) = 12(100 - V_1)$$

$$20V_1 - 6V_2 = 1200 \rightarrow (1)$$

at Node 2

$$\frac{V_2}{20} + \frac{V_2}{30} = \frac{V_1 - V_2}{10}$$

$$V_2 + 2V_2 = 2V_1 - 2V_2$$

$$5V_2 = 2V_1$$

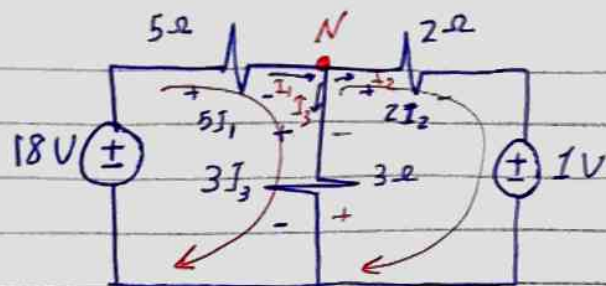
$$V_1 = 2,5V_2 \rightarrow (2)$$

$$V_2 = 27,27V$$

$$V_1 = 68,18V$$

Find  $I_1, I_2, I_3$

Sol ↓



KCL  $I_1 = I_2 + I_3 \rightarrow (1)$

① أولاً نفرض اتجاهات التيار  $I_1, I_2, I_3$  من المصدر ونوزع للدار النقطة

$$18 - 5I_1 - 3I_3 = 0 \Rightarrow 18 = 5I_1 + 3I_3 \rightarrow (2)$$

$$+3I_3 - 2I_2 - 1 = 0 \quad -1 = 2I_2 - 3I_3 \rightarrow (3)$$

① in ②

② نفرض اتجاه التيار في mesh مع عقارب الساعة

$$18 = 5(I_2 + I_3) + 3I_3$$

③ لو انه التيار  $I_1$  و  $I_2$  و  $I_3$  في نفس اتجاه التيار المفروض  
الاشارة  $(-)$

$$18 = 5I_2 + 8I_3$$

$$-1 = 2I_2 - 3I_3$$

لو انه اتجاه التيار  $I_1, I_2, I_3$  في عكس الاتجاه فانه اشارة العكس  $(+)$

$$36 = 10I_2 + 16I_3$$

$$+5 = -10I_2 - 15I_3$$

$$41 = I_3 \text{ A}$$

$$I_2 = \frac{-1 + 3I_3}{2} = \frac{-1 + 3 \times 41}{2} = 61 \text{ A}$$

$$I_1 = 102 \text{ A}$$



Example: use mesh analysis to find the currents in the circuit

Mesh 1

$$-2 - 4I_1 - 2(I_1 - I_2)$$

$$+ 6 = 0$$

$$6 - 2 = 2(I_1 - I_2) + 4I_1$$

$$4 = 6I_1 - 2I_2 \rightarrow (1)$$

Mesh (2)

$$-6 - 2(I_2 - I_1) - 5I_2 + 30V = 0$$

$$30 - 6 = 5I_2 + 2(I_2 - I_1)$$

$$24 = 7I_2 - 2I_1 \rightarrow (2)$$

Solving (1), (2)

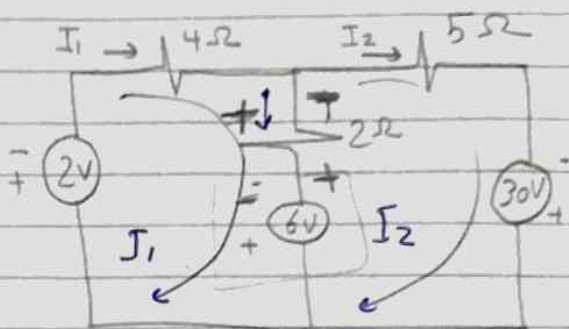
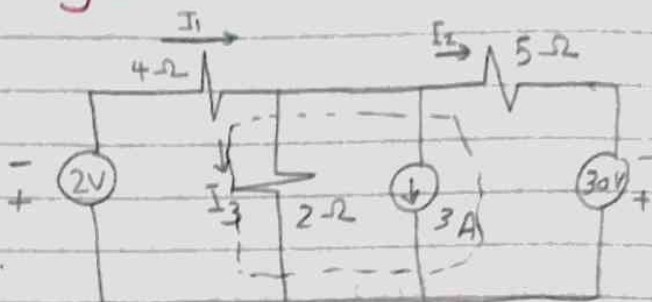
$$I_2 = 4A$$

$$I_1 = 2A$$

$$I_1 = I_3 + 3 + I_2$$

$$2 = I_3 + 3 + 4$$

$$I_3 = -5A \quad \uparrow$$



$$4 = 6I_1 - 2I_2$$

$$3 \times 24 = -2I_1 + 7I_2$$

$$72 = -6I_1 + 21I_2$$

$$76 = 19I_2$$

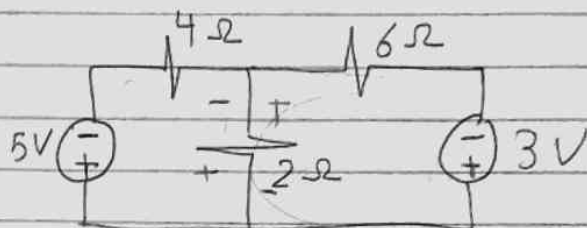
$$I_2 = 4A$$

معيار التوازن الكهربائي

Determine a network from which the following mesh equations might have come.

$$6I_1 - 2I_2 = 5$$

$$-2I_1 + 8I_2 = -3$$



$$6I_1 - 2I_2 - 5 = 0$$

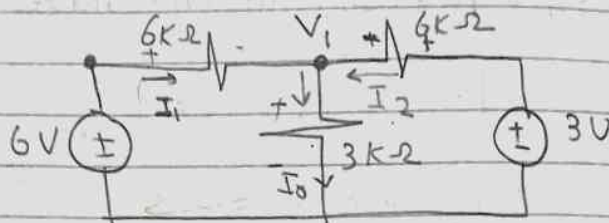
$$4I_1 + 2(I_1 - I_2) = 5$$

$$-2I_1 + 8I_2 + 3 = 0$$

$$6I_2 + 2(I_2 - I_1) = -3$$

Using Nodal analysis. Find  $V_1$  and  $I_0$

At Node  $V_1$



$$I_1 + I_2 = I_0$$

$$\frac{6 - V_1}{6} + \frac{3 - V_1}{6} = \frac{V_1}{3}$$

$$6 - V_1 + 3 - V_1 = 2V_1$$

$$9 - 2V_1 = 2V_1 \quad 9 = 4V_1 \quad V_1 = 9/4 \text{ V}$$

$$I_0 = \frac{9/4}{3K} = 3/4 \text{ mA}$$

اولاً يتم عمل Kcl معادلة

و يتم تحديد قطبية المقادرات بناء على مرور التيار بها  
التيار في المقادرات يمر من + الى -

يتم عمل تحديد النقطة موقع التحليل

يتم التحليل بناء على قطبية المقادرات والنقطة ناهية الاستاد +  
تكون موقع والنقطة ناهية الاستاد الى اليمين