Project 1 Report

Mass Efficient Key Component
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ME104
Mechanical Systems Design
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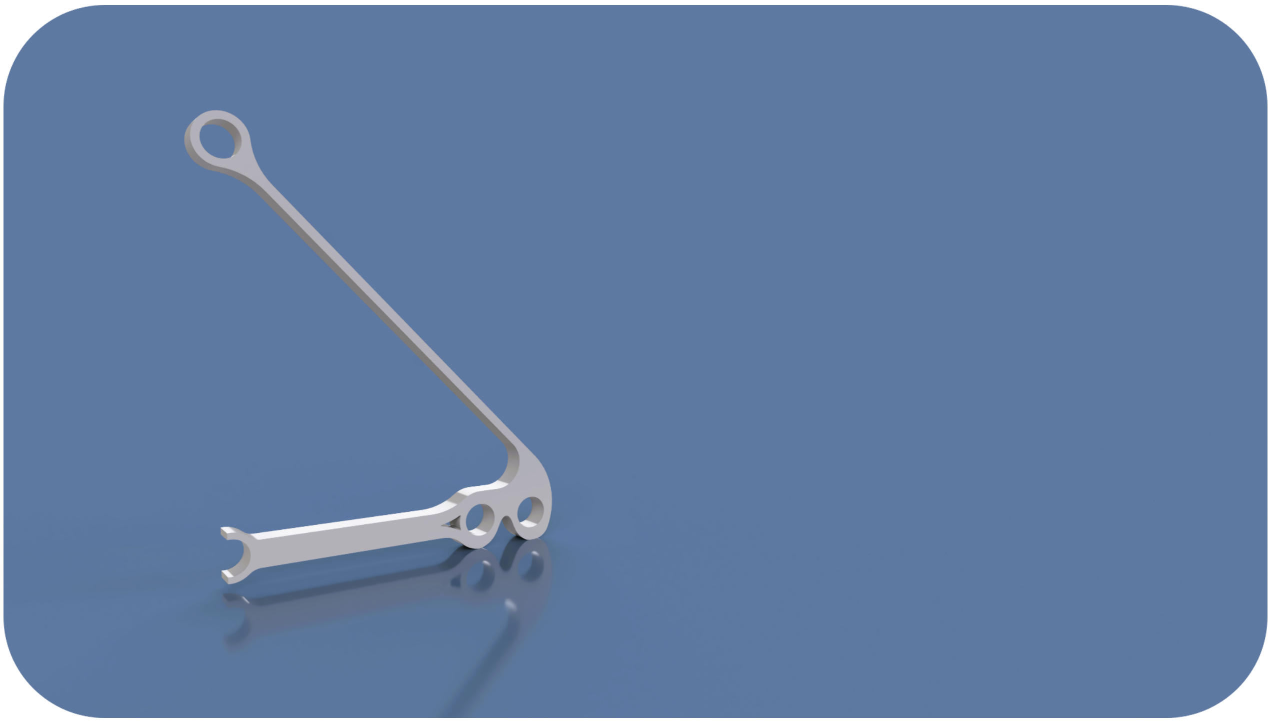
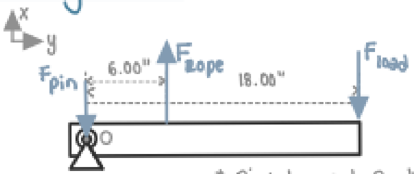
Summary



To support a vertical load at a given horizontal distance while optimizing mass efficiency, the key component consists of two arms whose projection is concurrent with the load's line of action. Coupled with limiting contact with the support plate to one peg per arm, effectively making them pins that cannot apply moments, bending moments through any major feature are eliminated. This allows for the both arms to be under pure axial tension or compression, distributing stress evenly through their respective cross-sections to optimize for mass efficiency. Considering the bottom arm as the critical member due to being under compression and potential buckling, mass efficiency is achieved by setting its length to be as short as possible by placing this arm horizontally and maximizing the angle of the top arm within the geometric limits. This fully defines the forces each arm experiences, facilitating cross-section analysis. The bottom arm has a solid square cross-section due to the shape's relatively high cross-sectional area moment of inertia, which minimizes thickness needed to prevent buckling in all planes. To facilitate printing without supports, the same thickness is used for the top arm, and since it is only in axial normal tension where only cross-sectional area matters, setting the thickness defines the height needed to support the expected load. The top arm lofts into the section that interfaces with the loading component to add support where stress distribution is highest in FEA and to avoid interference with the rope. To avoid stress concentrations, all crevices are filleted. Furthermore, to decrease mass, a triangular section that experienced very little stress around the left loading hole was cut out and half of the pegboard pinhole for the bottom arm was deleted as it did not bear any significant stress or have a structural purpose with the arm in pure compression.

Back-of-the-Envelope Analysis: FBD Diagrams

Loading lever



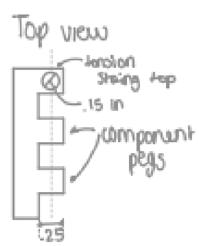
* Rigid and 2-dimensional (everything in plane)
* Negligible weight and friction at pin

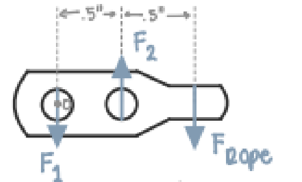
$$\Sigma F_x = 0$$
$$\Sigma F_y = F_{Rope} - F_{pin} - F_{Load} = 0$$
$$\Sigma M_{@_0} = F_{pin}(0) + F_{Rope}(6") - F_{Load}(18") = 0$$

$$F_{Rope} = 3F_{Load} \rightarrow F_{Rope} = 300\ N$$
$$F_{pin} = F_{Rope} - F_{Load} \rightarrow F_{pin} = 200\ N$$

Loading component

Top view



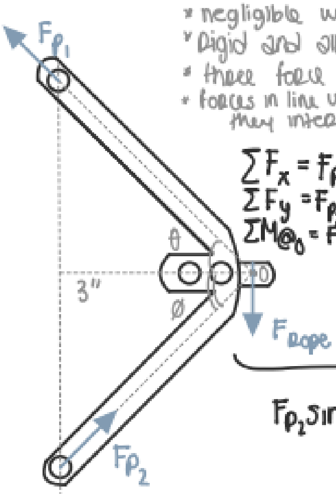


* Rigid
* negligible weight

$$\Sigma F_x = 0$$
$$\Sigma F_y = F_2 - F_1 - F_{Rope} = 0$$
$$\Sigma M_{@_0} = F_1(0) + F_2(.5") - F_{Rope}(1") = 0$$

$$F_2 = 2F_{Rope} \rightarrow F_2 = 600\ N$$
$$F_1 = F_2 - F_{Rope} \rightarrow F_1 = 300\ N$$

Key component and Loading Component



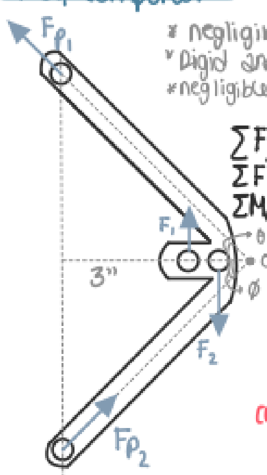
* negligible weight
* Rigid and all forces in plane
* three force member
* forces in line with arms so that they intersect at point O

$$\Sigma F_x = F_{p_2}\cos\phi - F_{p_1}\cos\theta = 0$$
$$\Sigma F_y = F_{p_2}\sin\phi + F_{p_1}\sin\theta - F_{Rope} = 0$$
$$\Sigma M_{@_0} = F_{Rope}(0) + F_{p_1}(0) + F_{p_2}(0) = 0$$

three force member, if forces meet here, no bending at this point

$$F_{p_2}\sin\phi + F_{p_1}\sin\theta = F_{Rope}$$
$$F_{p_2}\cos\phi = F_{p_1}\cos\theta$$

Key component



* negligible weight
* Rigid and all forces in plane
* negligible friction at pins

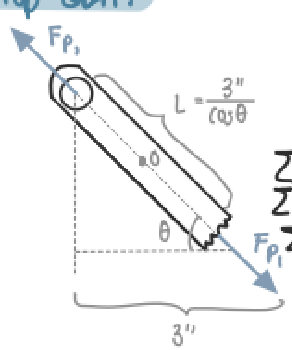
$$\Sigma F_x = F_{p_2}\cos\phi - F_{p_1}\cos\phi = 0$$
$$\Sigma F_y = F_1 - F_2 + F_{p_1}\sin\theta + F_{p_2}\sin\phi = 0$$
$$\Sigma M_{@_0} = F_{p_1}(0) + F_{p_2}(0) - F_1(1") + F_2(.5") = 0$$

confirmed no moment if F_{p_2} and F_{p_1} meet on axis of Rope tension

$$F_{p_1}\sin\theta + F_{p_2}\sin\phi = F_2 - F_1 \rightarrow 300\ N$$

FEA confirmed this tension / compression loading scenario

Top arm

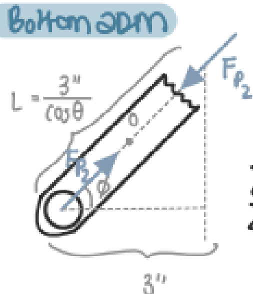


minimum distance to rope loading point → minimize mass

* Rigid, negligible weight
* purely in tension as drawn in component FBD's

$$\Sigma F_x = F_{p_1}\cos\theta - F_{p_1}\cos\theta = 0$$
$$\Sigma F_y = F_{p_1}\sin\theta - F_{p_1}\sin\theta = 0$$
$$\Sigma M_{@_0} = F_{p_1}(0) - F_{p_1}(0) = 0$$

Bottom arm



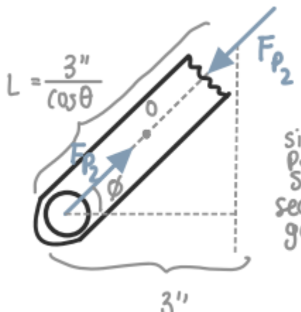
minimum distance to rope loading point → minimize mass

* Rigid, negligible weight
* purely in compression as drawn in component FBD's

$$\Sigma F_x = F_{p_2}\cos\phi - F_{p_2}\cos\phi = 0$$
$$\Sigma F_y = F_{p_2}\sin\phi - F_{p_2}\sin\phi = 0$$
$$\Sigma M_{@_0} = F_{p_2}(0) - F_{p_2}(0) = 0$$

Back-of-the-Envelope Analysis: Inverse Failure Analysis

Critical member: bottom arm because it is in compression thus more likely to fail in different modes and particularly geometry dependent. Top arm is in tension and therefore can be made significantly thinner before any failure.



minimum distance to rope loading point → minimize mass

simple and minimal-parameter square cross section that has a good moment of inertia to resist multiple modes of buckling

Stress:

$$\sigma_{max} = \frac{F_{P_2}}{A} = \frac{F_{P_2}}{a^2} = \frac{S_y}{FOS} \rightarrow a^2 = \frac{F_{P_2}\,FOS}{S_y}$$

$$m_s = \underbrace{\rho A L = \frac{\rho F_{P_2} FOS\, L}{S_y} = \frac{\rho F_{P_1}\cos\theta\, FOS\,(3'')}{\cos^2 Ø\, S_y}}$$

Stress analysis to minimize mass:

- minimize F_{P_2}
- minimize length of arm
- minimize F_{P_1}
- maximize θ
- minimize Ø ← dominating term

Buckling:

↓ since forces can move out of line, conservative mode is $C = \frac{1}{4}$

$$F_{CR} = \frac{(\frac{1}{4})(E)(\pi^2)(\frac{1}{12}a^4)}{L^2} = \frac{E\pi^2 a^4}{48L^2} = F_{P_2}\cdot FOS \Rightarrow a^2 = \sqrt{\frac{48F_{P_2}\,FOS}{E}}\frac{L}{\pi}$$

$$m_b = \underbrace{\rho A L = \rho\sqrt{\frac{48F_{P_2}FOS}{E}}\frac{L^2}{\pi} = \rho\sqrt{\frac{48F_{P_1}\cos\theta\, FOS}{\cos Ø\ \ E}}\frac{(3'')^2}{\pi\cos^2 Ø}}$$

buckling analysis to minimize mass:

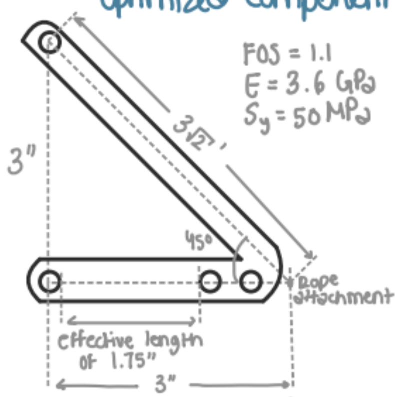
- minimize F_{P_2}
- minimize length
- minimize F_{P_1}
- maximize θ
- minimize Ø

coupled dominating terms

Both analyses suggest minimizing Ø along with maximizing θ, defining component geometry and forces.

$Ø = 0°$
$θ = 45°$ ⇒ $F_{P_1} = 300\sqrt{2}$ N, $F_{P_2} = 300$ N

Optimized component geometry



FOS = 1.1
E = 3.6 GPa
S_y = 50 MPa

Bottom arm – dominating failure mode when minimizing geometry is buckling:

from above:

$$a^2 = \sqrt{\frac{48F_{P_2}\,FOS}{E}}\frac{L^2}{\pi^2} = \sqrt{\frac{48(300N)(1.1)(\frac{1.75}{39.3701})^2}{(3.6\times10^9 Pa)(\pi^2)}} = 2.9679\times10^{-5}\,m$$

a = 5.448 mm
↳ ≈ .214 in

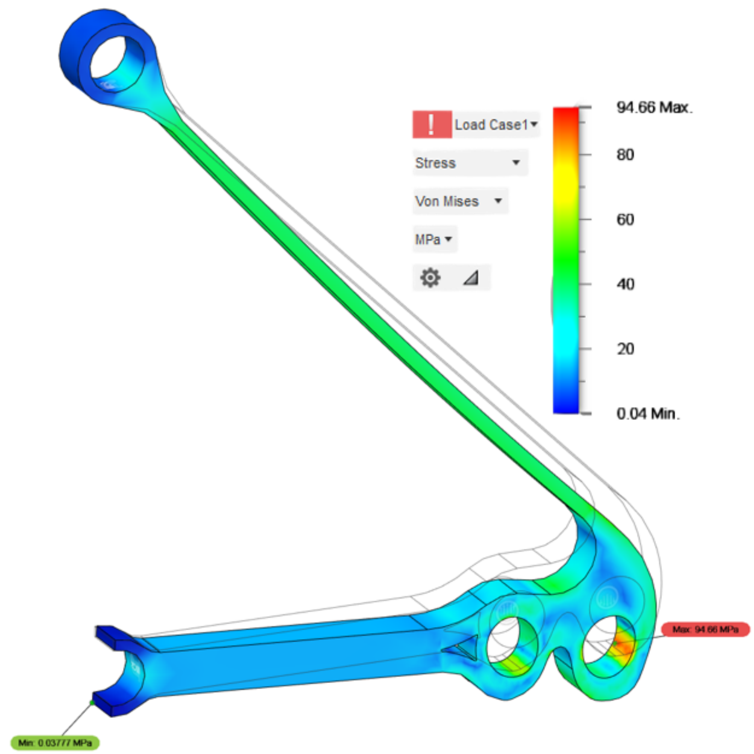
Top arm – In pure tension so only dependent on cross sectional area. Thus, set t = 5.448 mm to have even thickness in the component (facilitate printing with no supports) and solve for h.

$$\sigma_{max} = \frac{F_{P_1}}{th} = \frac{S_y}{FOS} \Rightarrow h = \frac{F_{P_1}\,FOS}{t(S_y)} = \frac{(300\sqrt{2}\,N)(1.1)}{(5.448\times10^{-3}m)(50\times10^6 Pa)}$$

h = 1.713 mm
↳ .067 in

NOTE: assuming isotropic and solid cross-sections (print at 100% infill)

Computation Analysis: Stress Distribution



Loading and Constraints:

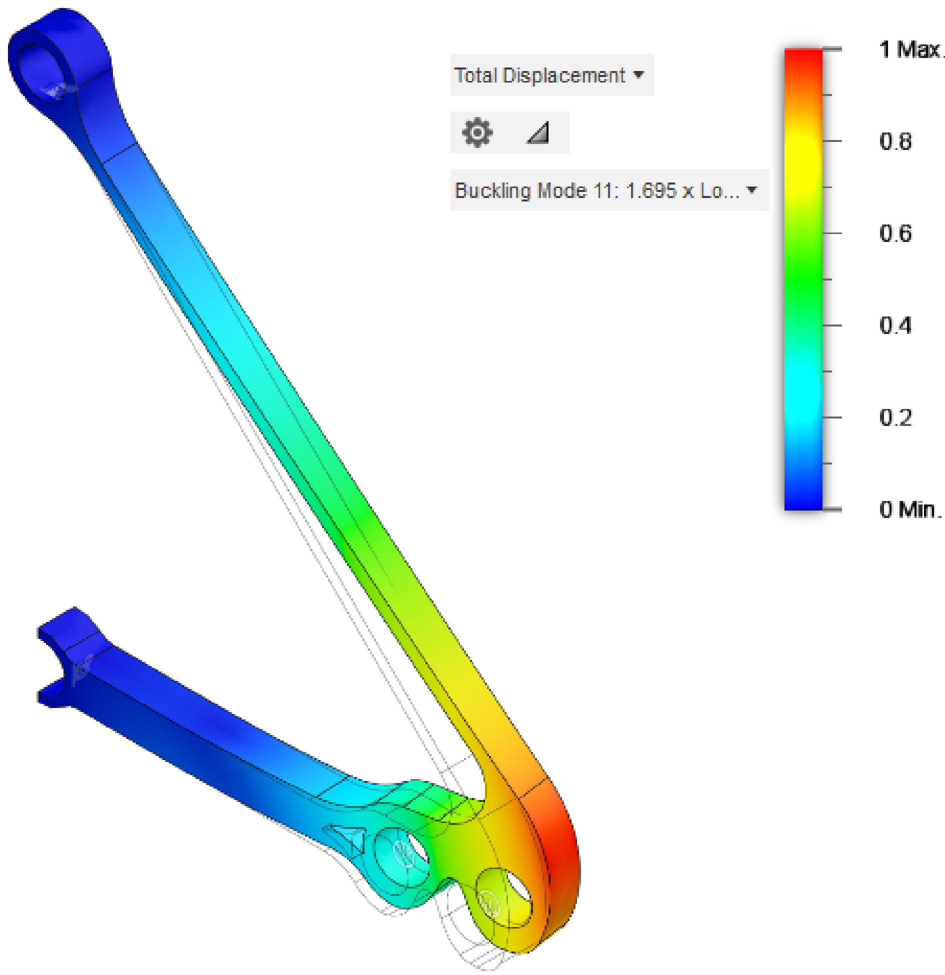
The constraints applied are Fusion 360's pin constraints where the component cylindrical surfaces contact the pegs in the support plate. The pins allow rotation while restricting radial and axial displacement. Assuming that the mating surfaces would not penetrate, deform, or introduce significant friction, this constraint allowed for solving the simulation while keeping the most relevant degree of freedom open for analysis.

The loading was applied on the cylindrical surfaces where the component interfaces with the loading component by using Fusion 360's bearing loads. Bearing loads represent a parabolic compressive load distribution across the compressed half of a cylindrical surface, imitating the contact expected in this scenario by the cylindrical "shafts" extending from the loading component. The magnitude of the bearing loads were 300 N upwards for the left hole and 600 N downwards for the right hole, as determined by the FBD analysis.

Stress Patterns:

As expected by FBD analysis, the stress through both arms is consistent, meaning that there is no bending moment through either, only axial tension or compression. However, my back-of-the-envelope analysis did not take into account the stress patterns formed where the component and the loading component interface. This area had the most varying stress distribution, with the highest stress located where the contact happens. Although not calculated, this stress and deformation over the small area is expected when using a bearing load, and sufficient thickness around this surface prevented failure starting here to spread across and cause ultimate failure.

Computational Analysis: Buckling Results



Loading and Constraints:

To maintain consistency across analyses, and since the loading scenario was the same, my loading and constraints are the same as described in the previous section. Furthemore, constraining the pin joints axially (as previously done) allowed use of the buckling model boundaries considered in the back-of-the-envelope buckling analysis.

Buckling Results:

The first buckling mode is what was expected from BOTEA, but with a higher factor than predicted. The safety factor for buckling used in BOTEA was 1.1, but the buckling simulation shows a factor of 1.695. The discrepancy is most likely due to using a different material and a different component geometry. Firstly, acrylic, which was used for simulation, has a significantly different elastic modulus than PLA in its strongest loading direction. Secondly, while analyzing buckling, only the bottom arm was considered and modeled as a long column fixed at the bottom and free at the top. However, the geometry of the entire component has a top arm that connects to the top of the bottom arm, which could alter the factor for this mode of buckling by adding lateral rigidity not considered in the free-top column simplification.