Section 3.5 #8, 17, 42, 48, 58, 66(b). Section 3.6 #4, 8, 12, 16, 32, 44, 50.
Section 4.2 #8

3.5 #8 Let S be the collection of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ that satisfy the given property. Either prove that S forms a subspace of $\mathbb{R}^3$ or give a counterexample to show that it does not.

$$|x - y| = |y - z|$$

∎

3.5 #17

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Give bases for $row(A)$, $col(A)$, and $null(A)$.

$\blacksquare$

3.5 #42 If A is a 4×2 matrix, what are the possible values of $nullity(A)$?

■

3.5 #48 Do $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ form a basis for $\mathbb{R}^4$?

■

3.5 #58 If A and B are $n \times n$ matrices of rank n, prove that AB has rank n.

$\blacksquare$

3.5 # 66(b) Let A be a skew-symmetric $n \times n$ matrix (see page 162). Prove that $I + A$ is invertible. [Hint: Show that $null(I + A) = \{0\}$].

■

3.6 #4 Prove that the transformation is linear, using the definition (or the Remark following Example 3.55):

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x+2y \\ 3x-4y \end{bmatrix}$$

.

■

3.6 #8 Give a counterexample to show that the transformation is not linear:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} |x| \\ |y| \end{bmatrix}$$

.

■

3.6 #12 Find the standard matrix of the linear transformation in Exercise 4.

■

3.6 #16 Show that the transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ is linear by showing that it is a matrix transformation:
R rotates a vector 45° counterclockwise about the origin.

■

3.6 #32 Verify Theorem 3.32 by finding the matrix of $S \circ T$ (a) by direct substitution and (b) by matrix multiplication of $[S][T]$:

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$$

,

$$S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + 3y_2 \\ 2y_1 + y_2 \\ y_1 - y_2 \end{bmatrix}$$

∎

3.6 #44 Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ (or $\mathbb{R}^3$ to $\mathbb{R}^3$). Prove that T maps a straight line to a straight line or a point. [Hint: use the vector form of the equation of a line.]

■

3.6 #50 Let ABCD be the square with vertices $(-1,1)$, $(1,1)$, $(1,-1)$, and $(-1,-1)$. Use the results in Exercises 44 and 45 to find and draw the image of ABCD under the given transformation:

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ -3x_1 + x_2 \end{bmatrix}$$

.

■

4.2 #8 Compute the determinant using cofactor expansion along any row or column that seems convenient:

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

.

∎