Name: ______________
Math 40, Section _____
HW02 Dot Products, Cross Products, Lines and Planes
January 26, 2018

Section 1.2 (15, 26, 46, 60, 62, 68b)
Section 1.3 (6, 10, 16ace, 18, 46)
Problem on page 49 (Exploration): 4 (cross products)
Section 2.1 (34, 42)

1 Section 1.2

15: **In Exercises 13-16,find the distance $d(\vec{u}, \vec{v})$ between $\vec{u}$ and $\vec{v}$ in the given exercise.*
Exercise 3:

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

26: **In Exercises 24-29, find the angle between $\vec{u}$ and $\vec{v}$ in the given exercise.*
Exercise 20:

$$\vec{u} = \begin{bmatrix} 4, & 3, & -1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1, & -1, & 1 \end{bmatrix}$$

46:

**Figure 1.39 suggests two ways in which vectors may be used to compute the area of a triangle. The area $\mathcal{A}$ of the triangle in part (a) is given by*

$$\frac{1}{2}\|\vec{u}\| \; \|\vec{v} - proj_{\vec{u}}(\vec{v})\|$$

and part (b) suggests the trigonometric form of the area of a triangle:

$$\mathcal{A} = \frac{1}{2}\| \vec{u}\| \; \|\vec{v}\| sin\theta$$

(We can use the identity $sin\theta = \sqrt{1 - cos^2\theta}$ to find $sin\theta$.)
In Exercises 46 and 47, compute the area of the triangle with the given vertices using both methods.

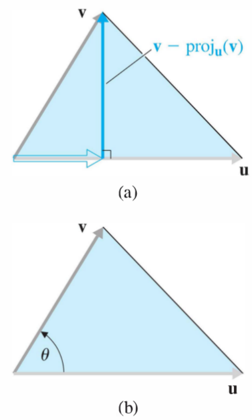

Figure 1.39

Exercise 46:

$$A = (1, -1), B = (2, 2), C = (4, 0)$$

60: Suppose we know that $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$. Does it follow that $\vec{v} = \vec{w}$? If it does, give a proof that is valid in $\mathbb{R}^n$; otherwise, give a counterexample (i.e., a specific set of vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ for which $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$ but $\vec{v} \neq \vec{w}$).

62:
(a) Prove that

$$\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2 = 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2$$

for all vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^n$.
(b) Draw a diagram showing $\vec{u}, \vec{v}, \vec{u}+\vec{v},$ and$\vec{u}-\vec{v}$ in $\mathbb{R}^2$ and use (a) to deduce a result about parallelograms.

68b:
Prove that if $\vec{u}$ is orthogonal to both $\vec{v}$ and $\vec{w}$, then $\vec{u}$ is orthogonal to s$\vec{v}$ + t$\vec{w}$ for all scalars s and t.

2 Section 1.3

6: **In Exercises 3-6, write the equation of the line passing through P with direction vector $\vec{d}$ in (a) vector form and (b) parametric form.*

$$P = (3, 0, -2), \vec{d} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

10: **In Exercises 9 and 1 0, write the equation of the plane passing through P with direction vectors $\vec{u}$ and $\vec{v}$ in (a) vector form and (b) parametric form.*

$$P = (6, -4, -3), \vec{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

16ace:
Consider the vector equation
$$\vec{x} = \vec{p} + t(\vec{q} - \vec{p})$$
where $\vec{p}$ and $\vec{q}$ correspond to distinct points P and Q in $\mathbb{R}^2$ or $\mathbb{R}^3$
(a) Show that this equation describes the line segment $\vec{PQ}$ as t varies from 0 to 1.
(c) Find the midpoint of $\vec{PQ}$ when P = (2, - 3) and Q = (0, 1).
(e) Find the two points that divide $\vec{PQ}$ in part (c) into three equal parts.

18:

The line ℓ passes through the point P = (1, - 1, 1) and has direction vector $\vec{d} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$.

For each of the following planes $\mathcal{P}$ determine whether ℓ and $\mathcal{P}$ are parallel, perpendicular, or neither:

(a) $2x + 3y - z = 1$
(b) $4x - y + 5z = 0$
(c) $x - y - z = 3$
(d) $4x + 6y - 2z = 0$

46: **In Exercises 45-46, show that the plane and line with the given equations intersect, and then find the acute angle of intersection between them.*
The plane given by 4x - y - z = 6 and the line given by:

$$\begin{aligned} x &= t \\ y &= 1 + 2t \\ z &= 2 + 3t \end{aligned} \tag{1}$$

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4: Use the cross product to help find the normal form of the equation of the plane.

(a) The plane passing through $P = (0, -1, 1)$, parallel to $\vec{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

(b) The plane passing through $P = (0, -1, 1)$, $Q = (2, 0, 2)$, and $R = (1, 2, -1)$

4 Section 2.1

34: **For Exercises 33-38, solve the linear systems in the given exercises.*
Exercise 28:

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 1 \\ x_1 \qquad\quad + x_3 &= 0 \\ -x_1 + 2x_2 - 2x_3 &= 0 \end{aligned} \tag{2}$$

42: **In Exercises 4 1 -44, the systems of equations are nonlinear. Find substitutions (changes of variables) that convert each system into a linear system and use this linear system to help solve the given system.*
Exercise 28:

$$\begin{aligned} x^2 + 2y^2 &= 6 \\ x^2 - y^2 &= 3 \end{aligned} \tag{3}$$