Name: ____________________
Math 40, Section _____
HW07
2/28/18

Homework 07: EigenThings and Diagonalization
4.3 #10, 22, 25, 33, 37
4.4 #7, 19, 42, 49, 51

4.3.10 Compute (a) the characteristic polynomial of A, (b) the eigenvalues of A, (c) the basis for each eigenspace of A, and (d) the algebraic and geometric multiplicity of each eigenvalue.

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

■

4.3.22 If $\vec{v}$ is an eigenvector of A with corresponding eigenvalue λ and c is a scalar, show that $\vec{v}$ is an eigenvector of $A - cI$ with a corresponding eigenvalue $\lambda - c$.

$\blacksquare$

4.3.25 If A and B are two row equivalent matrices, do they necessarily have the same eigenvalues? Either prove that they do or give a counterexample.

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4.3.33 Verify the Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$. That is, find the characteristic polynomial $c_A(\lambda)$ of A and show that $c_A(A) = O$

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4.3.37 For the matrix A in Exercise 33, use the Cayley-Hamilton Theorem to compute A^{-1} by expressing it as a linear combination of I, A, and A^2.

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4.4.7 A diagonalization of the matrix A is given in the form $P^{-1}AP = D$. List the eigenvalues of A and bases for the corresponding matrices.

$$\begin{bmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ \frac{5}{8} & -\frac{3}{8} & -\frac{3}{8} \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 2 & 0 & 2 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

■

4.4.19 Give A^k as a product of three matrices.

$$\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}^k$$

■

4.4.42 Prove that if A is similar to B, then A^T is similar to B^T.

■

4.4.49 Prove that if A is a diagonalizable matrix such that every eigenvalue of A is either 0 or 1, then A is idempotent (that is, $A^2 = A$).

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4.4.51 Suppose that A is a 6×6 matrix with characteristic polynomial

$$c_A(\lambda) = (l + \lambda)(1 - \lambda)^2(2 - \lambda)^3$$

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(a) Prove that it is not possible to find linearly independent vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ in $\mathbb{R}^6$ such that $A\vec{v}_1 = \vec{v}_1$, $A\vec{v}_2 = \vec{v}_2$, and $A\vec{v}_3 = \vec{v}_3$.

(b) If A is diagonalizable, what are the dimensions of the eigenspaces E_{-1}, E_1, and E_2?

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