Name: ___________________
Math 40, Section _____
HW05 - Subspaces and Determinants
February 16, 2018

Section 3.5 Numbers: 8, 17, 42, 48, 58, 66b
Section 3.6 Numbers: 4, 8, 12, 16, 32, 44, 50
Section 4.2: 8

3.5.8 *Let S be the collection of vectors* $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ *in* $\mathbb{R}^3$ *that satisfy the given property. Either prove that S forms a subspace of* $\mathbb{R}^3$ *or give a counterexample to show that it does not.*

$$|x-y| = |y-z|$$

∎

3.5.17 *Give bases for row(A), col(A), and null(A).*

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

■

3.5.42 *If A is a 4×2 matrix, what are the possible values of $nullity(A)$?*

■

3.5.48 *Do* $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, *and* $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ *form a basis for* $\mathbb{R}^4$*?*

■

3.5.58 *If A and B are $n \times n$ matrices of rank n, prove that AB has rank n.*

■

3.5.66 (b) *Let A be a skew-symmetric $n \times n$ matrix. Prove that $I + A$ is invertible.*
[Hint: Show that $null(I + A) = \{0\}$]

∎

3.6.4 *Prove that the transformation is linear, using the definition:*

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x+2y \\ 3x-4y \end{bmatrix}$$

.

$\blacksquare$

3.6.8 *Give a counterexample to show that the transformation is not linear:*

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} |x| \\ |y| \end{bmatrix}$$

.

■

3.6.12 *Find the standard matrix of the linear transformation in Exercise 4.*

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x + 2y \\ 3x - 4y \end{bmatrix}$$

■

3.6.16 *Show that the transformation from* $\mathbb{R}^2$ *to* $\mathbb{R}^2$ *is linear by showing that it is a matrix transformation:*

R rotates a vector 45° counterclockwise about the origin.

$\blacksquare$

3.6.32 *Verify Theorem 3.32 by finding the matrix of $S \circ T$*

(a) by direct substitution and

(b) by matrix multiplication of $[S][T]$.

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$$

,

$$S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + 3y_2 \\ 2y_1 + y_2 \\ y_1 - y_2 \end{bmatrix}$$

■

3.6.44 *Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ (or $\mathbb{R}^3$ to $\mathbb{R}^3$). Prove that T maps a straight line to a straight line or a point.* [Hint: use the vector form of the equation of a line.]

■

3.6.50 *Let ABCD be the square with vertices*$(-1,1)$, $(1,1)$, $(1,-1)$, and $(-1,-1)$. *Use the results in Exercises 44 and 45 to find and draw the image of ABCD under the given transformation:*

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ -3x_1 + x_2 \end{bmatrix}$$

.

■

4.2.8 *Compute the determinant using cofactor expansion along any row or column that seems convenient:*

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

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■