Box #______
Math 60 Section 1
Homework 2
16 May 2018

Collaborators:

Colley 2.3 #22 Find the gradient $\nabla f(\mathbf{a})$ where

$$f(x, y) = e^{xy} + \ln(x - y), \quad \mathbf{a} = (2, 1).$$

$\blacksquare$

Colley 2.3 #33 Find the matrix $D\mathbf{f}(\mathbf{a})$ of partial derivatives, where

$$\mathbf{f}(s,t) = (s^2, st, t^2), \quad \mathbf{a} = (-1, 1).$$

$\blacksquare$

Colley 2.3 #38 Find an equation for the plane tangent to the graph of $z = 4\cos(xy)$ at the point $(\pi/3, 1, 2)$.

$\blacksquare$

Colley 2.3 #42 Suppose that you have the following information concerning a differentiable function f:

$$f(2,3) = 12, \quad f(1.98,3) = 12.1, \quad f(2,3.01) = 12.2.$$

(a) Give an approximate equation for the plane tangent to the graph of f at $(2,3,12)$.

(b) Use the result of part (a) to estimate $f(1.98,2.98)$.

$\blacksquare$

Colley 2.4 #2 Verify the sum rule for derivative matrices for the following pair of functions.

$$\mathbf{f}(x,y) = (e^{x+y}, xe^{y}), \quad \mathbf{g}(x,y) = (\ln(xy), ye^{x}).$$

■

Colley 2.4 #14 For the following function, determine all second-order partial derivatives (including mixed partials).

$$f(x,y) = e^{x^2+y^2}.$$

■

Colley 2.4 #22 Consider the function $F(x, y, z) = 2x^3y + xz^2 + y^3z^5 - 7xyz$.

(a) Find F_{xx}, F_{yy}, F_{zz}.

(b) Calculate the mixed second-order partials $F_{xy}, F_{yx}, F_{xz}, F_{zx}, F_{yz}, F_{zy}$, and verify Theorem 4.3.

■

Colley 2.4 #29a The three-dimensional **heat equation** is the partial differential equation

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\frac{\partial T}{\partial t},$$

where k is a positive constant. It models the temperature $T(x,y,z,t)$ at the point (x,y,z) and time t of a body in space.

(a) We examine a simplified version of the heat equation. Consider a straight wire "coordinatized" by x. Then the temperature $T(x,t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial t}.$$

Show that the function $T(x,t) = e^{-kt}\cos x$ satisfies this equation. Note that if t is held constant at value t_0, then $T(x,t_0)$ shows how the temperature varies along the wire at time t_0. Graph the curves $z = T(x,t_0)$ for $t_0 = 0, 1, 10$, and use them to understand the graph of the surface $z = T(x,t)$ for some $t \geq 0$. Explain what happens to the temperature of the wire after a long period of time.

■