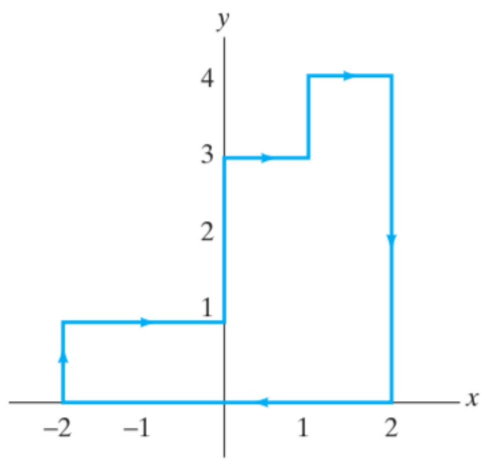
Box #______
Math 60 Section 1
Homework 11
30 May 2018

Collaborators:

Colley 6.2 #13 Evaluate $\oint_C \left(x^4y^5 - 2y\right) dx + \left(3x + x^5y^4\right) dy$, where C is the oriented curve pictured below



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Colley 6.2 #15

(a) Sketch the curve given parametrically by $\mathbf{x}(t) = \left(1 - t^2, t^3 - t\right)$.

(b) Find the area inside the closed loop of the curve.

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Colley 6.3 #1 Consider the line integral $\int_C z^2\,dx + 2y\,dy + xz\,dz$.

(a) Evaluate this integral, where C is the line segment from $(0,0,0)$ to $(1,1,1)$.

(b) Evaluate this integral, where C is the path from $(0,0,0)$ to $(1,1,1)$ parametrized by $\mathbf{x}(t) = (t, t^2, t^3), 0 \le t \le 1$.

(c) Is the vector field $\mathbf{F} = z^2\mathbf{i} + 2y\mathbf{j} + xz\mathbf{k}$ conservative? Why or why not?

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Colley 6.3 #3 Determine whether the given vector field

$$\mathbf{F} = e^{x+y}\mathbf{i} + e^{xy}\mathbf{j}$$

is conservative. If it is, find a scalar potential function for **F**.

$\blacksquare$

Colley 6.3 #4 Determine whether the given vector field

$$\mathbf{F} = 2x \sin y \mathbf{i} + x^2 \cos y \mathbf{j}$$

is conservative. If it is, find a scalar potential function for $\mathbf{F}$.

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Colley 6.3 #25 Let $\mathbf{F} = x^2\mathbf{i} + \cos y \sin z\mathbf{j} + \sin y \cos z\mathbf{k}$.

(a) Show that $\mathbf{F}$ is conservative and find a scalar potential function f for $\mathbf{F}$.

(b) Evaluate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ along the path $\mathbf{x} : [0, 1] \to \mathbb{R}^3$, $\mathbf{x}(t) = (t^2 + 1, e^t, e^{2t})$.

$\blacksquare$

Colley 6.3 #33

(a) Determine whether the vector field

$$\mathbf{F} = \frac{x + xy^2}{y^2}\mathbf{i} - \frac{x^2+1}{y^3}\mathbf{j}$$

is conservative.

(b) Determine a scalar potential for $\mathbf{F}$.

(c) Find the work done by $\mathbf{F}$ in moving a particle along the parabolic curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.

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Colley 7.2 #6 Find $\int\int_S (x^2 + y^2)\, dS$, where S is the lateral surface of the cylinder of radius a and height h whose axis is the z-axis.

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Colley 7.2 #14 Let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 4$, and lateral surface given by the equation $x^2 + y^2 = 9$. Orient S with outward normals. Determine

$$\int\int_S (x\mathbf{i} + y\mathbf{j}) \cdot d\mathbf{S}.$$

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Colley 7.2 #22 Find the flux of
$$\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + xz\mathbf{k}$$
across the upper hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$. Orient the hemisphere with an upward-pointing normal.

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Colley 7.2 #28 The glass dome of a futuristic greenhouse is shaped like the surface $z = 8 - 2x^2 - 2y^2$. The greenhouse has a flat dirt floor at $z = 0$. Suppose that the temperature T, at points in and around the greenhouse, varies as

$$T(x, y, z) = x^2 + y^2 + 3(z-2)^2.$$

Then the temperature gives rise to a **heat flux density field H** given by $\mathbf{H} = -k\nabla T$. (Here k is a positive constant that depends on the insulating properties of the particular medium.) Find the total heat flux outward across the dome and the surface of the ground if $k = 1$ on the glass and $k = 3$ on the ground.

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