Box #______
Math 65 Section 1
Homework 6
24 May 2018

Collaborators:

1. The general linear n^{th}-order constant-coefficient homogeneous DE for a function $u(t)$ has the form

$$a_n u^{(n)} + a_{n-1} u^{(n-1)} + a_2 u'' + a_1 u' + a_0 u = 0 \tag{1}$$

Write this as a system of first-order ODEs, in matrix form.

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2. Find the general solution of $\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix} \mathbf{x}$. Express your answer in the form $\mathbf{x}(t) = \Psi(t)\mathbf{c}$ where $\Psi(t)$ is a fundamental matrix.

$\blacksquare$

3. Find the general solution for the linear system

$$\mathbf{x}' = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \mathbf{x}. \tag{2}$$

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4. Suppose $\lambda \in \mathbb{C}$ and $\mathbf{v} \in \mathbb{C}^n$. Determine the real and imaginary parts of the function

$$\mathbf{x}(t) = e^{\lambda t}\mathbf{v}.$$

For consistency in grading, assume $\lambda = \alpha + i\beta$ ($\alpha, \beta \in \mathbb{R}$) and $\mathbf{v} = \mathbf{p} + i\mathbf{q}$ ($\mathbf{p}, \mathbf{q} \in \mathbb{R}^n$).

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5. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$. If the eigenvalues are complex, express your eigenvectors in the form $\mathbf{p} + i\mathbf{q}$, where $\mathbf{p}, \mathbf{q} \in \mathbb{R}^2$.

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6. Find the general real-valued solution for the system

$$\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \mathbf{x}. \tag{3}$$

Express your answer in the form $\mathbf{x}(t) = \Psi(t)\mathbf{c}$ where $\Psi(t)$ is a fundamental matrix.

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7. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \mathbf{x} \qquad (4)$$

with initial condition $\mathbf{x}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and determine $\lim_{t \to \infty} \mathbf{x}(t)$ for your solution.

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8. Consider the inhomogeneous linear system

$$\mathbf{x}' = A\,\mathbf{x} + \mathbf{g}(t) \tag{5}$$

where $A \in M_{nn}(\mathbb{R})$ and $\mathbf{g} : \mathbb{R} \to \mathbb{R}^n$. Suppose $\mathbf{x}_p$ is a particular solution of the inhomogeneous linear system and $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ is a basis for the space of solutions of the associated homogeneous system $\mathbf{x}' = A\,\mathbf{x}$. Prove that if $\mathbf{y}$ is any solution of the inhomogeneous equation (5) then there must exist constants $c_1, \ldots, c_n$ such that

$$\mathbf{y}(t) = c_1\mathbf{x}_1(t) + \cdots + c_n\mathbf{x}_n(t) + \mathbf{x}_p(t).$$

Therefore, if you know *one* solution to the inhomogeneous problem you know them *all*, up to something in the homogeneous solution space.

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