Box #_____
Math 60 Section 1
Homework 6
22 May 2018

Collaborators:

Colley 3.4 #4 Calculate the divergence of the vector field

$$\mathbf{F} = z \cos e^{y^2} \mathbf{i} + x\sqrt{z^2+1}\,\mathbf{j} + e^{2y} \sin 3x\mathbf{k}.$$

$\blacksquare$

Colley 3.4 #7 Find the curl of the vector field

$$\mathbf{F} = x^2\mathbf{i} - xe^y\mathbf{j} + 2xyz\mathbf{k}.$$

$\blacksquare$

Colley 3.4 #12

(a) Consider the vector field

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

and its curl. Sketch the vector field and use your picture to explain geometrically why the curl is what you calculated.

(b) Use geometry to determine $\nabla \times \mathbf{F}$ where

$$\mathbf{F} = \frac{(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})}{\sqrt{x^2 + y^2 + z^2}}.$$

(c) for $\mathbf{F}$ in part (b), verify your intuition by explicitly computing $\nabla \times \mathbf{F}$.

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Colley 3.4 #13 Can you tell in what portions of $\mathbb{R}^2$, the following vector fields have positive divergence? Negative divergence?

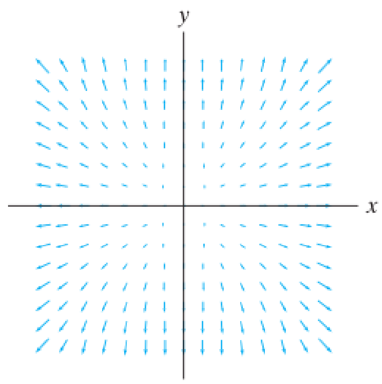

Figure 3.43 Vector field for Exercise 13(a).

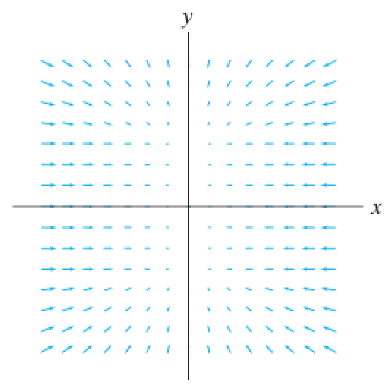

Figure 3.44 Vector field for Exercise 13(b).

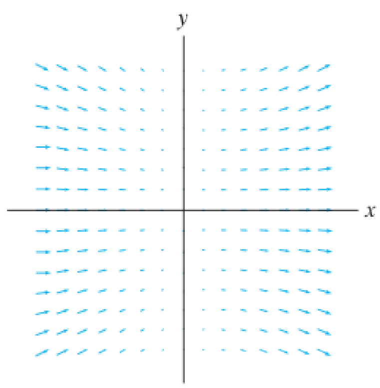

Figure 3.45 Vector field for Exercise 13(c).

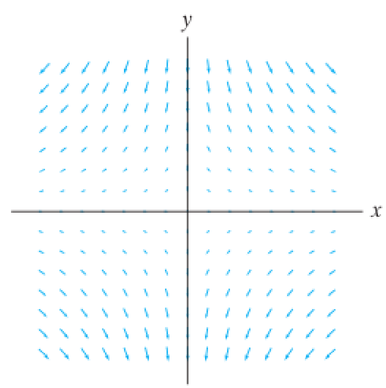

Figure 3.46 Vector field for Exercise 13(d).

Colley 3.4 #16 Prove Theorem 4.4: Let $\mathbf{F} : X \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field of class C^2. Then

$$\text{div}\,(\text{curl}\,\mathbf{F}) = 0.$$

That is, curl $\mathbf{F}$ is an incompressible vector field.

$\blacksquare$

Colley 3.4 #23 Establish

$$\nabla \cdot (f\mathbf{F}) = f\nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla f.$$

(You may assume that any functions and vector fields are appropriately differentiable.)

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