Box #______
Math 65 Section 1
Homework 3
17 May 2018

Collaborators:

Poole 6.3 #16 Let $\mathcal{B}$ and $\mathcal{C}$ be bases for $\mathcal{P}_2$. If $\mathcal{B} = \{x, 1+x, 1-x+x^2\}$ and the change-of-basis matrix from $\mathcal{B}$ to $\mathcal{C}$ is

$$\mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}.$$

Find $\mathcal{C}$.

$\blacksquare$

Poole 6.3 #22 Let V be an n-dimensional vector space with basis $\mathcal{B} = \{\mathbf{v}_1, ..., \mathbf{v}_n\}$. Let P be an invertible $n \times n$ matrix and set

$$\mathbf{u}_i = p_{1i}\mathbf{v}_1 + ... + p_{ni}\mathbf{v}_n$$

for $i = 1, ..., n$. Prove that $\mathcal{C} = \{\mathbf{u}_1, ..., \mathbf{u}_n\}$ is a basis for V and show that $P = P_{\mathcal{B} \leftarrow \mathcal{C}}$.

$\blacksquare$

Poole 6.4 #20 Show that there is no linear transformation $T : \mathbb{R}^3 \to \mathcal{P}_2$ such that

$$T \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 1 + x, \quad T \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} = 2 - x + x^2, \quad T \begin{bmatrix} 0 \\ 6 \\ -8 \end{bmatrix} = -2 + 2x^2.$$

■

Poole 6.4 #24 Let $\mathbf{v}_1, ..., \mathbf{v}_n$ be vectors in a vector space V and let $T : V \to W$ be a linear transformation.

(a) If $\{T(\mathbf{v}_1), ..., T(\mathbf{v}_n)\}$ is linearly independent in W, show that $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$ is linearly independent in V.

(b) Show that the converse of part (a) is false. That is, it is not necessarily true that if $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$ is linearly independent in V, then $\{T(\mathbf{v}_1), ..., T(\mathbf{v}_n)\}$ is linearly independent in W. Illustrate this with an example $T : \mathbb{R}^2 \to \mathbb{R}^2$.

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Poole 6.4 #32 Let $T : V \to V$ be a linear transformation such that $T \circ T = I$.

(a) Show that $\{\mathbf{v}, T(\mathbf{v})\}$ is linearly dependent if and only if $T(\mathbf{v}) = \pm\mathbf{v}$.

(b) Give an example of such a linear transformation with $V = \mathbb{R}^2$.

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Poole 6.5 #4 Let $T : \mathcal{P}_2 \to \mathcal{P}_2$ be the linear transformation defined by $T(p(x)) = xp'(x)$.

(a) Which, if any, of the following are in $\ker(T)$?
(i) 1 (ii) x (iii) x^2

(b) Which, if any, of the polynomials in part (a) are in range(T)?

(c) Describe ker(T) and range(T).

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Poole 6.5 #28 Show that $T : \mathcal{P}_n \to \mathcal{P}_n$ defined by $T(p(x)) = p(x-2)$ is an isomorphism.

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Poole 6.5 #34 Let $S : V \to W$ and $T : U \to V$ be linear transformations.

(a) Prove that if $S \circ T$ is one-to-one, so is T.

(b) Prove that if $S \circ T$ is onto, so is S.

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