

**Collaborators:**

**Colley 6.1 #2** Calculate  $\int_{\mathbf{x}} f \, ds$  where

$$f(x, y, z) = xyz, \quad \mathbf{x}(t) = (t, 2t, 3t), \quad 0 \leq t \leq 2.$$

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**Colley 6.1 #9** Find  $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$  where

$$\mathbf{F} = (y + 2)\mathbf{i} + x\mathbf{j}, \quad \mathbf{x}(t) = (\sin t, -\cos t), \quad 0 \leq t \leq \pi/2.$$

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**Colley 6.1 #21** Let  $\mathbf{F} = (x^2 + y)\mathbf{i} + (y - x)\mathbf{j}$  and consider the two paths

$$\mathbf{x}(t) = (t, t^2), \quad 0 \leq t \leq 1$$

$$\mathbf{y}(t) = (1 - 2t, 4t^2 - 4t + 1), \quad 0 \leq t \leq \frac{1}{2}$$

(a) Calculate  $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$  and  $\int_{\mathbf{y}} \mathbf{F} \cdot d\mathbf{s}$ .

(b) By considering the image curves of the paths  $\mathbf{x}$  and  $\mathbf{y}$ , discuss your answers in part (a).

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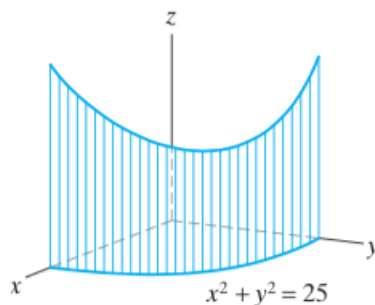
**Colley 6.1 #31** Evaluate

$$\int_C yz \, dx - xz \, dz + xy \, dy,$$

where  $C$  is the line segment from  $(1, 1, 2)$  to  $(5, 3, 1)$ .

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**Colley 6.1 #34** Tom Sawyer is whitewashing a picket fence. The bases of the fenceposts are arranged in the  $xy$ -plane as the quarter circle  $x^2 + y^2 = 25, x, y \geq 0$ , and the height of the fencepost at point  $(x, y)$  is given by  $h(x, y) = 10 - x - y$  (units are feet). Use a scalar line integral to find the area of one side of the fence.



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**Colley 6.1 #40** You are traveling through Cleveland, famous for its lake-effect snow in winter that makes driving quite treacherous. Suppose that you are currently located 20 miles due east of Cleveland and are attempting to drive to a point 20 miles due west of Cleveland. Further suppose that if you are  $s$  miles from the center of Cleveland, where the weather is the worst, you can drive at a rate of at most  $v(s) = 2s + 20$  miles per hour.

- (a) How long will the trip take if you drive on a straight-line path directly through Cleveland? (Assume that you always drive at the maximum speed possible.)
- (b) How long will the trip take if you avoid the middle of the city by driving along a semicircular path with Cleveland at the center? (Again, assume that you drive at the maximum speed possible.)
- (c) Repeat parts (a) and (b), this time using  $v(s) = (s^2/16) + 25$  miles per hour as the maximum speed that you can drive.

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**Colley 6.2 #4** Verify Green's Theorem for the given vector field

$$\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$$

and region  $D$  by calculating both

$$\oint_{\partial D} M dx + N dy \quad \text{and} \quad \iint_D (N_x - M_y) dA.$$

where

$$\mathbf{F} = 2y\mathbf{i} + x\mathbf{j},$$

and  $D$  is the semicircular region  $x^2 + y^2 \leq a^2, y \geq 0$ .

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