Box #_____
Math 65 Section 1
Homework 4
18 May 2018

Collaborators:

Poole 6.6 #2 Find the matrix $[T]_{\mathcal{C}\leftarrow\mathcal{B}}$ of the linear transformation $T : V \to W$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$ of V and W, respectively. Verify Theorem 2.26 for the vector $\mathbf{v}$ by computing $T(\vec{v})$ directly and using the theorem.

$T : \mathcal{P}_1 \to \mathcal{P}_1$ defined by $T(a + bx) = b - ax$, $\mathcal{B} = \{1 + x, 1 - x\}$, $\mathcal{C} = \{1, x\}$, $\mathbf{v} = p(x) = 4 + 2x$.

$\blacksquare$

Poole 6.6 #4 Find the matrix $[T]_{\mathcal{C} \leftarrow \mathcal{B}}$ of the linear transformation $T : V \to W$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$ of V and W, respectively. Verify Theorem 2.26 for the vector $\mathbf{v}$ by computing $T(\vec{v})$ directly and using the theorem.

$T : \mathcal{P}_2 \to \mathcal{P}_2$ defined by $T(p(x)) = p(x+2)$, $\mathcal{B} = \{1, x+2, (x+2)^2\}$, $\mathcal{C} = \{1, x, x^2\}$, $\mathbf{v} = p(x) = a + bx + cx^2$.

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Poole 6.6 #12 Find the matrix $[T]_{\mathcal{C}\leftarrow\mathcal{B}}$ of the linear transformation $T : V \to W$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$ of V and W, respectively. Verify Theorem 2.26 for the vector $\mathbf{v}$ by computing $T(\vec{v})$ directly and using the theorem.

$T : M_{22} \to M_{22}$ defined by $T(A) = A - A^T$, $\mathcal{B} = \mathcal{C} = \{E_{11}, E_{12}, E_{21}, E_{22}\}$, $\mathbf{v} = A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

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Poole 6.6 #14 Consider the subspace W of $\mathcal{D}$, given by $W = \text{span}\left(e^{2x}, e^{-2x}\right)$.

(a) Show that the differential operator D maps W into itself.

(b) Find the matrix of D with respect to $\mathcal{B} = \left\{e^{2x}, e^{-2x}\right\}$.

(c) Compute the derivative of $f(x) = e^{2x} - 3e^{-2x}$ indirectly, using Theorem 6.26, and verify that it agrees with $f'(x)$ as computed directly.

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Poole 6.6 #18 $T : U \to V$ and $S : V \to W$ are linear transformation and $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are bases for U, V, and W, respectively. Compute $[S \circ T]_{\mathcal{D} \leftarrow \mathcal{B}}$ in two ways:

(a) By finding $S \circ T$ directly and then computing its matrix

(b) by finding the matrices of S and T separately and using Theorem 6.27.

$T : \mathcal{P}_1 \to \mathcal{P}_2$ defined by $T(p(x)) = p(x+1)$, $S : \mathcal{P}_2 \to \mathcal{P}_2$ defined by $S(p(x)) = p(x+1)$, $\mathcal{B} = \{1, x\}$, $\mathcal{C} = \mathcal{D} = \left\{1, x, x^2\right\}$.

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Poole 6.6 #22 Determine whether the linear transformation T is invertible by considering its matrix with respect to the standard bases. If T is invertible, use Theorem 6.28 and the method of Example 6.82 to find T^{-1}.

$$T : \mathcal{P}_2 \to \mathcal{P}_2 \text{ defined by } T(p(x)) = p'(x).$$

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Poole 6.6 #32 a linear transformation $T : V \to V$ is given. If possible, find a basis $\mathcal{C}$ for V such that the matrix $[T]_{\mathcal{C}}$ of T with respect to C is diagonal.

$$T : \mathbb{R}^2 \to \mathbb{R}^2 \text{ defined by } T \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a - b \\ a + b \end{bmatrix}.$$

■

Poole 6.6 #34 a linear transformation $T : V \to V$ is given. If possible, find a basis $\mathcal{C}$ for V such that the matrix $[T]_{\mathcal{C}}$ of T with respect to C is diagonal.

$$T : \mathcal{P}_2 \to \mathcal{P}_2 \text{ defined by } T \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a - b \\ a + b \end{bmatrix}.$$

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