Box #_____
Math 65 Section 1
Homework 2
16 May 2018

Collaborators:

Poole 6.2 #30 Let $\mathcal{B}$ be a set of vectors in a vector space V with the property that every vector in V can be written uniquely as a linear combination of the vectors in $\mathcal{B}$. Prove that $\mathcal{B}$ is a basis for V.

$\blacksquare$

Poole 6.2 #36 Find the dimension of the vector space V and give a basis for V.

$$V = \{p(x) \text{ in } \mathcal{P}_2 : xp'(x) = p(x)\}$$

■

Poole 6.2 #40 Find a formula for the dimension of the vector space of symmetric $n \times n$ matrices.

■

Poole 6.2 #42 Let U and W be subspaces of a finite-dimensional vector space V. Prove **Grassmann's Identity**:

$$\dim(U+W) = \dim U + \dim W - \dim(U \cap W).$$

[*Hint:* The subspace of $U+W$ is defined in Exercise 48 of Section 6.1. Let $\mathcal{B} = \{\mathbf{v}_1, ..., \mathbf{v}_k\}$ be a basis for $U \cap W$. Extend $\mathcal{B}$ to a basis $\mathcal{C}$ of U and a basis $\mathcal{D}$ of W. Prove that $\mathcal{C} \cup \mathcal{D}$ is a basis for $U+W$.]

$\blacksquare$

Poole 6.2 #44 Prove that the vector space $\mathcal{P}$ is infinite-dimensional. [*Hint*: Suppose it has a finite basis. Show that there is some polynomial that is not a linear combination of this basis.]

■

Poole 6.2 #58 Let $\{\mathbf{v}_1, ..., \mathbf{v}_n\}$ be a basis for a vector space V. Prove that

$$\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, ..., \mathbf{v}_1 + ... + \mathbf{v}_n\}$$

is also a basis for V.

■

Poole 6.3 #6

(a) Find the coordinate polynomials $p(x)_{\mathcal{B}}$ and $p(x)_{\mathcal{C}}$ of $p(x)$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$, respectively.

(b) Find the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.

(c) Use your answer in part (b) to compute $p(x)_{\mathcal{C}}$ and compare your answer with the one found in part (a).

(d) Find the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{C}$ to $\mathcal{B}$.

(e) Use your answers to part (c) and (d) to compute $p(x)_{\mathcal{B}}$, and compare your answer with the one found in part (a).

$$p(x) = 1 + 3x, \quad \mathcal{B} = \{1 + x, 1 - x\}, \quad \mathcal{C} = \{2x, 4\} \text{ in } \mathcal{P}_1.$$

■

Poole 6.3 #8

(a) Find the coordinate polynomials $p(x)_{\mathcal{B}}$ and $p(x)_{\mathcal{C}}$ of $p(x)$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$, respectively.

(b) Find the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.

(c) Use your answer in part (b) to compute $p(x)_{\mathcal{C}}$ and compare your answer with the one found in part (a).

(d) Find the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{C}$ to $\mathcal{B}$.

(e) Use your answers to part (c) and (d) to compute $p(x)_{\mathcal{B}}$, and compare your answer with the one found in part (a).

$$p(x) = 4 - 2x - x^2, \quad \mathcal{B} = \left\{x, 1 + x^2, x + x^2\right\}, \quad \mathcal{C} = \left\{1, 1 + x, x^2\right\} \text{ in } \mathcal{P}_2.$$

■