Box #______
Math 65 Section 1
Homework 5
21 May 2018

Collaborators:

Poole 5.3 #4 The given vectors

$$\mathbf{x}_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}1\\1\\0\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}1\\0\\0\end{bmatrix},$$

form a basis for $\mathbb{R}^3$. Apply the Gram-Schmidt Process to obtain an orthogonal basis. Then normalize this basis to obtain an orthonormal basis.

■

Poole 5.3 #6 The given vectors

$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

form a basis for a subspace W of $\mathbb{R}^4$. Apply the Gram-Schmidt Process to obtain an orthogonal basis for W.

$\blacksquare$

Poole 5.3 #18 The columns of Q were obtained by applying the Gram-Schmidt Process to the columns of A. Find the upper triangular matrix R such that $A = QR$.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ -1 & -1 \\ 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{3} \\ 2/\sqrt{6} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} \end{bmatrix}.$$

■

Poole 7.1 #32 $\langle \mathbf{u}, \mathbf{v} \rangle$ is an inner product. Prove that

$$\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + 2\langle \mathbf{u}, \mathbf{v} \rangle + \|\mathbf{v}\|^2$$

is an identity.

■

Poole 7.1 #34 $\langle \mathbf{u}, \mathbf{v} \rangle$ is an inner product. Prove that

$$\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{4} \|\mathbf{u} + \mathbf{v}\|^2 - \frac{1}{4} \|\mathbf{u} - \mathbf{v}\|^2$$

is an identity.

$\blacksquare$

Poole 7.1 #38 Apply the Gram-Schmidt Process to the basis $\mathcal{B}$ to obtain an orthogonal basis for the inner product space V relative to the given inner product.

$$V = \mathbb{R}^2, \quad \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \quad \langle \mathbf{u}, \mathbf{v} \rangle = 4u_1v_1 - 2u_1v_2 - 2u_2v_1 + 7u_2v_2.$$

$\blacksquare$

Poole 7.1 #40 Apply the Gram-Schmidt Process to the basis $\mathcal{B}$ to obtain an orthogonal basis for the inner product space V relative to the given inner product.

$$V = \mathcal{P}_2[0,1], \quad \mathcal{B} = \left\{1, 1+x, 1+x+x^2\right\}, \quad \langle f, g \rangle = \int_a^b f(x)g(x)\,dx$$

∎

Poole 7.1 #42 If we multiply the Legendre polynomial of degree n by an appropriate scalar, we can obtain a polynomial $L_n(x)$ such that $L_n(1) = 1$.

(a) Find $L_0(x), L_1(x), L_2(x)$, and $L_3(x)$.

(b) It can be shown that $L_n(x)$ satisfies the recurrence relation

$$L_n(x) = \frac{2n-1}{n} x L_{n-1}(x) - \frac{n-1}{n} L_{n-2}(x)$$

for all $n \geq 2$. Verify this recurrence for $L_2(x)$ and $L_3(x)$. Then use it to compute $L_4(x)$ and $L_5(x)$.

■