Lecture Notes 1: Vector Operators

Contents

1 Introduction

A vector operator:

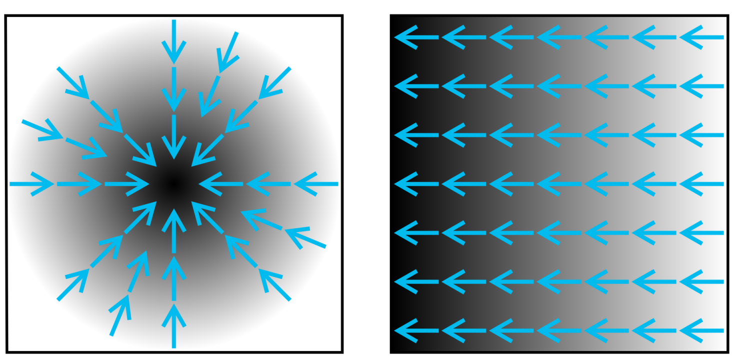
- A differential operator in vector calculus
- It can be used to move from the integral form of the equation to its differential one.
- The most important operators:
  - Gradient
  - Divergence
  - Curl

2 Gradient

It's a vector that represents the direction and the magnitude of the maximum space rate of increase of a scalar function.

$$\nabla V = \frac{\partial v}{\partial x}\underline{u_x} + \frac{\partial v}{\partial y}\underline{u_y} + \frac{\partial v}{\partial z}\underline{u_z}$$

2.1 Visualization



The gradient, represented by the blue arrows, denotes the direction of the greatest change of a scalar function. The values of the function are represented in greyscale and increase in value from white (low) to dark (high).

3 Divergence

The divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

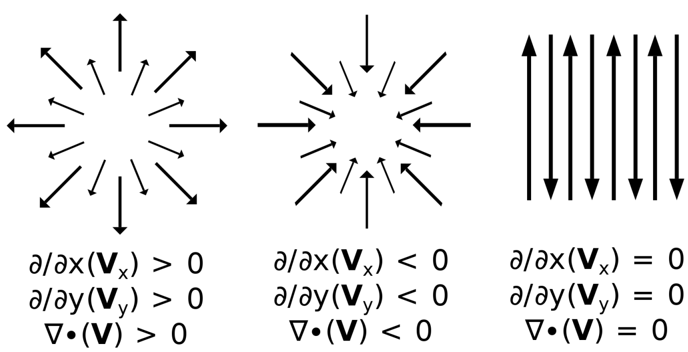
$$\nabla \cdot \underline{A} = \lim_{\Delta v \to 0} \frac{\oint_s \underline{A} \cdot ds}{\Delta v} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

3.1 Physical meaning

Divergence produces a scalar quantity which can be:

- **+ve**: This means that the point acts as a source of flux,(e.g: +ve charge)
- **-ve**: This means that the point acts as a sink of flux, (e.g: -ve charge)
- **zero**: This means that the point is:
  - source free (or number of source = number of sink)
  - $\underline{A}$ is called solenoidal field (e.g: Magnetic field)

3.2 Visualization



4 Curl

Circulation: It is the amount of force(vortex source) that pushes along a closed boundary or path, such as a circle.

Curl: It's simply the circulation per unit Area when the area tends to zero, and its direction is the normal to the area

$$\nabla \times \underline{A} = \lim_{\Delta s \to 0} \frac{\underline{u_n} \oint_c \underline{A} \cdot dl}{\Delta s}$$

$$\nabla \times \underline{A} = \begin{vmatrix} u_x & u_y & u_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

4.1 Physical meaning

The magnitude of the curl vector at a point measures how quickly the surrounding particles rotate around this point, or in other words, the curl at a point is a measure of the vector field 's "spin" at that point. and its direction follows the right hand rule (check visualization below):

- **+ve:** the pushing force (vortex source) is along u_n
- **-ve:** the pushing force (vortex source) is opposite to u_n
- **zero:**
  - no pushing force (vortex source) is crossing Δs
  - $\underline{A}$ is called irrotational field (e.g: Electrostatic field)

4.2 Visualization