Lecture Notes 2: Basic laws of static electricity

Contents

1 Coulomb's law

It's an experimental law of physics that measures the force between two stationary, electrically charged particles.

1.1 Observations

- There's an attraction/repulsion force between electrically charged particles
- This force is $\propto \frac{Qq}{r^2}$
- The force is in the direction $\underline{u_r}$
- It's analogous to newton's law of gravitational force

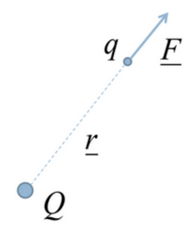


Figure 1: coulomb's law

1.2 Mathematical Modeling

$$\underline{F} = \frac{Qq}{4\pi\epsilon_0 r^2} \quad \underline{u_r}$$

where ϵ_0 is the permittivity in free space $\left(\frac{10^{-9}}{36\pi} F/m\right)$

2 Electrostatic field intensity vector

Any electric charge has an electric field that surrounds it, and it depends on:

- distance from the charge
- the medium
- the intensity of electric charge

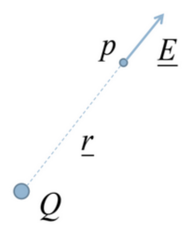


Figure 2: Electrostatic field at p

2.1 Mathematical Modeling

$$\underline{E_p} = \frac{Q}{4\pi\epsilon_0 r^2}\,\underline{u_r} \;= \frac{\underline{F}}{q}$$

We assume that we are calculating the force acting on a charge (q) with magnitude equal to 1 at point p.

2.1.1 Can electric field be equal to infinity?

In the above equation if we placed $r = 0$, then $E = \infty$, But actually "r" can never be 0, as when we get very close to a point charge, it acts as a spherical charge with radius "a" as shown in the figure below 4, and "a" can never be 0

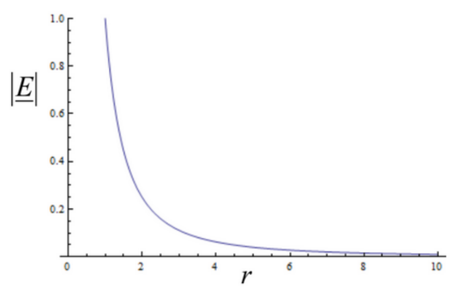


Figure 3: E and r graph

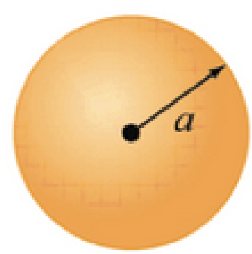


Figure 4: spherical charge

3 Superposition of electrostatic field

Every charge in space creates an electric field at point independent of the presence of other charges in that medium. The resultant electric field is a vector sum of the electric field due to individual charges.

$$\underline{E_p} = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0 r_i^2}\quad \underline{u_i}$$

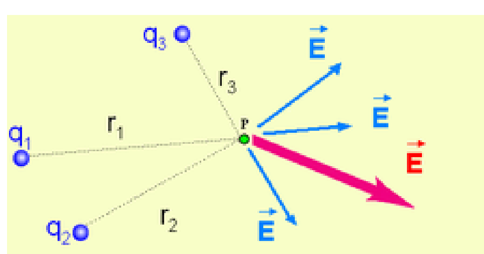


Figure 5: superposition of electrostatic field

4 Electrostatic field of a continuum of charges

Until this moment, we have been talking about point charges, so let's try to generalize what we have achieved, Let's consider a random 3D shape, as shown in the figure below 6.

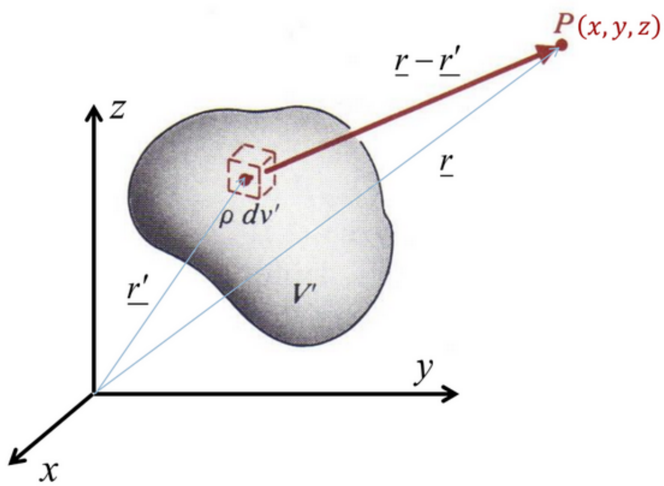


Figure 6: Random 3D shape of charges

We can divide this shape into infinitesimal charge elements (dq) each charge element of them will be defined in terms of the volume it acquires$(dq = \rho\, dv`)$

4.1 Mathematical Modeling

$$E_p = \int_V \frac{dq}{4\pi\epsilon_0\,(\underline{r} - \underline{r}`)^2} \quad u_{\underline{r}-\underline{r}`}$$

$$\because u_{\underline{r}-\underline{r}`} = \frac{\underline{r} - \underline{r}`}{|\underline{r} - \underline{r}`|} \qquad \text{and} \qquad dq = \rho\, dv`$$

$$\boxed{\therefore E_p = \int_V \frac{\rho\, dv`}{4\pi\epsilon_0\,(\underline{r} - \underline{r}`)^3} \quad (\underline{r} - \underline{r}`)}$$

5 Electrostatic field lines

They are visual representation of electric field, and they have the following properties:

- The direction of a field Line must be in the direction of the total field at the point.
- The lines depart from positive charges and terminate on negative charges.
- The lines are open and non-intersecting.
- The surface density of the lines crossing normally an element of surface area at a point in space gives the magnitude of the field at this point.

5.1 visualization

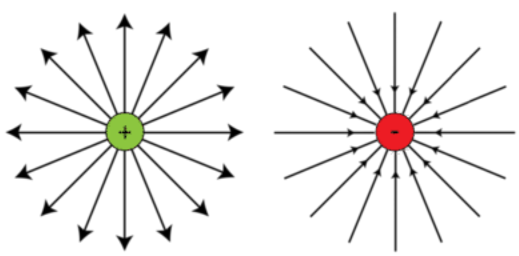


Figure 7: point charges field lines

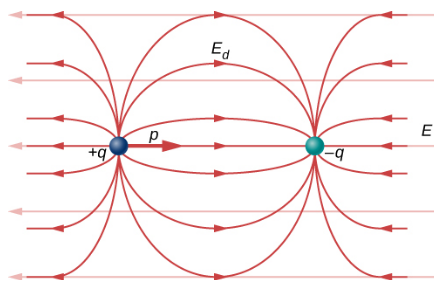


Figure 8: dipole field lines

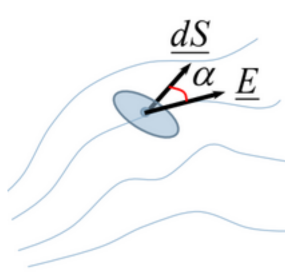
6 Electrostatic flux

Electrostatic flux(ψ): The number of field lines crossing normally an area of space in the field of electrostatic system of charges

6.1 Mathematical modeling

$$\begin{aligned}
|\underline{E}| &= \frac{d\psi}{ds_\perp} \\
d\psi &= |\underline{E}| ds_\perp \\
&= |\underline{E}||\underline{ds}|cos(\alpha) \\
&= \underline{E} \cdot \underline{ds}
\end{aligned}$$

$$\boxed{\psi = \int_s \underline{E}.\underline{ds}}$$



7 Electrostatic flux leaving a point charge

How can you determine the electrostatic flux lines leaving a point charge? The idea is simple. If you put this point charge inside any closed surface -as shown in the figure below 9- then you can use the mathematical equation we derived above to calculate the flux lines.

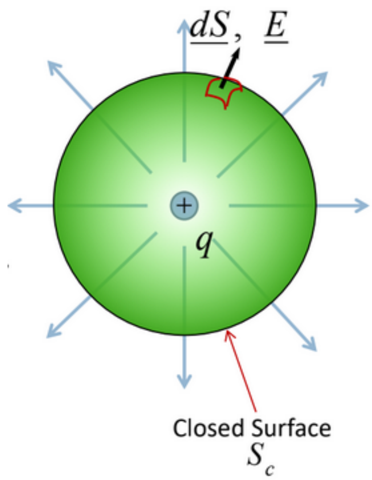


Figure 9: Closed surface

7.1 Why spherical coordinates

As we said, you can put the point charge inside any closed surface, so why did we choose a sphere? It's because a point charge is like a tiny sphere, so the math will be a lot easier -as we are going to see- if we used spherical coordinates. Choosing cylindrical coordinates, or cartesian ones should still give the correct answer, however, the integration will just be difficult.

7.2 Mathematical modeling

$$\psi = \oint_{s_c} \underline{E} \cdot \underline{ds}$$

$$\psi = \int_0^{\pi} \int_0^{2\pi} \frac{q}{4\pi\epsilon_0 r^2}\, \underline{ur} \cdot r^2 sin(\theta)\, d\theta d\phi\, \underline{u_r}$$

$$\psi = \frac{q}{4\pi\epsilon_0}(2)(2\pi)$$

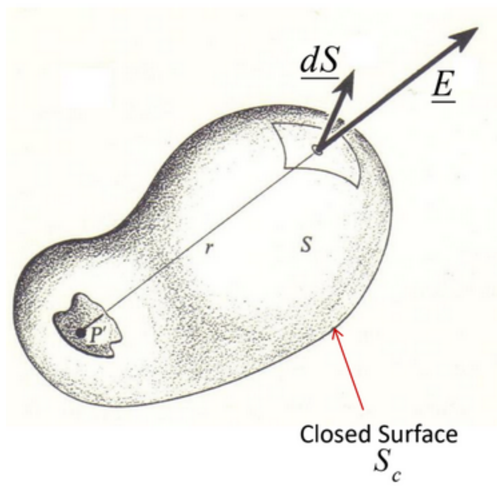
$$\boxed{\psi = \frac{q}{\epsilon}}$$

8 Gauss's law for electrostatics field

The total outward flux of the E-field over any closed surface in free space is equal to the total charge enclosed by the surface divided by ϵ_0

8.1 Mathematical modeling

$$\boxed{\psi = \oint_s \underline{E}.\underline{ds} = \frac{1}{\epsilon_0} \sum_{\text{inside } s_c} Q}$$



8.2 If the Charges are outside the surface

If there were no charges inside the closed surface, but there were some charges outside, then if we applied the above equation ($\psi = \frac{1}{\epsilon_0} \sum_{\text{inside } s_c} Q$) we get that ($\psi = 0$), so you may say: "How is that possible? Isn't there an electric field due to the outer charges?" so you should notice that:

- The flux lines due to outer charges will enter the surface ,but it will also leave it, so the total outward flux on the closed surface is 0
- As you may have guessed, even if the total outward flux is zero, if we consider only one point on the surface, It won't necessarily be zero

9 Gauss's law for distributed charges

9.1 Volumetric charge distribution

$$\psi = \oint_s \underline{E}.\underline{ds} = \frac{1}{\epsilon_0} \int_{V'} \rho_v \, dv'$$

9.2 surface charge distribution

$$\psi = \oint_s \underline{E}.\underline{ds} = \frac{1}{\epsilon_0} \int_{S'} \rho_v \, ds'$$

9.3 Linear charge distribution

$$\psi = \oint_s \underline{E}.\underline{ds} = \frac{1}{\epsilon_0} \int_{L'} \rho_v \, dl'$$