Drowning in the sea of Non-identity: sides and diagonals

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By "the incommensurability of side and diagonal," Beckett refers to the fact that the ratio of a square's diagonal to its side, 2, is not a fraction, as mentioned in our discussion of the Meno. The'secret' or'scandal' of the irrationals was popularised in the twentieth-century history of mathematics. This is where his pub classicists most likely first encountered the story. In a more respectable form, this story is still widely believed. Pythagoras founded an initiatory sect whose secret knowledge was at least partially mathematical. He is said to have taught that "all is number," where "number" refers to whole numbers (1, 2, 3,...), and he or his sect also attributed religious/magical significance to the regular solids (see Fig. 1).

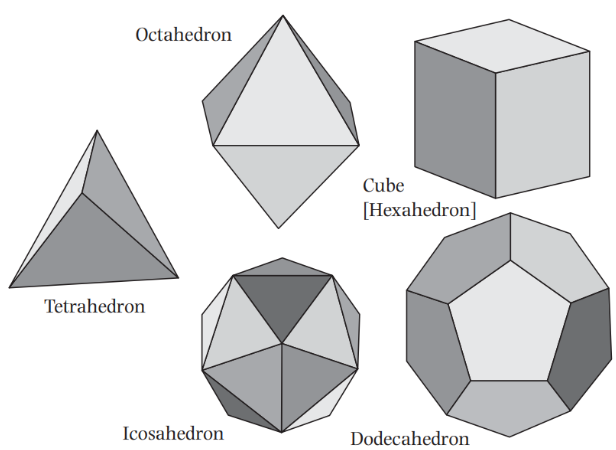

Figure 1: The five regular ('Platonic') solids.

However, it is impossible to construct regular solids, at least those involving pentagons, without introducing irrational ratios—see Appendix B. This, along with the problem of the side and the diagonal, suggests a difficulty or, to put it more strongly, a "scandal" for Pythagoras' alleged programme; for if "all is number," then the ratio of the side to the diagonal should be the ratio of two numbers.

The Pythagoreans allegedly kept this a secret (though this is not explicitly stated) because it posed a problem; however, the secrecy was eventually broken. Iamblichus, a commentator of the late third century ce (seven centuries after the events), describes

Hippasos of Metapontum as a Pythagorean who was expelled and drowned at sea (or suffered a similar fate) for revealing a secret—in one version, the structure of one of the solids, and in another, the nature of the rational and irrational. As Beckett's characters did, the extent to which you accept this story depends on your estimation of Iamblichus as a source, and he does not go out of his way to inspire confidence. The next step for historians of the twentieth century was to deduce that the secret or scandalous nature of the irrationals for the Pythagoreans extended to the Greek mathematical community as a whole, and that this explains why they avoided measurement.

In the 1920s, Hasse and Scholz produced the definitive version of this story. They referenced a 'crisis of foundations' in Greek mathematics to explain Euclid's use of proportions: Given that the Greeks, as is commonly believed, were born geometers, it must be concluded with absolute certainty that, following such a foundational crisis, they needed to develop a purely geometric mathematics. Using such mathematics,

inevitably encounter a theory of proportions in which no arithmetical components remain.

It is not a coincidence that they were writing during the time of Russell's paradox, when (modern) pure mathematics was undergoing precisely this crisis. Extrapolating backwards, they interpret ancient Greek anxiety as a version of turn-of-the-century anxiety about mathematical certainty.

Although the Hasse-Scholz thesis was criticised in the years following its publication (most notably by Freudenthal in 1966) and never became the only accepted view, it has survived well, in part because it adequately explains the aforementioned issues and because it is simple to adapt and revise. The most recent sustained assault was launched by David Fowler (1999). Plato and Aristotle, who are the closest to contemporary sources, refer to the irrational without implying that it was a problem; the subject is not even mentioned by Proclus, who is supposed to be summarising an earlier history; the time of the alleged 'crisis' coincides with the time when many major mathematical discoveries were made, apparently without difficulty; and Iamblichus is a well-known figure. (In passing, this fundamental disagreement emphasises the difficulty of drawing conclusions about the time period.) In one of numerous allusions to the issue, Fowler contrasts Aristotle's logical assessment:A geometer, for instance, would wonder at nothing so much as that the diagonal should prove to be commensurable

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