Zeno's Paradoxes: innovation of Ancient Greeks

Omar Mohamed Hussien Kotb

June 11, 2022

Ancient Greeks bequeathed us a tremendously rich intellectual legacy. Thales was the originator of both the science of geometry and the entire trajectory of Western philosophy. Both benefited from the phenomenal development of in antiquity, his early successors achieved a surprising degree of perfection. Aristotle provided the first systematic development of formal logic during the same time period. But the fertile soil from which all of this grew also spawned a series of enigmas that have challenged successive generations of philosophers and scientists right up until the present day. These are the celebrated paradoxes of Zeno of Elea, who lived around 500 B.C.

Zeno was a devoted disciple of the philosopher Parmenides, who believed that reality consisted of a single, undifferentiated, immutable, and partless whole. According to him, motion, change, and plurality were merely illusions. Few philosophers were able to accept this viewpoint, and Parmenides was apparently ridiculed by those who disagreed. According to reports, Zeno's primary objective was to refute those who made fun of his master. His goal was to demonstrate that those who believed in motion, change, and plurality were even more absurd. Less than ten of the approximately forty such puzzles that he posed have been passed down to us, but they involve some extremely subtle difficulties. These paradoxes strike at the heart of our concepts of space and time, as motion involves the occupation of different locations at different times.

Bertrand Russell once remarked, "Zeno's arguments, in some form, have provided the basis for virtually all theories of space, time, and infinity constructed from his time to the present day." This assertion was made in 1914, in an essay containing a penetrating analysis of the paradoxes. However, as we shall see, Russell was oblivious to problems inherent in these puzzles. In fact, these difficulties have a direct bearing on our previous discussions of space and geometry, revealing profound issues that we have barely touched upon.

Imagine that Achilles, the fastest of the Greek warriors, is competing against a tortoise in a footrace. It is just to give the tortoise the advantage. Zeno argues that Achilles can never catch the tortoise under these conditions, regardless of how fast he runs. Achilles must run from his starting point A to the tortoise's original starting point T0 in order to pass it (see Figure 1). During this time, the tortoise will have advanced to T_1. Now Achilles must travel to location T_1. While Achilles travels this additional distance, the tortoise advances even further to T_2.

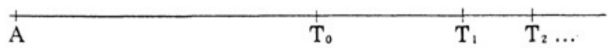

Figure 1:

Again, Achilles must reach this new position of the tortoise. And so it continues; whenever Achilles arrives at a point where the tortoise was, the tortoise has already moved a bit ahead. Achilles can narrow the gap, but he can never actually catch up with him. This is the most famous of all of Zeno's paradoxes. It is sometimes known simply as "The Achilles.

This paradox comes in two forms, progressive and regressive. According to the first, Achilles cannot get to the end of any racecourse, tortoise or no tortoise; indeed, he cannot even reach the original starting point T0 of the tortoise in the previous paradox. Zeno argues as follows. Before the runner can cover the whole distance he must cover the first half of it (see Figure 2).

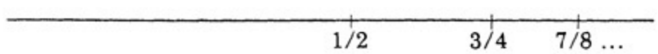

Figure 2:

Then he must travel the remaining distance in increments of one-half, etc. In other words, he must first run one-half, then another one-fourth, then another one-eighth, etc., always falling short of his objective. Zeno concludes, therefore, that he can never attain it. This is the progressive form of the paradox, and it has almost the same force as Achilles and the Tortoise. The only difference is that in the Dichotomy, the goal is stationary, whereas in Achilles and the Tortoise, it moves, but at a much slower speed than Achilles. The regressive form of the Dichotomy attempts to demonstrate, to make matters worse, that the runner cannot even begin. Before he can finish the entire course, he must run half of it (see Figure 3). Before he can complete the first half, however, he must first run the first quarter.

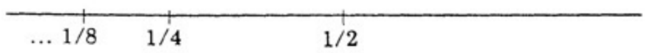

Figure 3:

He must run the first eighth before he can finish the first quarter. And so forth. Zeno concludes that in order to cover any distance, no matter how short, the runner must have already completed an infinite number of runs. Due to the fact that the sequence of runs he must have already completed has the form of a regression,... $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$, which has no first member, the runner cannot even begin.

References:

1. E. Drumwright, "Avoiding Zeno's paradox in impulse-based rigid body simulation," 2010 IEEE International Conference on Robotics and Automation, 2010, pp. 5127-5132, doi: 10.1109/ROBOT.2010.5509433.

2. L. John, J. Boardman and B. Sauser, "Technology and policy: Opposite ends of the paradox spectrum?," 2008 IEEE International Conference on System of Systems Engineering, 2008, pp. 1-6, doi: 10.1109/SYSOSE.2008.4724162.

3. D. Jacquette, "The Soundness Paradox," in Logic Journal of the IGPL, vol. 11, no. 5, pp. 547-556, Sept. 2003, doi: 10.1093/jigpal/11.5.547.