The Principia and its problems: Newton's picture

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After some time together, the Dr asked him what he thought the Curve would be that the Planets would describe if the force of attraction towards the Sun was reciprocal to the square of their distance from it. Sr Isaac immediately replied that it would be an ellipsis, and the Doctor, filled with joy and amazement, asked him how.

He knew it, and he says, "I calculated it." Dr Halley then asked him for his calculations without further delay, and Sr Isaac looked through his papers but couldn't find it, but he promised to renew it and then send it to him.

The above breathless quote (all those commas) is by Newton's friend Halley. account of how Halley's question about the paths of planets threw Newton into a three-year frenzy mathematical work published in Principia (1687). If the three massive volumes had They were certainly more impressive, and backed up, despite being as strange and novel as Leibniz's work. backed up by a formidable proof apparatus As stated in the quote above, the work deduces (using As a starting point, Newton's three laws) that the observed paths of the planets are compatible with an inverse-square law of attraction; and, even more speculatively, that such a law is the only one. which can explain the observations. Whatever he may have said about his calculus being used to deduce his results, as stated in the chapter's opening quote, the book presents itself as new physics demonstrated by means of old (i.e., Greek) mathematics. This is due to a variety of factors. As Hall has pointed out, if the work had used both a new physical and a new mathematical language, it would have been doubly unfamiliar to its readers:

To the few skilled mathematicians in Europe around 1685 who were capable of understanding Newton's mathematical arguments at all, however expressed, the form he actually adopted in Principia was far more convenient and familiar than either the method of fluxions—known to no one but Newton—or the Leibnizian differential calculus.

And there was already enough evidence that they could and did reject physics, most notably the concept of gravity. of a gravitational force acting at a distance through a vacuum, which appeared to be pure mystery in contrast to Descartes' theories The

Principia is intended to follow Archimedes' model. Although it is much longer, physical texts—the Statics and On Floating Bodies—are included. principles, as well as rigorous deductions from them Furthermore, as previously stated, Newton In the 1660s, he was an enthusiastic'modernist' and Descartes follower, but he had changed his mind. radically, for reasons that appear to be primarily concerned with his interests outside of physics and mathematics. mathematics. He now used every opportunity (regardless of his private practise) to criticise modernity. He scribbled 'Not Geometry' in the algebraic school of geometry to which he had once belonged. margin of his Descartes copy During the 1670s, he was deeply immersed in his studies on alchemy, as well as the meaning and chronology of the Old Testament, on which he held strong views. unorthodox. Similarly to Stevin 100 years before, he had come to believe in a golden age of 'firsts.' knowledge', which the Greeks had tainted; for example, the rotation of the earth round the sun. The Egyptians and Pythagoras both knew about the sun.

He had plans to expand the Principia. which would have explained this ancient knowledge (he had worked hard to reconstruct lost knowledge). If the Greek texts had been published, the work would have been regarded as clearly eccentric, and would have probably elicited far less admiration For instance, he wrote an explanation of the 'occult' gravitational force that would have persuaded none of the sceptics: Thus far I have explained the properties of gravity. But by no means do I consider its cause. However I will say in what sense the Ancients theorized about it. Thales held that all bodies were animate, inferring this from magnetic and electrical attractions ... He taught that everything was full of Gods, and by Gods he meant animate bodies.

We need to look at one example to see how the Principia might have appeared—and how it appears to us now that we can read it. As Appendix C, I have reproduced the crucial deduction of Kepler's area law, which is frequently cited as an example. The statement is Newton's interpretation of the area law; as he discovered, the law (equal areas are swept out in equal times) follows simply from the assumption that the body—always assumed to be a 'particle' concentrated at a point—moves under a force directed to the immovable centre S from which the areas are calculated. The proof is divided into two parts.

The first part is correct Greek geometry, albeit physically incorrect. We believe of the body being moved in a series of jerks at equal time intervals ('by a great impulse'), Instead of moving smoothly, the impulses are directed to the centre S. We discover (as in Fig. 1) that it moves in a polygon and that each triangle has an equal area We now assume the The number of triangles increased 'infinitum,' which is no longer Greek geometry at all, but the now-famous 'infinitum' triangle. The well-known argument that an infinite-sided polygon is a curve Newton claims that the force is now acting. Areas remain proportional to time indefinitely.

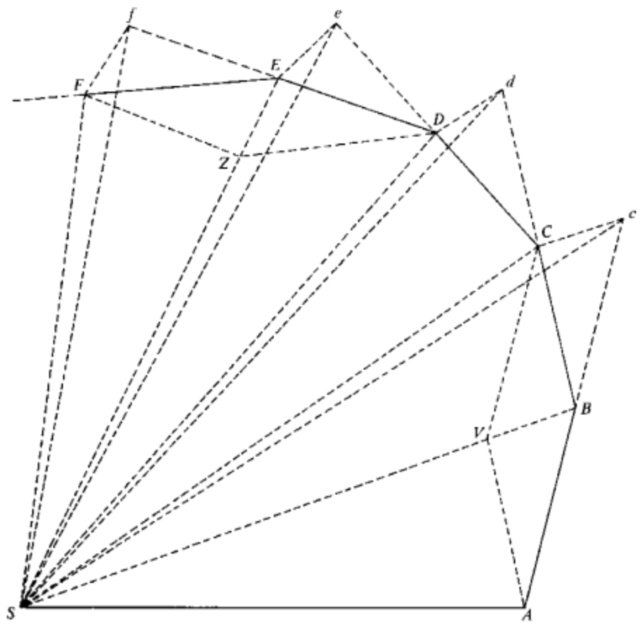

Figure 1: Newton's picture for Principia I, proposition 1.

This argument does not employ Newton's calculus; rather, it employs a more cautious version of the calculus. the infinitesimal arguments used by many of his predecessors (Pascal, Huygens, etc.) Wallis, amongst others). This, however, does not make matters any better. True, I skipped over the introductory material, particularly Cor. IV, Lem. III, which states that a polygon's limit is a curve; and that it contains the theory that justifies the passage to a point. Such infinitesimal geometry results in The Principia's arguments have a superficial robustness. What does Leibniz's (and Newton's) calculus have in common? The Principia lacks the security and ease of calculation provided by algebra. In this regard, Newton's decision to abandon Descartes' algebraic methods made things more difficult for him and his readers. 9 And the choice of exposition method obscured the extent to which the two obscure new works, Leibniz's calculus and Newton's physics, were related.

References:

1. Li Zenghua and Wang Jingye, "Weapon-Target Assignment research based on Genetic Algorithm mixed with damage simulation," 2010 International Conference on Computer Application and System Modeling (ICCASM 2010), 2010, pp. V15-460-V15-463, doi: 10.1109/ICCASM.2010.5622555.

2. A. Wendt et al., "Work in progress — engineering case studies inspired by society's Grand Challenges for Engineering: An integrated approach to middle school stem instruction," 2012 Frontiers in Education Conference Proceedings, 2012, pp. 1-2, doi: 10.1109/FIE.2012.6462261.

3. Y. Jilai and L. Zhuo, ”Ponderings and Strategies of Power Loss Allocation - Application of United Electrical Dissection of ac Branch and Bus,” 2006 International Conference on Power System Technology, 2006, pp. 1-7, doi: 10.1109/ICPST.2006.321453.