The Fibonacci Sequence: uncovering Art of Mathematics and the Mathematics of Art

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September 13, 2022

There has been a complex relationship between art and mathematics since the dawn of time. "The universe is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures," wrote Galilei[1]. The artist who wishes to study nature must first fully understand mathematics. Mathematics can be found in many arts today, including music, dance, painting, architecture, and sculpture. Unsurprisingly, many mathematicians have expressed themselves not only through theorems but also through the creation of art. We mention Helaman Ferguson, a sculptor who developed the PSLQ algorithm, an integer relation detection algorithm used to find a relation between whole numbers. Along with many other great scientists who dedicated their lives to discovering connections between math and nature, Fibonacci stood out.

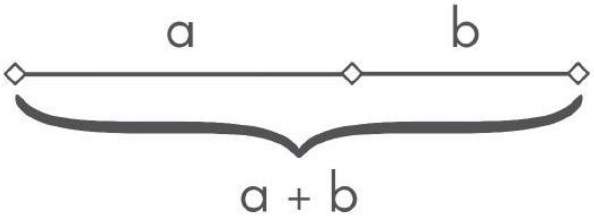
The Fibonacci Numbers are defined by the recursive relation defined by the equations $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$ where $f_1 = 1; f_2 = 1$. Although it is based on a rigorous recursion, it is found is found in nature, as in the sunflower's seeds, as shown below.

Michael Mästlin[2] after well studying of Fibonacci's theorem in nature he come to discover the golden ratio. First he came with a segment and divided it into two parts so that the ratio between the segment with the bigger part is equal to the ration of the bigger part to the smaller one. Then he called the ration of proportion The golden ration. The golden ration is also equal to the limit of ratio between two following terms in Fibonacci sequence as the term's number approaches infinity. To sum up, noticed that the golden ratio is equal to the follow.

The golden ratio was later found to be applied every day of your life. Even to see your face as a well-proportioned face must lie within the range of the golden ratio. The golden ratio is applied in every detail of life, starting from shells of snails to spiral galaxies in space.

Mathematicians didn't settle here, they started working on to find a relation between the Fibonacci theories and the outer space. Until few decades ago, the distances of moons from their parent planet have not been known to follow any sequence. So, assuming the that the mean distance of moons of planet x to them to the planet x itself is Z_n, the scientist starts to model to find a linear relation in the form $z_n = mf_n + b$. The greater

$$\phi = \lim_{x\to\infty} \frac{f_{n+1}}{f_n} = \frac{a+b}{a} = \frac{a}{b} = \frac{1+\sqrt{5}}{2} \approx 1.618...$$



number of moons of planet the more accurate the value of real distance of moon from planet to that plotted on the graph, so you will find that the Least accurate planets are Mars and Neptune as they have two Moons only.

Figure 1: Golden ration in Snail and Milky way galaxy

Through out the history, scientist have tried to understand the world around them. Math was one of the tools that they used to illustrate the phenomena in Natural around them as in flowers that we see in our daily life. even they exceeded the limit by looking into space and find relation between the planets and its moons. Finally, they realize that whatever advances in science made, our understanding of nature will be still far from complete. But one thing all scientists have agreed on is that without mathematics there is no art.

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[1] Galileo di Vincenzo Bonaiuti de' Galilei (15 February 1564 - 8 January 1642) was an Italian astronomer, physicist and engineer, sometimes described as a polymath. He was born in the city of Pisa, then part of the Duchy of Florence. Galileo has been called the "father" of observational astronomy, modern physics, the scientific method, and modern science.

[2] Michael Maestlin (30 September 1550 - 26 October 1631) was a German astronomer and mathematician, known for being the mentor of Johannes Kepler. He was a student of Philipp Apian and was known as the teacher who most influenced Kepler. Maestlin was considered to be one of the most significant astronomers between the time of Copernicus and Kepler.

Primary	No. of Moons	(10^3 km) m	b
Mars	2	14	-4.8
Jupiter	12	240	-50
Saturn	9	50	140
Uranus	5	70	60
Neptune	2	420	-66

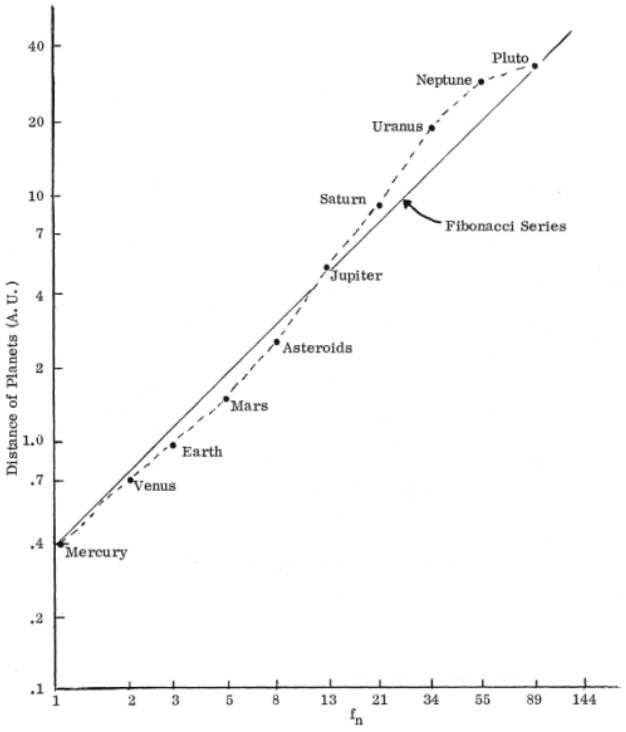

Figure 2: Relating the distance and moons number of a specific planet to Fibonacci sequence through linear equation

References

1. Jollanda Shara. 2014. "MATHEMATICS AND ART." Conference: 8th International Week Dedicated to Maths 2016, March 2016,Thessaloniki, Greece, ISBN 978-960-89672-6-7 https://www.researchgate.net/publication/358210455$_M ATHEMATICS_A ND_A RT$.

2. Clement E.March 2005" Falbo.The Golden Ratio—A Contrary Viewpoint"The College Mathematics Journal 36(2) DOI:10.2307/30044835 https://www.researchgate.net/publication/247892441$_T he_G olden_R atio - A_C ontrary_V iewpoint$

3. B. A. READ Captain, Canadian Armed Forces U. S. Army Satellite Communications Agency, Fort Monmouth, New Jersey "FIBONACCI SERIES IN THE SOLAR SYSTEM"

   https://www.fq.math.ca/Scanned/8-4/read-a.pdf