Space and infinity: Seperation that persisted nearly 1600 year

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It has previously been shown that the axioms of Euclid's geometry do not properly apply to what the ancient Greeks considered to be'space,' and this separation between the two fields of study seems to have persisted until approximately 1600. Marvell seems to have thought that lines may actually be limitless in the preceding quotation (around 1650), which is a poetic articulation of postulate 5. Something had altered the manner in which space was conceived, posing new challenges. Not only must geometry be "self-evident" in terms of some notion of common sense, but it must also explain the world.

Obviously, such speculations coexisted with more extreme ones, such as whether God could create a triangle whose angles did not add up to two right angles; however, in terms of a unification of geometric space with the actual universe, the concept gained traction through more radical early modern thinkers such as Giordano Bruno (16th century), Descartes, and finally Newton. Was Descartes' yearning for endless space ('the extent of the globe is limitless') connected to his revolution in geometry? It does not seem to be the case, given that Descartes's plane is still an abstract replica of Euclid's with numbers added. [And if you review the Geometry chapter 6 excerpts, you will notice that he uses parallelism to present the numbers.] Rather, it is a consequence of his physical law, equivalent to Newton's First Law, that a freely moving body will always continue to move in a straight line, thus perpetually performing the construction required by Euclid's second postulate, which would be impossible if every distance in the world were less than or equal to a given magnitude.

In fact, Newton went farther than Descartes by developing a comprehensive plan for the behavior of all matter in the cosmos. Here, the world was openly equated with the space of Euclidean geometry, whereby straight lines extend indefinitely. (Infinite, if you want a looser translation.) Again, it was impossible for the physical laws to function without this identification, but it is crucial to note that this was a comparatively recent development. If Newton had in some way inherited the concept from Descartes, he definitely made it far more obvious throughout the Principia's geometric and logical framework. With Descartes and Newton, the application of geometry to physical space is greatly advanced. Regarding the application of postulate 5, it implies that all inclined lines (in the same plane) will intersect someplace in the universe. This has the disadvantage that, at least for certain geometers, questions about geometry may become equivalent to queries about the world. 4 Obviously, in a sense, this has always been the case; nonetheless, the notion that the study of geometry was drawn from knowledge of the world, whether inherent (part of our mental structure) or empirical (derived from observation), grew prominent throughout the course of the subsequent centuries. The contrast with Plato's opinion that geometrical concepts were superior to practice is stark.

The most significant innovations in geometry after Newton's time, and this is significant when considering the relative significance of the 'new' geometries, concerns the increased use of coordinates and calculus as tools. To study curves and surfaces required a grasp of their equations, even if pictures were utilized to facilitate comprehension. In the late eighteenth century, the French mathematician Gaspard Monge invented 'descriptive geometry,' which became a central topic at the very famous École Polytechnique. The study of three-dimensional figures via their projections—plans, elevations, etc.; the dissection of a figure into its projections, and its reconstruction from them (Fig. 1); and its more advanced parts relied heavily on calculus. Descriptive geometry was popular during the nineteenth century and remains an essential part of practical training, despite being unknown in the majority of mathematics departments. It was not preoccupied with the issue of the world, in part due to the centrality of the use of coordinates and the significance of practical application. Michel Chasles, a student of Monge, praised him in 1837 for eschewing the diagrams that were required for thinking, for instance, about parallelism.

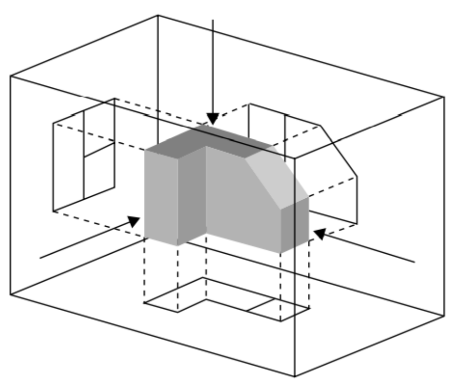

Figure 1: Traditional perspective generates the 'ideal line' at infinity XY of projective geometry. The lines CA, DB are parallel and meet at infinity at X in the 'extended plane'; similarly AB, CD meet at Y.

In fact, the study of projective geometry, which is still taught as a highly abstract topic, began as an extension of descriptive geometry5 and was seen as the study of space enhanced by the ideal points and lines 'at infinity' that are found in perspective drawings (Fig. 2). In spite of postulate 5, parallel lines might cross as long as their point of intersection was inside the imaginary exclusion zone known as the 'line at infinity' The Euclidean structure of space was not questioned, however bizarre that may appear. Bolzano's 1817 attack on the use of geometry to prove results in analysis was perhaps the strongest expression of the prevalent orthodoxy. However, it is clear that it is an intolerable offense against correct method to derive truths of pure (or general) mathematics (i.e. arithmetic, algebra, analysis) from considerations which belong to a merely applied (or special) part, namely geometry.

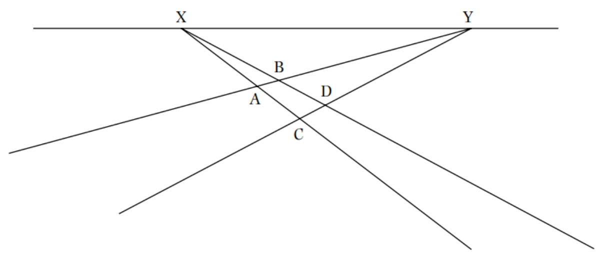

Figure 2: Classical descriptive geometry as it is still practised today. The three projections are united to give a general view, using the algebra of vectors.

Geometry was an applied discipline since its truth was drawn from our worldly understanding. Even if they did not agree with Bolzano's findings, his readers would have almost certainly agreed with this premise. It was not disputed by Lobachevsky and Bolyai, nor would it be fought for almost sixty years. Only in retrospect do we see that non-Euclidean geometry points toward a democracy of geometries in which all may have equal status and truth claims are no longer relevant.

References:

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