Descartes: a real-time specification language

Omar Mohamed Hussien Kotb

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I have spent considerable time describing how a "modern" perspective on numbers can be traced to the late sixteenth century. Because they are written in a language in transition between that of the mediaeval world and our own, the texts in which the work is done do not have a modern appearance. Descartes' Geometry, on the other hand, appears contemporary and is relatively simple to read—for us; his contemporaries found it challenging because it was novel. This is because he invented the common notation of modern algebra. Of course, this could also be viewed in a different light: if his terminology has endured, it is because he had the intelligence to create one that was straightforward and simple to employ. As a result of this, and more specifically of his 'coordinate' representation for geometric curves, French historians of mathematics in the eighteenth century regarded Descartes as the revolutionary who had liberated them from the tedious methods of the ancient Greeks by reducing difficult geometric problems to simple algebraic ones. This is a lookout

which is now frequently viewed with some scepticism, despite Descartes' promotion of it: I have provided these extremely simple [methods] to demonstrate that it is possible to solve all problems in standard geometry using only the information presented in the four figures. [These are the numbers that make up a sum, a product, a quotient, and a square root.] This is something that I believe ancient mathematicians did not observe, for if they had, they would not have spent so much time writing so many books in which the very sequence of propositions demonstrates that they did not have a reliable method of finding, but rather compiled propositions on which they had stumbled upon by accident.

Notably, Descartes does not claim to have rediscovered an ancient technique here. In fact, he claims that the simplicity of his methods proves that the ancients did not have them, or they would have discovered his results. Sometimes it is asserted that he lacked originality, as the graphical representation was derived from Oresme and the algebra from Viète. Descartes did acknowledge his debt to Viète, specifically defending himself against accusations of difficulty by asserting (which he makes no mention of in the Géométrie) that he assumed his readers were familiar with the Analytic Art. His project was unique and specific: the relationship between geometry and algebra. A typical contemporary textbook criticises Descartes for his lack of practicality:

> Our description of Descartes' geometry should make it clear how far removed the author's ideas were from the practical considerations that are now so prevalent when using coordinates. He did not use a coordinate frame to locate points, as a surveyor or geographer might, nor did he think of his coordinates as number pairs... La géométrie was as much a triumph of impractical theory in its day as Apollonius' Conics were in antiquity.

This critique is intriguing, but I believe it is misplaced. Even today, coordinate geometry is not "intrinsically" applicable; neither the statistician studying whether points in a scatter plot lie near a straight line $y = ax + b$nor the geometer attempting to visualise the curve $y^2 = x^3 + x^2$ (Fig. 2) are thinking as surveyors or geographers. Newton and Leibniz were able to comprehend, however, that the new ideas were very well adapted to certain practical tasks. Galileo goes to great lengths to demonstrate using Apollonius' Conics that a projectile describes a parabola, a fact which follows very easily by finding its equation; and while Descartes does not deal with results of this kind (his physics was too different from Galileo's, and largely confused), they are simplified and clarified by employing the methods contained in his book. To demonstrate this and to demonstrate how, unlike Viète, he avoided the Euclidean legacy of formal definitions, propositions, and proofs, I have included Appendix B, which demonstrates the basic construction in which coordinates first appear. The idea is to draw a curve using a simple machine (a pivoting ruler, subject to constraints) and to determine the curve's equation. The description of the machine is more complex than it actually is, and deriving the equation is not difficult. In conclusion, the curve is said to be "of the first kind," which Descartes defines as a conic section, because its equation in x and y is of the second degree (quadratic). Note that the use of machines for drawing curves could be considered a practical innovation typical of the Renaissance; however, it has a long history, both Greek (Eratosthenes) and Islamic, neither of which Descartes acknowledges.

In addition to inventing a new method and notation, Descartes also introduced a new style of writing mathematics, which was to have a significant impact. All previous books in Europe, including Stevin's, had been formally organised either on the Greek model (sequence of propositions and proofs) or on the model of the abbacus schools, which was also in part that of Diophantus and the Chinese (sequence of problems and solutions). It could be said that the same structure underlies the Géométrie (for example, the excerpt I have provided poses a question and provides an answer); however, the entire work is absorbed into a smooth narrative that appears to flow from one 'discovery' to the next, pausing only for comments, explanations, or excuses for avoiding them: But I will not stop to explain this in greater detail, as doing so would deprive you of the pleasure of mastering it on your own, as well as the benefit of training your mind through practise,

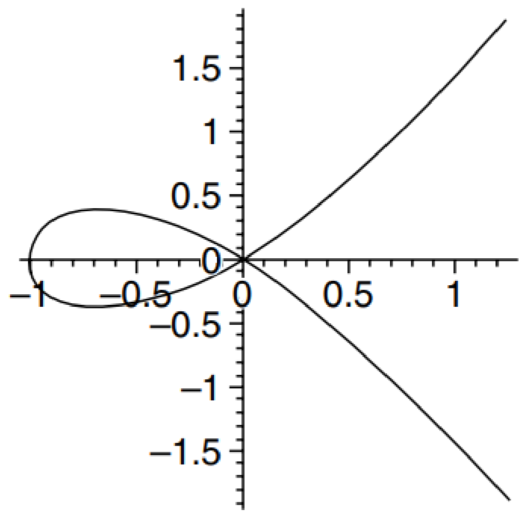

Figure 1: Graph of a cubic curve.

which is, in my opinion, the most important benefit of this science.

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