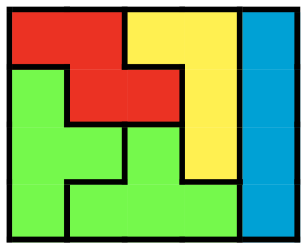
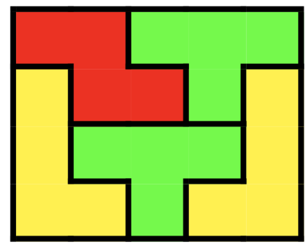
Part 2.

Let R denote the rectangular region $[0,5] \times [0,4] \subset \mathbb{R}^2$. A tetromino is a union T of 4 distinct 1×1 closed squares such that the interior of T is connected. A tiling of R is a collection $\{T_i\}_{i=1}^{k}$ of tetrominos such that $R = \bigcup_{i=1}^{k} T_i$ and $T_i \cap T_m = \emptyset$ for $i \neq m$. For example, the two images below depict two different tetromino tilings of R by five tetrominos.



Tiling B



Tiling A

Two tetromino tilings of R

Suppose the rectangle R represents a plot of land and each tetromino T in a given tetromino tiling of R represents land rented to one of five individuals. Due to unspecified rules and regulations, the amount of monthly rent $R(T)$ assigned to a tetromino $T \subset R$ is

$$\text{rent of } T = R(T) := \min \{f_0(x,y) : (x,y) \in T\},$$

where

$$f_0(x,y) = (x-2)^2 + 3(y-1)^2 + e^{2x-3y}.$$

For a given tiling $\{T_1, \ldots, T_k\}$ of R, the total monthly rent R_{total} obtained by the landlord is then

$$R_{\text{total}} = \sum_{i=1}^{k} R(T_i).$$

Suppose the landlord is only able to choose from one of the two tilings given above (i.e., Tiling A or Tiling B). The greedy landlord would like to maximize the total monthly rent R_{total} they receive and have asked you to determine which of the above two tilings will achieve a greater value for R_{total}.

Assignment:

- Determine which of the two tilings above has a greater value for R_{total}.
- To calculate each R_{total} and $R(T)$ for each tetromino, you must rely only on your code from part 1 of the final and on elementary hand calculations using only addition.
- Write one additional .py file to implement your code from part 1.
- Submit the additional .py file and a pdf report describing your methods and conclusions in a way that should convince the landlord that you are correct.

Optional Problems

(1) Do Part 2 of the final for higher dimensional tetrominos or polyminos (choose your own tilings etc).
(2) Write python code which takes in as an argument ASCII art representing a 5×4 tetromino tiling of R and outputs the total rent for a given convex smooth objective f_0.
(3) Let $Z = R \cap (\mathbb{Z} \times \mathbb{Z})$ denote the integer points in R, let $r_1, \ldots, r_k$ be positive real numbers and let $x_1, \ldots, x_k \in Z$. Fix a convex smooth objective $f_0 : R \to \mathbb{R}$. For a closed disk $B_j := B(x_j, r_j) \subset R$ of center x_j and radius r_j, define its rent to be

$$R(B_j) := \min \{f_0(x) : x \in B(x_j, r_j)\} .$$

For user specified $x_1, \ldots, x_k, r_1, \ldots, r_k$, find a configuration of the disk B_j so that
- $B_j \subset R$.
- $B_j \cap B_m = \emptyset$ for $j \neq m$.
- The total rent

$$R_{\text{total}} = \sum_{j=1}^{k} R(B_j)$$

is maximized.

Of course, certain values of r_j will prevent existence of possible configurations.
(4) Do the previous problem with B_j replaced by the closed annuli

$$A_j = B(x_j, r_j) \setminus \operatorname{int} B(x_j, \frac{r_j}{2}).$$