Information Diffusion

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Abstract

Social networks are the systems of complex social interaction motivating a diverse set of important mathematical models for the flow of information, rumors, trends, and beliefs, e.g. support for a particular political candidate. The dynamics of these beliefs are often prescribed by network models or systems of differential equations. We are concerned with advancing a model involving a system of differential equations [1]. The system treats the spread of mutually exclusive and exhaustive beliefs as the spread of disease, using a modified SIR model. The goal was to understand the support for two opinions in time under different initial conditions and parameters. Analytical solutions to this model are highly complicated or computationally inefficient, so an adaptive Runge-Kutta method has been used to numerically compute solutions to the system. The importance of the different parameters in the system was explored via sensitivity analysis. We also explore fitting the model to real world data, and discuss limitations and potential improvements. In this project, we use numerical approximations of a system of oridnary differential equations to draw conclusions about how competing ideas spread through a population.

1 Introduction

Rumor spreading can have huge impacts on people's lives: re-imagining scientific facts and influencing political opinions. With technologies that have significantly expanded access to the production and reproduction of information, the potential rate of spreading misinformation has increased significantly. Research into rumor spreading has primarily been based on either models of social and biological contagion, e.g. SIR, or upon models of opinion dynamics.

Here, "rumor spreading" will be extended to more than modeling gossip. Rather, we are interested in modeling a system in which two competing ideas are spreading through a population. The model used will only consider the existence of two mutually exclusive opinions, an assumption made to best model the two party political system in the United States. Furthermore, focusing on short-term dynamics is useful, thus by assuming that opinion support is experienced across the entire population, with no individuals entering or leaving the system.

Conclusions about the dynamics of the support for two ideas have key applications in politics. If parameters or initial conditions are found to greatly improve the support of one opinion over the other, campaigns may utilize this model to strengthen platforms and target key supporters.

Human behavior is largely unpredictable. As such, modeling human social networks can be extremely computationally expensive. Less restrictive assumptions could lead to a more realistic model, but the computational cost can easily reduce the usefulness of the model in applications. With this in mind, we will proceed with the assumptions above, and leave room for further improvement in the quality of the model in the future.

2 Existing Models

In "Epidemics and Rumors," Daley and Kendall discuss two models for rumor spread. They stratify the population into the following mutually exclusive and exhaustive classes:

X: Has not heard the rumor

Y: Actively spreading the rumor

Z: No longer spreading the rumor.

These classes are analogous to susceptible, infected, and recovered groups in an epidemic spread model. The first rumor model presented is exactly the basic SIR model:

$$\begin{cases} \frac{dX}{dt} = -\beta XY \\ \frac{dY}{dt} = \beta XY - \gamma Y \\ \frac{dZ}{dt} = \gamma Y \end{cases}$$

In this model, the mechanism of transition from Y to Z is forgetting the information. Another possibility presented in the paper is that people don't want to spread "stale" information. In this case, interaction between a spreader (Y) and someone who knows the information (Y or Z) causes a transition to Z. If the interaction is between two Y people, they both move to Z, and if the interaction is between a Y and a Z, the Y becomes a Z. The transition is thus proportional to $2Y + Z$, so the model becomes:

$$\begin{cases} \frac{dX}{dt} = -\beta XY \\ \frac{dY}{dt} = \beta XY - \gamma Y - \delta Y(2Y + Z) \\ \frac{dZ}{dt} = \gamma Y + \delta Y(2Y + Z) \end{cases}$$

The authors note that there are many ways to incorporate unwillingness to spread stale information, this is just one possible approach.

3 Our Contribution

The study is interested in modeling a system where two opposing viewpoints are being spread simultaneously. This could be two political candidates with opposite opinions, two contradictory rumors, etc. This would require adding a few more groups to the model. The chosen stratification was:

Neutral (N): Has not heard either opinion
Swayable 1 (S_1): Believes Opinion 1 on but can be swayed to opinion 2
Unswayable 1 (U_1): Believes Opinion 1 cannot be swayed
Swayable 2 (S_2): Believes Opinion 2 on but can be swayed to opinion 1
Unswayable 2 (U_2): Believes Opinion 2 cannot be swayed

With this stratification, people would move from neutral into swayable 1 and 2. From S_1 they could move to neutral, swayable 2, and unswayable 1, and vice versa. Once a person believes their position firmly, they can no longer be convinced to change groups. Transitions between N, S_1, and S_2 happen as a result of interactions between people, and transitions from S to U occur when a person has been in S for a long time and their opinion becomes firm. These transitions are visualized in Figure 1

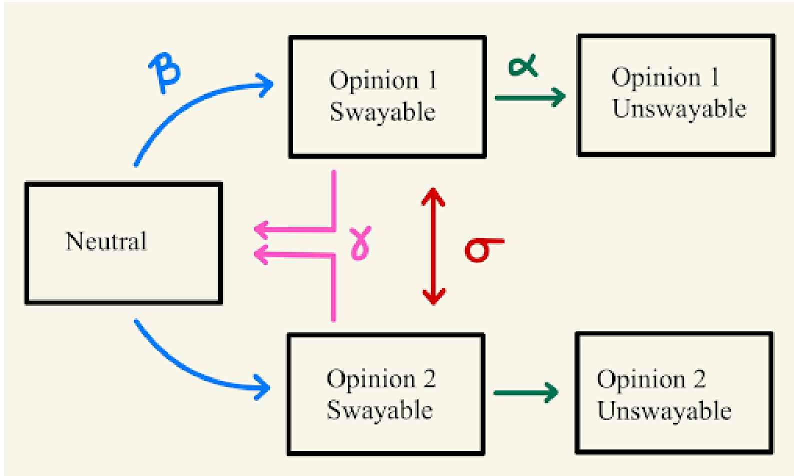

Figure 1: Transitions between population groups

The transfer of information is dictated by the use of law of mass action terms to represent interactions between groups. Thus, we derive the following model.

$$\begin{cases} N'(t) = \gamma_1 S_1 + \gamma_2 S_2 - \beta_1 N(S_1 + U_1) - \beta_2 N(S_2 + U_2) \\ S_1'(t) = \beta_1 N(S_1 + U_1) + \sigma_{21} S_2(S_1 + U_1) - \sigma_{12} S_1(S_2 + U_2) - \alpha_1 S_1 - \gamma_1 S_1 \\ U_1'(t) = \alpha_1 S_1 \\ S_2'(t) = \beta_2 N(S_2 + U_2) + \sigma_{12} S_1(S_2 + U_2) - \sigma_{21} S_2(S_1 + U_1) - \alpha_2 S_2 - \gamma_2 S_2 \\ U_2'(t) = \alpha_2 S_2 \end{cases}$$

4 Sensitivity Analysis

Understanding which parameters have the largest effect on the opinions of the population is important for anyone looking to predict or influence these dynamics. To determine which parameters are the most important, we employed a technique called sensitivity analysis. Using software provided by Dr. Mikucki, we calculated $\frac{\partial V_i}{\partial p_j}$ over time, where $V_i,\ i \in \{1, \cdots, 5\}$ are the variables representing the different groups $(N, S_1, U_1, \text{etc.})$, and $p_j,\ j \in \{1, \cdots, 8\}$ are the 8 parameters in the model $(\beta_1, \beta_2, \gamma_1,$ etc.). Comparing these partial derivatives allows us to analyze which parameters have the greatest impact on each variable.

The most important variables to investigate are U_1 and U_2, because in the long term, S_1, S_2, and N go to 0, so the unswayable categories determine which opinion ends up holding the majority. Figure 2 shows the plots of the partial derivatives of U_1 and U_2 with respect to the parameters over time.

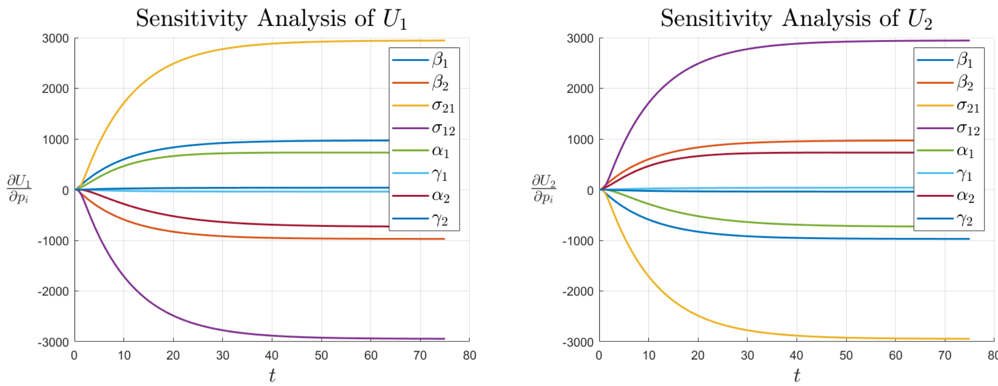

Figure 2: Sensitivity analysis of U_1 and U_2

The symmetrical nature of the model leads to nearly identical plots, the only difference being which of the paired parameters (β_1 and β_2, α_1 and α_2, etc.) is positive, and which is negative. In this format, it is hard to immediately distinguish the parameters. To help with this, we will consider a single point in time once equilibrium has been reached. Figure 3 shows the partial derivatives of U_1 with respect to the parameters at the final time ($t = 75$) in Figure 2. Note that an analogous plot for U_2 was omitted due to symmetry with U_1.

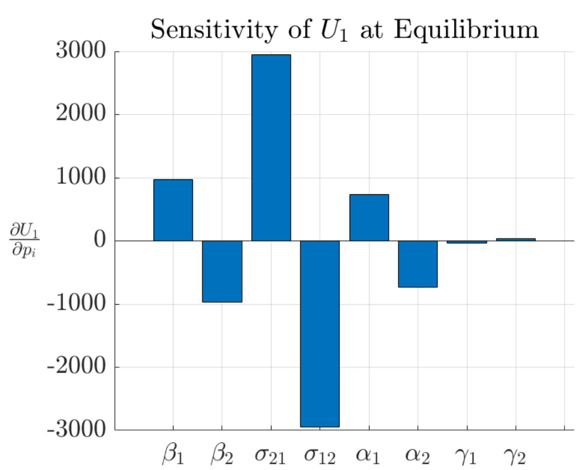

Figure 3: Sensitivity of U_1 at equilibrium

The magnitudes of the partial derivatives indicate the strength of the effect had by each parameter, and their signs represent if the effect drives U_1 up or down. From this analysis, the parameters can be ranked in order from most impactful to least impactful for the unswayable categories:

1. σ: Transitions between swayable groups
2. β: Transitions from neutral to swayable
3. α: Transitions from swayable to unswayable
4. γ: Transitions from swayable to neutral

Now that it is clear which parameters are the most important, we will perform some experiments to better understand their relationships. Changing one of the σ values while all of the other parameters are constant produces the expected effect. Figure 4 visualizes this simulation. Under these conditions, whichever opinion has an advantage in σ ends up gaining majority support.

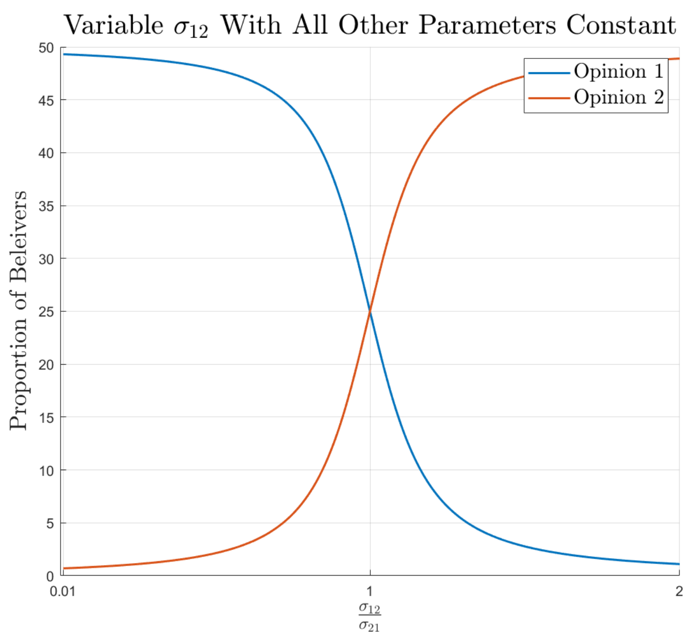

Figure 4: The effect of changing σ_{12} while keeping other paremeters equal

A more insightful test would be to determine how much more important the σ parameters are than the rest. From our earlier rankings, the β parameters are the second most impactful, so we will begin by comparing β and σ. To do so, we give opinion 2 a 5% advantage in σ. That is, we take $\sigma_{12} = 1.05\sigma_{21}$. Then, we repeatedly run the model while increasing β_1, which gives opinion 1 an advantage in β, until opinion 1 is able to win majority support in the equilibrium. Note that all parameters that aren't part of this experiment are held constant, and symmetric initial conditions are used.

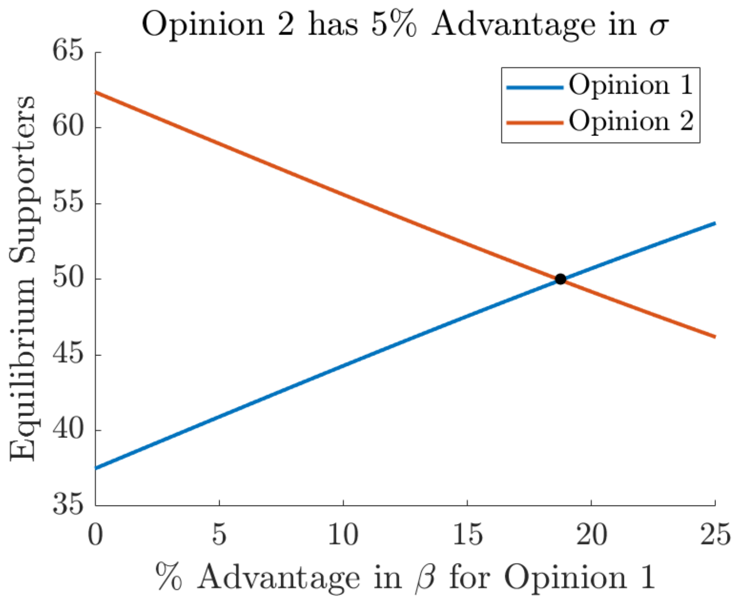

Figure 5: Comparing the impact of σ and β

Figure 5 indicates that to overcome a 5% disadvantage in σ, opinion 1 requires a 19% advantage in β. This gives us an idea of how much more impactful σ is than β.

Now we will perform the same experiment, but giving opinion 1 an advantage in α, which was the next most important parameter. This corresponds to increasing α_1. From Figure 6, we can conclude that to overcome a 5% disadvantage in β, opinion 1 needs a 65% advantage in α.

The results of our sensitivity analysis reveal that the σ parameters are profoundly more important than the rest when it comes to determining long term support. Recall that the σ parameters govern transitions between the two swayable groups. One area where this result could be useful is in an election; the best way for a candidate to ensure victory is to increase their σ. That is, they should focusing on converting their opponents moderate supporters, rather than targeting any other groups. Knowing this could help a candidate determine what advertisements to run, where to hold rallies, and which policies to build their campaign around.

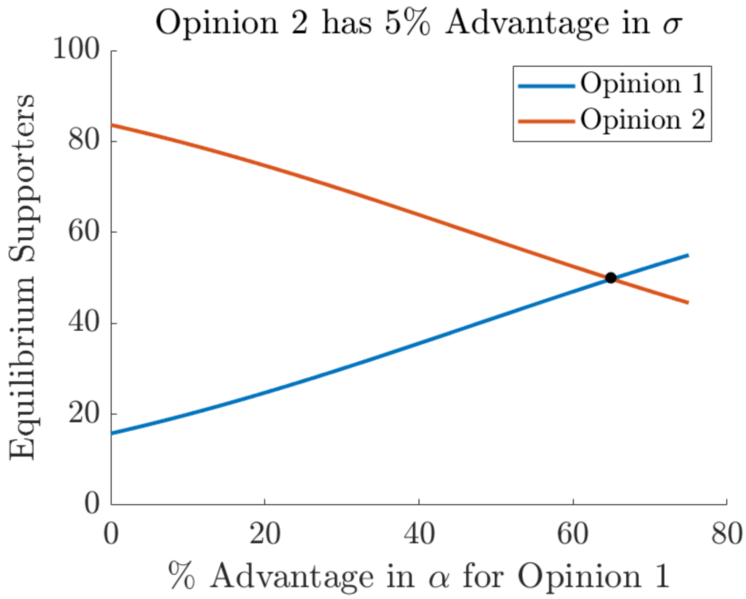

Figure 6: Comparing the impact of σ and α

5 Data Fitting

We decided to fit the model to data from a survey question: Do you think that global warming will pose a serious threat to you or your way of life in your lifetime?" (Question 23)[2]

Year	1992	1997	2001	2002	2006	2008	2009	2010	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Neutral	98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Swayable Non-Believing	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Swayable Believing 3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Unswayable Non-Believing	1	25	31	33	35	40	38	32	38	34	36	37	41	42	45	45	45	43
Unswayable Believing	1	69	66	65	62	58	60	67	61	64	64	62	57	57	54	55	54	57

Table 1: Data on belief of Climate opinions [2]

We started with the initial condition $y_0 = [98, 0, 0, 1, 1]$ (98 neutral and 1 in each Unswayable category). Since we are modeling over a long period of time we are interested in the long-term dynamics, so we added a 5-year gap to let the dynamics settle. This resulted in the following dynamics.

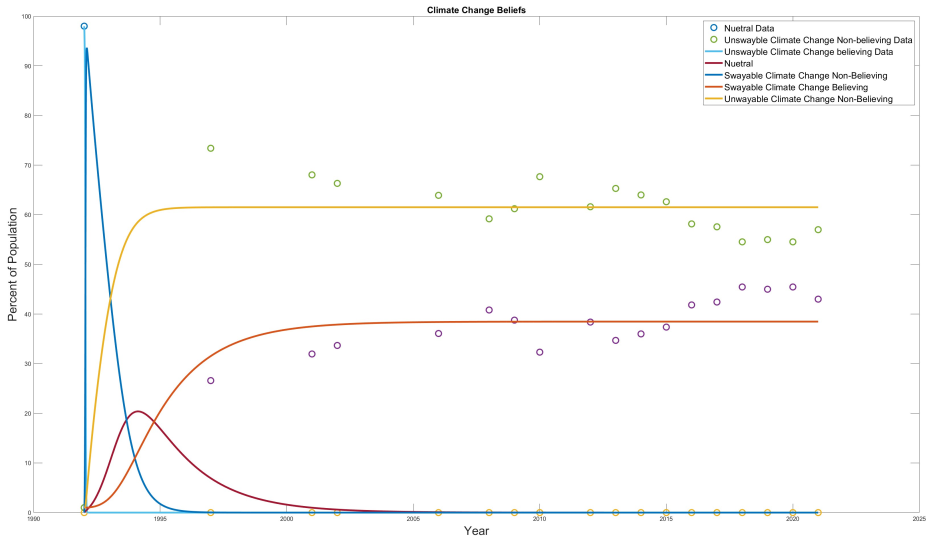

Figure 7: Model fitted to climate opinions

This was predictable as we are basically fitting the equilibrium to the average. This reveals an issue of the model: once an opinion is revealed, people will ultimately divide into two categories and not switch. This appears in the model due to the assumption that people are truly unswayable, but from the human condition we know that this is not true; when faced with convincing evidence opinions are bound to change.

So where can the model improve? Let us first perform a cursory analysis on some opinions.

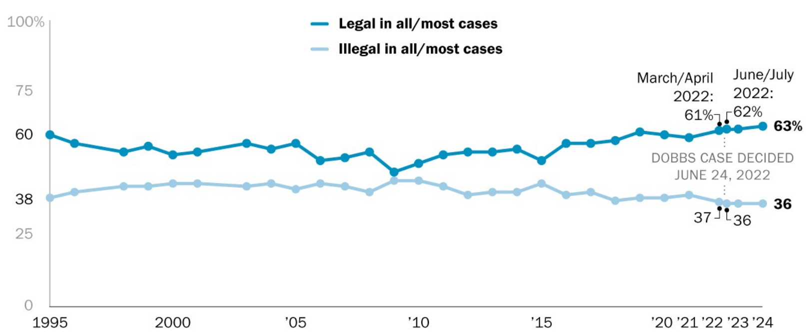

Figure 8: Public opinions on abortion. [5]

In the matter of abortion, we observe that opinions have remained relatively constant at around 60%. The model would likely have predicted these dynamics accurately. However, before concluding that the model is accurate, let us examine its predictions for climate change and gun control.

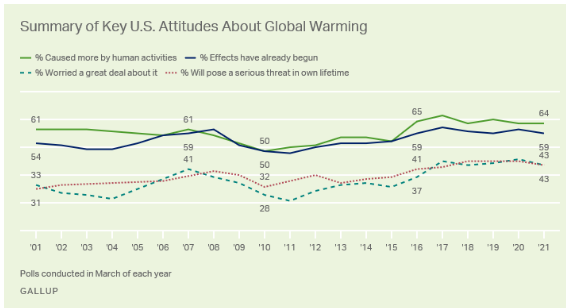

Figure 9: On the matter of climate change, the model would have predicted the dynamics before late 2005 very well. After this date, however, opinions became erratic and would not be predicted accurately. [4]

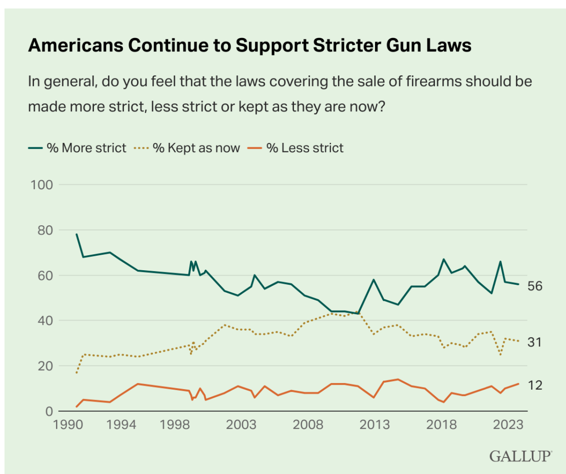

Figure 10: The model would have been a very poor predictor for Gun Control, which has been erratic since 1990. [3]

So, what makes the model perform so poorly for climate change and gun control compared to abortion? We believe the difference lies in the scale of evidence that influences public opinion. For abortion, opinion-changing events tend to be isolated.

Most of these events are likely personal experiences or interactions, which affect only a small portion of the population. In contrast, opinion-changing events related to climate change and gun control are often national in scale. Such events impact a significant portion of the population and are widely known. Examples of these larger-scale events include weather disasters and mass shootings.

How can we account for changes in response to these events? We suggest the creation of a new model in which the "unswayable" are not entirely unswayable. This new model includes the terms $\mu_1 U_1$ and $\mu_2 U_2$, which allow individuals to move from the "unswayable" state to neutral.

$$\begin{cases} N'(t) = \gamma_1 S_1 + \gamma_2 S_2 - \beta_1 N(S_1 + U_1) - \beta_2 N(S_2 + U_2) + \mu_1 U_1 + \mu_2 U_2 \\ S_1'(t) = \beta_1 N(S_1 + U_1) + \sigma_{21} S_2(S_1 + U_1) - \sigma_{12} S_1(S_2 + U_2) - \alpha_1 S_1 - \gamma_1 S_1 \\ U_1'(t) = \alpha_1 S_1 - \mu_1 U_1 \\ S_2'(t) = \beta_2 N(S_2 + U_2) + \sigma_{12} S_1(S_2 + U_2) - \sigma_{21} S_2(S_1 + U_1) - \alpha_2 S_2 - \gamma_2 S_2 \\ U_2'(t) = \alpha_2 S_2 - \mu_2 U_2 \end{cases}$$

Here, we will perform fitting on the new model using the earlier dataset. We will apply this new model in a piecewise fashion. Model 1 will be used from 1992 to 2005, covering the beginning of the dataset to the year Hurricane Katrina occurred. Model 2 will be applied from 2005 to end of the data, spanning the period from Hurricane Katrina to the present.

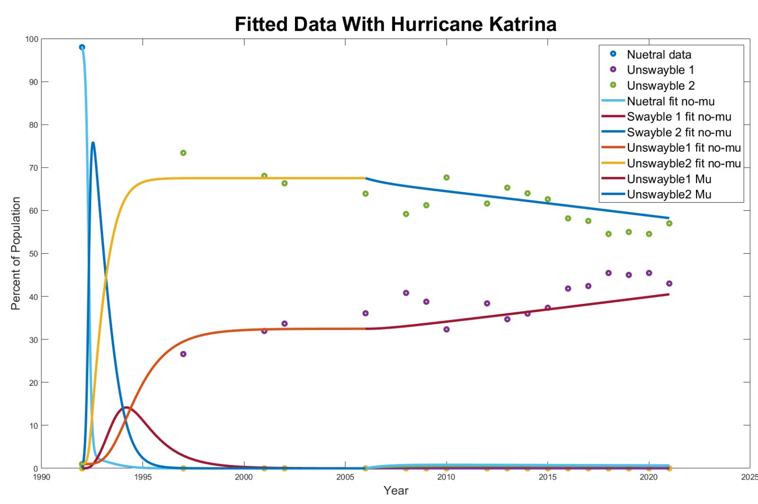

Figure 11: Figure with account for Hurricane Katrina

This model does capture the dynamics better and accounts for how opinions may change in response to large-scale opinion-changing events. However, it still doesn't fully capture the dynamics of the system. This could be due to the opinion-changing event term, $\mu_{(1/2)}U_{(1/2)}$, not being complex enough to sufficiently alter the dynamics to fit the model. We also could be missing key events that should have been accounted for in the model.

In further investigations, we would like to explore additional possibilities for this term. We would also investigate how to determine the opinion-changing event term, $\mu_{1/2}U_{1/2}$, based on factors such as the event's severity, scale, etc.

6 Conclusions and Future Work

In conclusion, the project aimed to model how two opposing ideas spread through a population. We started with a model that expanded from the basic SIR model by further stratifying the population. We then wanted to see and understand which parameters had the most influence on the dynamics. To do this, we performed sensitivity analysis where we were able to see that σ was the most impactful. With this information, we experimented to determine severity and relationships between the parameters and in doing so again found σ to be the most important. We then attempted to fit the model to real word data.

This approach was found to be problematic for subjects such as climate change and gun control, where opinions often exist on a broader spectrum and may shift over time, rather than remaining in a fixed "unswayable" category. To address this, we improved the basic model to include a μ that allows for the "unswayable" to have a possibility of becoming "neutral". With this improvement, we were able to see the model reflect better at how opinions shifted after events such as Hurricane Katrina.

Another thing we would like to explore with the model that allows individuals to move from the "unswayable" state to neutral is oscillatory dynamics. The way we can achieve this is by allowing for a smaller time step similar to what has been noted in the problems we have with the pulled data. This future work can then lend itself to more sensitivity analysis for after a "global event" occurs and see the shifts in the importance in some parameters.

References

[1] Daley, D., Kendall, D. *Epidemics and Rumours.* Nature 204, 1118 (1964). https://doi.org/10.1038/2041118a0

[2] Gallup. (2022, March 18) *Gallup Poll Social Series: Environment.* Timberline: 937614. https://cleanairact.org/wp-content/uploads/2022/04/220405Environment-1.pdf

[3] Jones, J. (2023, October 31). *Majority in U.S. Continues to Favor Stricter Gun Laws.* Gallup. https://news.gallup.com/poll/513623/majority-continues-favor-stricter-gun-laws.aspx

[4] Pew Research Center. (2024, May 13). *Public Opinion on Abortion. Pew Research Center's Religion & Public Life Project;* Pew Research Center. https://www.pewresearch.org/religion/fact-sheet/public-opinion-on-abortion/

[5] Saad, L. (2021, April 5). *Global Warming Attitudes Frozen Since 2016.* Gallup.com. https://news.gallup.com/poll/343025/global-warming-attitudes-frozen-2016.aspx

[6] Strogatz, S. *Exploring complex networks.* Nature 410, 268–276 (2001). https://doi.org/10.1038/35065725

[7] Tu, H. T., Phan, T. T., Nguyen, K. P. (2022). *Modeling information diffusion in social networks with ordinary linear differential equations.* Information Sciences, 593. https://doi.org/10.1016/j.ins.2022.01.063

[8] Zhang, Z. Liu, C. Zhan, X. Lu, X. Zhang, C. Zhang, Y. *Dynamics of information diffusion and its applications on complex networks.* Physics Reports, 651, 1-34. https://doi.org/10.1016/j.physrep.2016.07.002.