COSC 4370 – Homework 1

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September 2024

1 Objective

This assignment asks for the implementation of an algorithm for rasterizing ellipse. Where the ellipse to rasterize is $\left(\frac{x}{9}\right)^2 + \left(\frac{y}{16}\right)^2 = 24^2 \ where\ y \leq 0$.

2 Methods

There are two functions modified and utilized in the provided code: *writeEllipse* and *midpointEllipse*. In *writeEllipse* we only have to implement the lower quadrants of the ellipse as per the given requirement of $\left(\frac{x}{9}\right)^2 + \left(\frac{y}{16}\right)^2 = 24^2 \ where\ y \leq 0$. So, we plot the pixels in quadrant III where (-x, -y) and in quadrant IV where (+x, -y). The *midpointEllipse* function implements the midpoint ellipse algorithm which finds points in one quadrant, which is further divided into two regions: region 1 and region 2, to calculate pixels closer to the ellipse curve by taking midpoint between them. Taking advantage of the four-way symmetry of ellipse to replicate coordinates found in one quadrant to other quadrants to create a complete ellipse.

3 Implementation

Given the ellipse equation $\left(\frac{x}{9}\right)^2 + \left(\frac{y}{16}\right)^2 = 24^2 \ where\ y \leq 0$, the vertex point along the minor axis denoted as ‘a’ = 216 and the vertex point along major axis denoted as ‘b’ = 384. The basic equation of ellipse $f(x, y) = b^2x^2 + a^2y^2 - a^2b^2$ can be written as $b^2x^2 + a^2y^2 - a^2b^2 = 0$. The midpoint ellipse algorithm utilizes four-way symmetry, essentially only requiring finding coordinates in one quadrant. The quadrant is further divided into two regions: region 1 and region 2, due to the arc of the ellipse at a certain point. When a tangent line is drawn at the point perpendicular to the point between region 1 and region 2, the slope is -1 which is the point of boundary between region 1 and region 2. Thus, a slope of tangent > -1 is in region 1 and slope of tangent < -1 is in region 2. The $grad\ f(x,y) = \left(\frac{\partial f}{\partial x}\right)\hat{\imath} + \left(\frac{\partial f}{\partial y}\right)\hat{\jmath} = \frac{\partial}{\partial x}(b^2x^2 + a^2y^2 - a^2b^2) + \frac{\partial}{\partial y}(b^2x^2 + a^2y^2 - a^2b^2)$ which simplify to $grad\ f(x,y) = 2b^2x\hat{\imath} + 2a^2y\hat{\jmath}$. The magnitudes of $\hat{\imath} = \hat{\jmath}$ at the point of arc and tangent. So in region 1 where the midpoint is $\left(x_p + 1\,, y_p - \frac{1}{2}\right)$ substitute this into gradient $f\left(x_p + 1, y_p - \frac{1}{2}\right) = 2b^2(x_p + 1)\hat{\imath} + 2a^2\left(y_p - \frac{1}{2}\right)\hat{\jmath}$. So, when $\hat{\jmath} \geq \hat{\imath}$ it is region 1 and when $2b^2(x_p + 1) \geq 2a^2\left(y_p - \frac{1}{2}\right)$ is true it is region 2.

3.1 Region 1

The initial value for region 1 is $d_{start} = f\left(x_0 + 1, y_0 - \frac{1}{2}\right), so\ x_0 = 0\ and\ y_0 = b$

$$= f\left(1, b - \frac{1}{2}\right) = b^2 1^2 + a^2\left(b - \frac{1}{2}\right)^2 - a^2b^2$$

$$= -x^2y + \frac{x^2}{4} + y^2$$

When in region 1, $d = f\left(x_p + 1, y_p - \frac{1}{2}\right) = b^2(x_p + 1)^2 + a^2\left(y_p - \frac{1}{2}\right)^2 - a^2b^2 = d_{old}$. When d < 0 we select 'E' and when $d \geq 0$ we select 'SE'.

If 'E' is chosen:

$$d_{new} = f\left(x_p + 2, y_p - \frac{1}{2}\right)$$

$$\Delta\mathrm{E} = d_{new} - d_{old}$$

$$= b^2(2x_p + 3)$$

Else if 'SE':

$$d_{new} = f\left(x_p + 2, y_p - \frac{3}{2}\right)$$

$$\Delta\mathrm{SE} = b^2(2x_p + 3) + a^2(-2y_p + 2)$$

3.2 Region 2

The initial value for region 2 is $d_{start} = f\left(x + \frac{1}{2}, y - 1\right)$

$$= b^2\left(x+\frac{1}{2}\right)^2 + a^2(y-1)^2 - a^2b^2$$

The values of x and y are obtained from the final iteration of region 1. In region 2, (x_i, y_i) is your current pixel and $d_{old} = f\left(x_i+\frac{1}{2}, y_i-1\right) = b^2\left(x_i+\frac{1}{2}\right)^2 + a^2(y_i-1)^2 - a^2b^2$. When $d \geq 0$ select 'S' else if d < 0 choose 'SE'.

If 'S':

$$d_{new} = f\left(x_i+\frac{1}{2}, y_i-2\right)$$

$$\Delta S = a^2(-2y+3)$$

Else if 'SE':

$$d_{new} = f\left(x_i+\frac{3}{2}, y_i-2\right)$$

$$\Delta SE = b^2(2x+2) + a^2(-2y+3)$$

This concludes the implementation of *midpointEllipse* algorithm and the approach to rasterizing the ellipse $\left(\frac{x}{9}\right)^2 + \left(\frac{y}{16}\right)^2 = 24^2\ where\ y \leq 0.$

4 Results (.bmp file)