TDC (CBCS) Odd Semester Exam., 2024

PHYSICS

(1st Semester)

Course No. : PHSHCC-101T

(Mathematical Physics—I)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Find $A+B$, if

$$A=\begin{pmatrix}2 & 2 & 1\\ 1 & 5 & 0\\ 0 & 0 & 1\end{pmatrix} \text{ and } B=\begin{pmatrix}5 & 7 & 1\\ 0 & 3 & 0\\ 1 & 0 & 8\end{pmatrix}$$

(b) If A and B are symmetric matrices, then show that AB is symmetric if and only if A and B commute.

(c) What do you mean by 'order' and 'degree' of a differential equation?

2. Answer any *one* of the following : 6

(a) *(i)* What are Hermitian matrices? Show that the eigenvalues of Hermitian matrix are real. 1+3=4

(ii) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$$ 2

(b) *(i)* Solve the following by the method of integrating factor : 3

$$x\frac{dy}{dx} + y = x^3 + x$$

(ii) Solve the following equations by matrix method : 3

$$x + 5y + 3z = 1$$
$$3x + y + 2z = 1$$
$$x + 2y + z = 0$$

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$$

(b) Find the value of b for which the vector

$$\vec{A} = (2x + 3y)\hat{i} + (6y - 3z)\hat{j} + (bx - 12z)\hat{k}$$

is solenoidal.

(c) Find the area of the parallelogram, whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$.

4. Answer any *one* of the following : 6

(a) *(i)* Find m so that the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$, $\hat{i} - m\hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - 5\hat{k}$ are coplanar. 3

(ii) Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Find the vector $\vec{a} \times (\vec{b} \times \vec{c})$. 3

(b) What is gradient of a scalar function? Give its physical interpretation. Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$, where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$. 1+2+3=6

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) Evaluate

$$\int_{x=0}^{1} \int_{y=0}^{2} (x^2 + 3xy^2)\,dx\,dy$$

(b) Using Gauss divergence theorem, express the Gauss law in electrostatics in differential form.

(c) State Green's theorem.

6. Answer any *one* of the following : 6

(a) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k})d\vec{S}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant.

(b) If $\vec{F} = \vec{\nabla}\phi$ everywhere in a region R and ϕ is single-valued and has continuous derivatives in R, then show that $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of the path joining the points A and B.

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Write the expression for line element in spherical polar coordinate system.

(b) Write the values of scale factors h_1, h_2, h_3 of curvilinear coordinate system in spherical polar coordinate system.

(c) Write the expression for Laplacian operator in spherical polar coordinate system.

8. Answer any *one* of the following : 6

(a) Derive the expression for curl of a vector in orthogonal curvilinear coordinates.

(b) Evaluate $I = \int_0^1 \frac{1}{1+x} dx$ correct to the three decimal places using Simpson rule with $h = 0 \cdot 125$.

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Define the term 'constant error' and give a suitable example.

(b) What do you mean by least square fit method?

(c) Define Poisson distribution. Mention its importance in Physics.

10. Answer any *one* of the following : 6

(a) *(i)* What is conditional probability? 2

(ii) State and prove Bayes' theorem. 4

(b) *(i)* What is hypothesis? Explain, with examples, 'null hypothesis' and 'alternative hypothesis'. 1+3=4

(ii) Discuss the different types of systemic errors associated with a measurement. 2

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