1 Overview of exercises (PART I)

1. limb-darkening scattering exercise we did during the course. — You can look into your notes from that, and I attach here also a sample program which you can use a base. After you have familiarised yourself with this, you can start to think bout how you would go about to extend this to a 3D setting (assuming isotropic scattering).

2. (As prep for Monte-Carlo school) here is a script computing a UV resonance P-Cygni line in spherically symmetric wind with v beta-law. At top of routine, a few exercises are given, where you can modify and play around with code. Monte-Carlo program which computes a UV resonance spectral line from a fast outflowing spherically symmetric stellar wind (if you were not cc'd on that email, let me know so that I can send you the files as well). At the top of that little script, there are a few suggestions for exercises (additions) you could do to that program, in order to learn a bit more about the general workings of Monte-Carlo radiative transfer in this context. — So that might be a good idea for you to do as well ! (And you can also ask the others in the group for some tips etc. then.)

3. Some background reading:
   - Attached mc manual by Puls.
   - Paper by Sundqvist+ 2010 (Appendix, I think).

2 Overview of exercises (PART II)

1. calculate the probability distribution to sample from in the case of Eddington limb darkening for the initial distribution (see 4.3.4).

2. calculate analytical solution for simplified problem in 4.3.2 in the case that `mu = 1`.

3. perform convergence analysis

3 Limb darkening

3.0.1 2D Case

We again have $\mu = \cos(\theta)$. The solution of the radiative transfer equation in plane-parallel syummetry with frequency-independent absorption and emission, is

$$I(\mu) = I_1(0.4 + 0.6\mu) \tag{1}$$

In the Monte Carlo code, the photons are sorted according to the direction that they leave the atmosphere.

Goal Calculates the angular dependence of photon's emitted from a plane-parallel, grey atmosphere of radial optical depth `taumax`. The value of `tau` determines the position of the photon

Variables and Algorithm

- `muarray` contains emergent photons
- `na` number of channels
- `dmu` = 1/`na` width of channels
- `nphot` number of photons
- `taumax` maximum optical depth

Algorithm 1 Limb darkening: compute quantitiy of photons

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initialization
  radial optical depth τ
  direction μ
for all photons **do**
  $\boxed{\tau = \tau_{max}}$
  while `tau` ≥ 0 **do**
    compute scattering angle `mu`
    if tau $\geq$ taumax **then** $\boxed{mu = sqrt(x)}$ (initial distribution)
    else $\boxed{mu = 2 * x = 1}$ (isotropic scattering)
    tau_i = -log(x2)
    tau = tau - tau_i*mu
  end while
  now we know that the photon has left the photosphere
  compute the distribution of all angles `mu` at which the photon left the photosphere
end for
visualisation:

- plot photon numbers from $\mu d\mu$ against `mu`
- plot specific intensity from $d\mu$ against `mu` against

Figure 1 is according to what is expected $I = I_0(0.4 + 0.6\mu)$

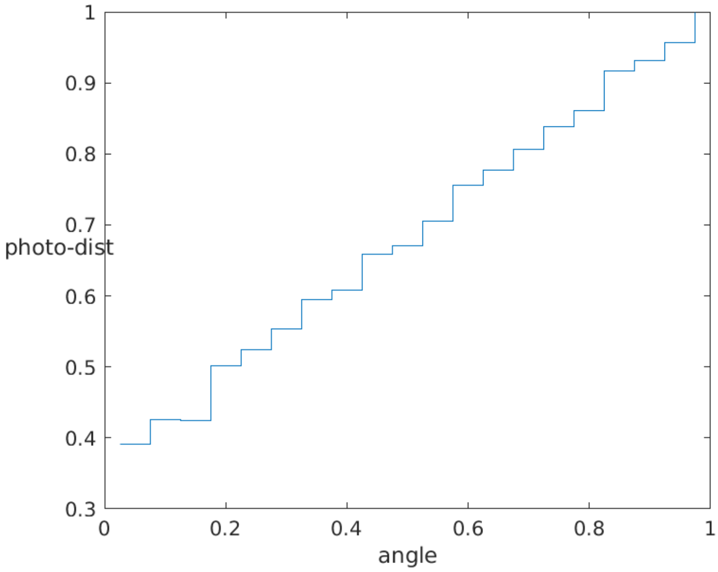

Figure 1: histogram for `mu`

3.0.2 3D Code

What changes is this:

- introduction of a new angle ϕ
- the optical depth has to be updated according to ϕ also

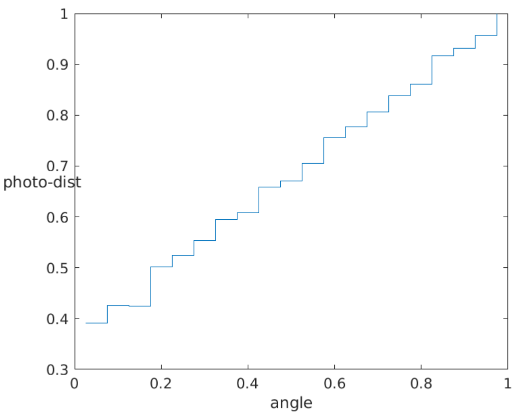

Figure 2: histogram for `mu`

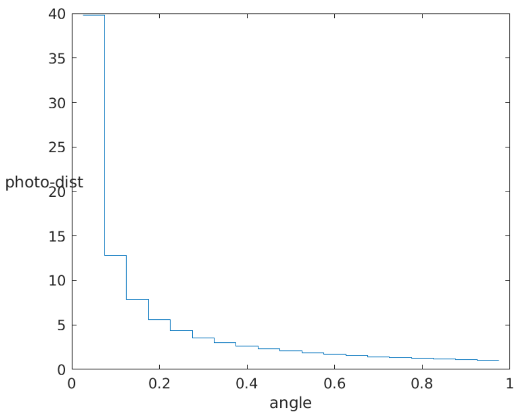

Figure 3: histogram for `phi`

Figure 2 and Figure 3 are according to what is expected, namely $I = I_0(0.4 + 0.6\mu)$ and a uniform distribution for *phi*, which corresponds to a $I \sim \dfrac{1}{\phi}$

4 Investigation of program: pcyg.f90

4.1 Overview of variables

name	explanation	scope
paramaters		
xk0		
alpha		
beta		
start frequency of the photon		
xstart	start frequency	
vmin		
vmax		
angle of the photon		
xmuestart	start angle	
xmuein	incident angle	
xmueou	outward angle	
pstart	impact parameter	
xnew	new photon frequency	
optical depth		
tau	optical depth	
number of photons admin		
nphot	number of photons	
nin	photons scattered back into core	
nout	photons escaped	
functions		
func	velocity profile	
	r	distance from center of star
xmueout	sign of outwards angle	
	xk0	
	alpha	
	r	
	v	
	sigma	

4.2 Mathematical things that are noteworthy

General working

$$\xrightarrow{\quad\texttt{xstart}\quad} \boxed{\texttt{pcyg.f90}} \xrightarrow{\quad\texttt{xnew}\quad}$$

The photons are sorted according to `xnew`.

Practical formula

- emission angle $\mu = \cos(\theta)$
- according p-ray $p = \sqrt{1 - \mu^2} = \sin(\theta)$
- incident angle $xmuin = \sqrt{1 - \left(\frac{pstart}{r}\right)^2}$

Geometry & Symmetry assumptions

- spherical geometry

4.3 Exercises

4.3.1 Investigation of original code

In original version of the code, all photons are released isotropially from the photosphere.

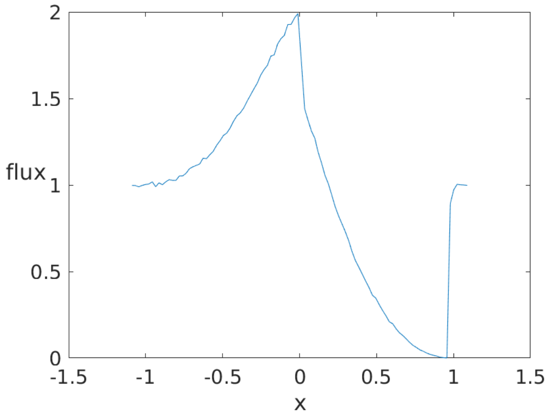

Figure 4: Original version of the code

4.3.2 First adaptation: what if all photons are released radially from photosphere?

Release photons radially: experiments What would happen with line-profile, if you assumed all photons were released radially from photopshere?

- In other words `xmuestart` $= 1$. Results in Figure 5.
- This is implemented under the test case `test_number=1`.

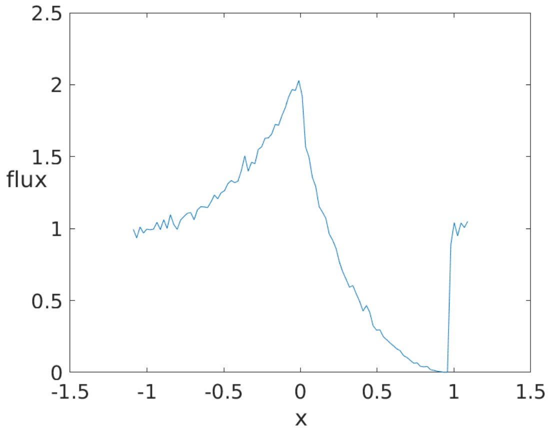

Figure 5: First adaptation

Derive analytic expression See also slide Sundqvist 26/49.

- since `mu = 1` we have for the velocity profile that $v = v_\infty(1 - b/r)^\beta$
- a scaled version of the above yields

$$u = \frac{v(r)}{v_\infty} = \left(1 - \frac{r_\infty}{r}\right)^\beta \in [0..1] \tag{2}$$

- Doppler shift: $x_{CMF} = x_{REF} - \mu u$
- condition for resonance from Sobolov approximation (to be studied later): $x_{CMF} = 0$ thus

$$x_{RF} = \mu u \tag{3}$$

  or thus $u_{int} = x_{REF}$ and than solve Equation 2 for r_{int}

- Equation 3 can be solved for the frequency, namely

$$x = \left(1 - \frac{r_\infty}{r}\right)^\beta \tag{4}$$

- thus

$$x^{-\beta} = 1 - \frac{r_\infty}{r}$$

$$r(1 - x^{-\beta}) = r_\infty$$

  thus

$$r = \frac{r_\infty}{1 - x^{-\beta}} \tag{5}$$

- then the calculation of the optical depth proceeds as follows:

$$\tau = \frac{\texttt{xk0}}{rv^{2-\alpha}(1 + \texttt{muein}^2\sigma)} \tag{6}$$

  where

  - $v = \left(1 - \frac{b}{r}\right)^{\beta}$
  - $\frac{dv}{dr} = \beta \frac{b}{r^2} \left(1 - \frac{b}{r}\right)^{\beta - 1}$
  - $\sigma = \frac{dv}{dr}\frac{r}{v} - 1$

4.3.3 Second adaptation: isotropic scattering

What would happen to line-profile, is you assumed scattering !was isotropic (i.e., NOT following Sobolev-distrobution)

- test case number 2

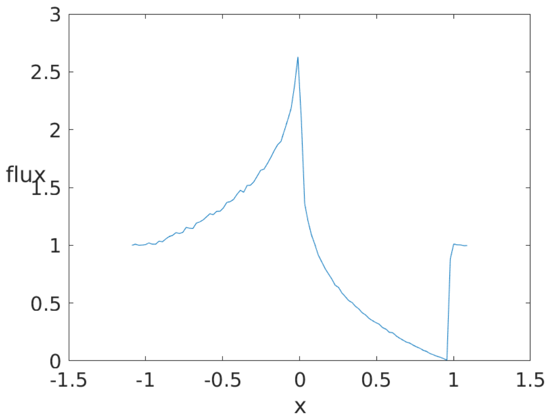

Figure 6: Second adaptation

The *grosso modo* form has not changed, although the scaling has changed.

4.3.4 Third adaptation: introduction of Eddington limb-darkening

Put Eddington limb-darkening in. What happens?

General discussion: Eddington limb darkening The data are taken from Christensen, 2015.

- the source function $S =< I >= a + b\tau_\nu$ with $a = \frac{\sigma}{2\pi}T^4_{eff}$ and $b = \frac{3\sigma}{4\pi}T^4_{eff}$
- solve the equation
- this yields $\frac{I(\theta)}{I(0)} = \frac{a + b\cos(\theta)}{a+b} = \frac{2}{5} + \frac{3}{5}\cos(\theta)$

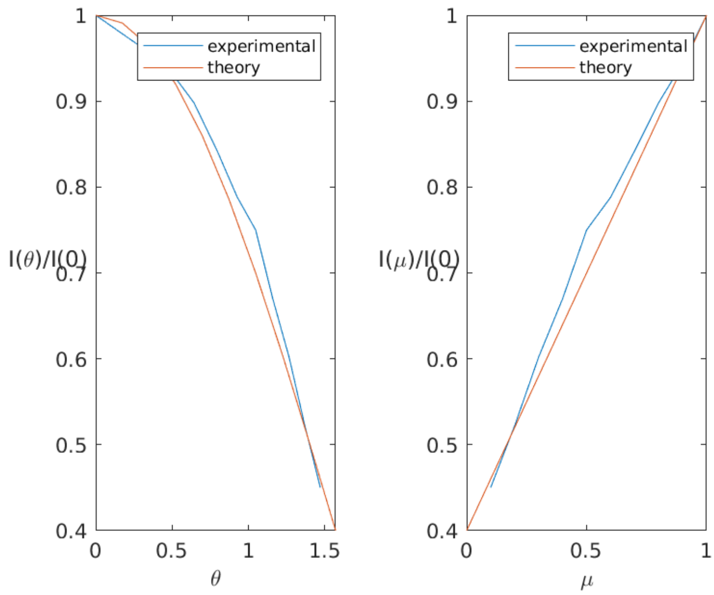

Figure 7: Eddington limb darkening (two times the same plot with $\mu = \cos(\theta)$

for which star are the exerpimental data and what assumptions are used in the theory?

Let us thus first review the emmission case where the flux in each directino is isotropic (paragraph 4.3.3) $I(\theta) = I$

- the specific intensity $I_\nu(\mu) = \frac{dE_\nu}{\cos(\theta)dAdtd\nu d\Omega} = \frac{dE_\nu}{\mu dAdtd\nu d\Omega}$
- the flux $F_\nu = \int_\Omega I_\nu \cos(\theta)d\Omega$ is in this case isotropic thus

$$\xi = \int_0^\mu F_\nu d\mu = \int_0^\mu \int_\Omega I_\nu \cos(\theta)d\Omega d\mu = A\int_0^\mu \mu d\mu \tag{7}$$

with the condition that μ satisfies a probability distribution:

$$1 = \int_{-1}^1 F_\nu d\mu = \int_{-1}^1 \int_\Omega I_\nu \cos(\theta)d\Omega d\mu = \frac{A}{2} \tag{8}$$

thus $A = 2$.

Application to exercise Now we look at a new case where the photons need to be emitted following a distrubution that corresponds to $I(\theta) = I(0)(0.4 + 0.6\cos(\theta))$. In the code this corresponds to `test_number = 3`.

- in this case the flux $F_\nu = \int_\Omega I_\nu \cos(\theta) d\Omega$ is also isotropic but satisfies

$$F_\nu = \int_\Omega I_\nu(0)[0.4 + 0.6\cos(\theta)]\cos(\theta) d\Omega \tag{9}$$

$$\xi = \int_0^\mu F_\nu d\mu = A \int_0^\mu (0.4 + 0.6\mu)\mu d\mu \tag{10}$$

subject to the condition -very similar to Equation 8 - that

$$1 = \int_{-1}^1 F_\nu d\mu = \frac{2A}{5} + \frac{A}{3} = \frac{11A}{15} \tag{11}$$

thus $A = \frac{15}{11}$

4.3.5 Fourth adaptaion: photospheric line-profile

Challening: Put photospheric line-profile (simple Gaussian) in !What happens? Test on xk0=0 (opacity =0) case.

- test case number 4 (not yet implemented)

4.3.6 Convergence analysis

5 Mass loss from inhomogeneous hot star winds (Sundqvist)

- GOAL: synthesis of UV resonance lines from inhomogeneous 2D winds
  - clumped in density
  - clumped in velocity
  - effects of non-void inter-clump medium
- WIND MODELS
  - symmetry assumptions
    * 1D: spherical symmetry
    * 2D: symmetry in Φ
  - models
    1. time-dependent radiation-hydrodynamic from Puls and Owocki (POF)
       * 1D
       * isothermal flow
       * perturbations triggered by photospheric sound waves
    2. time-dependent radiation-hydrodynamic from Feldmeier (FPP)
       * 1D
       * treatment of energy equation
       * perturbations triggered by photospeheric sound waves or Langevin perturbagions (photospheric turbulence)
    3. stochastic model, clumped in density
       * smooth winds with $v_\beta = (1 - b/r)^\beta$ with $\beta = 1$
       * clumping factor f_{cl}
    4. stochastic model, clumped in density and in velocity (non-monotonic velocity field)
       * smooth winds with $v_\beta = (1 - b/r)^\beta$ with $\beta = 1$
       * clumping factor f_{cl}
- RADIATIVE TRANSFER (MC-2D)

6 Asymptotic preserving Monte Carlo methods for radiative transfer equation in diffusion limit (Dimarco+ 2018)

6.1 Goldstein-Taylor

6.2 Radiative transfer