1 overview

- limb-darkening scattering exercise we did during the course. — You can look into your notes from that, and I attach here also a sample program which you can use a base.
- After you have familiarised yourself with this, you can start to think bout how you would go about to extend this to a 3D setting (assuming isotropic scattering).
- (As prep for Monte-Carlo school) here is a script computing a UV resonance P-Cygni line in spherically symmetric wind with v beta-law. At top of routine, a few exercises are given, where you can modify and play around with code. Monte-Carlo program which computes a UV resonance spectral line from a fast outflowing spherically symmetric stellar wind (if you were not cc'd on that email, let me know so that I can send you the files as well). At the top of that little script, there are a few suggestions for exercises (additions) you could do to that program, in order to learn a bit more about the general workings of Monte-Carlo radiative transfer in this context. — So that might be a good idea for you to do as well ! (And you can also ask the others in the group for some tips etc. then.)
- Some background reading:
  - Attached mc manual by Puls.
  - Paper by Sundqvist+ 2010 (Appendix, I think).
- organise meeting

2 questions

- pcyg.f90: what is the paramter p?
- what is Sobo-distribution?
- what is meant by Eddington limb-darkening?
- Sundqvist+2009: slice: what is the geometry?

3 Solved questions

- Sundqvist + 2009: what is thermal velocity (see Wikipedia)
- what is line force, appears in Sundqvist + 2009 (see explanation Dylan)
- what is a flux limiter? (see course notes)
- what is cross section of scattering (see Wikipedia)
- Puls: p.26: how does the Milne equation appear? (see library book)

4 pcyg.f90

4.1 Overview of variables

name	explanation	scope
paramaters		
xk0		
alpha		
beta		
start frequency of the photon		
xstart	start frequency	
vmin		
vmax		
angle of the photon		
xmuestart	start angle	
xmuein	incident angle	
xmueou	outward angle	
pstart	impact parameter	
xnew	new photon frequency	
optical depth		
tau	optical depth	
number of photons admin		
nphot	number of photons	
nin	photons scattered back into core	
nout	photons escaped	
functions		
func	velocity profile	
	r	distance from center of star
xmueout	sign of outwards angle xk0 alpha r v sigma	

Finally the plot contains flux(i)-freq(i)

4.2 Exercises

4.2.1 Original code

In original version of the code, all photons are released radially from photosphere, thus `xmuestart` $= 1$

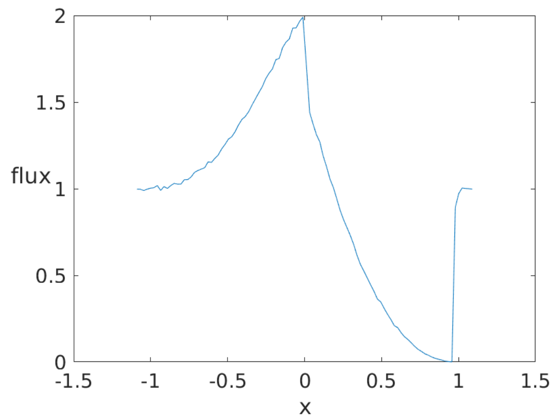

Figure 1: Original version of the code

4.2.2 First adaption: what if all photons are released radially from photosphere?

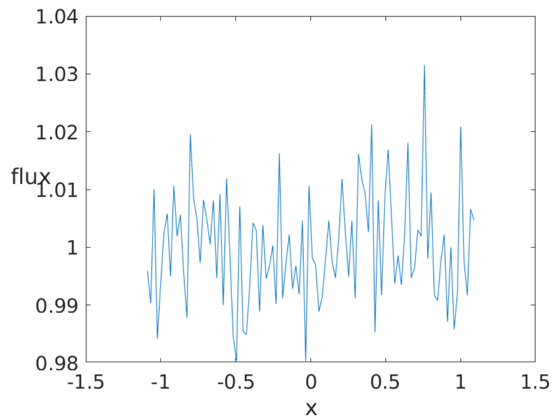

Figure 2: First adaption

4.2.3 Second adaption: isotropic scattering

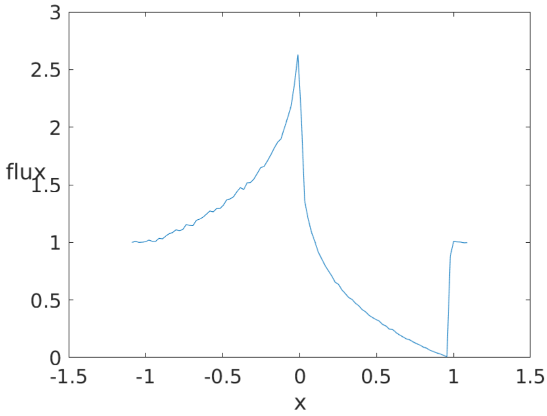

Figure 3: Second adaption

4.2.4 Third adaption: introduction of Eddington limb-darkening

Eddington limb darkening

- the source function $S =< I >= a + b\tau_\nu$ with $a = \frac{\sigma}{2\pi}T_{eff}^4$ and $b = \frac{3\sigma}{4\pi}T_{eff}^4$
- solve the equation
- this yields $\frac{I(\theta)}{I(0)} = \frac{a + b\cos(\theta)}{a + b} = \frac{2}{5} + \frac{3}{5}\cos(\theta)$

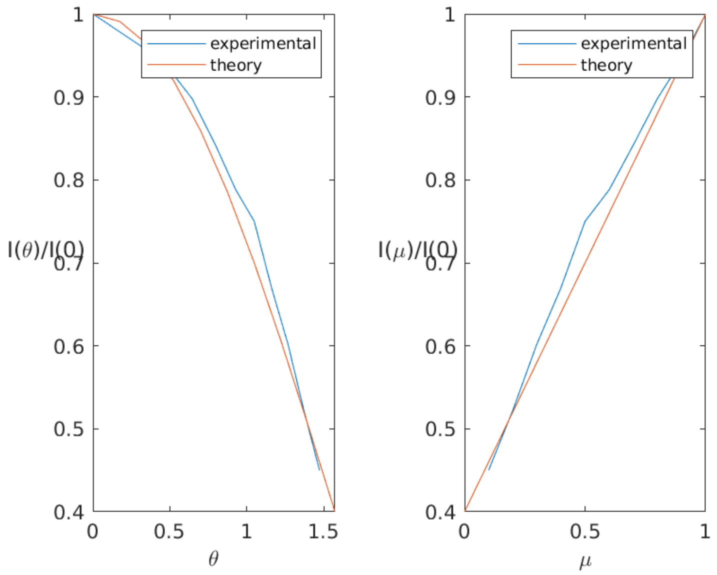

Figure 4: Second adaption

4.2.5 Fourth adaptaion: photospheric line-profile

.

5 Glossary

- (spectral) line-force:
- SED (spectral energy distribution)

6 Very broad introduction: Radiation Hydrodynamics

The material here originates from the master thesis of Nicolas Moens and the course notes *Introduction to numerical methods for radiation in astrophsyics* from professor Sundqvist.

Heat flux diffusion equation $u_t = u_{xx}$. The flux

Specific intensity and its angular moments

specific intensity	$\Delta\epsilon = \boxed{I_\nu} A_1 A_2 / r^2 \Delta\nu \Delta t$
energy density	$E = \frac{1}{c} \iint I_\nu d\nu d\Omega$
flux vector	$F = \iint I_\nu n d\nu d\Omega$
pressure tensor	$P = \iint I_\nu n n d\nu d\Omega$
mean intensity	$J_\nu = \frac{c}{4\pi} E_\nu$
Eddington flux	$H_\nu = \frac{1}{4\pi} F_\nu$
Eddington's K	$K_\nu = \frac{c}{4\pi} P_\nu$

RHD equations The full RHD equations consist of

- five partial differential equations
- one HD closure equation, e.g. (i) variable Eddington tensor method or (ii) flux limited diffusion

Eddington factor In general, the Eddington factor is a tensor, for 1D systems it is reduced to a scalar.

$$f_\nu = \frac{K_\nu}{J_\nu} = \frac{P_\nu}{E_\nu} \tag{1}$$

- isotropic radiation field
- radiation field stronly peaked in radial (i.e. vertical in cartesian) direction

Radiation transport equations, diffusion, equilibrium

- black body radiation (Planck function $I_\nu = J_\nu = B_\nu$
- in general, extinction(absorption,scattering) and emission

$$\frac{dI_\nu}{ds} = j_\nu - k_\nu I_\nu \tag{2}$$

  – Cartesian coordinates:

$$\boxed{\frac{\partial I_{n,\nu}}{\partial t} \frac{1}{c} + n \nabla I_{n,\nu} = j_\nu - k_{n,\nu} I_{n,\nu}} \tag{3}$$

  – spherical coordinates

  – 1D-problem with only variation along z-axis $\mu \frac{dI}{dz} = j - kI$

  – spherical symmetry $\mu \frac{\partial I}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I}{\partial \mu} = j - kI$

- plane-parallel approximation

$$\boxed{\mu \frac{dI}{dr} = j - kI} \tag{4}$$

The angle μ is constant throughout the computational domain. Dividing by k_ν, this yields

$$\mu \frac{dI}{k_\nu dr} = \mu \frac{dI}{k_\nu dz} = S - I \tag{5}$$

- Oth moment equation: integrate Equation (3) over ν and Ω, i.e. $\int d\nu d\Omega$. Conservation of energy
- first multiply Equation (3) with $\frac{n}{c}$ and then do integration

Radiative Diffusion Approximation

1. Black-body radiation in perfect equilibrium
2. Radiative transfer equation in the *near-surface* limit.

The approximation is the following: replace $\boxed{I = B}$ or $I_\nu = B_\nu$, once but not twice.

$$I_\nu = B_\nu - \mu \frac{dB_\nu}{k_\nu dz} \tag{6}$$

Derive this equation as a random walk of photons!

6.1 Examples of radiation (diffusion equation)

1. Temperature structure in a static stellar atmosphere
2.

6.2 Applications and approximations for radiative forces

- definition of general radiative acceleration vector $g = \frac{1}{\rho c} \int \int n k_\nu I_\nu d\Omega d\nu$
  - continuum Thomson scattering
  - spectral line with extinction
    - furhtermore assume central continuum source
    - then $g_{line} = \frac{F_\nu^0 k_L}{\rho c}$
- Sobolev approximation
- CAK theory

6.3 Recap

optical depth	optical depth along ray
	$\tau_{\mu,\nu} = \int_z^{z_{max}} \frac{\alpha_{nu}(z')}{\mu} dz' = \frac{\tau_\nu(z)}{\mu}$

7 Introduction: course material from CMPAA (Sundqvist)

7.1 EXERCISES: Introduction to numerical methods for radiation in astrophysics

1. introduction
2. radiation quantities
    - exercise p.3:
        - on one hand, we know that $\Delta\epsilon \sim C/r^2$
        - on the other hand, from the definition we know that $\Delta\epsilon = I_\nu A_1 A_2/r^2 \Delta\nu\Delta t$
        - combining these equations shows that I_ν is independent from r
    - exercise p.4:
        -
    - exercise 1:
        - $F_x = \int_0^\pi \left[I_\nu(\theta)\sin^2(\theta) \int_0^{2\pi} \cos(\phi) \right] d\theta d\phi = 0$
        - the same reasoning for $F_y = 0$
    - exercise 2:
        - the equation follows from $d\mu = d\cos(\theta) = \sin(\theta)d\theta$
    - exercise 3:
        - isotropic radiation field (i.e. $I(\mu) = I$) then we have $F_\nu = 2\pi \int_{-1}^{1} I\mu d\mu = 2\pi I \left.\frac{x^2}{2}\right|_{-1}^{1} = 0$
    - exercise 4:
        - $F_\nu = 2\pi \int_{-1}^{1} I(\mu)\mu d\mu = 2\pi \int_{-1}^{0} I_\nu^- \mu d\mu + 2\pi \int_0^1 I_\nu^+ \mu d\mu = 2\pi I_\nu^+$
    - exercise p.7:
        - isotropic radiation field:
            * although the radiation pressure is a tensor, we will denote it as a scalar $P_\nu = \frac{4\pi I_\nu}{c}$
            * the radiation energy density $E_\nu = \frac{12\pi I_\nu}{c}$
            * thus $f_\nu = \frac{1}{3}$
        - very strongly peaked in radial direction (beam): $I_\nu = I_0\delta(\mu - \mu_0)$ with $\mu_0 = 1$
            * pressure tensor $P_{nu} = \frac{1}{c} \int I_0\delta(\mu - \mu_0)nnd\Omega$
            * energy density $E_\nu = \frac{1}{c} \int I_\nu d\Omega$
            * in this case $P_\nu = E_\nu$ thus $f_\nu = 1$
3. radiation transport vs. diffusion vs. equilibrium
    - exercise p. 12: 1D, Cartesian geometry, plane-parallel, frequency-independent and isotropic emission/extinction
        - radiation energy equation
            * The equation follows by integrating Equation (4)
            * By definition, $E = \frac{1}{c} \iint I_\nu d\nu d\Omega$
            * thus $\frac{dE}{dr} = \int (j - kI) d\nu d\Omega$ thus $\boxed{\frac{dE}{dr} = \frac{(j - kI)4\pi(\nu_1 - \nu_0)}{c}}$

- work out the integral taking into account frequency-independent and isotropic coefficients:

- zeroth momentum equations
  - One must also take into account the specific form of the flux vector $F = \iint I_\nu n d\nu d\Omega = 2\pi \int_{-1}^{1} I_\nu(\mu)\mu d\mu$
  - thus $\frac{dF}{dr} = \frac{1}{c}\int (j - kI) n d\nu d\Omega$ thus $\boxed{\frac{dF}{dr} = \frac{(j-kI)4\pi(\nu_1 - \nu_0)n}{c}}$
- first moment equation
  - similar reasoning
  - $\frac{dP}{dr} = \int (j - kI) n.n d\nu d\Omega$ thus $\boxed{\frac{dF}{dr} = \frac{(j-kI)4\pi(\nu_1 - \nu_0)n}{c}}$

- first exercise p. 15
  - $P = \frac{1}{c}\iint I_\nu \mu^2 d\Omega d\nu = \frac{2\pi}{c}\int_\nu \int_{-1}^{1} I_\nu \mu^2 d\mu d\nu = \frac{4\pi}{3c}\int B_\nu d\nu = \frac{aT^4}{3} = \frac{E}{3}$
- second exercise p.15
  - assuming the diffusion limit,
  - flux-weighted mean opacity $\kappa_F = \frac{\int F_\nu \kappa_\nu d\nu}{\int F_\nu d\nu}$
  - Rosseland mean opacity $\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$.
    - in the diffusion limit, $F_\nu = -\frac{4\pi}{3}\frac{dB_\nu}{k_\nu dz}$ thus $\frac{dB_{nu}}{dT} =$
    -
- third exercise p.15

4. the equations of radiation-hydrodynamics
5. numerical techniques for the radiative diffusion approximation
6. applications and approximations for a dynamically important radiative force in supersonic flows
   - exercise p.27: $L_{SOB} = \Delta r = \frac{v_{th}}{dv/dr} = \frac{10[km/s]}{1000[km/s]/R_*} = 0.01R_*$
7. Appendix A: properties of equilibrium black-body radiation
   - exercise p. 29
     - this should be satisfied: $B_\nu d\nu = -B_\lambda d\lambda$ and also $\nu = \frac{c}{\lambda}$
     - this is equivalent to saying that $0 = \nu d\lambda + \lambda d\nu$ or $d\lambda = -\frac{\lambda}{\nu} d\nu$ thus $B_\lambda = \frac{\nu}{\lambda} B_\nu$
     - $B_\lambda(T) = \frac{\nu}{\lambda}\frac{2h\nu^3}{(\lambda\nu)^2}\frac{1}{e^{hc/\lambda kT} - 1} = \frac{2h\nu^2}{\lambda^3}\frac{1}{e^{hc/\lambda kT} - 1} = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$
   - first exercise p.31
     - derive that $\lambda_{max} T = 2897.8[\mu m K]$
     - ...
   - second exercise p.31
     - this is about the spectra of (unknown) stars
   - first exercise p.32
     - see exercise 7
   - second exercise p.32

- BB radiation: $I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kt}-1}$
- the radiative flux for isotropic BB radiation is zero. See also exercise 3. This dus also holds for BB radiation.

- exercise p. 33
  - HR-diagram

8. Appendix B: Simple examples to the radiative transfer equation

- first exercise p. 34
  - start from radiative transport equation $\mu\frac{dI}{ds} = \alpha - \eta I$ in which $\eta = 0$ thus $\boxed{\mu\frac{dI}{ds} = \alpha}$
  - solving the ODE in the general case that $\alpha(s)$ is not constant:
    - integrate the equation $\mu I = \int_0^D \alpha ds$
    - ...
  - second exercise p. 34
    - case $\tau(D) >> 1$: then $I(D) \approx S$
    - case $\tau(D) << 1$: then $I(D) \approx I(0) + S(1-1) = I(0)$
  - first exercise p.35
    - is the plane-parallel approximation valid for the solar photosphere?
  - second exercise p.35
    - goal: find a solution to the equation $\mu\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$ where $I(\tau,\mu)$
    - solution
- second exercise p.35

9. Appendix C: connecting random walk of photons with radiative diffusion model

- exercise p. 38. Computing the average photon mean-free path inside the Sun. $l = \frac{1}{\kappa\rho} = \frac{V_o}{\kappa M_o}[cm]$
- exercise p.39. Computing the random-walk time (diffusion time) for photons

7.2 Implicit 1D solver (20-11-2018)

7.3 ADI 2D Solver

7.4 Area of a circle

7.5 Limb Darkening

See Section 9.1.

8 Computational Methods in Astrophysics: MC and RT (Puls)

8.1 basic definitions and facts

8.2 about random numbers

8.3 MC integration

8.4 MC simulation

Radiative transfer in stellar atmospheres

- GOAL: spatial radiation energy density $E(\tau)$ in an atmospheric layer
  - only photon-electron scattering
  - τ is the optical depth
- Milne's integral equation $\boxed{E(\tau) = \frac{1}{2}\int_0^\infty E(t)E_1(|t-\tau|)dt}$
  - analytical solution $\frac{E(\tau)}{E(0)} = \sqrt{3}(\tau + q(\tau))$
  - MC simulation
    * emission angle
    * optical depth until next scattering event
    * scattering angle
- HOW DOES THIS WORK?

Algorithm 1 Limb darkening: compute quantitiy of photons

create photons
probability distribution for emission angle $\mu = \cos(\theta)$: $\boxed{p(\mu)d\mu = \mu d\mu}$
optical depth until next scattering event: $\boxed{p(\tau)dt \approx e^{-\tau}d\tau}$
isotropic scattering angle at low energies: $\boxed{p(\mu)d\mu \approx d\mu}$
follow all photons until they leave the atmosphere or are scattered back into stellar interior

8.5 Exercise 1: RNG

8.6 Exercise 2: Planck-function

1. analytical method
2. MC method

8.7 limb darkening

See section 9.1.

9 Monte Carlo Radiation Transport

9.1 Limb Darkening

9.1.1 1D Code

We again have $\mu = \cos(\theta)$. The solution of the radiative transfer equation in plane-parallel syummetry with frequency-independent absorption and emission, is

$$I(\mu) = I_1(0.4 + 0.6\mu) \tag{7}$$

In the Monte Carlo code, the photons are sorted according to the direction that they leave the atmosphere.

Goal Calculates the angular dependence of photon's emitted from a plane-parallel, grey atmosphere of radial optical depth `taumax`. The value of `tau` determines the position of the photon

Variables and Algorithm

- `muarray` contains emergent photons
- `na` number of channels
- `dmu` = 1/`na` width of channels
- `nphot` number of photons
- `taumax` maximum optical depth

Algorithm 2 Limb darkening: compute quantitiy of photons

---

initialization
  radial optical depth τ
  direction μ
for all photons **do**
  $\boxed{\tau = \tau_{max}}$
  while `tau` ≥ 0 **do**
    compute scattering angle `mu`
    if tau $\geq$ taumax **then** $\boxed{mu = sqrt(x)}$ (initial distribution)
    else $\boxed{mu = 2 * x = 1}$ (isotropic scattering)
    tau_i = -log(x2)
    tau = tau - tau_i*mu
  end while
  now we know that the photon has left the photosphere
  compute the distribution of all angles `mu` at which the photon left the photosphere
end for
visualisation:

- plot photon numbers from $\mu d\mu$ against `mu`
- plot specific intensity from $d\mu$ against `mu` against

9.1.2 3D Code

What changes is this:

- introduction of a new angle ϕ
- the optical depth has to be updated according to ϕ also

9.2 Introduction to Monte Carlo Radiation Transfer

- (Wood, Wittney, Bjorkman, Wolff - 2001)
- (Wood, Wittney, Bjorkman, Wolff - 2013)

9.2.1 Elementary principles

specific intensity	I_ν
radiant energy	dE_ν
surface area	dA
angle	θ
solid angle	$d\Omega$
frequency range	$d\nu$
time	dt
flux	F_ν
cross section	σ
scattering angle	χ $\mu = \cos(\chi)$
mean intensity	J
flux	H
radiation pressure	K

intensity	$I_\nu(l) = I_\nu(0)e^{n\sigma l}$
angular phase function of the scattering particle	$P(\cos(\chi))$

inverse method	$\xi = \int_0^{x_0} P(x)dx$ with $\xi \in \mathcal{U}(0,1)$
rejection method	

9.2.2 Eddington factors

9.2.3 Example: plane parallel atmosphere

1. emission of photons: select two angles (3D space). In isotropic scattering
   - θ met $\mu = \cos(\theta)$
     - $\mu = 2\xi - 1$ (isotropic scattering)
     - $\mu = \sqrt{\xi}$ (A slab is heated from below. Then $P(\mu) = \mu$)
   - $\phi = 2\pi\xi$
2. propagation of photons
   - sample optical depth from $\tau = -\log(\xi)$
   - distance travelled $L = \frac{\tau z_{max}}{\tau_{max}}$
3. conclusion of emission and propagation

$$\begin{aligned} x &= x + L\sin(\theta)\cos(\phi) \\ y &= y + L\sin(\theta)\sin(\phi) \\ z &= z + L\cos(\theta) \end{aligned} \tag{8}$$

4. Binning: once the photon exists the slab. Produce histograms of the distribution function. Finally, we wish to compute the output flux or the intensity.

I have seen that a newer version of the paper is available, which was also used in these notes (which contains amongst other up-to-date references to code fragments).

A Plane Parallel, Isotropic Scattering Monte Carlo Code

9.3 Monte Carlo Radiative Transfer

From a macroscopic perspective, RT calculations rest on the transfer equation

- emissivity η (how much energy is added to radiation field due to emission)
- opacity χ (how much energy is removed due to absorption)
- the source function $S = \frac{\eta}{\chi}$
- optical depth τ captures the opaqueness of a medium

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \nabla.n\right) I = \eta - \chi I \tag{9}$$

$$d\epsilon = I d\nu dt d\Omega dA.n \tag{10}$$

9.4 P Cygni profile for beta-velocity law and given opacity Monte Carlo simulation

9.4.1 Structure of the code

- module common
- module my_inter
- program pcyg
  - INPUT xk0, alpha, beta
  - OUTPUT
  - PROGRAM FLOW: loop over all photons
    - get xstart and vstart
    -
  - then do normalisation
- function func(r)
- function xmueout(xk0,alpha,r,v,sigma)
- function rtbis(func,x1,x2,xacc)

10 Mass loss from inhomogeneous hot star winds (Sundqvist)

- GOAL: synthesis of UV resonance lines from inhomogeneous 2D winds
  - clumped in density
  - clumped in velocity
  - effects of non-void inter-clump medium
- WIND MODELS
  - symmetry assumptions
    * 1D: spherical symmetry
    * 2D: symmetry in Φ
  - models
    1. time-dependent radiation-hydrodynamic from Puls and Owocki (POF)
       * 1D
       * isothermal flow
       * perturbations triggered by photospheric sound waves
    2. time-dependent radiation-hydrodynamic from Feldmeier (FPP)
       * 1D
       * treatment of energy equation
       * perturbations triggered by photospeheric sound waves or Langevin perturbagions (photospheric turbulence)
    3. stochastic model, clumped in density
       * smooth winds with $v_\beta = (1 - b/r)^\beta$ with $\beta = 1$
       * clumping factor f_{cl}
    4. stochastic model, clumped in density and in velocity (non-monotonic velocity field)
       * smooth winds with $v_\beta = (1 - b/r)^\beta$ with $\beta = 1$
       * clumping factor f_{cl}
- RADIATIVE TRANSFER (MC-2D)

11 The mathematics of Radiative Transfer

11.1 Auxiliary mathematics

- $\cos(\Theta) = cos(\theta)\cos(\theta') + \sin(\theta)\sin(\theta')\cos(\phi - \phi')$
- phase function $\boxed{p(\mu, \phi, \mu', \phi', \tau) = \sum_{n=0}^{N} \omega_n P_n(\cos(\Theta))}$
  - isotropic scattering $p(\tau) = \omega_0(\tau)$
- equation of transfer $\boxed{\mu \frac{\partial I(\tau, \mu, \phi)}{\partial \tau} = I(\tau, \mu, \phi) - \mathcal{S}(\tau, \mu, \phi)}$
  with $\mathcal{S}(\tau, \mu, \phi) = B_1(\tau) + \frac{1}{4\pi} \int_{-1}^{1} d\mu' \int_0^{2\pi} I(\tau, \mu', \phi') p(\mu, \phi, \mu', \phi') d\phi'$
  - axially symmetric with isotropic scattering
    $\mathcal{S}(\tau) = \frac{\omega_0(\tau)}{2} \int_{-1}^{1} I(\tau, \mu') d\mu' = B_1(\tau) + \frac{\omega_0(\tau)}{2} \int_0^{\tau_1} \mathcal{S}(t) E_1(|t - \tau|) dt$
  - the Milne equation of the problem $(1 - \omega_0 \bar{\Lambda})\{$
    $mahtcal S(t)\} = B(\tau)$
    * solve for $\mathcal{S}(t)$
    * then find $I(\tau, \mu)$

11.2 The H-functions

- characteristic equation

12 Overview of symmetry assumptions

plane-parallel	1D atmosphere bounded by horizontal surfaces	

13 Equation meetings

- 10 April 2019
- 17 April 2019
- 14 August 2019

14 Challenges in Radiative Transfer (Ivan Milic)

14.1 Overview of the problem

$$\xrightarrow{\quad I^*_\lambda \quad} \boxed{T(\tau)\ ,\ \rho(\tau)\ ,\ \vec{B}(\tau)\ ,\ \vec{v}(\tau)} \xrightarrow{\quad I^+_\lambda \quad}$$

Forward problem

The forward problem is schematically represented

$$\xrightarrow{\quad \vec{T}, \rho, \vec{B}, \vec{v} \quad} \boxed{\begin{array}{c}\text{forward problem}\\ I^+_\lambda = F(\vec{T}, \rho, \vec{B}, \vec{v})\end{array}} \xrightarrow{\quad I^+_\lambda \quad}$$

In fact solve for intensity vector $\vec{I} = \begin{pmatrix} I \\ Q \\ \alpha \\ V \end{pmatrix}$ obeying the equation

$$\frac{d\vec{I}}{d\tau} = -X(\vec{T}, \rho, \vec{B}, \vec{v})\vec{I} - \vec{j}(\vec{T}, \rho, \vec{B}, \vec{v}) \tag{11}$$

and the solution

$$I^+_\lambda = I^+_0 e^{-\int} + \int \vec{j} e^{-\int} d\tau \tag{12}$$

> **Example** Source function $S = a\tau + b$ then $\int_0^{\tau_{max}} (a\tau + b)e^{-\tau} d\tau = ...$

Inverse problem

The inverse problem is schematically represented

$$\xrightarrow{\quad I^+_\lambda \quad} \boxed{\begin{array}{c}\text{inverse problem}\\ F^{-1}(I^{obs}_\lambda)\end{array}} \xrightarrow{\quad \vec{T}, \rho, \vec{B}, \vec{v} \quad}$$

Via least-squares approximation

$$\min_{\vec{T}, \rho, \vec{B}, \vec{v}} \sum \left(I^{obs}_\lambda - I_\lambda(\vec{T}, \rho, \vec{B}, \vec{v}) \right)^2 \tag{13}$$

14.2 Challenging domains of application

- Lyman alpha in Galaxy Halos
- Dusty torii (AGD)
- protoplanetary disks
- circumstellar disks
- athmospheres

15 Vragen aan professor Samaey

- what is the difference between Monte Carlo and equation-free computing?