1 Overview of exercises (PART I)

1. limb-darkening scattering exercise we did during the course. — You can look into your notes from that, and I attach here also a sample program which you can use a base. After you have familiarised yourself with this, you can start to think bout how you would go about to extend this to a 3D setting (assuming isotropic scattering).

2. (As prep for Monte-Carlo school) here is a script computing a UV resonance P-Cygni line in spherically symmetric wind with v beta-law. At top of routine, a few exercises are given, where you can modify and play around with code. Monte-Carlo program which computes a UV resonance spectral line from a fast outflowing spherically symmetric stellar wind (if you were not cc'd on that email, let me know so that I can send you the files as well). At the top of that little script, there are a few suggestions for exercises (additions) you could do to that program, in order to learn a bit more about the general workings of Monte-Carlo radiative transfer in this context. — So that might be a good idea for you to do as well ! (And you can also ask the others in the group for some tips etc. then.)

3. Some background reading:
   - Attached mc manual by Puls.
   - Paper by Sundqvist+ 2010 (Appendix, I think).

2 Overview of exercises (PART II)

1. Calculate the probability distribution to sample from in the case of Eddington limb darkening for the initial distribution (see 4.3.4).

2. Calculate analytical solution for simplified problem in 4.3.2 in the case that `mu = 1`.

3. Perform convergence analysis. See Section 4.3.6

3 Limb darkening

3.0.1 2D Case

We again have $\mu = \cos(\theta)$. The solution of the radiative transfer equation in plane-parallel syummetry with frequency-independent absorption and emission, is

$$I(\mu) = I_1(0.4 + 0.6\mu) \tag{1}$$

In the Monte Carlo code, the photons are sorted according to the direction that they leave the atmosphere.

Goal Calculates the angular dependence of photon's emitted from a plane-parallel, grey atmosphere of radial optical depth `taumax`. The value of `tau` determines the position of the photon

Variables and Algorithm

- `muarray` contains emergent photons
- `na` number of channels
- `dmu` = 1/`na` width of channels
- `nphot` number of photons
- `taumax` maximum optical depth

Algorithm 1 Limb darkening: compute quantitiy of photons

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initialization
  radial optical depth τ
  direction μ
for all photons **do**
  $\boxed{\tau = \tau_{max}}$
  while `tau` ≥ 0 **do**
    compute scattering angle `mu`
    if tau ≥ taumax **then** $\boxed{mu = sqrt(x)}$ (initial distribution)
    else $\boxed{mu = 2 * x = 1}$ (isotropic scattering)
    tau_i = -log(x2)
    tau = tau - tau_i*mu
  end while
  now we know that the photon has left the photosphere
  compute the distribution of all angles `mu` at which the photon left the photosphere
end for
visualisation:

- plot photon numbers from $\mu d\mu$ against `mu`
- plot specific intensity from $d\mu$ against `mu` against

Figure 1 is according to what is expected $I = I_0(0.4 + 0.6\mu)$

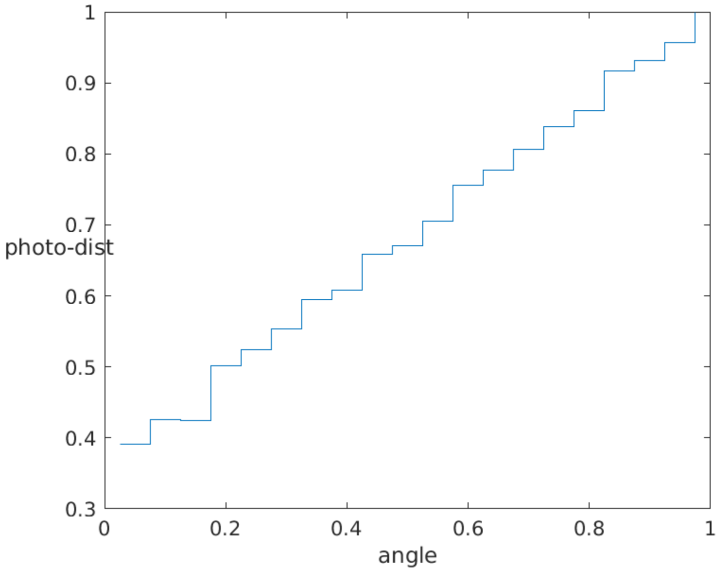


Figure 1: histogram for `mu`

3.0.2 3D Code

What changes is this:

- introduction of a new angle ϕ
- the optical depth has to be updated according to ϕ also

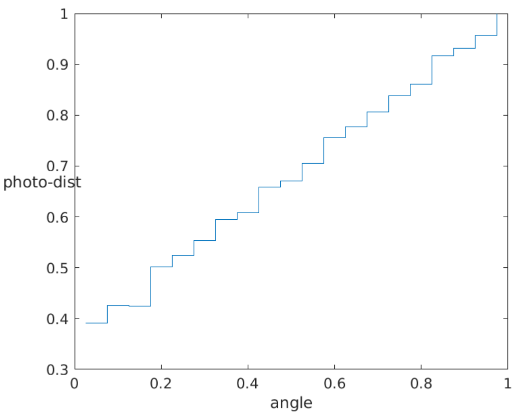


Figure 2: histogram for `mu`

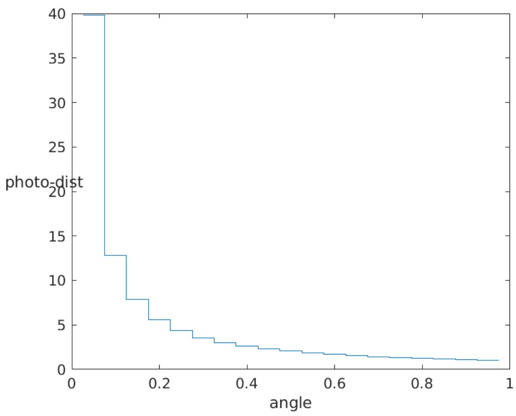


Figure 3: histogram for `phi`

Figure 2 and Figure 3 are according to what is expected, namely $I = I_0(0.4 + 0.6\mu)$ and a uniform distribution for *phi*, which corresponds to a $I \sim \dfrac{1}{\phi}$

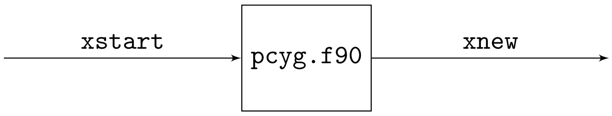
4 Investigation of program: pcyg.f90

4.1 Overview of variables

name	explanation
paramaters	
xk0	
alpha	velocity profile parameter
beta	velocity profile parameter
start frequency of the photon	
xstart	start frequency
vmin	
vmax	
angle of the photon	
xmuestart	start angle
xmuein	incident angle
xmueou	outward angle
pstart	impact parameter
xnew	new photon frequency
optical depth	
tau	optical depth
number of photons admin	
nphot	number of photons
nin	photons scattered back into core
nout	photons escaped
functions	
func	velocity profile distance from center of star r
xmueout	outwards (scattered) angle xk0 alpha r v sigma

4.2 Mathematical things that are noteworthy

4.2.1 General working



The photons are sorted according to `xnew`. In general, the flux is dependent on μ and the frequency x.

make formula

- I think that it satisfies $N(x)dx \sim I(x)xdx$
- We are thus interested in $F_\lambda = F_\nu$

4.2.2 Practical formula

- emission angle $\mu = \cos(\theta)$
- according p-ray $p = \sqrt{1-\mu^2} = \sin(\theta)$
- incident angle `xmuein` $= \sqrt{1 - \left(\frac{pstart}{r}\right)^2}$

4.2.3 Geometry & Symmetry assumptions

- spherical geometry

4.3 Exercises

4.3.1 Investigation of original code

In original version of the code, all photons are released isotropially from the photosphere.

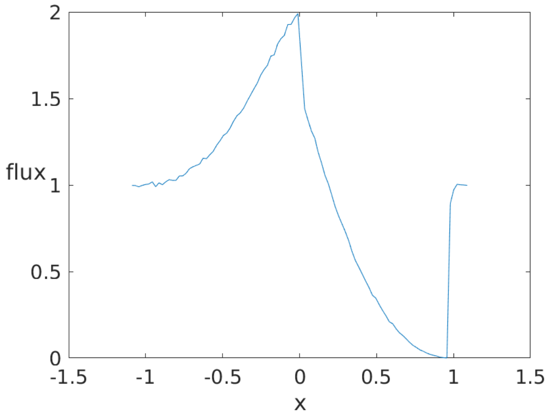
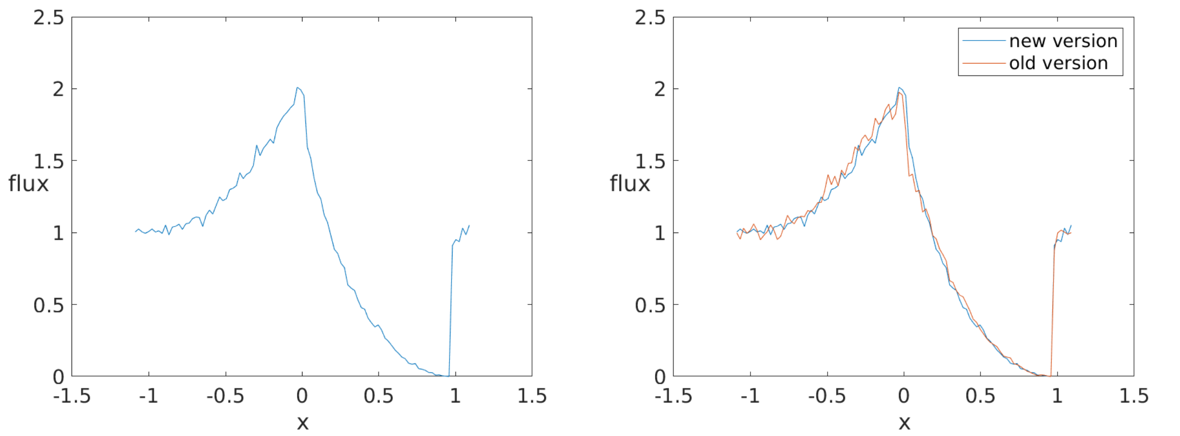


Figure 4: Original version of the code

4.3.2 First adaptation: what if all photons are released radially from photosphere?

Release photons radially: numerical MC experiments What would happen with line-profile, if you assumed all photons were released radially from photopshere?

- In other words `xmuestart` $= 1$. Results in Figure **??**.
- This is implemented under the test case `test_number=1`.



(a) First adaptation

(b) Same plot (together with output of initial version)

Figure 5: The number of photons equals 10^5

Derive analytic expression See also slide 26/49 [Sundqvist course material].

- since `xmuein = 1` we have for the velocity profile

$$v = v_\infty (1 - b/r)^\beta \tag{2}$$

A scaled version of the Equation (2) yields

$$u = \frac{v(r)}{v_\infty} = \left(1 - \frac{r_\infty}{r}\right)^\beta \tag{3}$$

with $u \in [0..1]$

- Doppler shift for the frequency of the photons: $x_{CMF} = x_{REF} - \mu u$.
- Condition for resonance from Sobolov approximation (to be studied later): $\boxed{x_{CMF} = 0}$ thus

$$x_{REF} = \mu u \tag{4}$$

or thus $x_{REF} = \boxed{u_{\text{interaction}}}$ and than solve Equation 3 for $r_{\text{interaction}}$

- If $\mu = 1$ then

$$x = \left(1 - \frac{r_\infty}{r}\right)^\beta \tag{5}$$

$$x^{-\beta} = 1 - \frac{r_\infty}{r}$$

$$r(1 - x^{-\beta}) = r_\infty$$

$$\boxed{r(x) = \frac{r_\infty}{1 - x^{-\beta}}} \tag{6}$$

- From the location of interaction r, the incident angle can be calculated

$$\texttt{xmuein} = \sqrt{1 - \left[\frac{\texttt{pstart}}{r}\right]^2} = \sqrt{1 - \left[\frac{\sqrt{1 - \texttt{xmuestart}^2}}{r}\right]^2} \tag{7}$$

Now also taking into account that `xmuestart = 1` then yields

$$\texttt{xmuein = 1} \tag{8}$$

- The calculation of the optical depth goes as follows:

$$\tau = \frac{\texttt{xk0}}{rv^{2-\alpha}(1 + \texttt{xmuein}^2\sigma)} \tag{9}$$

Now also taking into account that `xmuestart = 1` gives

$$\tau = \frac{\texttt{xk0}}{rv^2(1 + \sigma)} \tag{10}$$

where $\boxed{v(x) = \left(1 - \frac{b}{r}\right)^\beta}$ and $\frac{dv}{dr} = \frac{\beta b}{r^2}\left(1 - \frac{b}{r}\right)^{\beta-1}$

and $\sigma(x) = \frac{dv}{dr}\frac{r}{v} - 1$ thus $\boxed{\sigma(x) = \frac{\beta b}{r}\left(1 - \frac{b}{r}\right)^{-1}}$

- Conclusion: $\tau(x)$ is only dependent on x and not on `xmuestart` or `xmuein`.
- `xmueou` follows the distribution as given by the function `xmueout`, namely

$$p(x) = \frac{1 - e^{-\tau}}{\tau} \tag{11}$$

with $\tau = \frac{\texttt{tau0}}{1 + \texttt{X}^2\sigma}$ where `X` is a random number, so actually this comes down to

$$\boxed{p(x) = \frac{1 - e^{-\frac{\tau_0}{1+x^2\sigma(x)}}}{\frac{\tau_0}{1+x^2\sigma(x)}}} \tag{12}$$

- Finally one can combine these results to get the distribution of the photons according to the frequency x via the relation

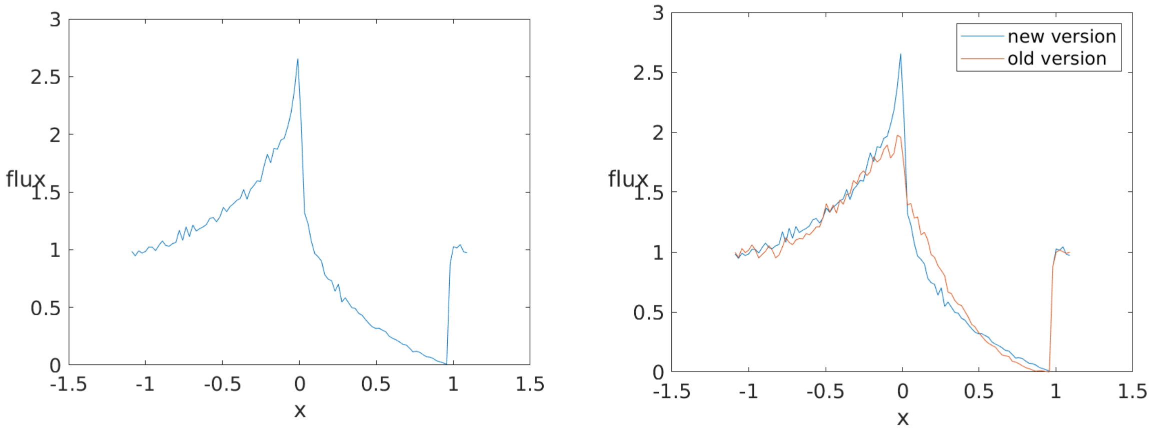
$$\texttt{xnew = xstart + v(xmueou-xmuein) = xstart + v(xmueou -1)} \tag{13}$$

Via this link, you can go back to the exercises overview: Section 2.

4.3.3 Second adaptation: isotropic scattering

What would happen to line-profile, is you assumed scattering was isotropic
(i.e., NOT following Sobolev-distrobution)

- test case number 2



(a) Second adaptation

(b) Same plot (together with output of initial version)

Figure 6: The number of photons equals 10^5

The *grosso modo* form has not changed, although the scaling has changed.

4.3.4 Third adaptation: introduction of Eddington limb-darkening

Put Eddington limb-darkening in. What happens?

General (introductory) discussion: Eddington limb darkening The data are taken from Christensen, 2015.

- the source function $S =< I >= a + b\tau_\nu$ with $a = \frac{\sigma}{2\pi}T_{eff}^4$ and $b = \frac{3\sigma}{4\pi}T_{eff}^4$
- solve the equation
- this yields $\frac{I(\theta)}{I(0)} = \frac{a + b\cos(\theta)}{a+b} = \frac{2}{5} + \frac{3}{5}\cos(\theta)$

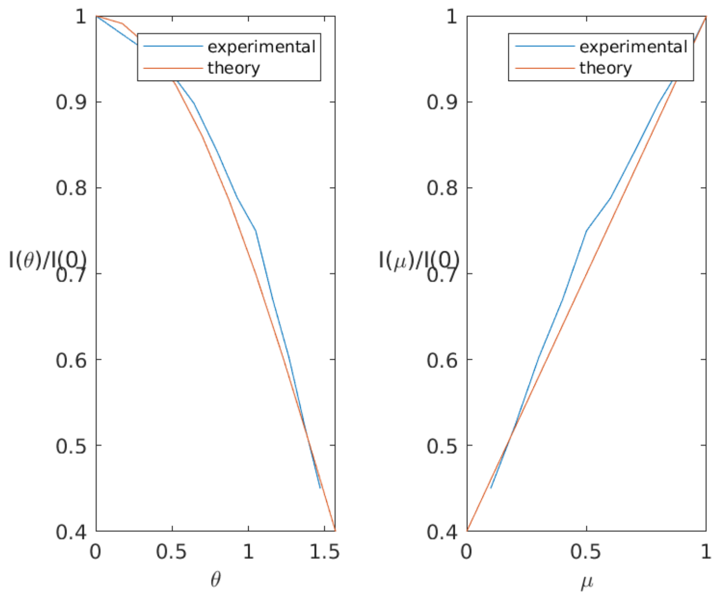


Figure 7: Eddington limb darkening (two times the same plot with $\mu = \cos(\theta)$

Construction of probability distribution corresponding to Eddington limb darkening

1. Let us thus first review the emmission case where the flux in each direction is isotropic i.e. $I(\theta) = I$ (as experimented in paragraph 4.3.3)

   - the specific intensity is defined as $I_\nu(\mu) = \frac{dE_\nu}{\cos(\theta)dAdtd\nu d\Omega} = \frac{dE_\nu}{\mu dAdtd\nu d\Omega}$
   - the flux $F_\nu = \int_\Omega I_\nu \cos(\theta)d\Omega$ is in this case isotropic thus

   $$\xi = \int_0^\mu F_\nu d\mu = \int_0^\mu \int_\Omega I_\nu \cos(\theta)d\Omega d\mu = A\int_0^\mu \mu d\mu \tag{14}$$

   together with the condition that μ satisfies a probability distribution:

   $$1 = \int_{-1}^1 F_\nu d\mu = \int_{-1}^1 \int_\Omega I_\nu \cos(\theta)d\Omega d\mu = \frac{A}{2} \tag{15}$$

   thus $A = 2$. Photons need to be sampled according to $\mu d\mu$.

2. Now we look at a new case where the photons need to be emitted following a distrubution that corresponds to $I(\theta) = I(0)(0.4 + 0.6\cos(\theta))$.

   - in this case the flux $F_\nu = \int_\Omega I_\nu \cos(\theta) d\Omega$ is isotropic but also satisfies

$$F_\nu = \int_\Omega I_\nu(0)[0.4 + 0.6\cos(\theta)]\cos(\theta) d\Omega \tag{16}$$

I am not sure about the correctness of the assumption of isotropy of the flux

$$\xi = \int_0^\mu F_\nu d\mu = A\int_0^\mu (0.4 + 0.6\mu)\mu d\mu \tag{17}$$

subject to the normalisation condition -very similar to Equation (15) - that

$$1 = \int_{-1}^{1} F_\nu d\mu = \frac{2A}{5} + \frac{A}{3} = \frac{11A}{15} \tag{18}$$

thus $A = \frac{15}{11}$. Photons need to be sampled according to

$$(0.4 + 0.6\mu)\mu d\mu \tag{19}$$

In the code `pcyg.f90` this corresponds to `test_number = 3` (not yet implemented).

The results of an accept-reject method that samples the probability distribution in Equation (19).

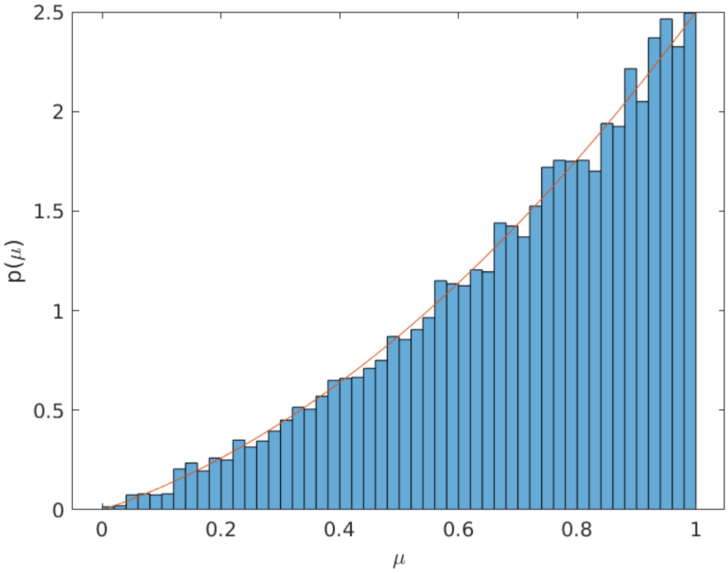


Figure 8: Accept-reject method for Eddington limb darkening

Via this link, you can go back to the exercises overview: Section 2.

4.3.5 Fourth adaptaion: photospheric line-profile

Challening: Put photospheric line-profile (simple Gaussian) in !What happens? Test on xk0=0 (opacity =0) case.

- test case number 4 (not yet implemented)

4.3.6 Convergence analysis

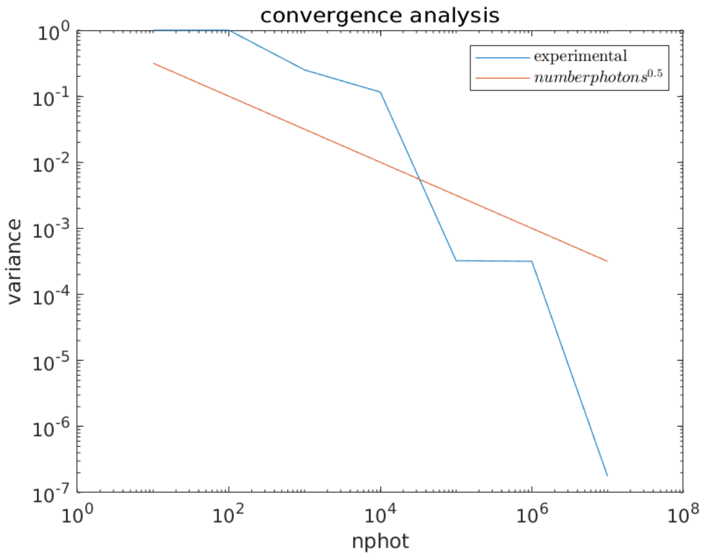


Figure 9: Original version of the code: convergence analysis (xk0=0)

Via this link, you can go back to the exercises overview: Section 2.

5 Milic Exercises

5.1 Lecture 7

1. Derive expressions for the emergent radiation when properties are the following:
   - optically thin slab at all wavelengths
   - wavelength-independent incident radiation

   Solution: see slide 14?

2. Derive ralations between Einstein coefficients.

3. Calculate electron density in atmosphere from FALC model

6 Mass loss from inhomogeneous hot star winds (Sundqvist)

- GOAL: synthesis of UV resonance lines from inhomogeneous 2D winds
  - clumped in density
  - clumped in velocity
  - effects of non-void inter-clump medium
- WIND MODELS
  - symmetry assumptions
    * 1D: spherical symmetry
    * 2D: symmetry in Φ
  - models
    1. time-dependent radiation-hydrodynamic from Puls and Owocki (POF)
       * 1D
       * isothermal flow
       * perturbations triggered by photospheric sound waves
    2. time-dependent radiation-hydrodynamic from Feldmeier (FPP)
       * 1D
       * treatment of energy equation
       * perturbations triggered by photospeheric sound waves or Langevin perturbagions (photospheric turbulence)
    3. stochastic model, clumped in density
       * smooth winds with $v_\beta = (1 - b/r)^\beta$ with $\beta = 1$
       * clumping factor f_{cl}
    4. stochastic model, clumped in density and in velocity (non-monotonic velocity field)
       * smooth winds with $v_\beta = (1 - b/r)^\beta$ with $\beta = 1$
       * clumping factor f_{cl}
- RADIATIVE TRANSFER (MC-2D)

7 Asymptotic preserving Monte Carlo methods for radiative transfer equation in diffusion limit (Dimarco+ 2018)

7.1 Goldstein-Taylor

7.2 Radiative transfer