DS 221 - Assignment 1

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Question 1

We will define the following terms:

- number of products as *pd*,
- the number of hashtags per product as *h*,
- the number of customers as *c*,
- and the number of purchases as *pur*.

For question 1, the time complexity for reading and populating the data structure is as follows. The code is:

```
vector<string> split(const string &s, char delimiter){
    vector<string> tokens;
    string token;
    istringstream tokenStream(s);
    while (getline(tokenStream, token, delimiter)){
        tokens.push_back(token);
    }
    return tokens;
}
```

Which is called in the following way:

```
vector<pair<string,vector<string>>> productHashtags;
while(hashtags.hasNext()){
    string line = hashtags.next();
    vector<string> tokens = split(line,',');
    if (tokens.size() > 1) { // #PROMTP# Ensure
    there are hashtags present
        string product = tokens[0];
        vector<string> prodHashtags(tokens.begin() + 1,
         tokens.end());
        //#PROMPT# Make sure all prodHashtags are unique
        sort(prodHashtags.begin(), prodHashtags.end());
        prodHashtags.erase(unique(prodHashtags.begin(),
        prodHashtags.end()), prodHashtags.end());
        productHashtags.push_back(make_pair(product, prodHashtags));
    }
}

// #PROMPT# Extract all the lines from the purchases file and
store them in an appropriate data structure
vector<pair<string,string>> customerPurchases;
while(purchases.hasNext()){
    string line = purchases.next();
    vector<string> tokens = split(line,',');
    if (tokens.size() == 2) { // #PROMPT# Ensure the line
     has both customer and product
        string customer = tokens[0];
        string product = tokens[1];
        customerPurchases.push_back(make_pair(customer, product));
    }
}
```

The analysis is as follows:

- Best case:
  We first iterate through all the poducts which is $O(pd)$. For each product, we call the split function which is $O(h)$. After this we sort the hashtags which is $O(h \log h)$. After sorting we remove duplicates which is $O(h^2)$, this is because I use the erase and unique function which is both $O(h)$. So the total time complexity for this part is $O(pd(h + h \log h + h^2))$. Next we iterate through all the purchases which is $O(pur)$. For each purchase we call the split function which is $O(1)$. So the total time complexity for this part is $O(pur)$. So the total time complexity for the entire code is $O(pd(h \log h + h^2) + pur)$.

- Worst case:
  The worst case is the same as the best case as the time complexity is the same.

- Average case:
  The average case is the same as the best case as the time complexity is the same.

We now analyze the time complexity of the compute part of the function which forms clusters. The code is as follows:

```
map<string,vector<pair<string,int>>> customerHashtags;
for(auto purchase : customerPurchases){
    string customer = purchase.first;
    string product = purchase.second;
    for(auto prodHashtags : productHashtags){
        if(prodHashtags.first == product){
            for(auto hashtag : prodHashtags.second){
                if(customerHashtags.find(customer) ==
                customerHashtags.end()){
                    vector<pair<string,int>> temp;
                    temp.push_back(make_pair(hashtag, 1));
                    customerHashtags[customer] = temp;
                }
                else{
                    bool found = false;
                    for(auto& tag : customerHashtags[customer]){
                        if(tag.first == hashtag){
                            tag.second++;
                            found = true;
                            break;
                        }
                    }
                    if(!found){
                        customerHashtags[customer].push_back
                        (make_pair(hashtag, 1));
                    }
                }
            }
        }
    }
}

// #PROMTP# Sort the hashtags based on the count and
if count is equal, then hashtags should be ordered
 lexiographically
for(auto& customer : customerHashtags){
    sort(customer.second.begin(), customer.second.end(),
    [](pair<string,int> a, pair<string,int> b){
        if(a.second == b.second){
            return a.first < b.first;
        }
        return a.second > b.second;
    });
}

// #PROMPT# For each customer, get the first k hashtags out
map<string,vector<string>> customerTopKHashtags;
for(auto customer : customerHashtags){
    vector<string> topK;
    for(int i = 0; i < k && i < customer.second.size(); i++){
        topK.push_back(customer.second[i].first);
    }
    customerTopKHashtags[customer.first] = topK;
}

/* #PROMPT# Create a map between the group id (int) and customer ids
(vector of ints). The group id should be unique for each
group. Add the first customer to group 1, check if the
top k hashtags of the second customer are the same as
the first customer, if yes, add the second customer to
group 1, else add the second customer to group 2 and
so on. The map should be group id to vector of
customer ids. */
map<int, vector<int>> customerGroup;
int group = 1;
for(auto customer : customerTopKHashtags){
    bool found = false;
    for(auto& cgroup : customerGroup){
        if(customerGroup[cgroup.first].size() > 0){
            //#PROMTP# sort the hashtags and compare
            vector<string> first = customerTopKHashtags
            [to_string(customerGroup[cgroup.first][0])];
            vector<string> second = customer.second;
            sort(first.begin(), first.end());
            sort(second.begin(), second.end());
            if(first == second){
                customerGroup[cgroup.first].push_back
                (stoi(customer.first));
                found = true;
                break;
        }
    }
    }
    if(!found){
        vector<int> temp;
        temp.push_back(stoi(customer.first));
        customerGroup[group] = temp;
        group++;
    }
}
```

The analysis is as follows:

- Best case:
  We first iterate through all the purchases which is $O(pur)$. For each purchase we iterate through all the products which is $O(pd)$. But in best case it will be $O(1)$ if the first product matches the purchase. For each product we iterate through all the hashtags which is $O(h)$. With each operation we do a find operation which is $O(\log(c))$ as I am using a map. Right after, we insert a value into a map which is $O(\log(c))$. For the best case the following two else and if conditions are constant time. So the total time complexity for this part is $O(pur * 1 * h(\log(c) + \log(c) + 1))$. Next we do a sort operation which is the number of customers * number of hashtags per customer * log(number of hashtags per customer). We can say that the number of hashtags per customer is $pd * h/c$. Hence the time complexity is $O(c * pd * h/c * \log(pd * h/c))$, this is constant in best case, average case, and worst case. Next we iterate through all the customers which is $O(c)$. For each customer we iterate throught the hashtags per customer which is $O(pd * h/c)$ and we then perform an insertion into a map which is $O(\log(c))$ in all cases. Hence the time complexity is $O(c * (pd * h/c + \log(c)))$. Finally we iterate through all the customers which is $O(c)$. For each loop we go through cgroup, which will in the best case be 1(all are part of the same group). We then sort the hashtags which is $O(pd * h/c * \log(pd * h/c))$. We perform another sort operation which is $O(pd * h/c * \log(pd * h/c))$.

Thus the complexity is $O(c*1*(pd*h/c*\log(pd*h/c)))$. We only create a group once which is constant time in the grand scheme of things. Therefore the total best case time complexity is $O(pur*1*h(\log(c)+\log(c)+1)+pd*h*\log(pd*h/c)+c*(pd*h+\log(c))+c*(pd*h/c*\log(pd*h/c)))$. This simplifies into

$$O(pur*h*\log(c)+pd*h*[\log(pd*h/c)+c]+c*\log(c))$$

.

- Average case:
In the average case we follow the same analysis as best case. Iterating through the all the purchases is $O(pur)$. For each purchase we iterate through all the products which is for the average case $O(pd/2)=O(pd)$. For each product we iterate through its hashtags which is $O(h)$. With each iteration we do a find operation which is $O(\log(c))$. We then insert into a map which is $O(\log(c))$. The map data structure has a $O(\log(n))$ time complexity for all cases. In the else operation we go through all the tags of a customer which is on average $O(pd*h/c)$. The total time complexity for this part is $O(pur*pd*h(\log(c)+\log(c)+pd*h/c+1))$. Next we sort the hashtags which is $O(c*pd*h/c*\log(pd*h/c))$. We iterate through all the customers which is $O(c)$. For each customer we iterate through the hashtags per customer which is $O(pd*h/c)$. We then insert into a map which is $O(\log(c))$. The total time complexity for this part is $O(c*(pd*h/c+\log(c)))$. Finally we iterate through all the customers which is $O(c)$. For each loop we go through cgroup, which will in the average case be $c/2$. We then sort the hashtags which is $O(pd*h/c*\log(pd*h/c))$. We perform another sort operation which is $O(pd*h/c*\log(pd*h/c))$. Thus the complexity is $O(c*c/2*(2*pd*h/c*\log(pd*h/c)))$. On the average case, there is $c/2$ times that we create a new group, hence the insertion will be $O(c/2*\log(c/2))$. The total average case time complexity is $O(pur*pd*h(\log(c)+\log(c)+pd*h/c+1)+pd*h*\log(pd*h/c)+c*(pd*h/c+\log(c))+c*c/2*(2*pd*h/c*\log(pd*h/c))+c/2*\log(c/2))$. This simplifies into $O(pur*pd*h(\log(c)+pd*h/c)+pd*h*c*\log(pd*h/c)+c*\log(c))$.

- Worst case:
In the worst case we follow the same analysis as best case. Iterating through the all the purchases is $O(pur)$. For each purchase we iterate

through all the products which is for the $O(pd)$ for all cases. For each product we iterate through its hashtags which is $O(h)$. With each iteration we do a find operation which is $O(\log(c))$. We then insert into a map which is $O(\log(c))$. The map data structure has a $O(\log(n))$ time complexity for all cases. In the else operation we go through all the tags of a customer which is on worst case $O(pd * h)$ as all the hashtags are different. The total time complexity for this part is $O(pur * pd * h(\log(c) + \log(c) + pd * h + 1))$. Next we sort the hashtags which is $O(c * pd * h/c * \log(pd * h/c))$. We iterate through all the customers which is $O(c)$. For each customer we iterate through the hashtags per customer which is $O(pd * h/c)$. We then insert into a map which is $O(\log(c))$. The total time complexity for this part is $O(c * (pd * h/c + \log(c)))$. Finally we iterate through all the customers which is $O(c)$. For each loop we go through cgroup, which will in the worst case be c. We then sort the hashtags which is $O(pd * h/c * \log(pd * h/c))$. We perform another sort operation which is $O(pd * h/c * \log(pd * h/c))$. Thus the complexity is $O(c * c * (pd * h/c * \log(pd * h/c)))$. On the worst case, there is c times that we create a new group, hence the insertion will be $O(c * \log(c))$.
The total worst case time complexity is $O(pur * pd * h(\log(c) + \log(c) + pd * h + 1) + pd * h * \log(pd * h/c) + c * (pd * h + \log(c)) + c * c * (2 * pd * h/c * \log(pd * h/c)) + c * \log(c))$. This simplifies into $O(pur * pd * h(\log(c) + pd * h) + pd * h * [c + c * \log(pd * h/c)] + c * \log(c))$.

Time taken to write the data structure:
Analyzing the code the time taken to write is as follows:

- Best case: $O(1 * T(w))$
- Average case: $O(c * T(w))$
- Worst case: $O(c * T(w))$

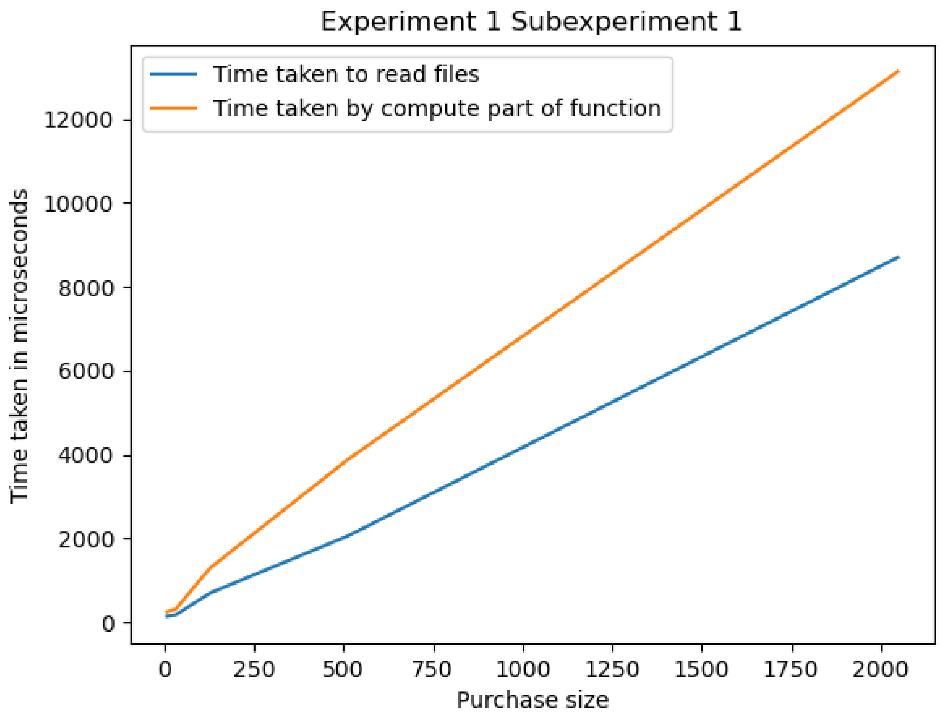
Where $T(w)$ is the time taken for the filewrite operation to write one group into a file.

Time Complexity Results
Tools Used:
I used a python script to generate csv files for the various experiments that I conducted. And to run the *main.cpp* file, I wrote a shell script which helps

me run the experiments for question 1.
To empirically verify these I conducted 5 experiments.
Experiment 1:
Subexperiment 1:

- No of unique products = 5
- No of hashtags per product = 2
- Total unique hashtags = 10
- No of customers = 6

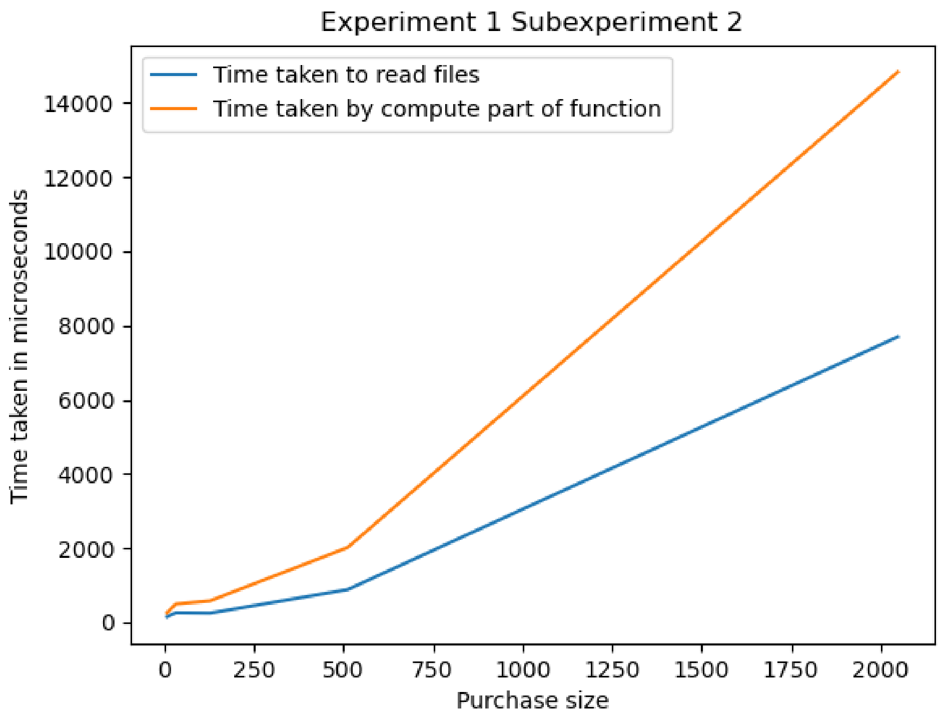
I varied number of purchases between 8 to 2048 to get the following graph:
As everything is constant, the graph is clearly linear.



Subexperiment 2:

- No of unique products = 5
- No of hashtags per product = 4
- Total unique hashtags = 10
- No of customers = 6

I varied number of purchases between 8 to 2048 to get the following graph: At lower purchases the other terms of the complexity are dominating such

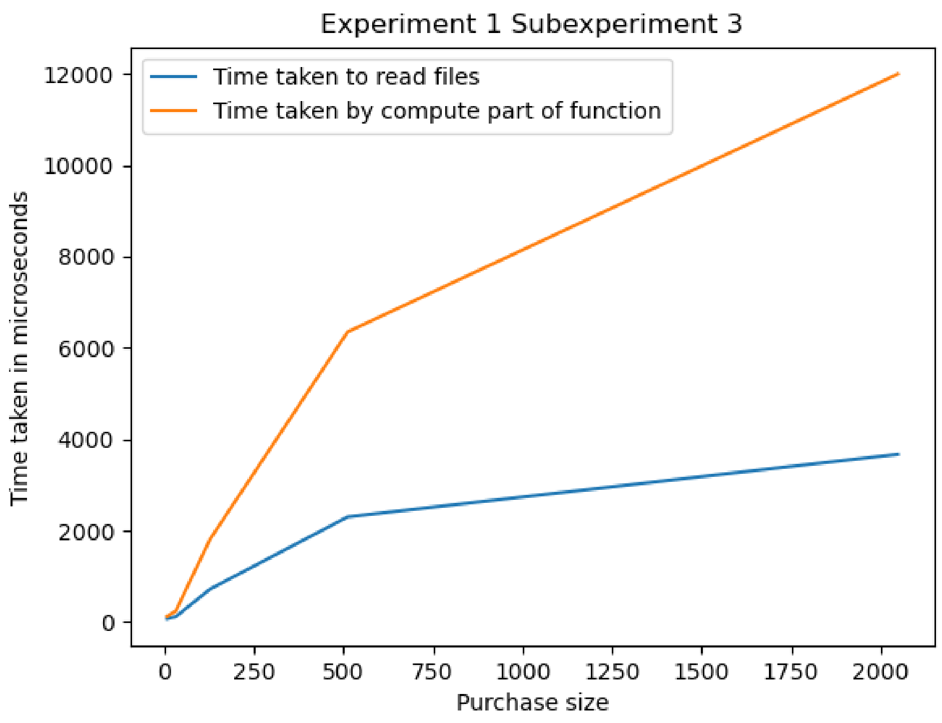


as the $O(pur * pd * h * (log(c) + pd * h/c))$ term, but after a point it becomes linear with purchases as it dominates.

Subexperiment 3:

- No of unique products = 10
- No of hashtags per product = 2
- Total unique hashtags = 10
- No of customers = 8

I varied number of purchases between 8 to 2048 to get the following graph: At lower purchases the other terms of the complexity are dominating, but

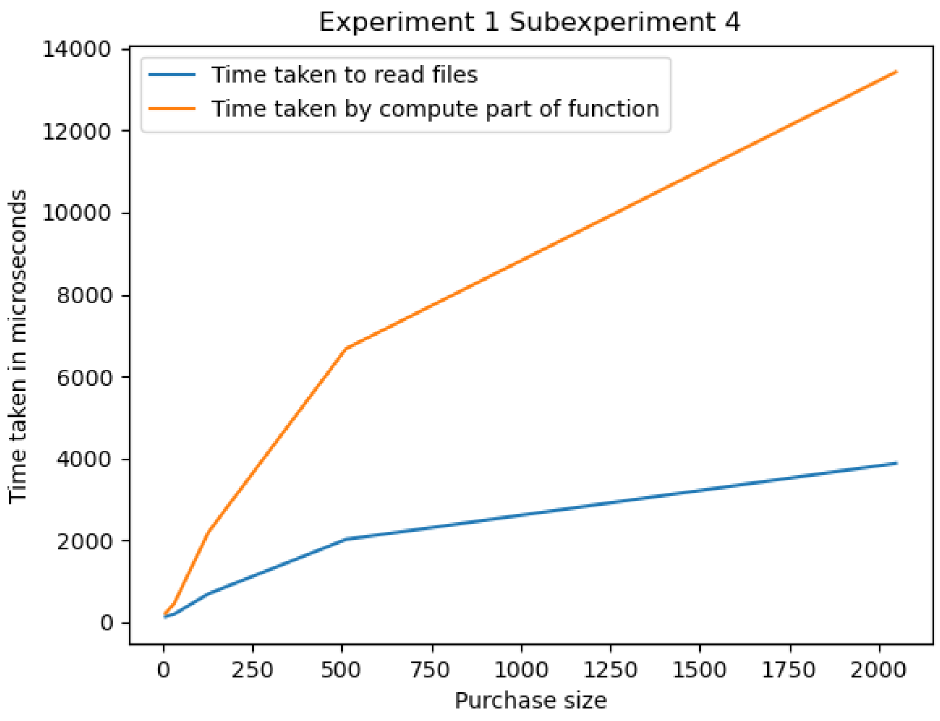


after a point it becomes linear with purchases as it dominates.

Subexperiment 4:

- No of unique products = 10
- No of hashtags per product = 4
- Total unique hashtags = 10
- No of customers = 6

I varied number of purchases between 8 to 2048 to get the following graph: This graph has almost 3 segments, as the number of purchases is less, the

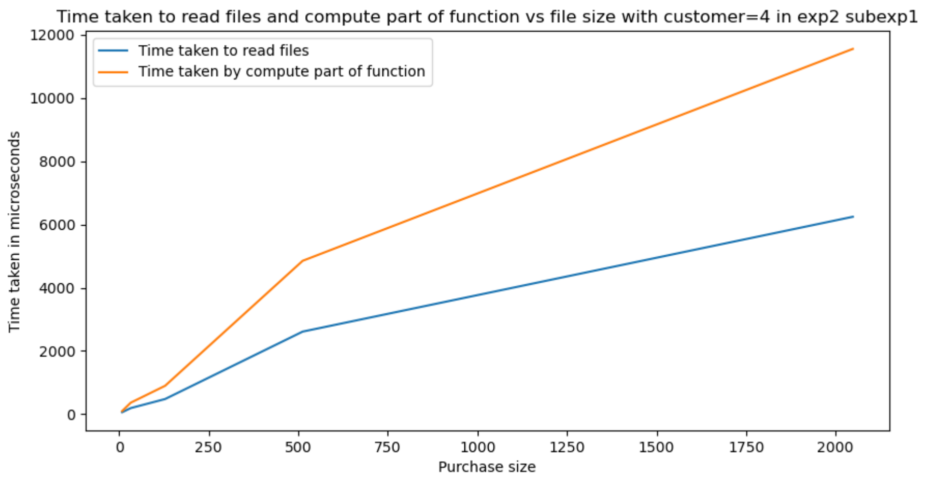


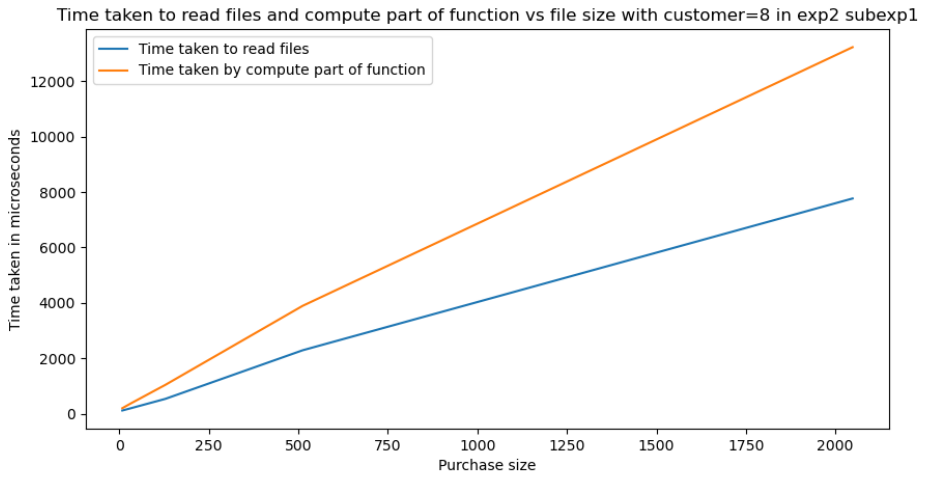
$pd * h * c * \log(pd * h/c)$ term dominates until a point, when it is taken over by the purchase term which makes the linear.

Experiment 2:
Subexperiment 1:

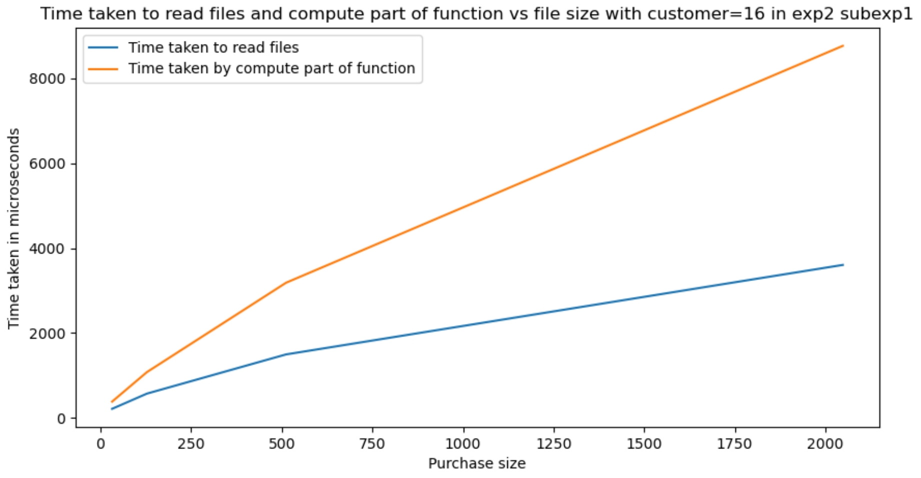
- No of unique products = 5
- No of hashtags per product = 2
- Total unique hashtags = 10
- Here I have varied the number of customers between 4 to 64 to get the following graphs.

I vary the number of purchases between 8 to 2048 for each number of customers.

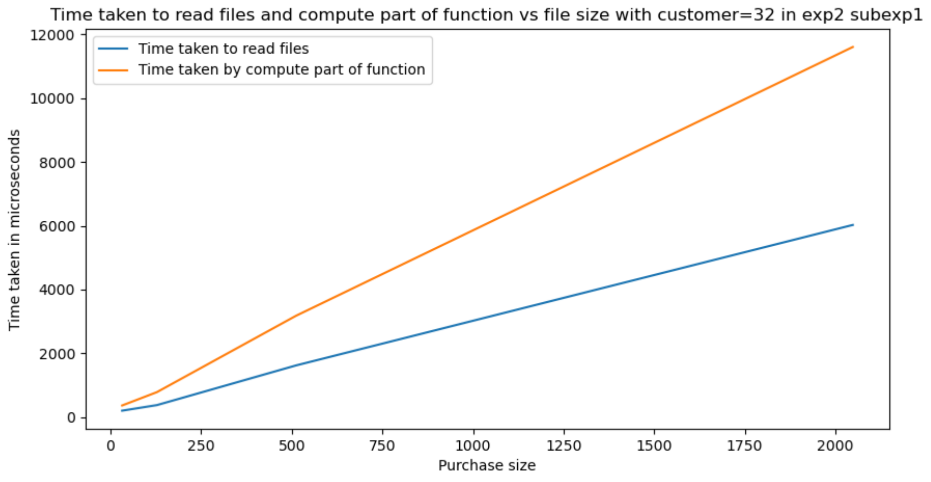


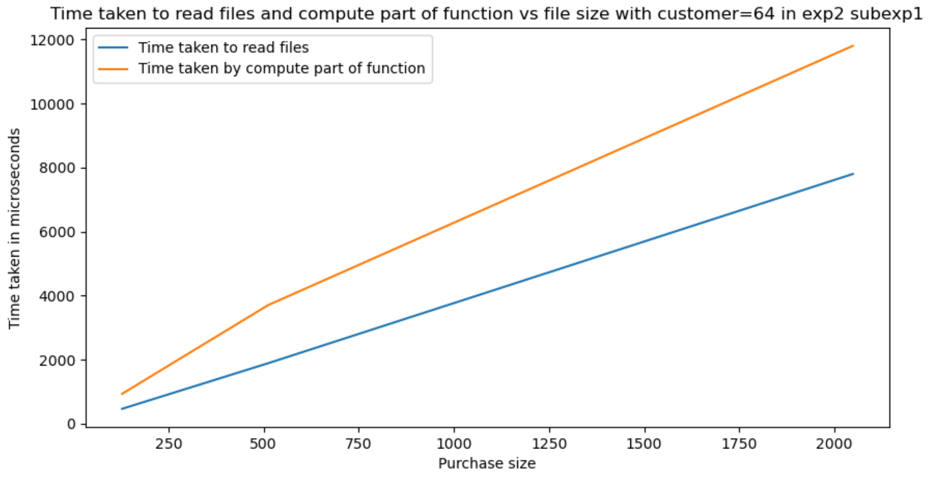


Time taken to read files and compute part of function vs file size with customer=8 in exp2 subexp1
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
10000
12000
0
250
500
750
1000
1250
1500
1750
2000

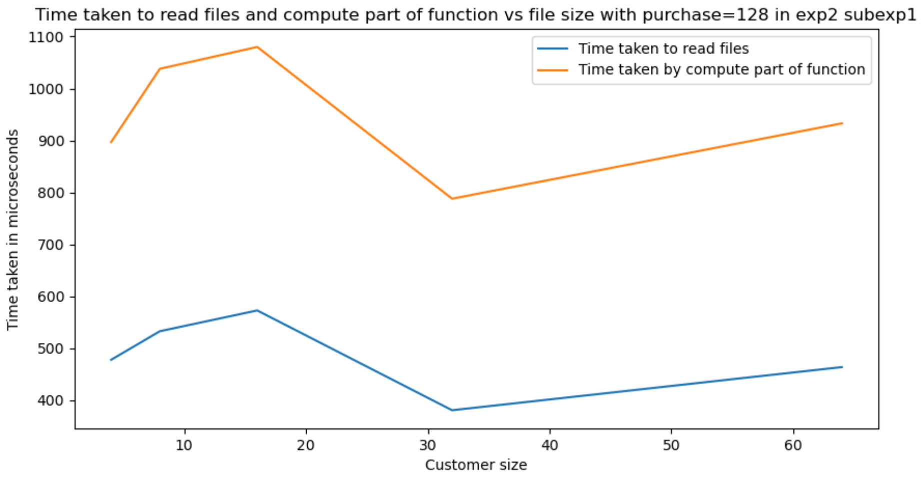


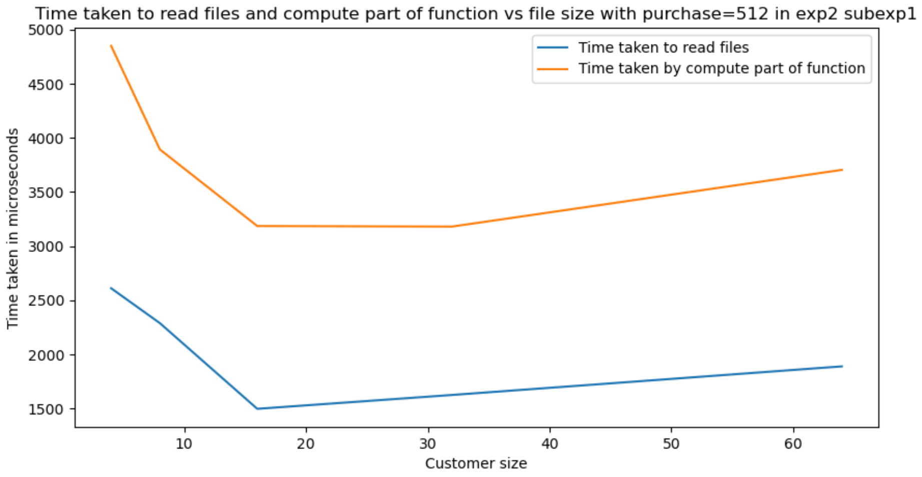
Time taken to read files and compute part of function vs file size with customer=16 in exp2 subexp1
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
0
250
500
750
1000
1250
1500
1750
2000

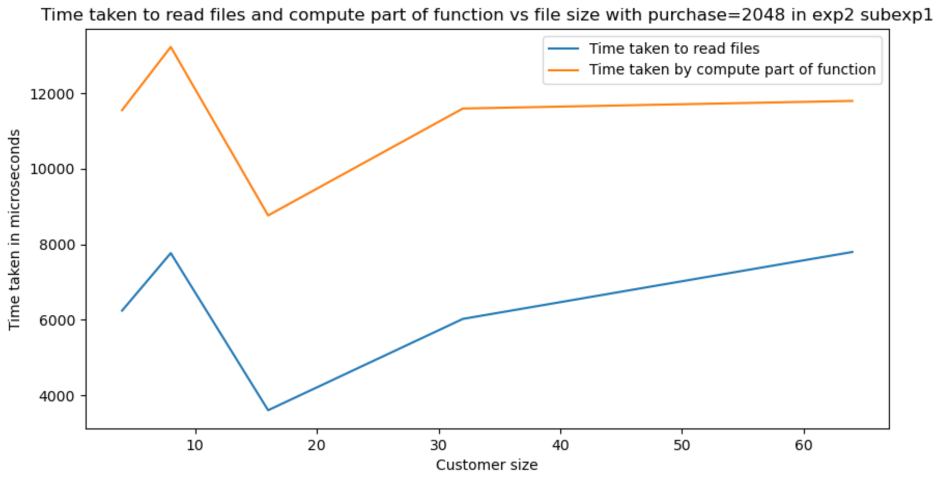




As we can see from these graphs for a constant number of products, hashtags per product, and customers, the time complexity is linear with the purchases. The times vary slightly with the number of customers, most likely due to the $O(c * \log(c))$ term.
I have also compared the time complexity for a constant number of purchases and products, but varying the number of customers.


Time taken to read files and compute part of function vs file size with purchase=128 in exp2 subexp1
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Customer size
1100
1000
900
800
700
600
500
400
10
20
30
40
50
60


Time taken to read files and compute part of function vs file size with purchase=512 in exp2 subexp1
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Customer size
5000
4500
4000
3500
3000
2500
2000
1500
10
20
30
40
50
60

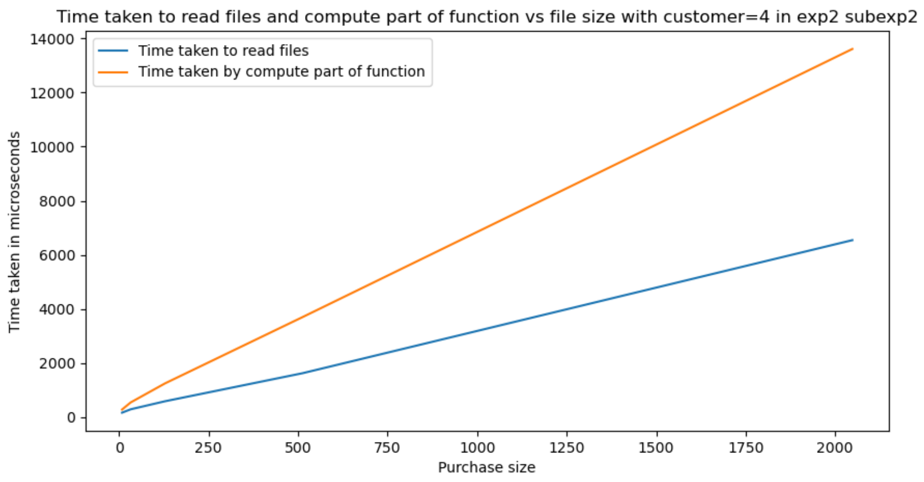


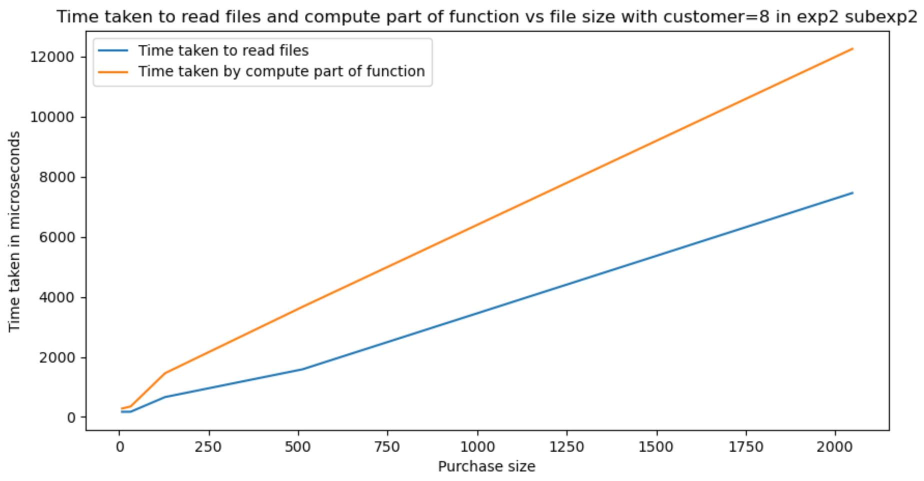
This is a mismatch of complexities as the c term is present in multiple terms of the complexity term. Hence we see a pathwork of complexities as c varies and starts to dominate.
But we can clearly see that as number of purchases increase, the time taken on average increases, which is expected.

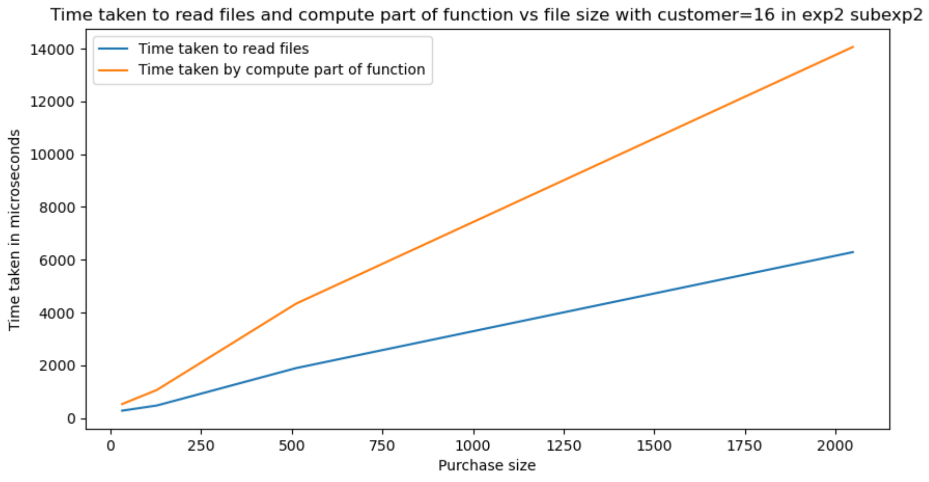
Subexperiment 2:

- No of unique products = 5
- No of hashtags per product = 4
- Total unique hashtags = 10
- Here I have varied the number of customers between 4 to 64 to get the following graphs.

I vary the number of purchases between 8 to 2048 for each number of customers.


Time taken to read files and compute part of function vs file size with customer=4 in exp2 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
10000
12000
14000
0
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=8 in exp2 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
10000
12000
0
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=16 in exp2 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
10000
12000
14000
0
250
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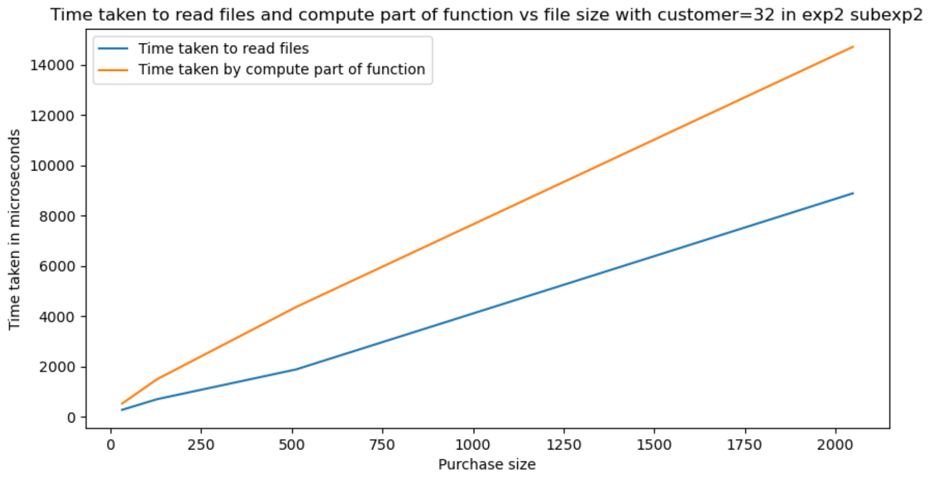
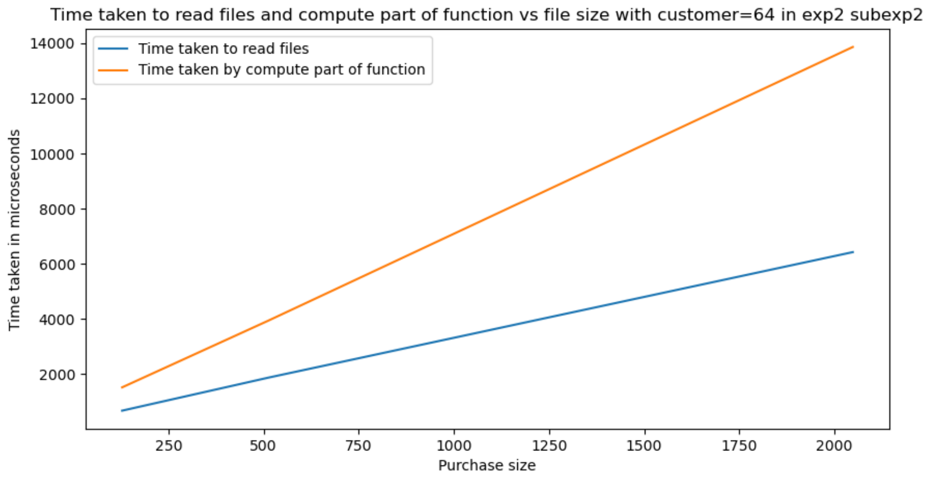
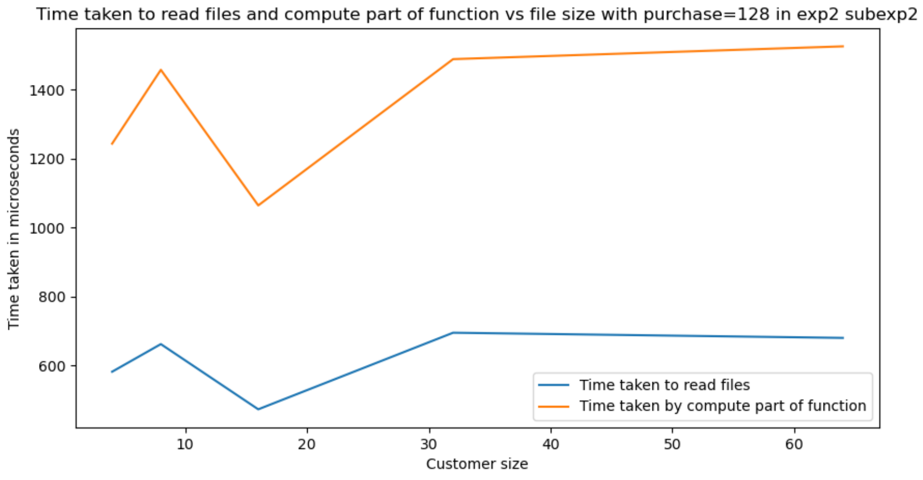
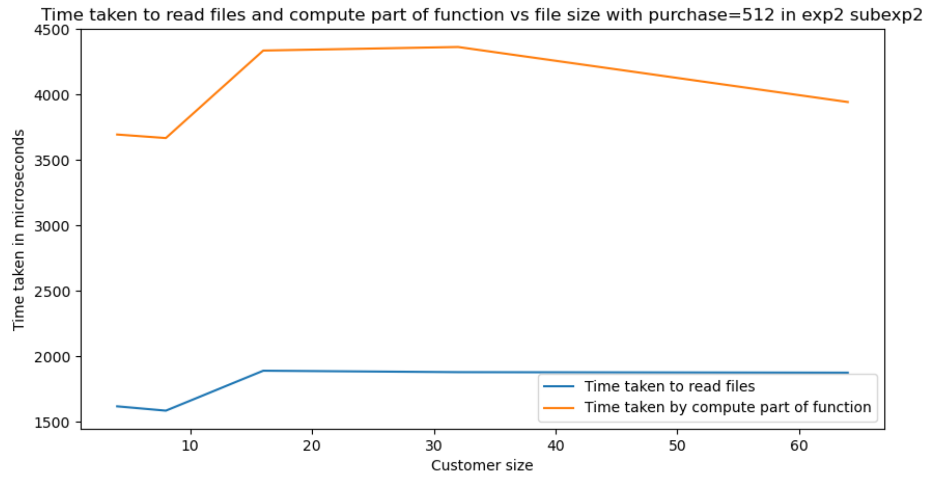
750
1000
1250
1500
1750
2000

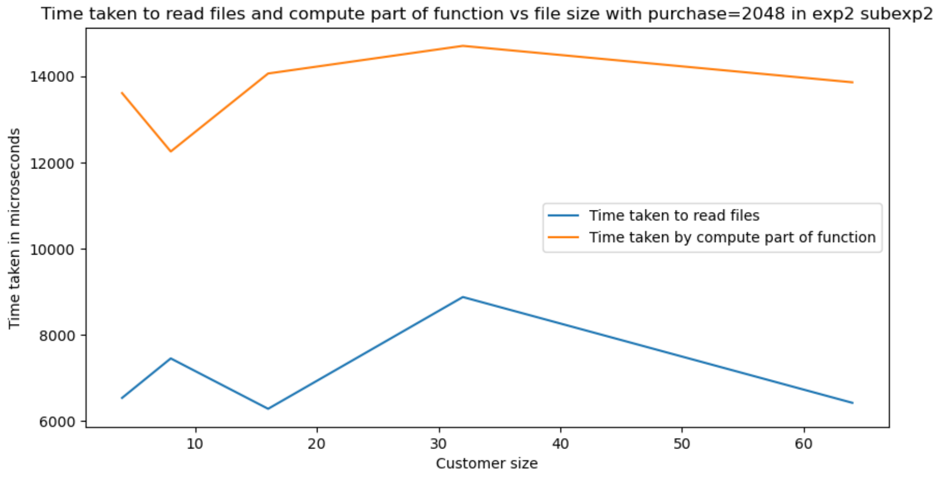

Time taken to read files and compute part of function vs file size with customer=32 in exp2 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
10000
12000
14000
0
250
500
750
1000
1250
1500
1750
2000



As we can see from these graphs for a constant number of products, hashtags per product, and customers, the time complexity is linear with the purchases. The times vary slightly with the number of customers, most likely due to the $O(c * \log(c))$ term.
I have also compared the time complexity for a constant number of purchases and products, but varying the number of customers.







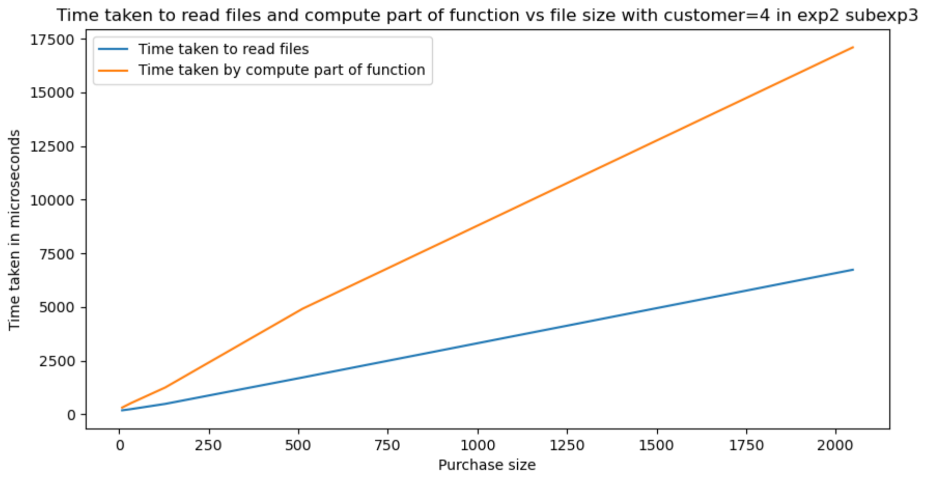
This is a mismatch of complexities as the c term is present in multiple terms of the complexity term. Hence we see a patchwork of complexities as c varies and starts to dominate.
But we can clearly see that as number of purchases increase, the time taken on average increases, which is expected.

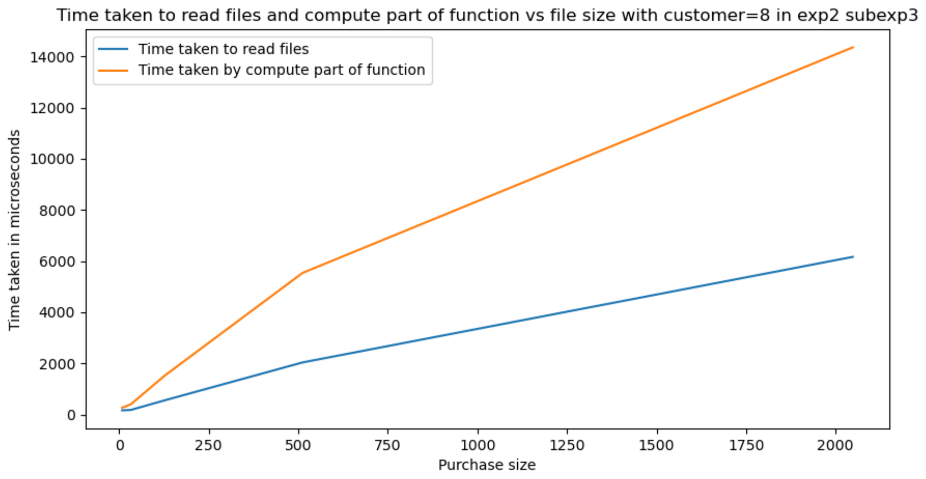
Subexperiment 3:

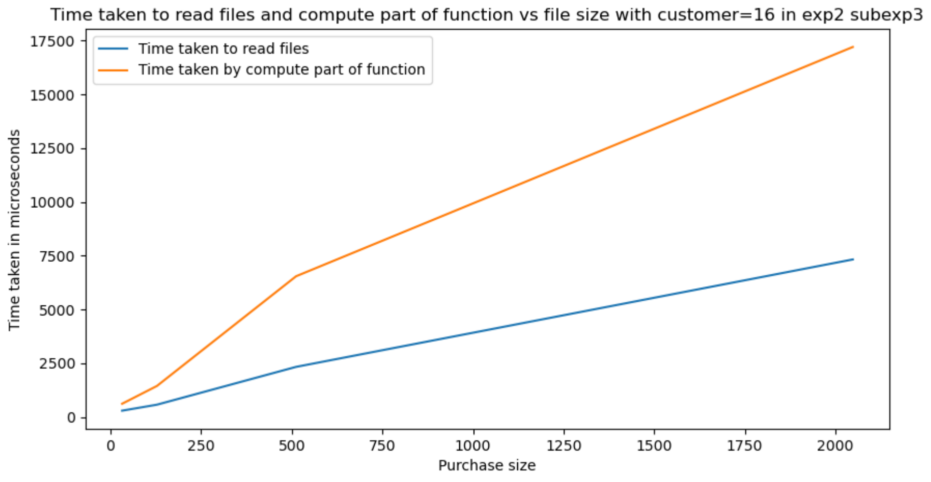
- No of unique products = 10
- No of hashtags per product = 2

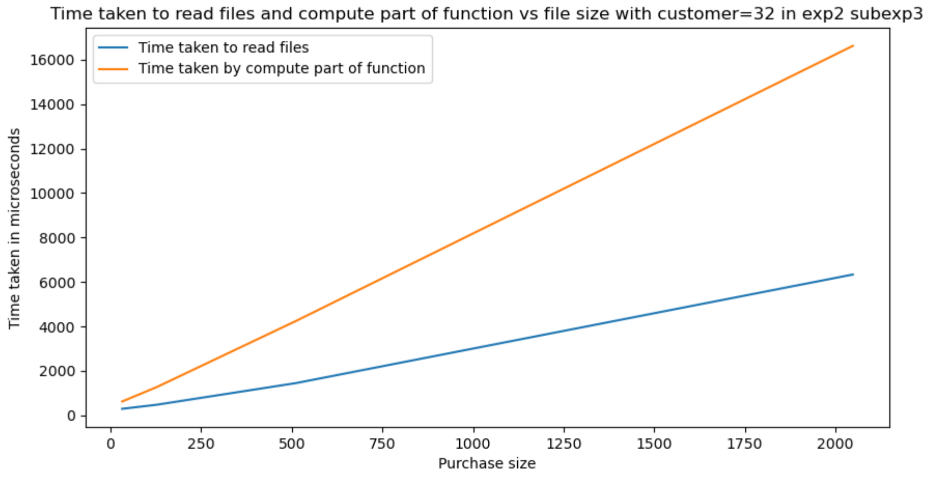
- Total unique hashtags = 10
- Here I have varied the number of customers between 4 to 64 to get the following graphs.

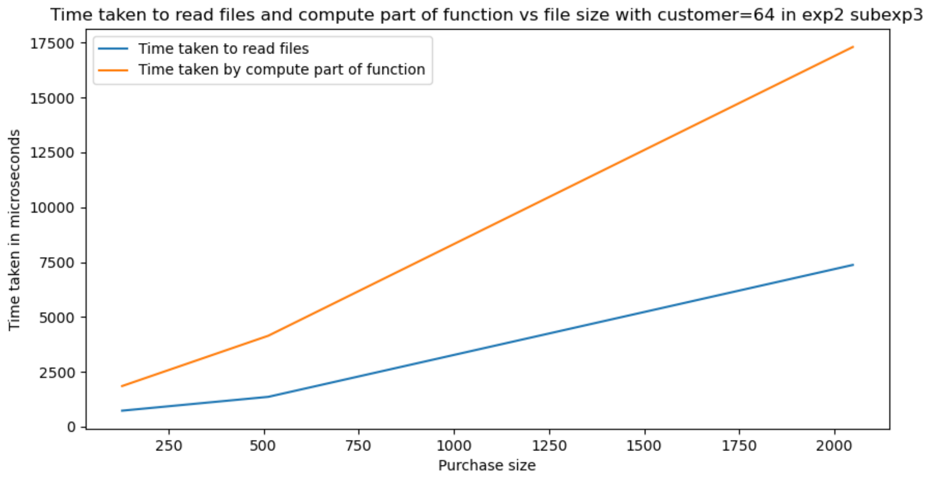
I vary the number of purchases between 8 to 2048 for each number of customers.



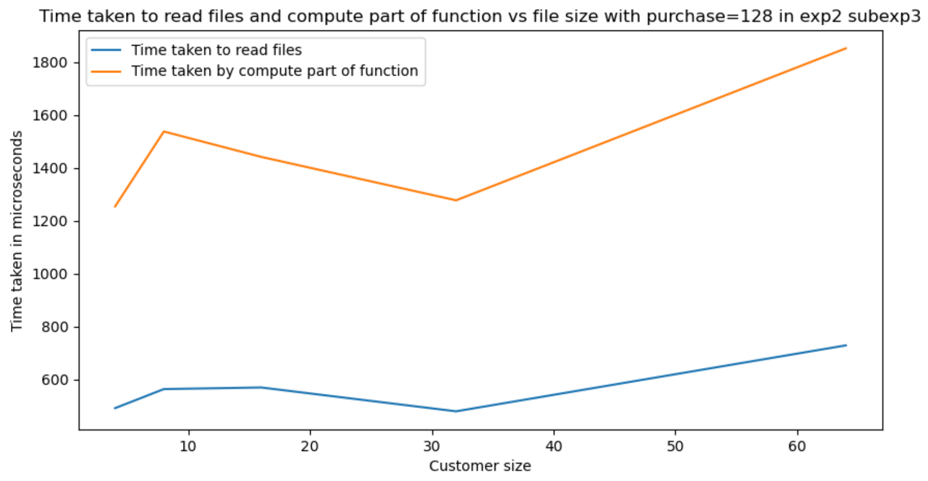


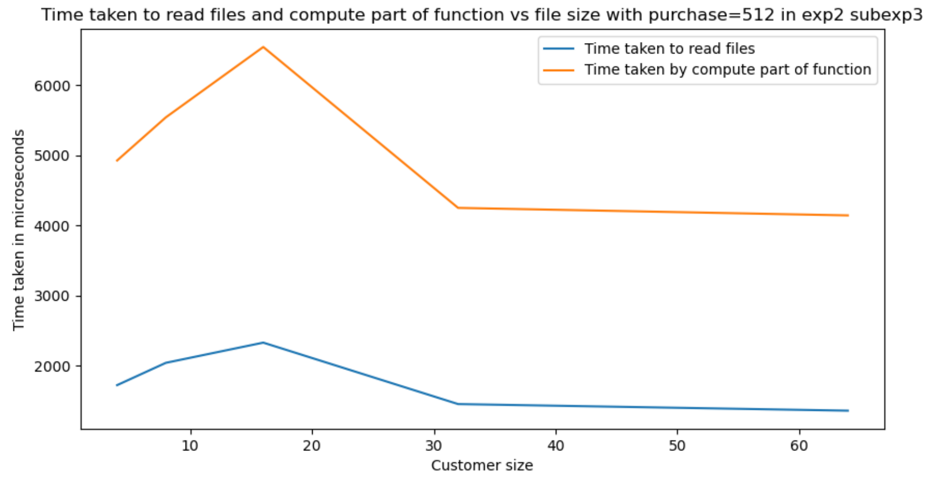

Time taken to read files and compute part of function vs file size with customer=16 in exp2 subexp3
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2500
5000
7500
10000
12500
15000
17500
0
250
500
750
1000
1250
1500
1750
2000

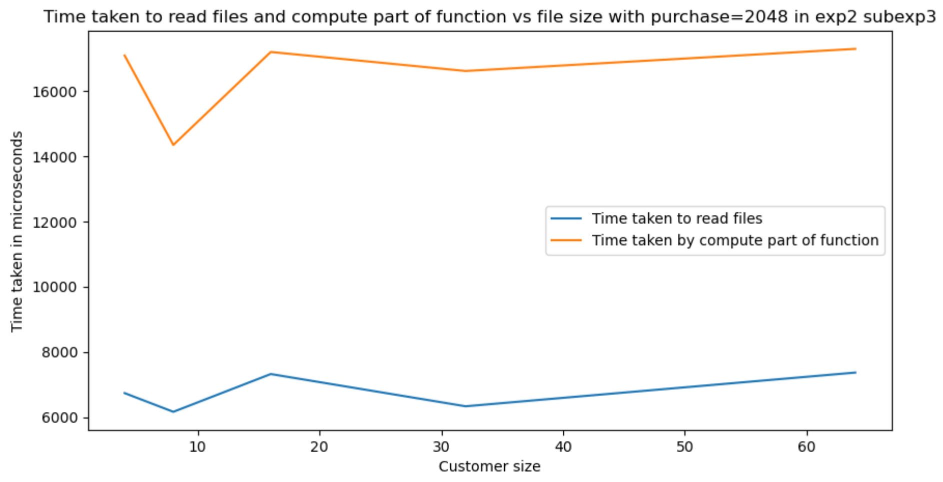

Time taken to read files and compute part of function vs file size with customer=32 in exp2 subexp3
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2000
4000
6000
8000
10000
12000
14000
16000
0
250
500
750
1000
1250
1500
1750
2000



As we can see from these graphs for a constant number of products, hashtags per product, and customers, the time complexity is linear with the purchases. The times vary slightly with the number of customers, most likely due to the $O(c * \log(c))$ term.
I have also compared the time complexity for a constant number of purchases and products, but varying the number of customers.







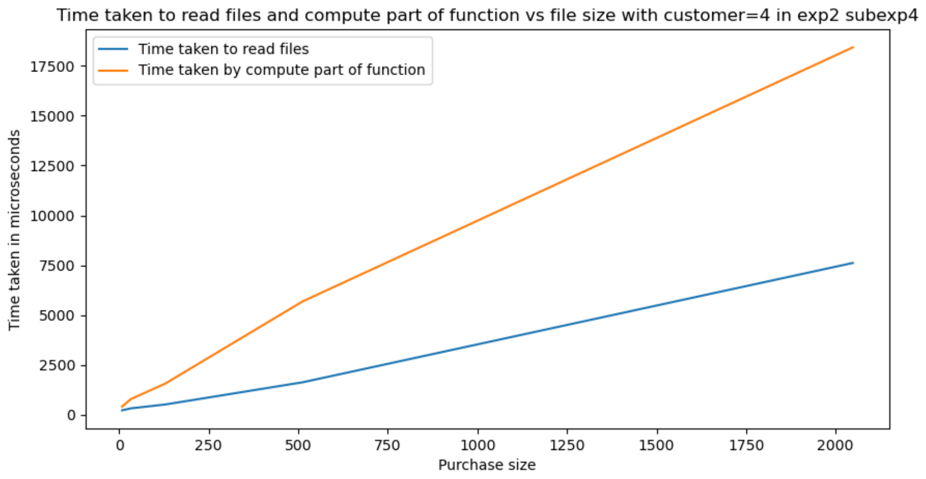
This is a mismatch of complexities as the c term is present in multiple terms of the complexity term. Hence we see a patchwork of complexities as c varies and starts to dominate.
But we can clearly see that as number of purchases increase, the time taken on average increases, which is expected.

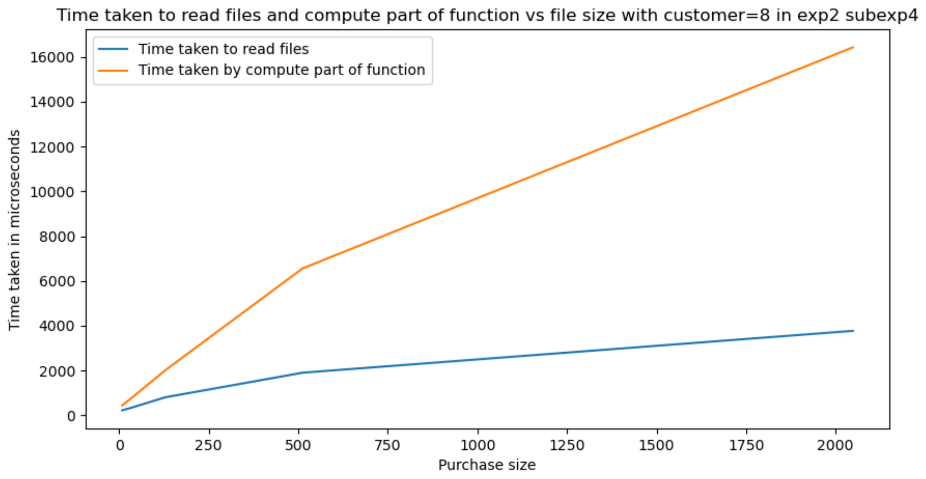
Subexperiment 4:

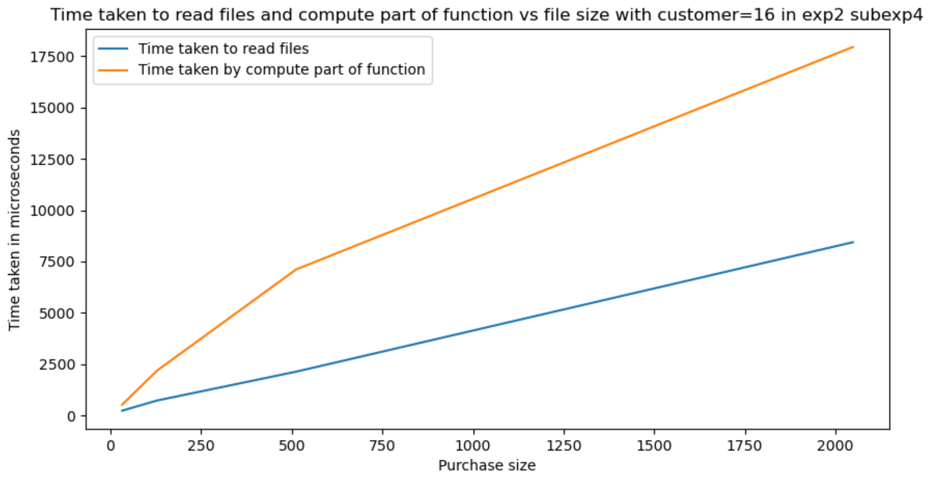
- No of unique products = 10
- No of hashtags per product = 4

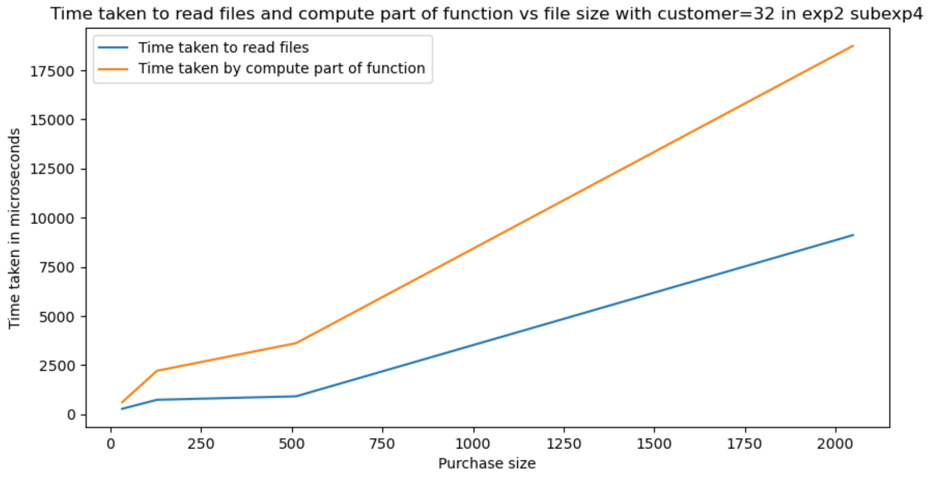
- Total unique hashtags = 10
- Here I have varied the number of customers between 4 to 64 to get the following graphs.

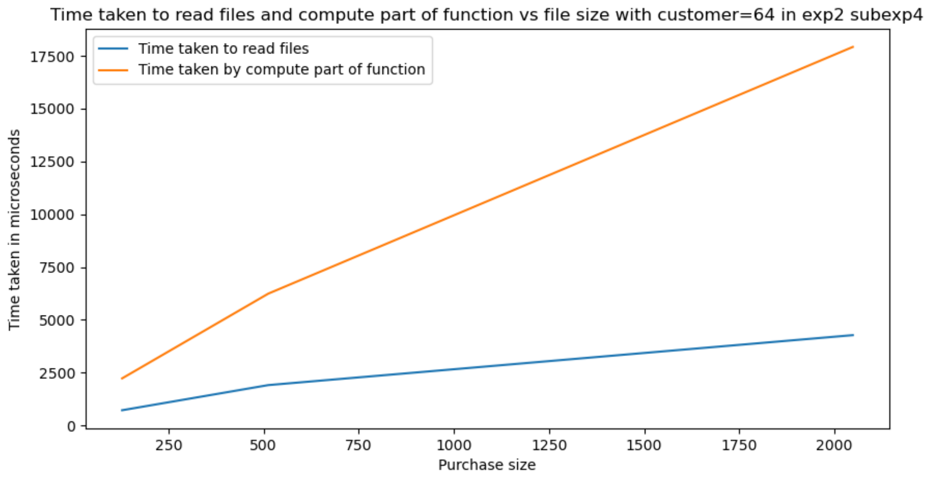
I vary the number of purchases between 8 to 2048 for each number of customers.



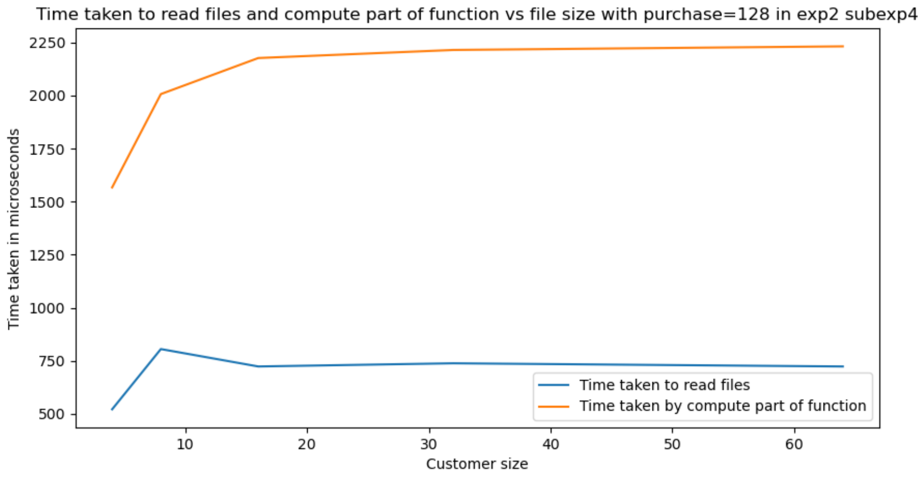


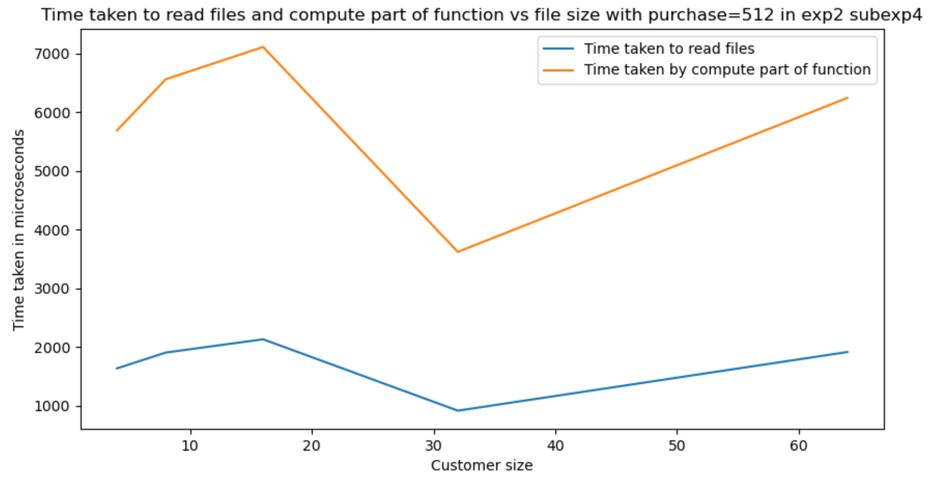

Time taken to read files and compute part of function vs file size with customer=16 in exp2 subexp4
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2500
5000
7500
10000
12500
15000
17500
0
250
500
750
1000
1250
1500
1750
2000

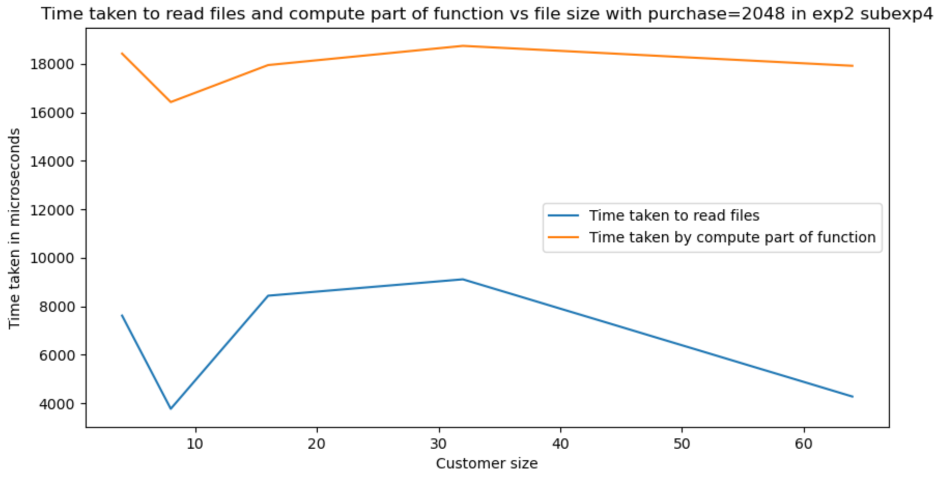

Time taken to read files and compute part of function vs file size with customer=32 in exp2 subexp4
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2500
5000
7500
10000
12500
15000
17500
0
250
500
750
1000
1250
1500
1750
2000



As we can see from these graphs for a constant number of products, hashtags per product, and customers, the time complexity is linear with the purchases. The times vary slightly with the number of customers, most likely due to the $O(c * \log(c))$ term.
I have also compared the time complexity for a constant number of purchases and products, but varying the number of customers.







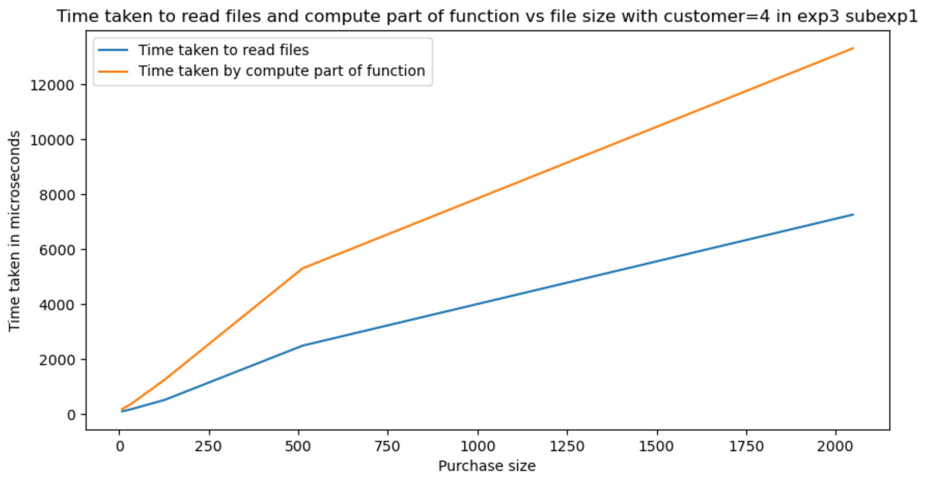
This is a mismatch of complexities as the c term is present in multiple terms of the complexity term. Hence we see a patchwork of complexities as c varies and starts to dominate.
But we can clearly see that as number of purchases increase, the time taken on average increases, which is expected.

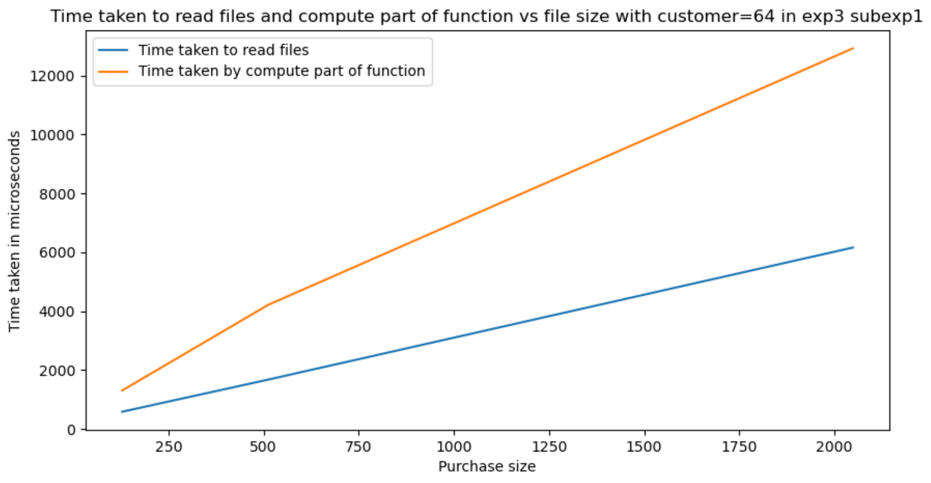
Experiment 3-5:
I randomized the number of hashtags per product, and ran the same experiments as done in experiment 2. The results were similar to the ones obtained in experiment 2.

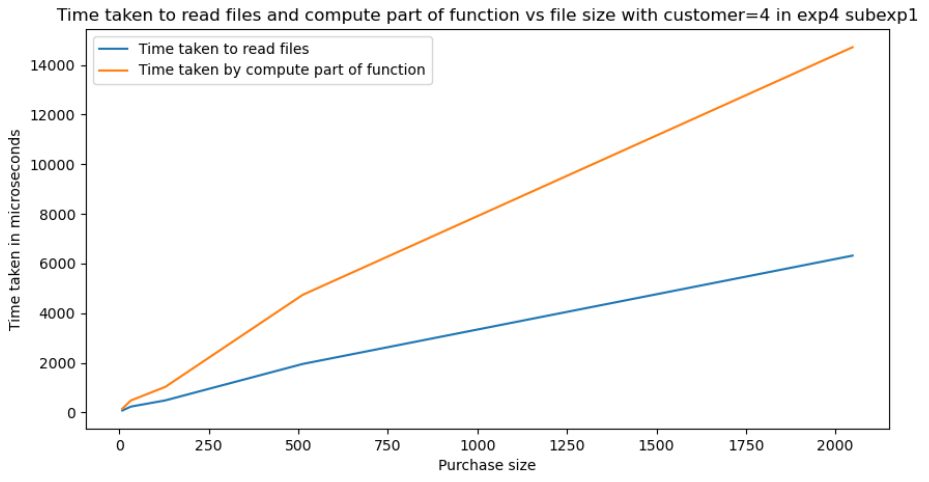
Subexperiment 1:

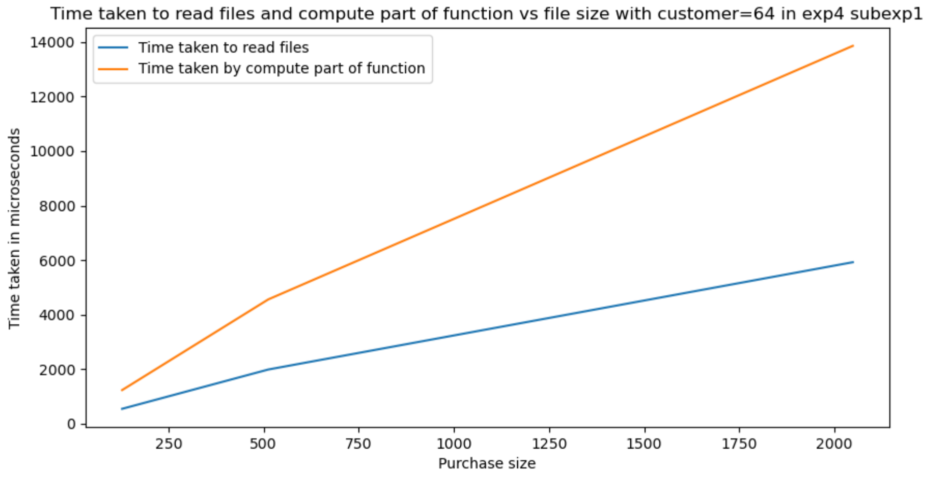
- No of unique products = 5
- Total unique hashtags = 10

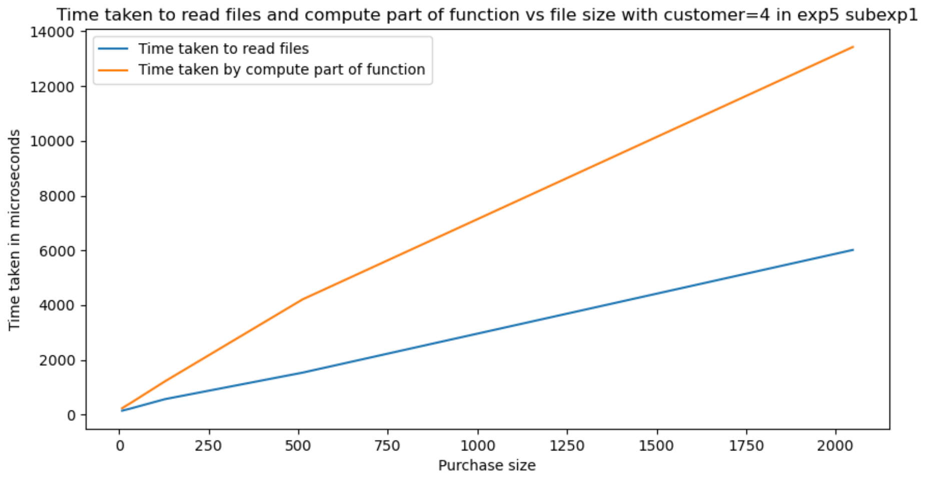
As the experiments I conducted are extensive, I will only show some sample graphs from here.

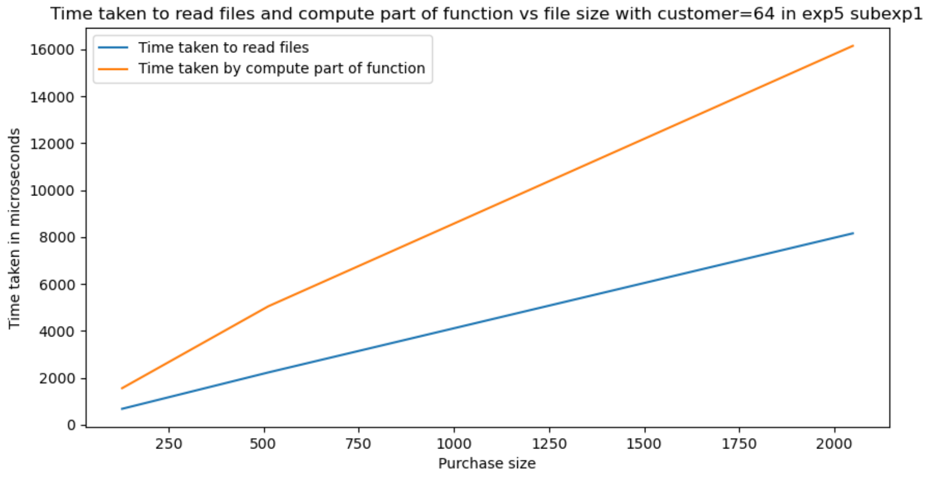





Time taken to read files and compute part of function vs file size with customer=4 in exp4 subexp1
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
14000
12000
10000
8000
6000
4000
2000
0
0
250
500
750
1000
1250
1500
1750
2000
Purchase size


Time taken to read files and compute part of function vs file size with customer=64 in exp4 subexp1
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
14000
12000
10000
8000
6000
4000
2000
0
250
500
750
1000
1250
1500
1750
2000
Purchase size

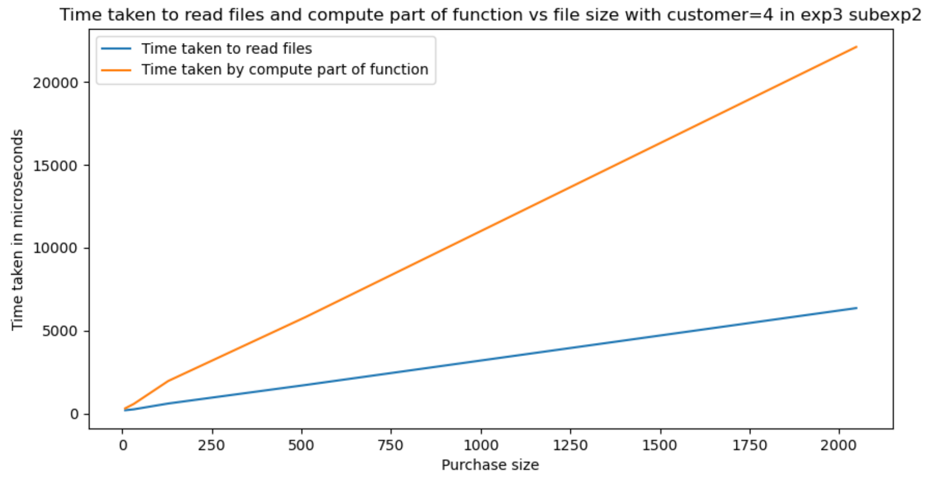


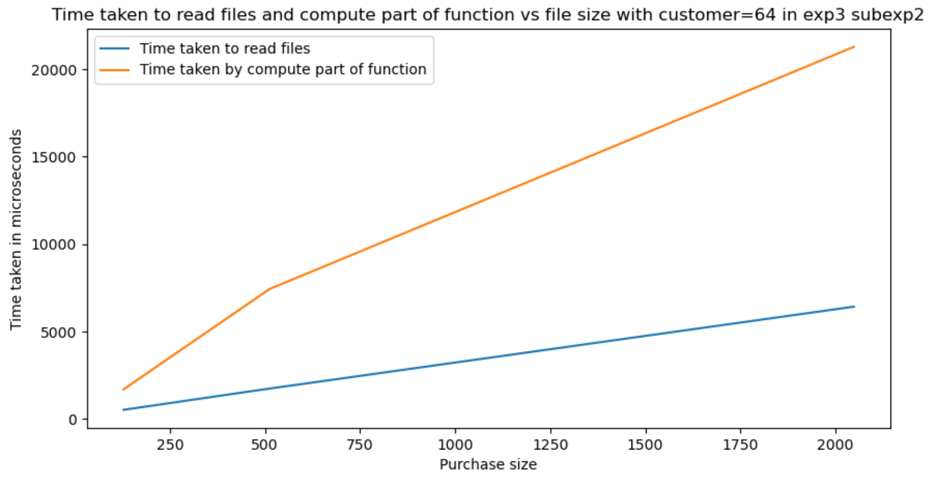


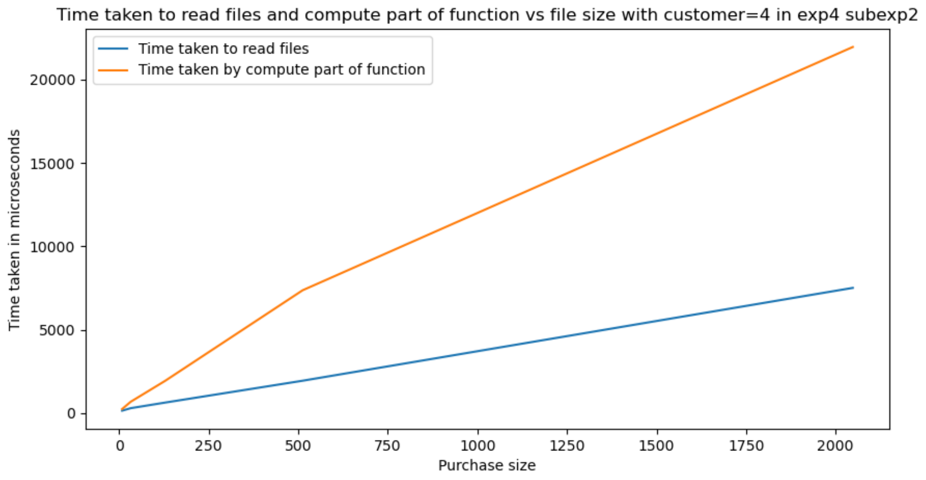
The time taken for each exp increases form subexp1 to subexp3 as the number of customers increase.

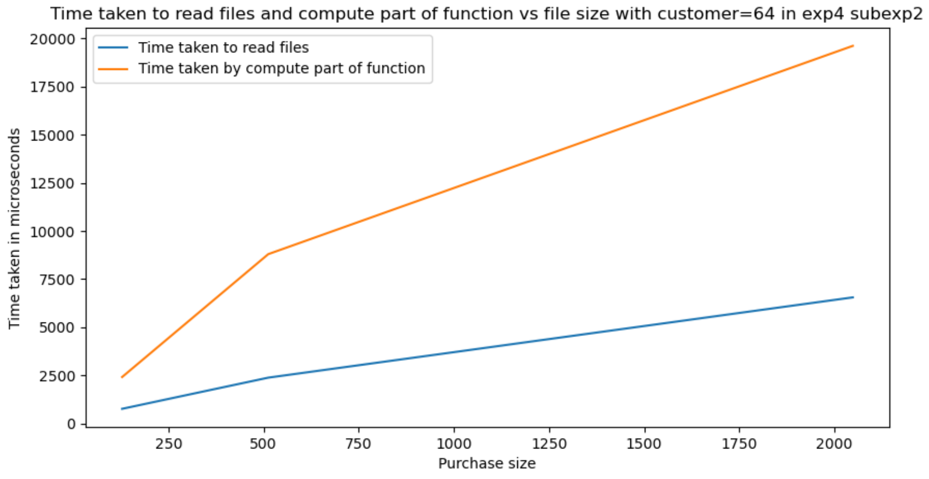
Subexperiment 2:

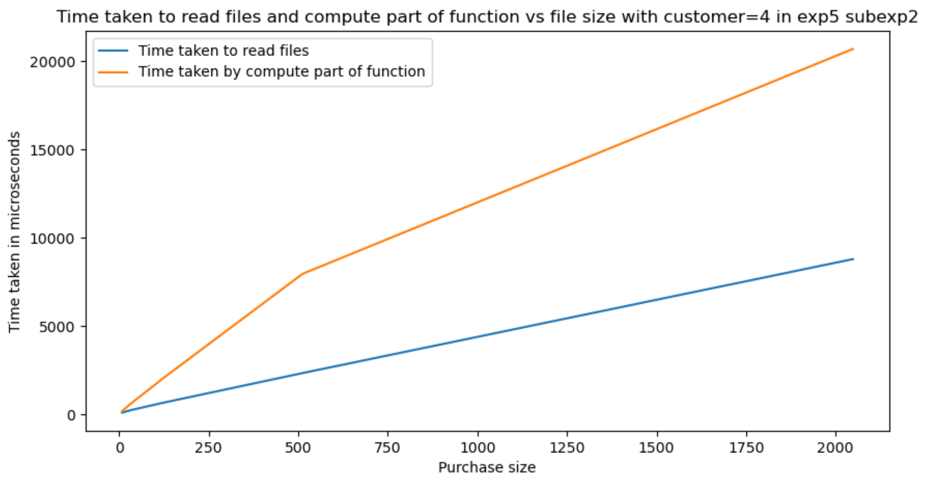
- No of unique products = 10
- Total unique hashtags = 10

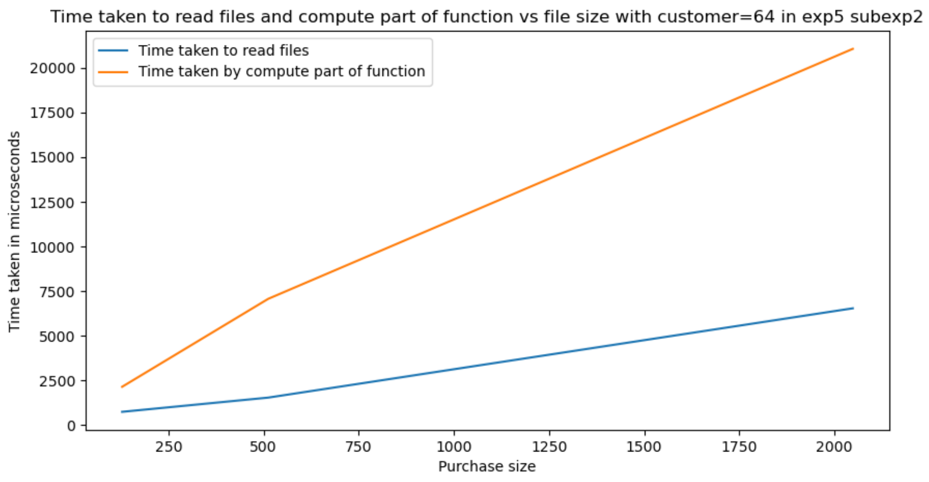

Time taken to read files and compute part of function vs file size with customer=4 in exp3 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
0
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=64 in exp3 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=4 in exp4 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
0
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=64 in exp4 subexp2
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
2500
5000
7500
10000
12500
15000
17500
20000
250
500
750
1000
1250
1500
1750
2000

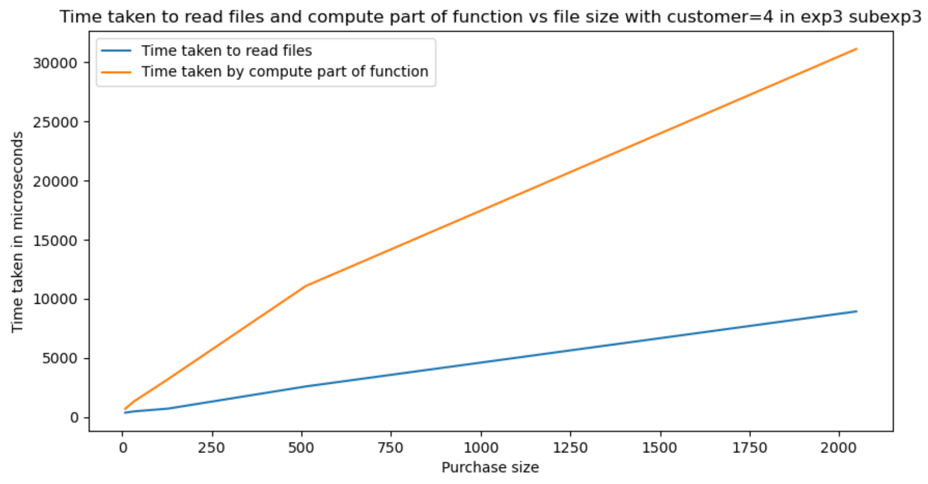


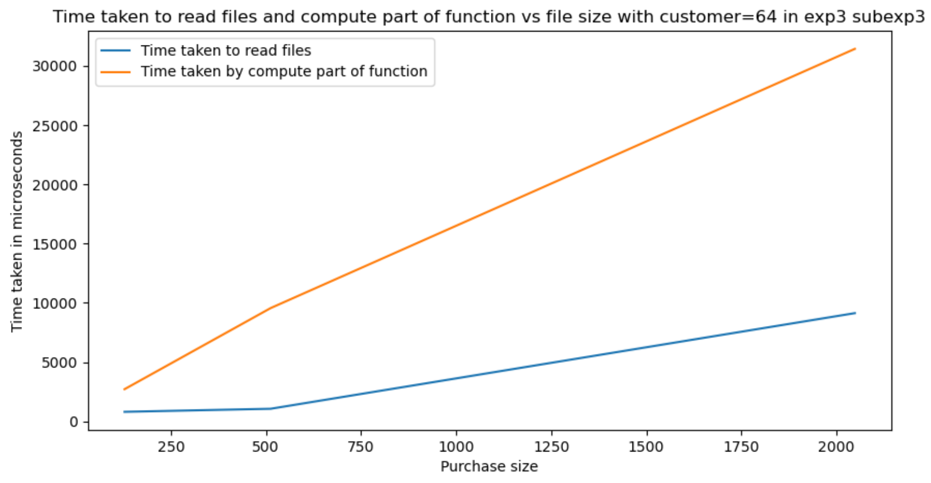


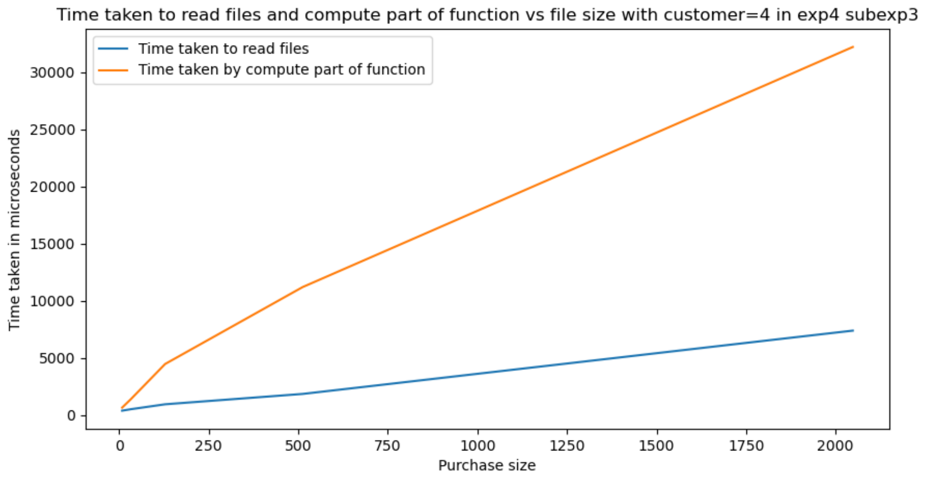
The time taken for each exp increases form subexp1 to subexp3 as the number of customers increase.

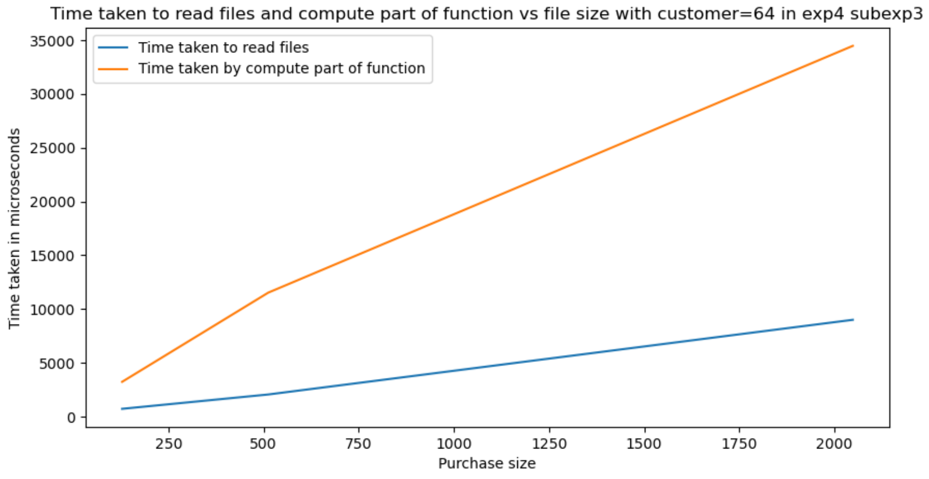
Subexperiment 3:

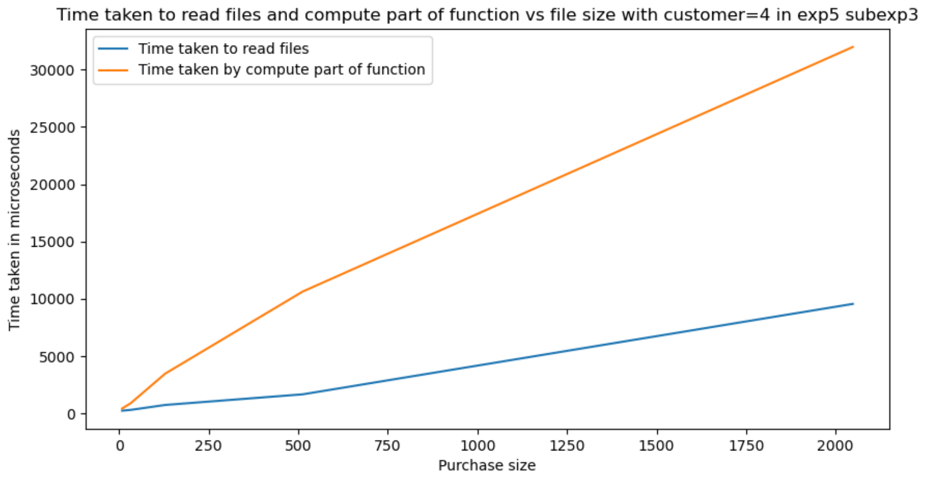
- No of unique products = 20
- Total unique hashtags = 10

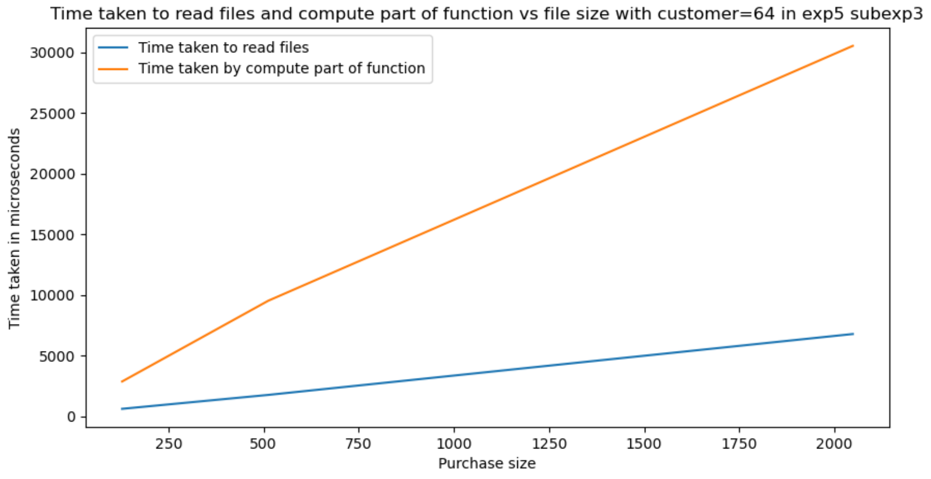

Time taken to read files and compute part of function vs file size with customer=4 in exp3 subexp3
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
25000
30000
0
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=64 in exp3 subexp3
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
25000
30000
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=4 in exp4 subexp3
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
25000
30000
0
250
500
750
1000
1250
1500
1750
2000


Time taken to read files and compute part of function vs file size with customer=64 in exp4 subexp3
Time taken to read files
Time taken by compute part of function
Time taken in microseconds
Purchase size
0
5000
10000
15000
20000
25000
30000
35000
250
500
750
1000
1250
1500
1750
2000





The time taken for exp increases form subexp1 to subexp3 as the number of customers increase.

Observations:
Overall, as the number of unique products increase, the time taken increases accross each experiment, as seen in the graphs. Each graph verifies the time complexity.

Time Complexity of Writing the groups to the output:

Unfortunately, I was unable to measure this time complexity empirically, as in the code the $fileIO$ variable was taken as 0 for all experiments that I was running. After looking into this for a while, I found out that beacuse C++ is so efficient, it caches the outputs and since the files are writing to the same output files, it is just skipping this operation altogether, hence making it 0. Thus for all my experiments the time taken by funciton and total time taken are the same. I have tried running this on my own system to see if there was any change, but the results were the same. Hence I chalk this up to a language specific feature of c++ and do not report any reults here.

Space Complexity Analysis:
Throughout my code, I use vectors and maps. The space complexity for the code is as follows:

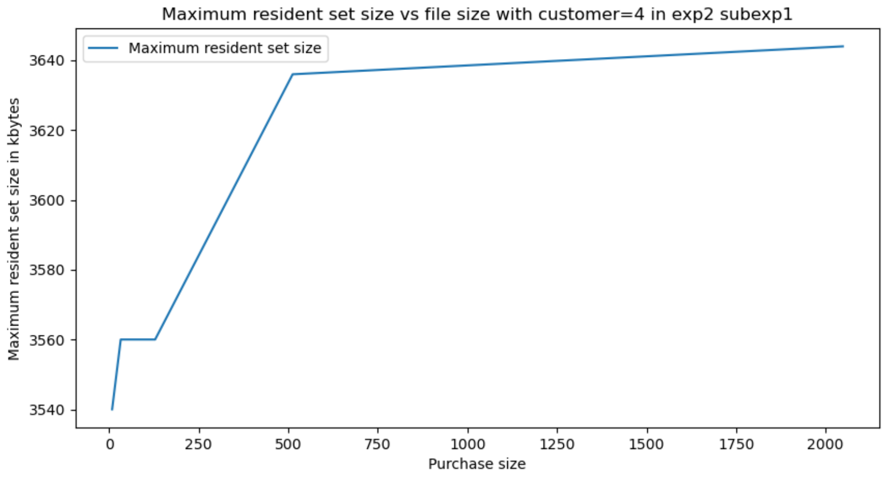
- productHashtags: $O(pd * h)$
- customerPurchases: $O(pur)$
- customerHashtags: $O(c * pd * h)$
- customerTopKHashtags: $O(pd * h)$
- customerGroup: $O(c)$

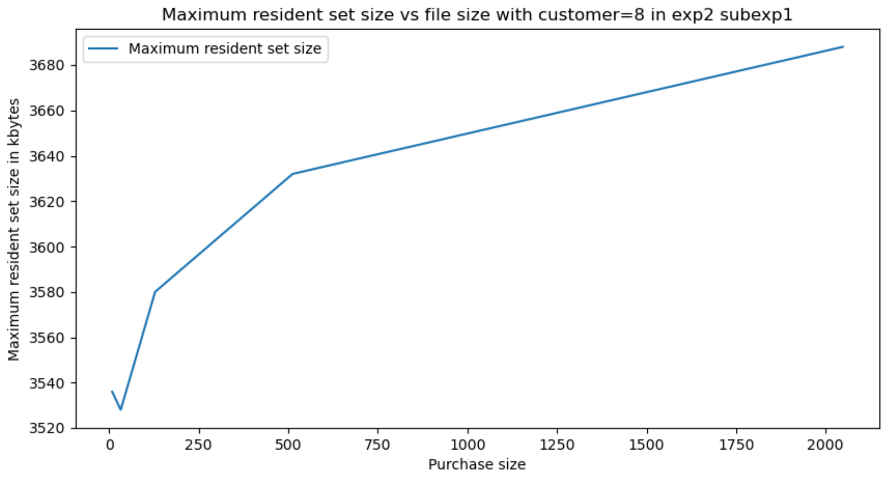
The total space complexity is $O(pd * h + pur + c * pd * h + c)$.

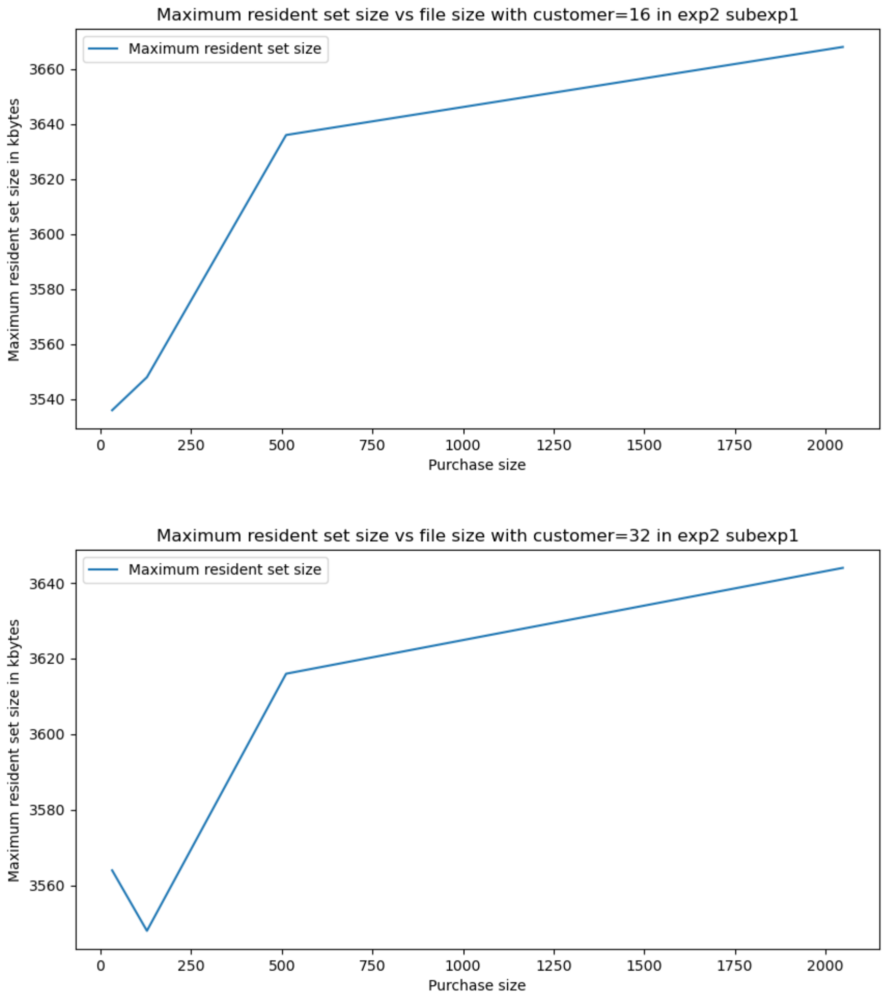
To measure the space complexity, we use maximum resident size, printed to terminal. This is the maximum memory accessed by the program during its run time.
I have measured the space complexity for the code in **experiment 2** which has the following results:
Subexperiment 1:

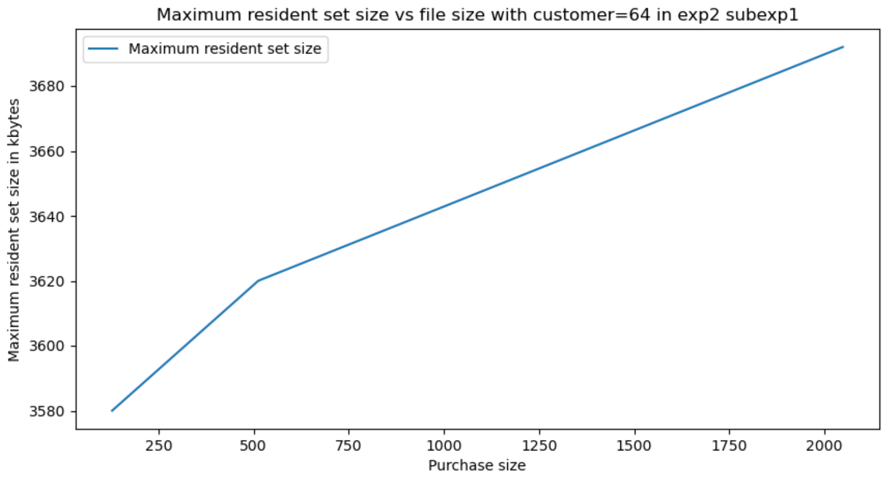
- No of unique products = 5
- No of hashtags per product = 2

I varied number of purchases between 8 to 2048 to get the following graphs:






Maximum resident set size vs file size with customer=16 in exp2 subexp1
Maximum resident set size
Maximum resident set size in kbytes
Purchase size
3540
3560
3580
3600
3620
3640
3660
0
250
500
750
1000
1250
1500
1750
2000
Maximum resident set size vs file size with customer=32 in exp2 subexp1
Maximum resident set size
Maximum resident set size in kbytes
Purchase size
3560
3580
3600
3620
3640

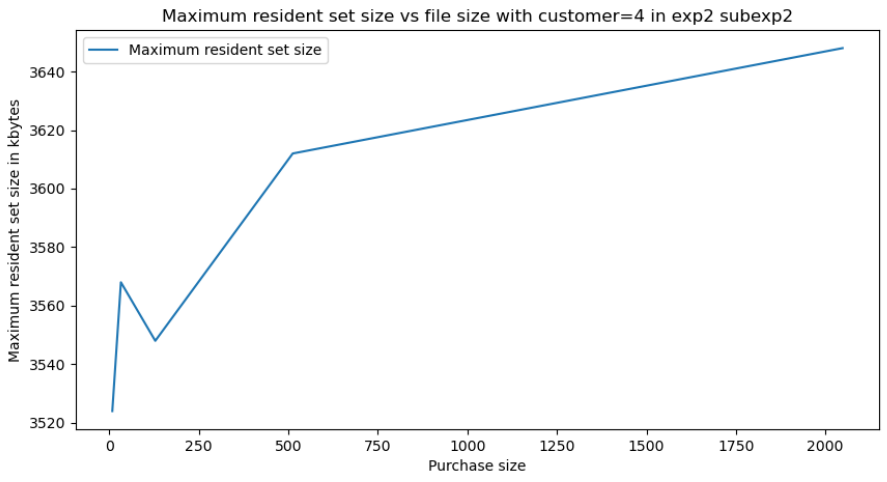


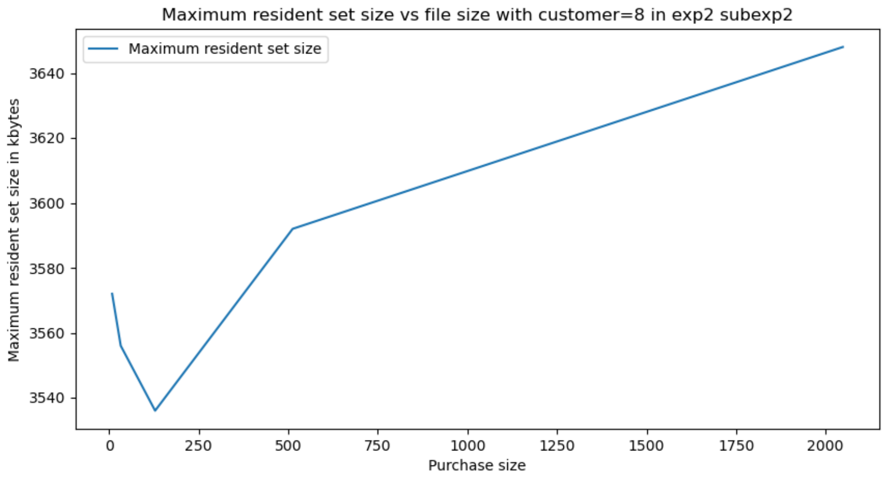
Here as *pur* increases, it becomes the dominating term, hence the graph is linear after a point.

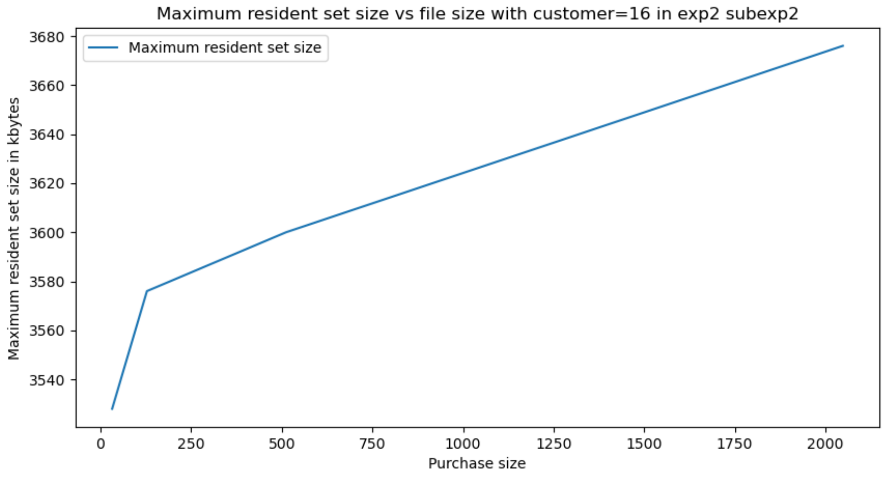
Subexperiment 2:

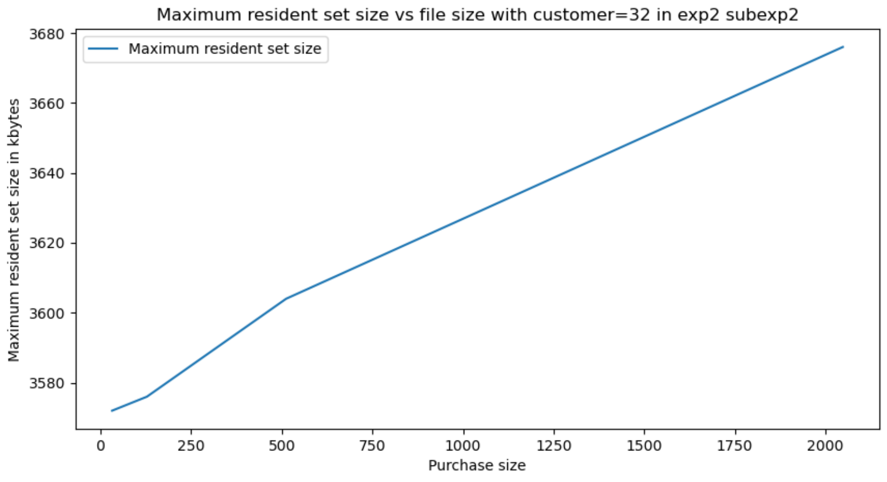
- No of unique products = 5
- No of hashtags per product = 4

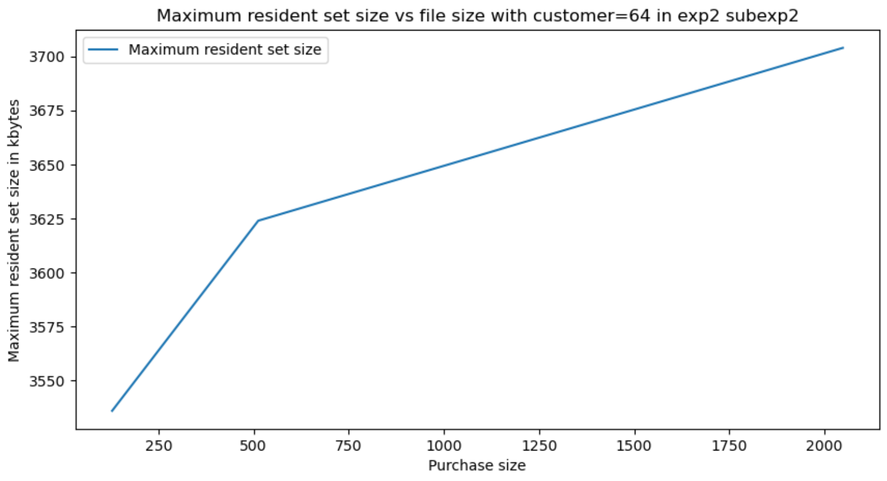
I varied number of purchases between 8 to 2048 to get the following graphs:




Maximum resident set size vs file size with customer=8 in exp2 subexp2
Maximum resident set size
Maximum resident set size in kbytes
Purchase size


Maximum resident set size vs file size with customer=16 in exp2 subexp2
Maximum resident set size
Maximum resident set size in kbytes
Purchase size





Here as *pur* increases, it becomes the dominating term, hence the graph is linear after a point.

The other two subexperiments also follow the same trends, hence I shall not show them here.

It is clear from the graphs that as the complexity increases, but as c++ is an efficient language, this doesn't differ too much from each case.

Question 2

There was no analysis of question 2 asked of me. The written code is in *user_code.h* file.

Question 3

The optimized grouping function is as follows:

```
if(!globalproductHashtagsInitialized){
    // #PROMPT# Extract all the lines from the hashtags
    file and store them in an appropriate data structure
    vector<pair<string,vector<string>>> productHashtags;
    while(hashtags.hasNext()){
        string line = hashtags.next();
        vector<string> tokens = split(line,',');
        if (tokens.size() > 1) { // #PROMPT# Ensure
        there are hashtags present
            string product = tokens[0];
            vector<string> prodHashtags(tokens.begin() + 1,
            tokens.end());
            // #PROMPT# Make sure all prodHashtags
            are unique
            sort(prodHashtags.begin(), prodHashtags.end());
            prodHashtags.erase(unique(prodHashtags.begin(),
            prodHashtags.end()),                prodHashtags.end());
            productHashtags.push_back(make_pair(product, prodHashtags));
        }
    }
    globalproductHashtags = productHashtags;
    globalproductHashtagsInitialized = true;
    for(auto line : newHashtags){
        vector<string> tokens = split(line,',');
        if (tokens.size() > 1) { // #PROMPT# Ensure
        there are hashtags present
            string product = tokens[0];
            vector<string> newprodHashtags(tokens.begin() + 1,
            tokens.end());
            // #PROMPT# Make sure all prodHashtags are unique
            sort(newprodHashtags.begin(), newprodHashtags.end());
            newprodHashtags.erase(unique(newprodHashtags.begin(),
            newprodHashtags.end()), newprodHashtags.end());
            for(auto& prod : globalproductHashtags){
                if(prod.first == product){
                    for(auto hashtag : newprodHashtags){
                        if(find(prod.second.begin(), prod.second.end(),
                        hashtag) == prod.second.end()){
                            prod.second.push_back(hashtag);
                        }
                    }
                }
            }
        }
    }
}
else{
    /* #PROMPT# Add the new hashtags to the globalproductHashtags vector.
    To do this iterate over the newHashtags vector
    and for each product in the newHashtags vector,
    add the hashtags to the product in the globalproductHashtags vector.
    The product will be present in the globalproductHashtags vector as
    it has been initialized in the previous iteration.*/
    for(auto line : newHashtags){
        vector<string> tokens = split(line,',');
        if (tokens.size() > 1) { // #PROMPT# Ensure
        there are hashtags present
            string product = tokens[0];
            vector<string> newprodHashtags(tokens.begin() + 1,
            tokens.end());
            // #PROMPT# Make sure all prodHashtags are unique
            sort(newprodHashtags.begin(), newprodHashtags.end());
            newprodHashtags.erase(unique(newprodHashtags.begin(),
            newprodHashtags.end()), newprodHashtags.end());
            for(auto& prod : globalproductHashtags){
                if(prod.first == product){
                    for(auto hashtag : newprodHashtags){
                        if(find(prod.second.begin(), prod.second.end(),
                        hashtag) == prod.second.end()){
                            prod.second.push_back(hashtag);
                        }
                    }
                }
            }
        }
    }
}
```

This is the only code that is different, all other function are the same.
The time complexity for this code is as follows: We will define the following terms:

- number of products as pd,
- the number of hashtags per product as h,
- the number of customers as c,
- and the number of purchases as pur.
- the number of new hashtags as nh.
- the number of total hashtags as h'.

We first check if the global product hashtags are initialized. If not, we initialize them. The time complexity for this part is $O(pd*(h*\log(h)+h^2))$. After this we set the global product hashtags to be initialized. Next we iterate through all the new hashtags which is $O(nh)$. For each newHashtag we do a sort, which has complexity $O(nh/p*\log(nh/p))$. For each new hashtag we iterate through all the products which is $O(pd)$. For each product we iterate through all the hashtags which is $O(h)$. We then perform a find operation which is $O(h)$. If the hashtag is not found, we insert it which is $O(1)$. The total time complexity for this part is $O(nh*(nh/p*\log(nh/p)+pd*h*(h)))$. Else, if the global product hashtags are initialized, we iterate through all the new hashtags which is $O(nh)$. For each newHashtag we do a sort, which has complexity $O(nh/p*\log(nh/p))$. Them, for each new hashtag we iterate

through all the products which is $O(pd)$. For each product we iterate through all the hashtags which is $O(h)$. We then perform a find operation which is $O(h)$. If the hashtag is not found, we insert it which is $O(1)$. The total time complexity for this part is $O(nh * (nh/p * \log(nh/p) + pd * h * (h)))$.
Therefore the average time complexity of this code is $O(pd * (h * \log(h) + h^2) + nh * (nh/p * \log(nh/p) + pd * h * (h)))$.
This time complexity then gets added to the time complexity of the rest of the code. The only difference being that the new h is $h_n = h'/p$. We use h_n everywhere else as the total number of hashtags, h', has been updated.

The space complexity of this code is as follows:

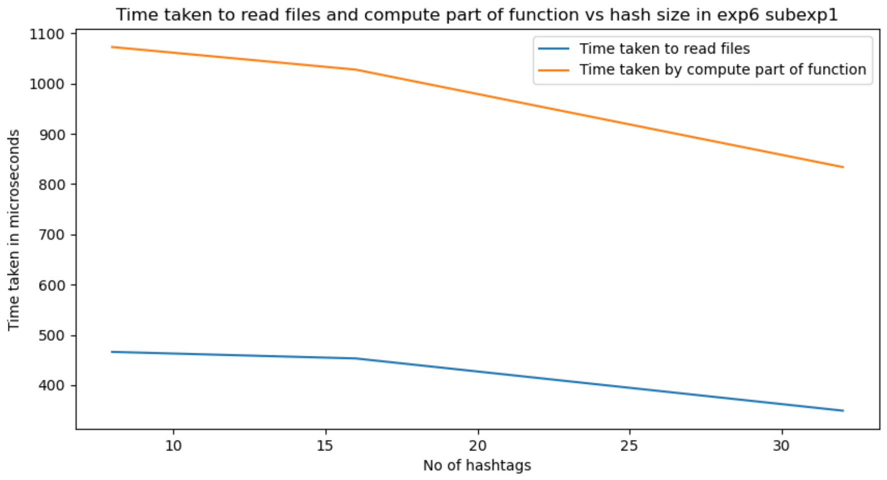
- productHashtags: $O(pd * h)$ (it is a vector)
- globalproductHashtags: $O(pd * h)$ (it is a map)
- newprodHashtags: $O(nh)$ (it is a vector)
- The rest of the variables are the same as before, using h_n instead of h.

The total space complexity is $O(pd * h + pd * h_n + nh + pur + c * pd * h_n + c)$.

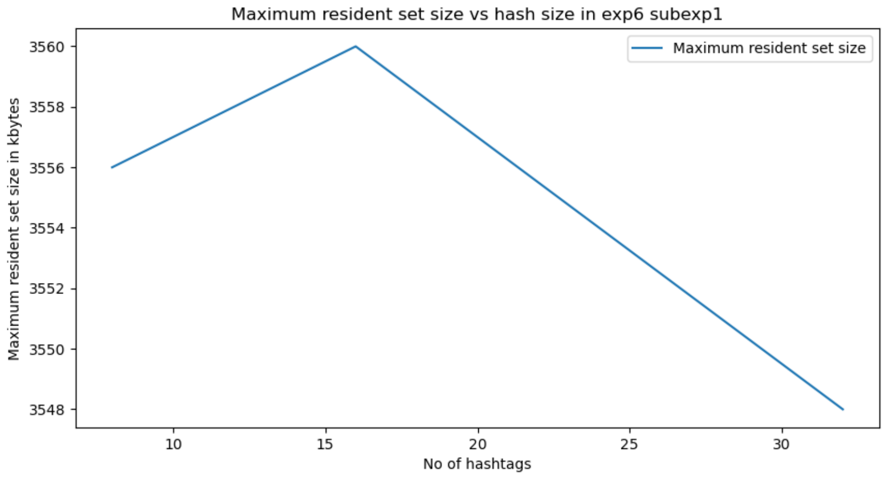
Empirical Results:
I have conducted experiment 6 just for question 3. I ran a different shell script to get the results here.
Subexperiment 1:

- No of unique products = 5
- Total unique hashtags = 10
- No of hashtags per product = 2
- No of customers = 16
- No of purchases = 64

I varied the number of new hashtags added between 8 to 32 to get the following graph:



Ad the number of hashtags increases, the time taken to compute and read both decrease.
The space graph is as follows:



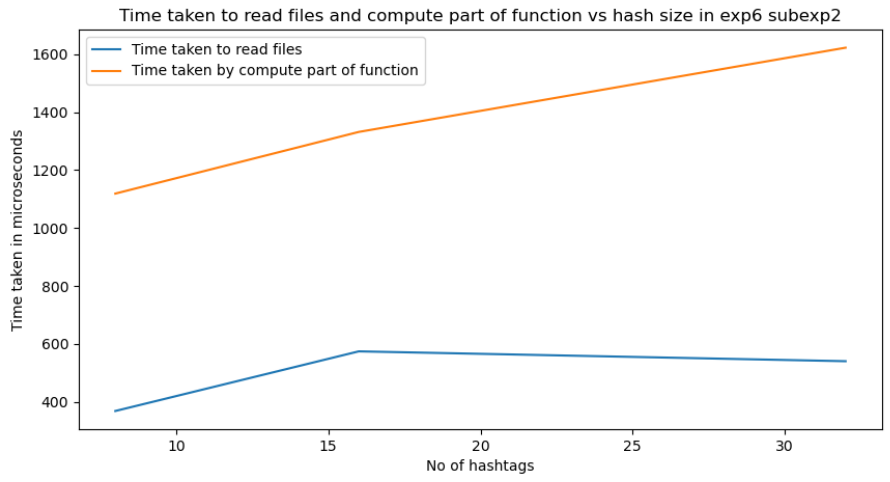
This graph increases and then starts to decrease.

Subexperiment 2:

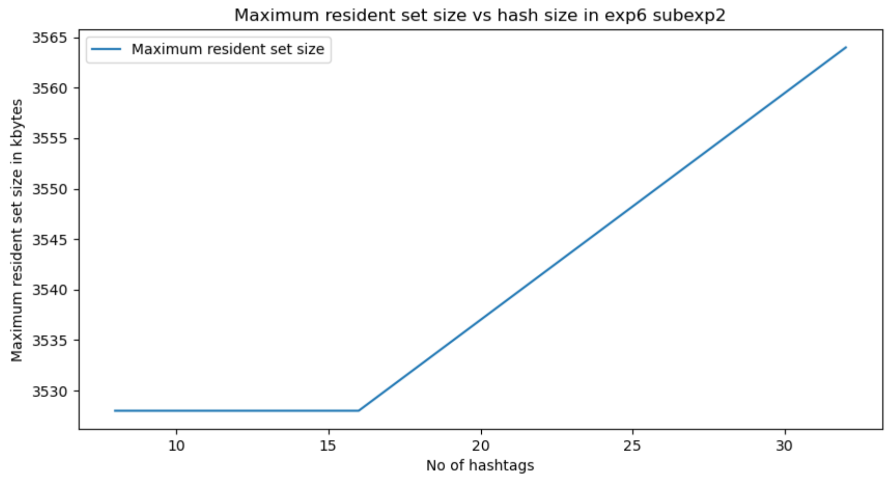
- No of unique products = 10
- Total unique hashtags = 10

- No of hashtags per product = 2
- No of customers = 16
- No of purchases = 64

I varied the number of new hashtags added between 8 to 32 to get the following graph:



While the time taken to compute increases, the time taken to read plateaus after a while.
The space graph is as follows:

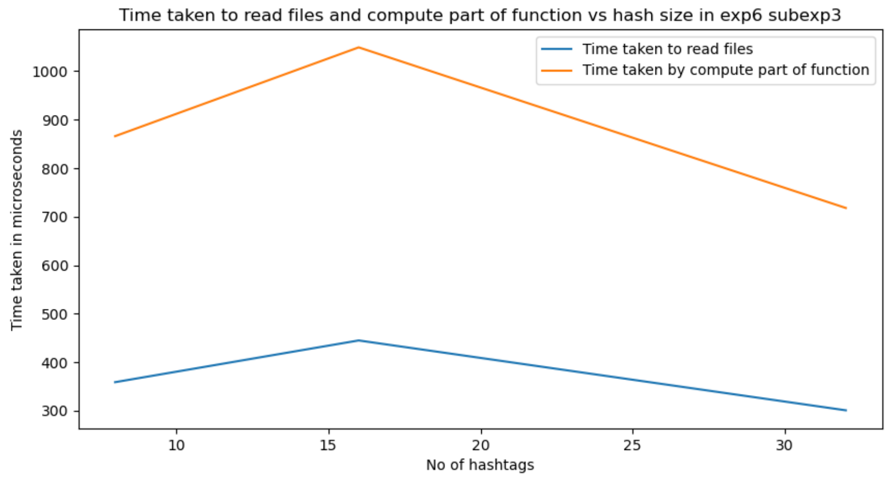


This graph stays the same and starts to increase.

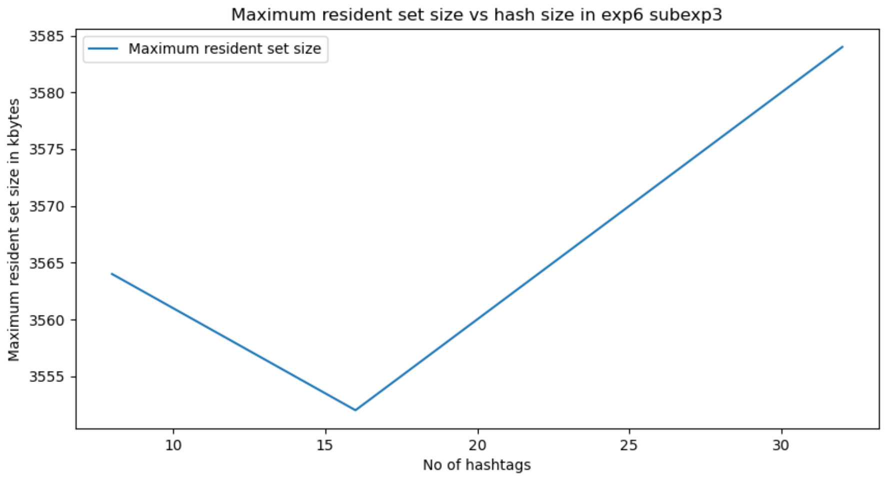
Subexperiment 3:

- No of unique products = 10
- Total unique hashtags = 20
- No of hashtags per product = 4
- No of customers = 16
- No of purchases = 64

I varied the number of new hashtags added between 8 to 32 to get the following graph:



Both the times increase, and start to decrease after a while.
The space graph is as follows:

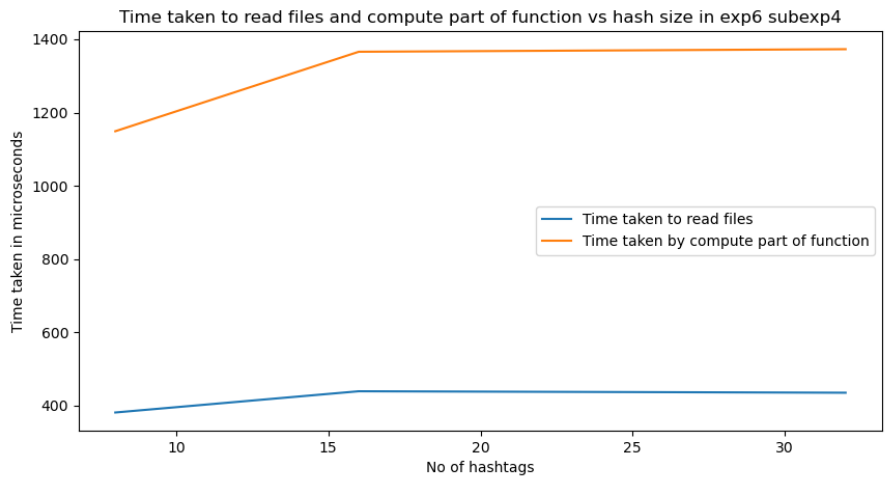


This graph starts to decrase for a short while before increasing again.

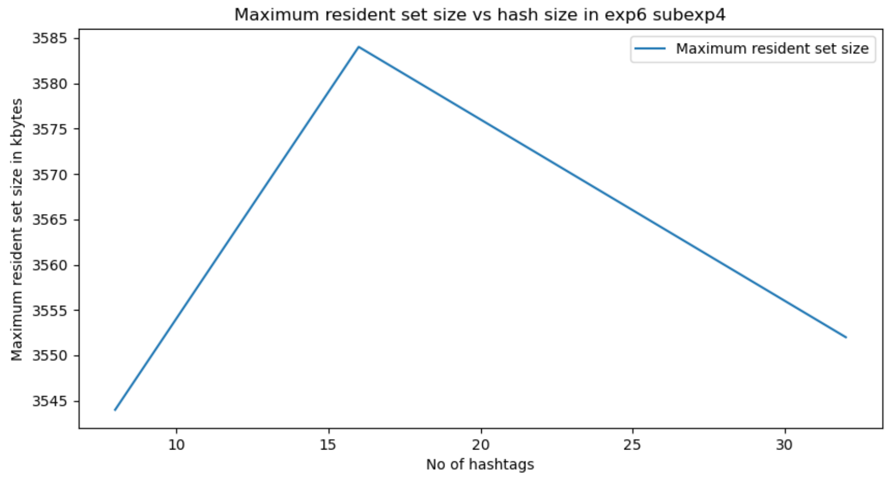
Subexperiment 4:

- No of unique products = 20
- Total unique hashtags = 20
- No of hashtags per product = 4
- No of customers = 16
- No of purchases = 64

I varied the number of new hashtags added between 8 to 32 to get the following graph:



Both the times plateu after increasing.
The space graph is as follows:



This graph increases and then starts to decrease.

Observations:
This deviation from expected results is likely due to a programming specific angle from c++. The language is highly optimized, and is likely caching many things, which causes the time taken to plateau or even decrease after some time. For further analysis, I would need to run the same experiment extensively and maybe use different languages to see if the results are the

same. But that is beyond the scope of this report.
For the space complexity of each subexperiment, as stated before, c++ is highly optimized, and is likely caching many things. For subexperiment 1 the space complexity increases for a short while before decreasing, which is consistent with the time complexity decreasing.
For the second subexperiment, the the compute time increases, which prompts us that the example is new and uncached, causing space complexity to increase.
For the third subexperiment, the time complexity increases and then decreases, but the behavious of the space complexity is exaxtly the opposite. While the time complexity is increases, the space complexity decreases and vise versa. This behaviour is unknown even to me, and I would have to run more experiments to understand this behaviour.
For the fourth subexperiment, the time complexity plateaus after increasing, and the space complexity increases and then decreases. This is consistent with the time complexity.

Approach:
No my approach did not change to solving the problem. I decided to keep a gloabl variable to store all the product hashtags, and then update them as and when new hashtags are added. Other than that, the rest of the goruping approach is the exact same.

Failed Prompts
The following are a list of failed prompts to get copilot to write the proper code for question 1.

```
/* #PROMPT# Write a code snippet that takes
customerPurchases and adds the product hashtags of
each product purchased by the customer to a
data structure. Write the code below:*/
vector<pair<string,vector<string>>> customerHashtags;
for(int i=0;i<customerPurchases.size();i++){
    string customer = customerPurchases[i].first;
    string product = customerPurchases[i].second;
    vector<string> hashtags;
    for(int j=0;j<productHashtags.size();j++){
        if(productHashtags[j].first == product){
            hashtags = productHashtags[j].second;
            break;
        }
    }
    customerHashtags.push_back(make_pair(customer,hashtags));
}

/* #PROMPT# Write a code snippet that takes
customerPurchases and adds the product
hashtags of each product purchased by the customer.
This new variable should be an array of customers
with all their hashtags and the count of each hashtag.
If the hashtags is repeated, the count
for the certain hashtag increases. Write the code below:*/
vector<pair<string,vector<string>>> customerHashtags;
for(int i=0;i<customerPurchases.size();i++){
    string customer = customerPurchases[i].first;
    string product = customerPurchases[i].second;
    vector<string> prodHashtags;
    for(int j=0;j<productHashtags.size();j++){
        if(productHashtags[j].first == product){
            prodHashtags = productHashtags[j].second;
            break;
        }
    }
    bool found = false;
    for(int j=0;j<customerHashtags.size();j++){
        if(customerHashtags[j].first == customer){
            found = true;
            for(int k=0;k<prodHashtags.size();k++){
                bool foundHashtag = false;
                for(int l=0;l<customerHashtags[j].second.size();l++){
                    if(customerHashtags[j].second[l] == prodHashtags[k]){
                        foundHashtag = true;
                        break;
                    }
                }
                if(foundHashtag){
                    continue;
                }
                customerHashtags[j].second.push_back(prodHashtags[k]);
            }
            break;
        }
    }
    if(!found){
        customerHashtags.push_back(make_pair(customer,prodHashtags));
    }
}

/* #PROMPT# Write a code snippet that takes customerPurchases and
iterates through it to create a
map of customers and their hashtags.
Write the code below:*/
map<string,vector<string>> customerHashtags;
for(auto purchase:customerPurchases){
    string customer = purchase.first;
    string product = purchase.second;
    for(auto prodHashtags:productHashtags){
        if(prodHashtags.first == product){
            for(auto hashtag:prodHashtags.second){
                customerHashtags[customer].push_back(hashtag);
            }
        }
    }
}


/* #PROMPT# Write a code snippet
that takes customerPurchases and productHashtags
and iterates through them.
Each entry in the array is a customer id followed by a map.
This map is the hashtag followed by the number of times it is
associated with the ceratin customer. Don't use a map for outer array as
customer id will not be unique. Write the code below:*/
```

```
vector<pair<string,map<string,int>>> customerHashtags;
for(int i=0;i<customerPurchases.size();i++){
    string customer = customerPurchases[i].first;
    string product = customerPurchases[i].second;
    map<string,int> hashtagCount;
    for(int j=0;j<productHashtags.size();j++){
        if(productHashtags[j].first == product){
            for(int k=0;k<productHashtags[j].second.size();k++){
                hashtagCount[productHashtags[j].second[k]]++;
            }
        }
    }
    customerHashtags.push_back(make_pair(customer,hashtagCount));
}


/* #PROMPT# Now that we have grouped the customers based on the top k
hashtags, we can now write the output to a file.
The output should contain the customer ids of the customers in each group.
Each group should be written on a new line in the file.
The customer ids in each
group should be separated by commas. Write the code below:*/
vector<vector<string>> groups(group);
for(auto customer:customerGroup){
    groups[customer.second-1].push_back(customer.first);
}
for(auto group:groups){
    string line = "";
    for(auto customer:group){
        line += customer + ",";
    }
    line.pop_back();
    writeOutputToFile(line,outputFilePath);
}
```

End