Discussion 6: Bayes' Theorem

Discussion Topic:

You work for an insurance company. An analyst comes into your office in a panic. They have been asked to implement AI in a new proof of concept and need to use Bayes' Theorem to help predict the results of someone having a life-threatening disease.

Help the analyst out. How would you explain Bayes' Theorem to them?
How would you connect it to the way insurance companies assess risk, and to this problem in particular?

My Post:

Hello Class,

To explain Bayes' Theorem to an analyst I will start by giving some background information about the Bayes' Theorem as follows:

In probability theory, Bayes' Theorem, also known as Bayes' Rule or Bayes' Law, is a mathematical formula that updates the probability of a hypothesis based on new evidence. In statistics, it is a way to revise or update existing predictions or theories based on new or additional evidence (Hayes, 2024), In other words, it is a mathematical formula for determining conditional probability. A conditional probability is the likelihood of an outcome occurring based on a previous outcome that occurred under similar circumstances. Thus, Bayes' Theorem can also be defined as a mathematical property that allows conditional probability to be expressed in terms of the inverse of the conditional (Data Science Discovery, n.d.).

The Bayes' Theorem formula:

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Where:
- $P(A)$ is the prior probability of an event A.
- $P(B)$ is the probability of an event B.
- $P(B|A)$ is the probability of an event B occurring given that A has occurred.
- $P(A|B)$ is the probability of an event A occurring given that B has occurred.

Therefore, the Bayes' Theorem computes the reverse of the conditional probability. That is the probability of a cause A given an effect $B - P(A|B)$, when the overall probability of the cause $- P(A)$, the probability of the effect $- P(B)$, and the probability of the effect occurring given that the cause has occurred $- P(B|A)$ are known.

Note: $P(B) = P(B \mid A)P(A) + P(B \mid \neg A)P(\neg A)$
Where:
- $P(B \mid \neg A)$ – the probability of event B occurring given that $\neg A$ event has occurred
- $P(B \mid A)$ – the probability of event B occurring given that A event has occurred

(Taylor, 2023)

Another way to formulate Bayes' Theorem is to base it on a hypothesis context:

$$P(H \mid E) = \frac{P(E \mid H) \cdot P(H)}{P(E)}$$

Where:
- H: The hypothesis
  (e.g., a person has a certain disease)
- E: Evidence
  (e.g., the person tests positive in a diagnostic test)
- $P(H)$: Prior probability is the known probability about the hypothesis or the initial belief about the hypothesis, that is the beliefs before observing the evidence.
  (e.g., the probability someone has a specific disease before considering specific symptoms or test results)
- $P(E)$: Marginal probability is the probability of observing the evidence under all possible scenarios. It may be thought of as an unconditional probability, or it is not conditioned on another event (Albright, n.d.).
  $$P(E) = P(E \mid H)P(H) + P(E \mid \neg H)P(\neg H)$$
  (e.g., the probability that a diagnostic test would show a positive result, whether or not the person actually has the disease).
- $P(E \mid H)$: Likelihood is the probability of observing the evidence (e.g., a positive test result) given that the hypothesis is true. In other words, it is the likelihood of E being true based on H being true.
  (e.g., the likelihood of a positive test result being accurate, knowing that the person has the disease; note that medical tests are not 100% accurate — false positive)
  On the other hand, $P(H \mid E)$ is likelihood of H being true based on E being true.
- $P(H \mid E)$: The posterior probability is the updated probability of the hypothesis given the observed evidence.
  (e.g., the probability that the person actually has the disease, given that they tested positive)

Then I will explain to an insurance analyst how the theorem can be used to predict risks:

The formulated Bayes' Theorem above can be used to assess risks by calculating the likelihood of an event, such as accidents, illnesses, or natural disasters. This information is very valuable for an insurance company, enabling it to better understand and predict potential risks.
The steps below describe how the theorem can be applied to predict the risk of a person developing a disease:

1. Prior Knowledge:
   Establish the disease hypothesis, such as the probability someone has a specific disease before considering specific symptoms or test results, then collect relevant data and compute the prior probability $P(H)$ from it.

   For example: If 5 in 1,000 people in the general population have the disease, therefore the prior is 0.005.

   $$P(H) = 0.005$$

2. Incorporate the Evidence:
   Incorporate evidence such as a positive result from a diagnostic test. That is the Likelihood — $P(E \mid H)$. That is the likelihood of a positive test result being accurate, knowing that the person has the disease.
   For example: The test is accurate 96% of the time.

   $$P(E \mid H) = 0.96$$

3. Account for False Positives:
   Compute The marginal probability $P(E)$. That is the probability that a diagnostic test would show a positive result, whether or not the person actually has the disease.
   ○ True Positives $P(E \mid H)P(H)$ — the test correctly identifies the disease.
   ○ False Positives $P(E \mid \neg H)P(\neg H)$ — the test incorrectly identifies the disease.
   ○ Then $P(E) = P(E \mid H)P(H) + P(E \mid \neg H)P(\neg H)$
   For example:

   $$P(E \mid \neg H) = 0.04 \ (4\% \ false \ positive \ rate).$$
   $$P(\neg H) = 1 - P(H) = 0.995.$$
   $$P(E) = P(E \mid H)P(H) + P(E \mid \neg H)P(\neg H) = (0.96)(0.005) + (0.04)(0.995)$$
   $$P(E) = 0.0048 + 0.0398 = 0.0446$$
   $$P(E) = 0.0446$$

4. Compute the Posterior:
   Use Bayes' Theorem to update the probability based on the evidence. That is the probability that the person actually has the disease, given that they tested positive.

   $$P(H \mid E) = \frac{P(E \mid H) \cdot P(H)}{P(E)}$$

   For example:

   $$P(H \mid E) = \frac{0.96 \cdot 0.005}{0.0446} \approx 0.1076$$

Thus, even though the test is 96% accurate the posterior probability $P(H \mid E)$ is equal to 10.76%. This is due to *P(H)* being only 0.5% (5 in 1,000 people in the general population have the disease). In other words, a person having a positive test result increases the person's probability of having the disease from 0.5% to about 10.76%; however, it is still more likely that the person does not have the disease. For the insurance, it means that it needs to consider both test accuracy, $P(E \mid H) = 0.96$, and disease prevalence, *P(H)*, to not overestimate risks, as risks are low even with a positive test, and to set insurance premiums that represent a fair and accurate assessment of the probability of a person having the diseases.

Finally, I will explain the advantages and risks of implementing Bayes' Theorem AI in insurance companies.

As shown from the example above, the Bayes' Theorem is a powerful tool that provides more accurate predictions than solely relying on simple probabilities or data alone. An Artificial Intelligence (AI) Bayes' Theorem implementation can transform and enhance the insurance company's decision-making processes by providing:
- More accurate predictions of risks, such as accidents, illnesses, or natural disasters real-time data processing.
- More personalized and refined prediction-based individual-level data.
- By analyzing vast amounts of data.
- By automating systems streamlining claims processing and fraud detection.

However, a Bayes' Theorem AI may invertedly create biases. Additionally, by using vast personal and private amounts of data such as medical history and financial information the AI system introduces privacy and security concerns, as well as data regulatory compliance challenges. These potential issues need to be carefully considered when using such systems.

To summarize, Bayes' Theorem is a mathematical formula that updates the probability of a hypothesis based on new evidence providing a way to incorporate prior knowledge and observed data in real time into predictions. It is a powerful tool that provides more accurate predictions than solely relying on simple probabilities or data alone and when implemented in an AI model can transform and enhance the insurance company's decision-making processes.

-Alex

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