



## IEL – protokol k projektu

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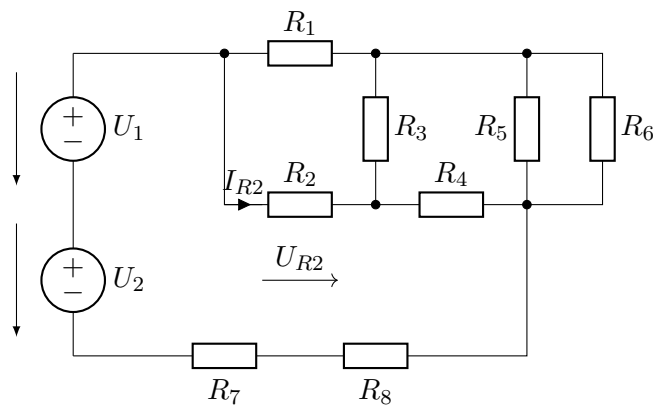
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## Příklad 1

Stanovte napětí  $U_{R2}$  a proud  $I_{R2}$ . Použijte metodu postupného zjednodušování obvodu.

sk.	$U_1$ [V]	$U_2$ [V]	$R_1$ [ $\Omega$ ]	$R_2$ [ $\Omega$ ]	$R_3$ [ $\Omega$ ]	$R_4$ [ $\Omega$ ]	$R_5$ [ $\Omega$ ]	$R_6$ [ $\Omega$ ]	$R_7$ [ $\Omega$ ]	$R_8$ [ $\Omega$ ]
G	130	60	380	420	330	440	450	650	410	275



Stav 1:

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{15960}{113} \Omega$$

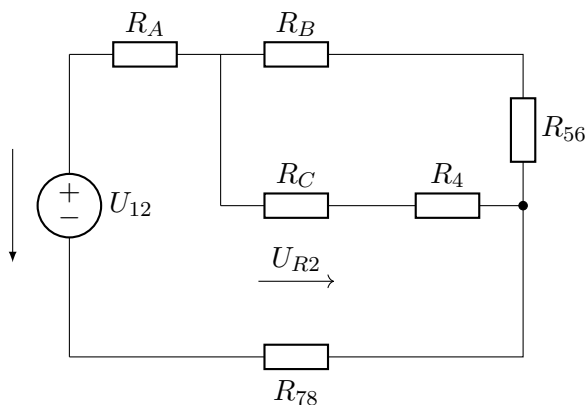
$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{12540}{113} \Omega$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{13860}{113} \Omega$$

$$R_{56} = \frac{R_5 R_6}{R_5 + R_6} = \frac{2925}{11} \Omega$$

$$R_{78} = R_7 + R_8 = 685 \Omega$$

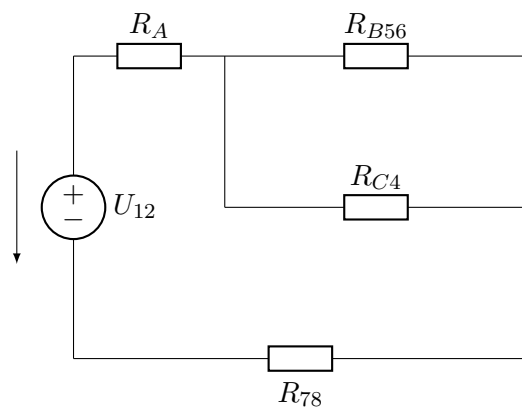
$$U_{12} = U_1 + U_2 = 190V$$



Stav 2:

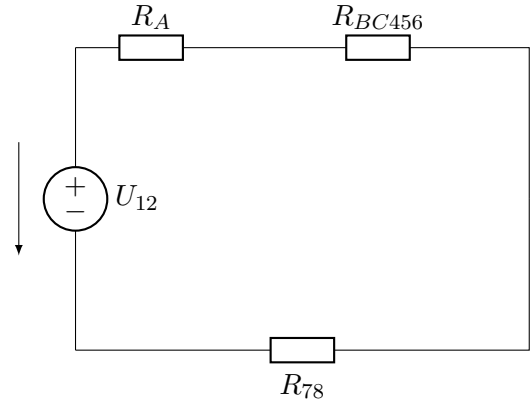
$$R_{B56} = R_B + R_{56} = \frac{468465}{1243} \Omega$$

$$R_{C4} = R_C + R_4 = \frac{63580}{113} \Omega$$



Stav 3:

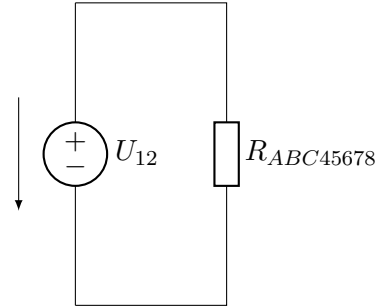
$$R_{BC456} = \frac{R_{B56}R_{C4}}{R_{B56} + R_{C4}} = \frac{5957000940}{26393297}\Omega$$



Stav 4:

$$R_{ABC45678} = R_A + R_{BC456} + R_{78} = \frac{245700625}{233569}\Omega$$

$$I = \frac{U_{12}}{R_{ABC45678}} = \frac{8875622}{49140125}A$$



4 -> 3:

$$\begin{aligned} I &= I_{RA} = I_{RBC456} = I_{R78} \\ U_{RA} &= IR_A = \frac{28330985424}{1110566825}V \\ U_{RBC456} &= IR_{BC456} = \frac{45273207144}{1110566825}V \\ U_{78} &= IR_{78} = \frac{1215960214}{9828025}V \end{aligned}$$

3 -> 2:

$$\begin{aligned} U_{RBC456} &= U_{RB78} = U_{RC4} \\ I_{R56} &= \frac{U_{RBC456}}{R_{B56}} = \frac{5315288}{49140125}A \\ I_{RC4} &= \frac{U_{RBC456}}{R_{C4}} = \frac{3560334}{49140125}A \end{aligned}$$

2 -> 1:

$$\begin{aligned} I_{RC4} &= I_{RC} = I_{RA} \\ U_{RC} &= I_{RC4}R_C = \frac{9869245848}{1110566825}V \end{aligned}$$

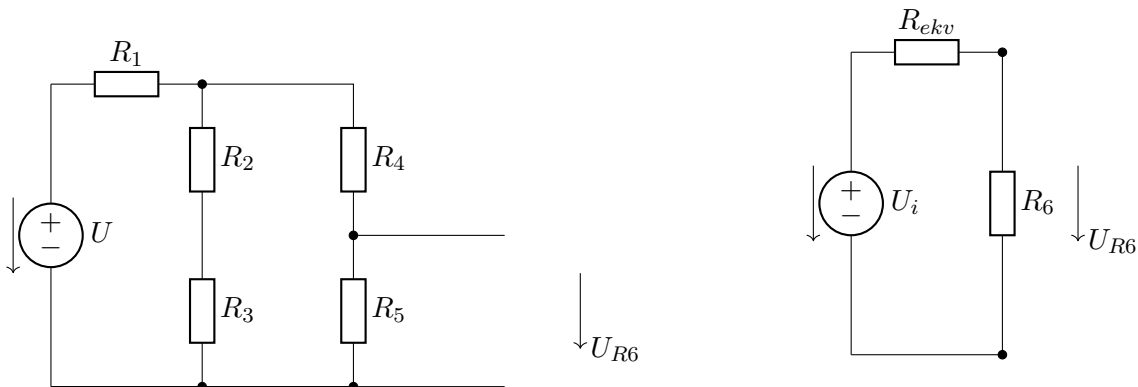
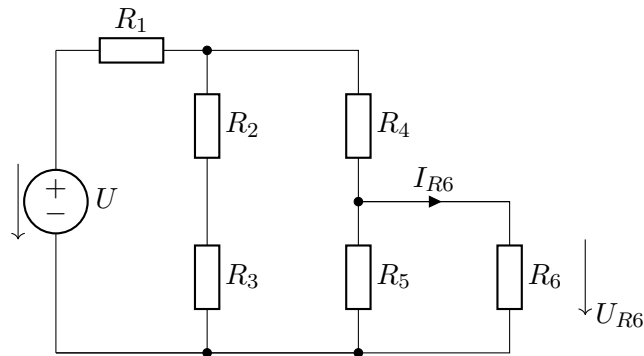
Konečný výpočet:

$$\begin{aligned}
U_{R2} &= U_{RA} + U_{RC} = IR_A + \frac{U_{RBC456}}{R_{B56}} R_C \\
&= IR_A + \frac{I \frac{R_{B56} R_{C4}}{R_{B56} + R_{C4}}}{R_B + R_{56}} R_C \\
&= IR_A + \frac{I \frac{(R_B + R_{56})(R_C + R_4)}{R_B + R_{56} + R_C + R_4}}{R_B + \frac{R_5 R_6}{R_5 + R_6}} R_C \\
&= \frac{U_{12}}{R_{ABC45678}} R_A + \frac{\frac{U_{12}}{R_{ABC45678}} \frac{(R_B + \frac{R_5 R_6}{R_5 + R_6})(R_C + R_4)}{R_B + \frac{R_5 R_6}{R_5 + R_6} + R_C + R_4}}{R_B + \frac{R_5 R_6}{R_5 + R_6}} R_C \\
&= \frac{338055144}{9828025} \doteq 34.39705V \\
I &= \frac{U_2}{R_2} = \frac{4024466}{49140125} \doteq 81.8978mA
\end{aligned}$$

## Příklad 2

Stanovte napětí  $U_{R6}$  a proud  $I_{R6}$ . Použijte metodu Théveninovy věty.

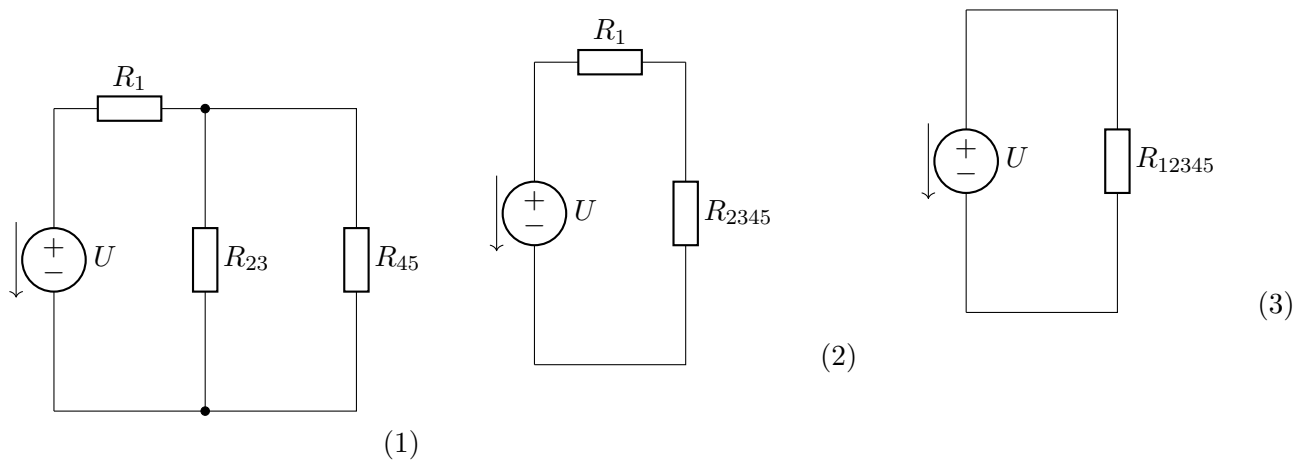
sk.	$U$ [V]	$R_1$ [ $\Omega$ ]	$R_2$ [ $\Omega$ ]	$R_3$ [ $\Omega$ ]	$R_4$ [ $\Omega$ ]	$R_5$ [ $\Omega$ ]	$R_6$ [ $\Omega$ ]
F	130	180	350	600	195	650	80



Ekvivalentní odpor  $R_{ekv}$

$$R_{ekv} = \frac{\left(\frac{R_1(R_2+R_3)}{R_1+R_2+R_3} + R_4\right)R_5}{\frac{R_1(R_2+R_3)}{R_1+R_2+R_3} + R_4 + R_5} = \frac{\left(\frac{180(950)}{1130} + 195\right)650}{\frac{180(950)}{1130} + 195 + 650} = \frac{5087550}{22517} \Omega$$

Napětí náhradního zdroje:



$$\begin{aligned}
R_{23} &= R_2 + R_3 & R_{2345} &= \frac{R_{23}R_{45}}{R_{23} + R_{45}} & R_{12345} &= R_1 + R_{2345} \\
R_{45} &= R_4 + R_5
\end{aligned}$$

$$\begin{aligned}
I &= \frac{U}{R_{12345}} \\
U_{R2345} &= IR_{2345} = U_{2345} \\
I_{2345} &= \frac{U_{R2345}}{R_{45}}
\end{aligned}$$

$$U_i = R_5 = I_{2345}R_5 = \frac{\frac{U}{R_1 + \frac{(R_2+R_3)(R_4+R_5)}{R_2+R_3+R_4+R_5}} \frac{(R_2+R_3)(R_4+R_5)}{R_2+R_3+R_4+R_5}}{R_4 + R_5} R_5 = \frac{1605500}{22517} V$$

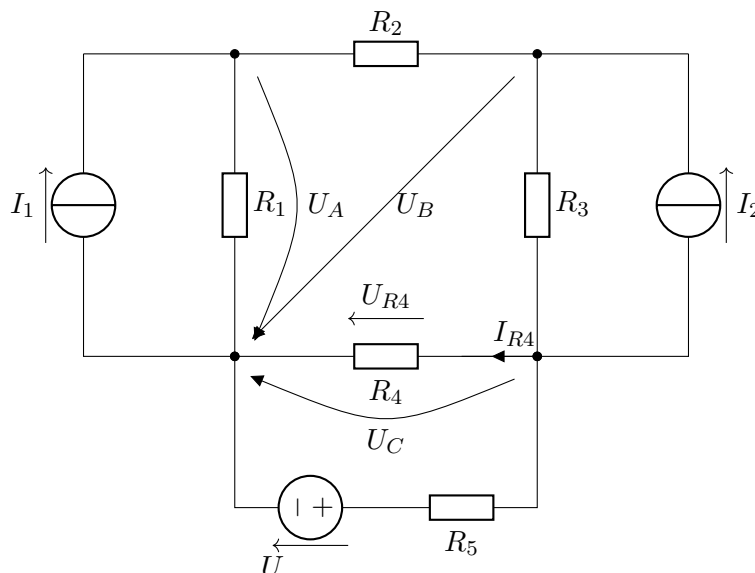
Dopočítání napětí a proudu na  $R_6$ :

$$\begin{aligned}
I_{R6} &= \frac{U_i}{R_{ekv} + R_6} = \frac{5087550}{22517} = 0.2331 A \\
U_{R6} &= I_{R6}R_6 = \frac{12844000}{688891} = 18.6445 V
\end{aligned}$$

### Příklad 3

Stanovte napětí  $U_{R4}$  a proud  $I_{R4}$ . Použijte metodu uzlových napětí ( $U_A, U_B, U_C$ ).

sk.	$U$ [V]	$I_1$ [A]	$I_2$ [A]	$R_1$ [ $\Omega$ ]	$R_2$ [ $\Omega$ ]	$R_3$ [ $\Omega$ ]	$R_4$ [ $\Omega$ ]	$R_5$ [ $\Omega$ ]
D	115	0.6	0.9	50	38	48	37	28



$$\begin{aligned}
 A : I_1 - I_{R1} + I_{R2} &= 0 & I_{R1} &= \frac{U_A}{R_1} \\
 B : I_2 - I_{R2} - I_{R3} &= 0 & I_{R2} &= \frac{U_B - U_A}{R_2} \\
 C : I_{R3} + I_{R5} - I_2 - I_{R4} &= 0 & I_{R3} &= \frac{U_B - U_C}{R_3} \\
 & & I_{R4} &= \frac{U_C}{R_4} \\
 & & I_{R5} &= \frac{U - U_C}{R_5}
 \end{aligned}$$

$$\begin{aligned}
 A : \frac{U_A}{50} - \frac{U_B - U_A}{38} &= 0.6 \\
 B : \frac{U_B - U_A}{38} + \frac{U_B - U_C}{48} &= 0.9 \\
 C : \frac{U_B - U_C}{48} - \frac{U_C}{37} + \frac{U - U_C}{28} &= 0.9
 \end{aligned}$$

$$\begin{bmatrix} \frac{1}{50} + \frac{1}{38} & -\frac{1}{38} & 0 \\ -\frac{1}{38} & \frac{1}{38} + \frac{1}{48} & -\frac{1}{48} \\ 0 & \frac{1}{48} & -\frac{1}{48} - \frac{1}{37} - \frac{1}{28} \end{bmatrix} \times \begin{bmatrix} U_A \\ U_B \\ U_C \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.9 \\ -\frac{449}{140} \end{bmatrix}$$

Cramerovo pravidlo:

$$U_3 = \frac{D_{UC}}{D} = 57.2099V$$

$$I_3 = \frac{U_C}{R_4} = 1.5462A$$

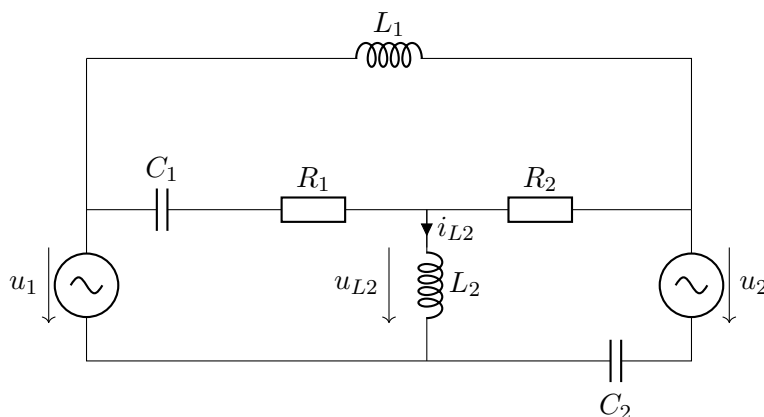
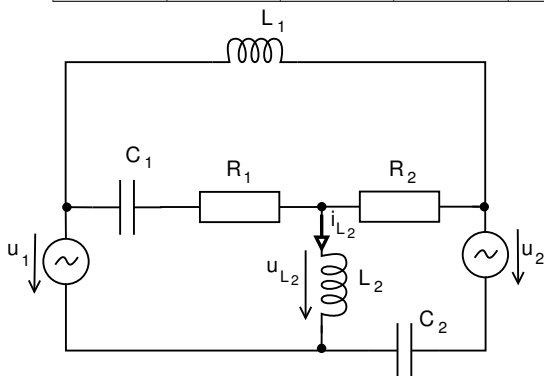
## Příklad 4

Pro napájecí napětí platí:  $u_1 = U_1 \cdot \sin(2\pi ft)$ ,  $u_2 = U_2 \cdot \sin(2\pi ft)$ .

Ve vztahu pro napětí  $u_{L_2} = U_{L_2} \cdot \sin(2\pi ft + \varphi_{L_2})$  určete  $|U_{L_2}|$  a  $\varphi_{L_2}$ . Použijte metodu smyčkových proudů.

Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ( $t = \frac{\pi}{2\omega}$ ).

sk.	$U_1$ [V]	$U_2$ [V]	$R_1$ [ $\Omega$ ]	$R_2$ [ $\Omega$ ]	$L_1$ [mH]	$L_2$ [mH]	$C_1$ [ $\mu$ F]	$C_2$ [ $\mu$ F]	$f$ [Hz]
G	5	5	13	12	140	60	160	80	60



$$\omega = 2\pi f = 120\pi$$

$$Z_{L1} = j\omega L_1 = j\frac{36\pi}{5}\Omega$$

$$Z_{L2} = j\omega L_2 = j\frac{84\pi}{5}\Omega$$

$$Z_{C1} = \frac{1}{\omega C_1} = -j\frac{625}{12\pi}\Omega$$

$$Z_{C2} = \frac{1}{\omega C_2} = -j\frac{625}{6\pi}\Omega$$

Smyčkové proudy:

$$I_A : U_1 = (Z_{C1} + R_1 + Z_{L2})I_A - (Z_{C1} + R_1)I_B - Z_{L2}I_C$$

$$I_B : 0 = -(Z_{C1} + R_1)I_A + (Z_{C1} + R_1 + Z_{L1} + R_2)I_B - R_2I_C$$

$$I_C : -U_2 = -(Z_{L2})I_A - (R_2)I_B + (Z_{L2} + R_2 + Z_{C2})I_C$$



$$\begin{bmatrix} 13 + j(\frac{84\pi}{5} - \frac{625}{12\pi}) & -13 + j\frac{625}{12\pi} & j\frac{84\pi}{5} \\ -13 + j\frac{625}{12\pi} & 25 + j(\frac{625}{12\pi} + \frac{36\pi}{5}) & -12 \\ j\frac{84\pi}{5} & -12 & 12 - j(\frac{84\pi}{5} + \frac{625}{6\pi}) \end{bmatrix} \times \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -5 \end{bmatrix}$$

$$D = 15557.8850643911 + 20990.4517653742i$$

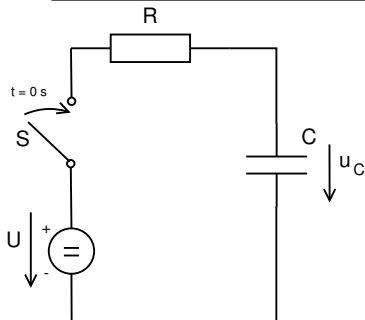
$$|U_{L2}| = Z_{L2}(I_A - I_C) = Z_{L2}(\frac{D_{IA}}{D} - \frac{D_{IC}}{D}) = 9.23081V$$

$$\varphi_{L_2} = 49.8443^\circ$$

## Příklad 5

V obvodu na obrázku níže v čase  $t = 0$  [s] sepne spínač  $S$ . Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení  $u_C = f(t)$ . Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	$U$ [V]	$C$ [F]	$R$ [ $\Omega$ ]	$u_C(0)$ [V]
F	22	10	5	8



$$1 : i = \frac{U_R}{R}$$

$$2 : U_R + U_C - U = 0$$

$$3 : u'_C = \frac{i}{C}$$

$$u'_C = \frac{U - U_C}{RC} = \frac{22 - U_C}{10 * 5}$$

$$u'_C + \frac{U_C}{RC} = \frac{U}{RC}$$

Charakteristická rovnice(  $\lambda = u'_C, u_C = 1$ ):

$$\lambda = -\frac{1}{RC} = -\frac{1}{50}$$

$$u_C(t) = f(t)e^{\lambda t} = f(t)e^{-\frac{t}{RC}}$$

$$u'_C(t) = f(t)'e^{-\frac{t}{RC}} + f(t)\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}$$

Dosadíme:

$$f(t)'e^{-\frac{t}{RC}} = \frac{U_C}{RC}$$

$$f(t) = \frac{U_C}{RC}e^{\frac{t}{RC}}$$

Integrace:

$$f(t) = \frac{U_C}{RC}e^{\frac{t}{RC}}$$

Dosazení počátečních podmínek:

$$u_C(t) = (Ue^{\frac{t}{RC}} + k)e^{-\frac{t}{RC}} = U + ke^{-\frac{t}{RC}}$$

$$k = ke^0 = u_C(0) - U = 8 - 22 = -14V \quad u_C = 22 - 14e^{-\frac{t}{50}}$$

Vyjádření:

$$u'_C + \frac{U_C}{RC} = \frac{U}{RC}$$
$$u'_C + \frac{22 - 14e^{-\frac{t}{50}}}{50} = \frac{22}{50}$$
$$u'_C = \frac{14e^{-\frac{t}{50}}}{50}$$

Kontrola pro  $t = 0$ :

$$u_C = 22 - 14e^{-\frac{t}{50}} = 8 = u_C(0)$$

## Shrnutí výsledků

Příklad	Skupina	Výsledky	
1	G	$U_{R2} \doteq 34.39705V$	$I_{R2} \doteq 81.8978mA$
2	F	$U_{R6} \doteq 18.6445V$	$I_{R6} \doteq 0.2331A$
3	D	$U_{R4} \doteq 57.2099V$	$I_{R4} \doteq 1.5462A$
4	G	$ U_{L2}  \doteq 9.23081V$	$\varphi_{L2} \doteq 49.8443^\circ$
5	F	$u_C = 22 - 14e^{-\frac{t}{50}}V$	