# LOGIC

## Propositional Logic:

The rules of logic give precise meaning to mathematical statements.

It has numerous applications in computer science. The rules are used in the design of computer circuits, the construction of computer program and in many ways, verification and correctness of programs.

**Proposition**: A proposition is a declarative sentence that is either true or false, but not both.

Examples:

1+1=2 (true)

2+2 =3 (false)

Mukalla ihe capital of Hadramout (true).

The following statements are not propositions,

$$x+1 = 2$$

$$x+y = z$$

**Note**: The first two may be made propositions if we assign values.

Letters can be used for denoting propositional variables (or statement variables) p,q,r,s ……

The tuth values of the propositions, true denoted by (T) and false denoted by (F).

**Logical operators**: Negation , Conjunction, Disjunction,Conditional,Biconditional and Exclusive OR.

**Definition:** Let p be a proposition. The **negation** of p, denoted by ($\neg$p) is the opposite of the truth values of p.

**Example 1**: Find the negation of the proposition.

p: Today is Friday

**¬p** : Today is **not** Friday

| **p** | **(¬p)** |
|---|---|
| T | F |
| F | T |

**Definition :** Let p and q be propositions. The conjunction of p and q denoted by $p \wedge q$ is true when both p and q are true and is false otherwise.

**Example**:

p: Today is Friday ,

q: It is raining today

**($p \wedge q$)** : Today is Friday **and** it is raining today,

| **p** | **q** | **$p \wedge q$** |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**Definition:** Let p and q be propositions. The disconjunction of p and q denoted by $p \vee q$ is false when both p and q are false and is true otherwise.

**Example**: p: Today is Friday,

q: It is raining today,

($p \vee q$): Today is Friday **or** it is raining today

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

**Definition:** Let p and q be propositions. The **conditional** statement $p \rightarrow q$ is false when p is true and q is false and is true otherwise. ($p \rightarrow q$ is if p then q). In the conditional statement $p \rightarrow q$, *p* is called the *hypothesis* (or *antecedent* or *premise)* and *q* is called the *conclusion* (or *consequence).*

| p | q | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

**Example**:

p: Get 100% in the final,

q: You will get A,

($p \rightarrow q$): **If** you get 100% in the final **then** you will get A.

**Example:** Let *p* be the statement "Maria learns discrete mathematics" and *q* the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

**Solution:**

p: Maria learns discrete mathematics,

q: Maria will find a good job,

($p \rightarrow q$): **If** Maria learns discrete mathematics **then** she will find a good job.

**Definition:** Let p and q be propositions. The **biconditional** statement $p \leftrightarrow q$ is true when p and q have same truth values and is false otherwise. ($p \leftrightarrow q$ is p iff q). Biconditional statements are also called *bi-implications*.

| p | q | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

**Example**: Let *p* be the statement "You can take the flight" and let *q* be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement.

**Solution:**

p: You can take the flight,

q: You can buy the ticket,

($p \leftrightarrow q$) :You can take a flight **iff** you can buy a ticket.

**Definition:** Let p and q be propositions. The **exclusive OR** of p and q denoted by $p \oplus q$ is a proposition that is true when exactly one of p and q is true and is false otherwise.

| p | q | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

**Precedence of logic operators**: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

| **p** | **q** | **$\neg$p** | **$\neg$q** | **p $\wedge$ q** | **p $\vee$ q** | **p $\rightarrow$ q** | **p $\leftrightarrow$ q** | **p$\oplus$q** |
|---|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T | T | F |
| T | F | F | T | F | T | F | F | T |
| F | T | T | F | F | T | T | F | T |
| F | F | T | T | F | F | T | T | F |

**Example:** Construct the truth table for the compound proposition

$$(\mathbf{p} \vee \neg\, \mathbf{q}) \rightarrow (\mathbf{p} \wedge \mathbf{q}).$$

**Solution:**

| **p** | **q** | **$\neg$q** | **p$\vee\neg$q** | **p $\wedge$ q** | **(p $\vee\neg$ q) $\rightarrow$(p $\wedge$ q)** |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | T | T | F | F |
| F | T | F | F | F | T |
| F | F | T | T | F | F |

**Example:** Construct the truth table for the compound proposition

$$(\mathbf{p} \rightarrow \mathbf{q}) \wedge (\neg \mathbf{p} \rightarrow \mathbf{q})$$

**Solution:**

| **p** | **q** | **$\neg$p** | **p$\rightarrow$q** | **$\neg$p$\rightarrow$ q** | **( p $\rightarrow$ q ) $\wedge$ ($\neg$p $\rightarrow$ q)** |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | F | T | T | F | F |

**Example:** Construct the truth table for the compound proposition

$$\neg \mathbf{p} \oplus \neg \mathbf{q}$$

## Solution:

| p | q | ¬p | ¬q | ¬p ⊕ ¬q |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | F |

# Exercises

**1.** How many rows appear in a truth table for each of these compound propositions?

a) $(q \to \neg p) \vee (\neg p \to \neg q)$

b) $(p \vee \neg t) \wedge (p \vee \neg s)$

c) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$

d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

**2.** Construct a truth table for each of these compound propositions.

a) $p \to \neg p$

b) $p \leftrightarrow \neg p$

c) $p \oplus (p \vee q)$

d) $(p \wedge q) \to (p \vee q)$

e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

**3.** Construct a truth table for each of these compound propositions.

a) $p \oplus p$

b) $p \oplus \neg p$

c) $p \oplus \neg q$

d) $\neg p \oplus \neg q$

e) $(p \oplus q) \vee (p \oplus \neg q)$

f) $(p \oplus q) \wedge (p \oplus \neg q)$