Between abstraction and composition...

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November 2021

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PL theory = advancing linguistic solutions to the contradiction between abstraction and composition (paraphrasing Reynolds, 1983).

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Thesis: need linguistic protocols to **smoothly interpolate** between different levels of abstraction.

We will tell our story in three parts.

- 1. Breaking abstraction
- 2. Enforcing abstraction
- 3. Prospects

1. Breaking abstraction

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Exploitation of representation *invariants*.

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- 4. All functions (string @ HighSecurity) \rightarrow (bool @ LowSecurity) are constant.

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Abstraction simplifies both programming and verification tasks.

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 - 1.1 Specializing polymorphic functions (e.g. C++ templates)
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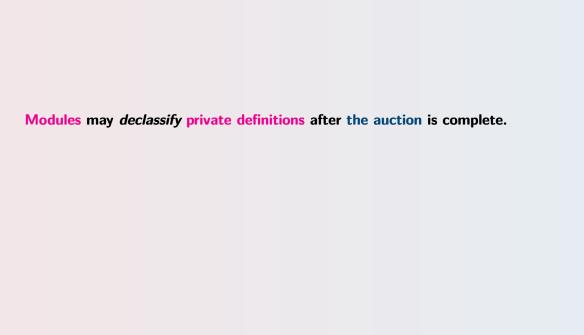
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Most languages treat abstraction as a binary choice, but our needs are more complex.

Using *cross-module inlining* as an example, I will illustrate a path forward employing recent advances in the understanding of type theory.









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Alternative: whole-program analysis à la MLton. Works great, but very slow and memory-intensive. **We want** to put the choice in the programmer's hands.

Program units and their interfaces

Programs are divided into compilation units; units are classified by an *interface* that represents their imports and exports.

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Example

A fragment of the (idealized) interface to OS.FileSys in SML's Basis Library:

```
import
 option : type \rightarrow type
 some : (\alpha : \mathbf{type}) \to \alpha \to \mathsf{option}(\alpha),
 none : (\alpha : \mathbf{type}) \rightarrow \mathsf{option}(\alpha),
 case : (\alpha, \beta : \mathsf{type}) \to (\alpha \to \beta) \to \beta \to \mathsf{option}(\alpha) \to \beta,
 . . .
export
 dirstream : type,
 opendir : string \rightarrow dirstream,
 readdir : dirstream \rightarrow option(string),
 . . .
```

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\begin{array}{l} \mathsf{dirpath} : \{ \mathbf{type} \hookrightarrow \mathsf{string} \}, \\ \mathsf{dirstream} : \mathbf{type}, \\ \mathsf{opendir} : \mathsf{dirpath} \rightarrow \mathsf{dirstream}, \\ \dots \end{array}
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dirpath : \{ \text{type} \hookrightarrow \text{string} \}, dirstream : \text{type}, opendir : \text{dirpath} \rightarrow \text{dirstream}, . . .
```

Inlining problem is similar, but we want to reveal representation details to the **compiler** but **not** the programmer. Need for *controlled abstraction breaking*.

The geometry of phase distinctions

A phase distinction is a protocol for breaking and enforcing abstraction. **Main moves:** "hide information until" and "redact information from".

Technically, phase distinctions are open/closed partitions in a space of program behaviors (*c.f.* Alpern and Schneider (1985)!).

New modal type structure to mediate between open and closed subspaces (Rijke, Shulman, and Spitters, 2020).

Let's see how it works for our running example!

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- ▶ Phases are a partial order $\mathcal{O} = \{\mathbf{C} \leq \top\}$ where \top represents "now". The (total) partial singleton $\{\tau \mid \top \hookrightarrow e\}$ is the singleton $\{\tau \hookrightarrow e\}$.

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- ▶ Judgments $\Gamma \vdash_{\varphi} e : \tau$ and $\Gamma \vdash_{\varphi} e \equiv e' : \tau$ are *contravariantly* indexed in phases $\varphi \in \mathcal{O}$.

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import

```
\label{eq:hid:prop} \begin{split} &\text{hid: } \{ \textbf{type} \mid \textbf{C} \hookrightarrow \text{unsigned short} \}, \\ &\text{qr, opcode, aa, } \ldots : \{ \textbf{type} \mid \textbf{C} \hookrightarrow \text{unsigned char} \}, \\ &\text{header: } \{ \textbf{type} \mid \top \hookrightarrow \text{hid} \times \text{qr} \times \text{opcode} \times \text{aa} \times \ldots \}, \\ &\textbf{export} \\ &\text{parseheader: bits} \rightarrow \text{option(header)} \times \text{bits} \end{split}
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hid : {type | C \hookrightarrow \text{unsigned short}},
qr, opcode, aa, . . . : {type | C \hookrightarrow \text{unsigned char}},
header : {type | T \hookrightarrow \text{hid} \times \text{qr} \times \text{opcode} \times \text{aa} \times . . .},
export
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Theorem. The parser does not observably depend on the reprs of opcode, etc.

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Theorem. The parser does not observably depend on the reprs of opcode, etc.

Compilation proceeds by pulling back along the phase transition $C \leq_{\mathcal{O}} T$; we have:

```
\vdash_{\mathsf{C}} header \equiv unsigned short \times unsigned char \times unsigned char \times . . .
```

⇒ unboxed repr. possible without breaking programmer-abstractions!

2. Enforcing abstraction

In the inlining example, we were hiding information *until* a phase, *e.g.* the *open modality* for phase **C**:

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 $\mathbf{C} \Rightarrow \mathsf{opcode} \equiv \mathsf{int}$

What about hiding information *from* a phase? *e.g.* stripping and noninterference of profiling data achieved by means of complementary *closed modality*:



(the smallest type containing int that becomes isomorphic to unit at compiletime)

Sealing: instrumentation sans interference

Poor man's profiling: add counter fields to some datatypes and keep track of how many times you call functions.

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Solution: seal the counter variables under the closed modality $\mathbb{C} \vee \tau$; this causes them to be erased by the default compiler target.

Noninterference / modal phase splitting automatically ensures that input-output behavior of compiled programs cannot depend on the values of counters.

 $\begin{array}{l} \textbf{fun} \ \mathsf{myfun} \ () = \\ \\ \mathsf{mybody}() \end{array}$

```
val counter : (C \vee int) ref =
ref (seal 0)
fun myfun () =
mybody()
```

```
val counter : (C \lefta int) ref =
  ref (seal 0)

fun myfun () =
  Ref.update (Seal.map Int.incr) counter;
  mybody()
```

the program at phase **C**

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val counter : unit ref =
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Relaxing noninterference with declassification seems possible by *mixing* open / closed modalities relative to authorization policy. (Stay tuned!)

3. Prospects

A menagerie of phase distinctions

open subspace	closed subspace	
observable properties	safety properties	Alpern and Schneider (1985)
syntax	semantics	Sterling and Angiuli (2021)
static code	dynamic code	Sterling and Harper (2021b)
compiletime	devtime	Sterling and Harper (2021a)
functions/behavior	algorithms/cost	Niu, Sterling, Grodin, and Harper (2021)

Payoff so far: syntax/semantics phase distinction was the key to *Synthetic Tait Computability*, a new kind of logical relations method that made it tractable to prove normalization for cubical type theory and representation independence for ML modules.

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Three magic weapons:

- $\qquad \qquad \textbf{Open modality} \ \overline{|\varphi\Rightarrow\tau|} \ \text{and partial singletons} \ \{\tau \ | \ \varphi\hookrightarrow \mathbf{e}\} \ \text{reveal data at phase} \ \varphi.$
- ► Closed modality $\varphi \lor \tau$ hides data at phase φ .
- Fracture theorem: any type τ is a subtype of $\varphi \Rightarrow \tau \times \varphi \vee \tau$. A complement to the Alpern–Schneider (1985) result on safety & liveness?

Prospects and future work

Several applications of the phase distinction metalanguage already developed:

- ▶ [POPL'21] A cost-aware logical framework (Niu, Sterling, Grodin, and Harper, 2021)
- ▶ [J.ACM] Logical relations as types (Sterling and Harper, 2021b)
- ▶ [LICS'21] Normalization for cubical type theory (Sterling and Angiuli, 2021)
- ► Normalization for multi-modal type theory (Gratzer, 2021)

Next steps:

- ▶ Develop connection to security typing and declassification (jww. Balzer and Harper)
- Generalize to support general recursion and realistic computational effects (jww. Birkedal)

Please join me! I'm looking for new collaborations.

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