Burning Man Shortest Path Algorithm

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1 Introduction

Burning Man is an annual event packed full of fun activities. Often one finds themselves wanting to travel across the large landscape quickly. Due to the peculiar structure of the landscape, it is difficult to intuitively tell what the shortest path between two points is. This document provides the optimal solution, proves its optimality, and provides a simple near-optimal solution that one can practically recall and apply.

2 The Landscape

Let's begin by taking a look at a map of the Burning Man landscape:

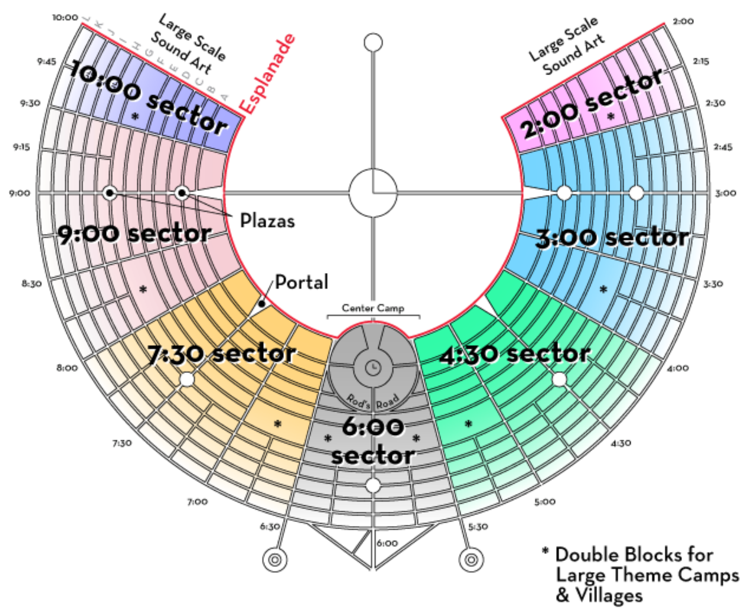

Figure 1: Burning Man landscape

The white center (the Playa) is traversable open space. The Playa's boundary is called the Esplanade. The colored blocks represent obstacles. The obstacles are organized into concentric rings labelled from A to L (see top left of image for labels). The white gaps between obstacles are traversable paths. Clock notation is used to describe position along each lettered concentric ring (2:00 to 10:00), and is punctuated by 15 minute distances.

An example query would be "what is the shortest path from 4:30F to 9:15G?".

3 Why This Is a Hard Problem

The key to appreciating the complexity of the decision problem is, it is difficult to tell whether one should cut into the Playa, or traverse a concentric ring. For example, to get from 2:30L to 9:00L, it is faster to cut into the Playa:

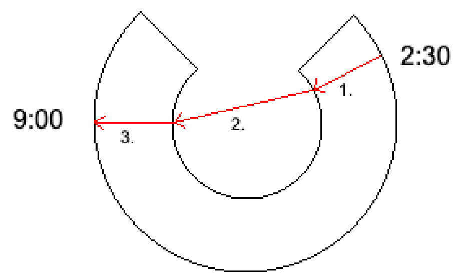

Figure 2: Example path where cutting into the Playa is optimal

1. Travel from 2:30L to 2:30 Esplanade
2. Travel from 2:30 Esplanade to 9:00 Esplanade
3. Travel from 9:00 Esplanade to 9:00L

Than to traverse the concentric L path:

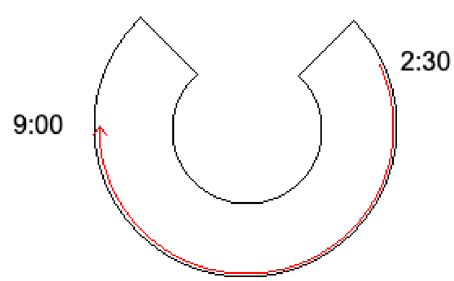

Figure 3: Example path traversing the concentric path is non-optimal

However, to get from 2:30L to 3:00L, it is faster to traverse the concentric L path:

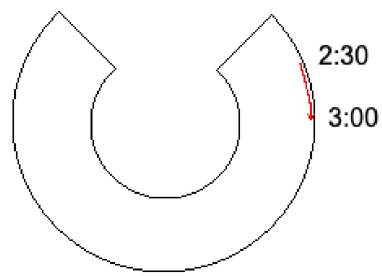

Figure 4: Example path traversing the concentric path is optimal

than to cut into the Playa:

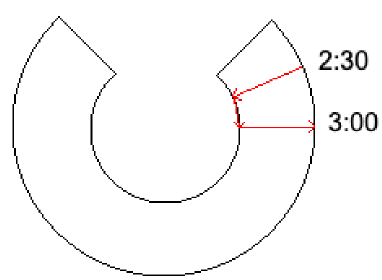

Figure 5: Example path where cutting into the Playa is non-optimal

4 Simple Near-Optimal Policy

If the difference in clock position between the source and destination point is less than or equal to 3:30 (e.g. 2:30 to 3:00 has difference 0:30 which is less than 3:30), traverse the concentric path. Otherwise, cut into the Playa.

A slight nuance for the concentric path case is that you always want to traverse the arc closest to the Playa. For example to get from 2:30A to 3:00L, the closest arc to the Playa out of A and L is A. So opt for this path:

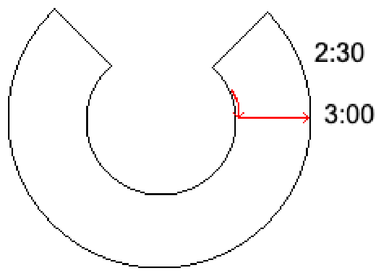

Figure 6: Example path for optimal concentric path selection

Over this path:

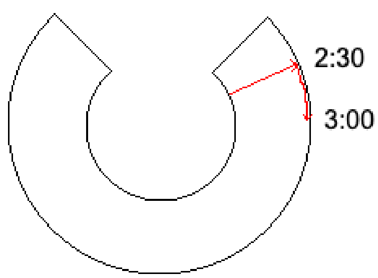

Figure 7: Example path for non-optimal concentric path selection

5 Proof

Imagine that you are at point P_1, and would like to travel to point P_2 while covering as little distance as possible. You would like to travel in a straight line, as this is the shortest distance connecting the points, but this direct path is obstructed. Given the constraints, there are a few ways to get from P_1 to P_2:

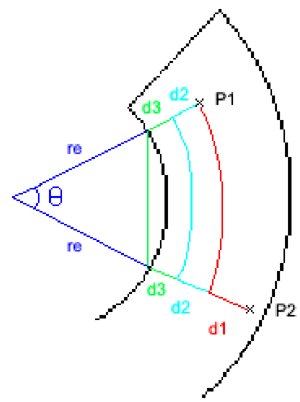

Figure 8: Possible paths from P_1 to P_2. The d terms are arbitrary. r_e is used as the distance between the Esplanade and the center of the Playa

The first way involves immediately traversing the concentric path of P_1 towards P_2:

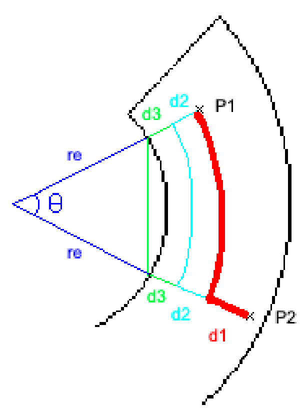

Figure 9: First path from P_1 to P_2

The second way involves traversing inwards towards the Playa some amount, to reduce the arc length of the concentric path, and then coming back out to P_2:

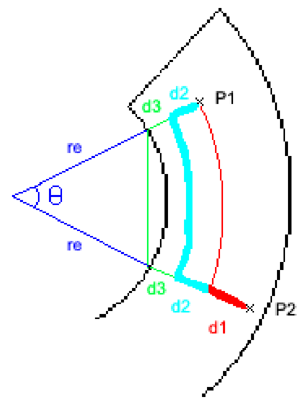

Figure 10: Second path from P_1 to P_2

The third way involves traversing inwards all the way to the Playa, replacing the need to traverse an arc with a straight line, before coming all the way back out to P_2:

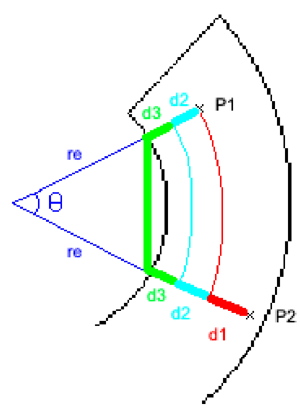

Figure 11: Third path from P_1 to P_2

For convenience, let's define $r = r_e + d_3 + d_2$. Now we can express all three paths in terms of variables:

1. $\theta r + d_1$

2. $d_2 + \theta(r - d_2) + d_2 + d_1$
3. $d_2 + d_3 + \sqrt{2r_e^2 - 2r_e^2 cos\theta} + d_3 + d_2 + d_1$

Note that the third path's square root term is derived from the cosine rule.

5.1 Comparing the First and Second Paths

Let's begin by comparing when we should traverse the first path vs. the second path. We can do this by seeing when the paths have an equivalent distance:

$$\begin{aligned}
\theta r + d_1 &= d_2 + \theta(r - d_2) + d_2 + d_1 \\
\theta r + d_1 &= 2d_2 + \theta(r - d_2) + d_1 \\
\theta r + d_1 &= 2d_2 + \theta r - \theta d_2 + d_1 \\
\theta r &= 2d_2 + \theta r - \theta d_2 \\
0 &= 2d_2 - \theta d_2 \\
\theta d_2 &= 2d_2 \\
\theta &= 2
\end{aligned}$$

Thus, the first and the second paths have equal distance when $\theta = 2$ radians.

The above also shows us that when $\theta < 2$ radians, the first path is shorter, and when $\theta > 2$ radians, the second path is shorter.

5.2 Understanding the Second Path

We know to use the second path when $\theta > 2$ radians. Note however that we have not established how far inwards towards the Playa we should traverse to minimize the length of the second path. We can solve this decision problem by investigating the partial derivative of the second path with respect to the distance inwards (d_2):

$$\begin{aligned}
&\frac{\partial \ second \ path}{\partial d_2} \\
&= \frac{\partial \ (2d_2 + \theta r - \theta d_2 + d_1)}{\partial d_2} \\
&= 2 + 0 - \theta + 0 \\
&= 2 - \theta
\end{aligned}$$

We are using the second path when $\theta > 2$ radians. For this bound, the partial derivative is negative. Thus, the length of the second path decreases as we increase the distance we go inwards towards the Playa.

Therefore, to minimize the length of the second path, we maximize the distance we go inwards towards the Playa. This is exactly what the third path captures. Thus, we can discard the second path as an option, and always opt for the third path.

5.3 Wrapping Up

The critical value of $\theta = 2$ radians does not perfectly apply to deciding between the first and third path, since the third path is slightly shorter than the best version of the second path.

The second path contains an arc of length $\theta(r - d_2)$. Maximizing d_2 makes this arc have length θr_e. This is always greater than the length of the straight line that the third path replaces this arc with. The straight line has length $\sqrt{2r_e^2 - 2r_e^2 cos\theta}$.

The algebra does not simplify nicely – the decision criterion between the first and third paths involves both r and θ.

For a computer program processing an input query, it is easiest to compute the distances for the first and third paths, and output the path that has the lowest distance (see https://github.com/OmerBaddour/BurningMan).

For a human, it is practically fine to approximate the third path with the second path to mitigate this dependency on r, and apply the $\theta = 2$ radians criterion. $\frac{2}{2\pi} \times 12$ hours $= 3.82$ hours ≈ 3.75 hours $=$ 3 hours and 45 minutes. Since this is a little less memorable than 3 hours and 30 minutes, I opt for the 3 hours and 30 minutes human heuristic.

6 Bonus: Understanding the Significance of 2 Radians

The prevalence of $\theta = 2$ radians is no coincidence. 2 radians means double the length of the radius:

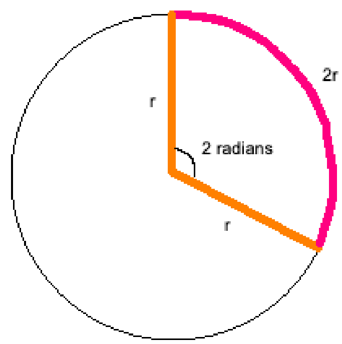

Figure 12: Understanding 2 radians

The length of all paths from the circumference of a circle to another point on the circumference of the circle is fixed at $2r$. Similarly, the length of an arc along the circumference of a circle that spans an angle of 2 radians has length $2r$. However, if the span of the angle is less than 2 radians, the length of the arc will be less than $2r$.

This is why comparing θ to 2 radians answers the "do we got inwards towards the Playa?" question.

7 Credits

Shoutout to Orson Rosetto and Finn Mackay for working on this problem with me :)