הטכניון – הפקולטה למדעי המחשב **חורף תשע"ה**
גרפיקה ממוחשבת – 234325 **02/02/2015**

מרצה: פרופ' מירלה בן חן
מתרגל: עומרי אזנקוט

מבחן סיום

שם: _____

מס' סטודנט: _____

הנחיות:

- בבחינה שלפניכם 4 דפים כולל דף זה. בדקו זאת.
- **עליכם לענות על כל 4 השאלות.**
- מומלץ לקרוא ראשית כל שאלה עד סופה, ורק אח"כ לענות.
- כתבו בקצרה. כל המאריך גורע!
- **משך הבחינה: 180 דקות.**
- יש לכתוב את כל התשובות במקום המתאים בטופס הבחינה ולהגיש טופס זה.
- יש להקפיד על כתיבה ברורה ומסודרת של התשובות.
- אם הנכם מוצאים צורך להניח הנחות כלשהן, ציינו אותן במפורש ונמקו.
- מותר השימוש בכל חומר עזר כתוב או מודפס (לא אלקטרוני).

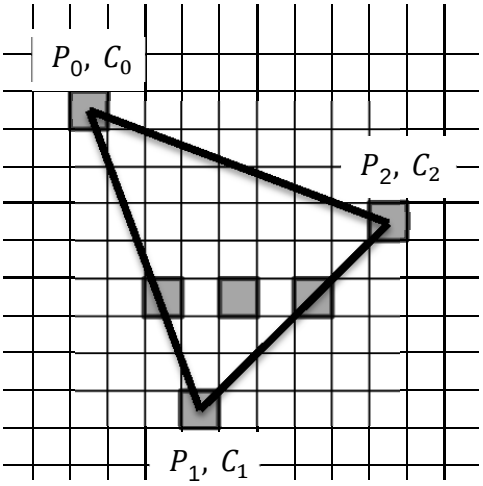
בהצלחה !

שאלה	נקודות	
1	25	
2	25	
3	25	
4	25	
סה"כ	**100**	

Question 1 – Shading (25 pts.)

(a) (5 pts.) You are given software for rendering a single polygon, using one of the shading methods – flat, Gouraud and Phong. What parameters should the software get as input? How can you know which of the shading methods the software uses? Give a specific example for corresponding input(s)/output(s) that will allow you to distinguish between the different shading methods.

(b) (10 pts.) The Gouraud shading method learnt in class can be described as follows: Given the colors C_0, C_1, C_2 at the vertices P_0, P_1, P_2, the color of a pixel on an edge is given by linear interpolation of the colors at the corresponding vertices, and the color of an interior pixel is given by linear interpolation along the scan line of the colors on the edges that intersect the scan line:



In this model, can the color of an interior pixel change as a result of rotating the triangle in the image plane? Assume the triangle is large enough, so that you can neglect truncation errors. If the color can change show a detailed example, if not explain exactly why not.

(c) (10 pts.) The model from (b) is also valid for quadrilateral polygons: the color on the edge is given by linear interpolation on the edge, and the color of an interior pixel is given by linear interpolation on the scan line. In this case, can the color of an interior pixel change as a result of rotating the quadrilateral in the image plane? If the color can change show a detailed example, if not explain exactly why not.

Question 2 – Animation (25 pts.)

(a) (4 pts.) An orientation in 3D is represented using the three Euler angles: $\theta_x = \frac{\pi}{3}, \theta_y = \frac{\pi}{2}, \theta_z = \frac{\pi}{6}$, where the corresponding rotation matrix is $R(\theta_z, \theta_y, \theta_x) = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$. Is there a different set of Euler angles $\beta_x, \beta_y, \beta_z$ that represent the same orientation? If so, give an example, if not explain why not.

(b) (5 pts.) At time *t* = 0 you are given a square with side length 1, centered at the origin. We want to generate a planar animation, such that at time *t* = 1 the center of the square will be at the point (5,5). Write a transformation matrix $T(t)$ (whose elements depend on *t*), such that applying it to the square will generate a smooth animation between these two key frames.

(c) (8 pts.) Repeat (b) for the case that at *t* = 1 the center of the square is at (5,5) and it is rotated by $\frac{\pi}{4}$ with respect to the *x* axis. The animation should be smooth, and specifically, the shape should remain a unit square for all times *t*.

(d) (8 pts.) What would change in your answer to (c) if we add an additional key frame at $t = \frac{1}{2}$, at which the center of the square should be at the point (2,3)?

Question 3 – Transformations (25 pts.)

Let T: $\mathbb{R}^3 \to \mathbb{R}^3$ be an affine transformation, namely transformation of the form $x \mapsto Ax + b$, where A is a linear transformation and b is a vector. Remember that given two points p, q, you can represent the straight line containing both using the expression $(1-t)p + tq, t \in \mathbb{R}$.

(a) (4 pts.) Show that T preserves distance ratios along a straight line. Specifically, given four collinear points x_1, x_2, x_3, x_4, show that $\frac{\|x_4 - x_3\|}{\|x_2 - x_1\|} = \frac{\|T(x_4) - T(x_3)\|}{\|T(x_2) - T(x_1)\|}$.

(b) (7 pts.) Does T preserve angles? Prove or give a counter example.

(c) (7 pts.) In which cases is T invertible? Explain, and give an expression for the inverse T^{-1}.

(d) (7 pts.) Does T preserve parallel lines? Prove or give a counter example.

Question 4 – Clipping (25 pts.)

(a) (10 pts.) What is the maximal number of vertices in the intersection (clipping) of a triangle and a square? Explain.

(b) (10 pts.) Given a convex polygon P, what are the properties of the intersection of P and a square? Explain.

(c) (5 pts.) Given a convex polygon P with *n* vertices, what is the maximal number of vertices in the intersection of P and a square? Explain.