הטכניון – הפקולטה למדעי המחשב **חורף תשע"ב**
גרפיקה ממוחשבת – 234325 **28/2/2013**

מרצה: פרופ' מירלה בן חן
מתרגל: רועי פורן

מבחן סיום

שם: _____

מס' סטודנט: _____

הנחיות:

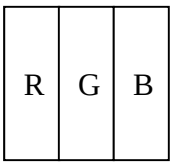
- בבחינה שלפניכם 4 דפים כולל דף זה. בדקו זאת.
- **עליכם לענות על כל 4 השאלות.**
- מומלץ לקרוא ראשית כל שאלה עד סופה, ורק אח"כ לענות.
- כתבו בקצרה. כל המאריך גורע!
- **משך הבחינה: 180 דקות.**
- יש לכתוב את כל התשובות במקום המתאים בטופס הבחינה ולהגיש טופס זה.
- יש להקפיד על כתיבה ברורה ומסודרת של התשובות.
- אם הנכם מוצאים צורך להניח הנחות כלשהן, ציינו אותן במפורש ונמקו.
- מותר השימוש בכל חומר עזר כתוב או מודפס (לא אלקטרוני).

בהצלחה !

שאלה	נקודות	
1	25	
2	25	
3	25	
4	25	
סה"כ	**100**	

Question 1 (25 pts.)

a) (5 pts.) In today's LCD screens, each pixel is a square, and is usually constructed out of three "sub-pixels" in R,G,B colors. The pixels are positioned in a square lattice and the structure of each pixel is as follows:



A student tried to draw a horizontal line where pixels were colored in the pattern RBRBR… After drawing the line, the student noticed that the line actually had the pattern MKMKMK… (M-Magenta, K-Black). Explain what happened.

b) (5 pts.) The phenomenon of part a) can be used, in certain situations, to draw lines with a higher resolution than the screen's pixel resolution. Explain how.

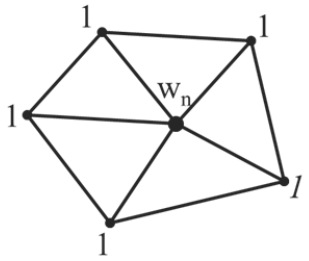
c) (5 pts.) An image that was displayed on a screen with a certain gamut (R_1, G_1, B_1) is given. The image needs to be displayed on a different screen, with different gamut (R_2, G_2, B_2), such that the images will look identical on both screens. What is the requirement on the two gamuts for this to be possible?

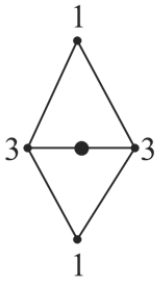
d) (5 pts.) Assuming this requirement is satisfied, what is the transformation that should be applied to the image colors, so that the images will look identical on both screens?

e) (5 pts.) What can be expected when the requirement is not satisfied, but the transformation is applied anyway?

Question 2 (25 pts.)

a) (10 pts.) Recall Loop subdivision:





Assume for simplicity that $w_n = n$.

Let M be a triangular mesh. A vertex $q \in$ M with $z = 1$ is connected to n other vertices, $p_1, \dots, p_n \in M$ all of which have $z = 0$.

1. Draw a sketch of the mesh.
2. Compute the z-coordinate for the vertex q after applying a single subdivision step.
3. Compute the z-coordinate of a **new** vertex that is created next to q after one subdivision step.
4. How will your answer to (3) change if n is changed?

b) (10 pts.) Show that Loop subdivision and affine transformations commute. That is, show that the result of applying Loop subdivision to a mesh followed by an affine transformation is equivalent to doing the same actions in reversed order.

c) (5 pts.) For subdivision schemes for triangular meshes: what is the number of neighbors of an old vertex and of a new vertex after applying one subdivision step?

Question 3 (25 pts.)

a) (5 pts.) Let $C(t)$ be a cubic Bezier curve:

$$C(t) = \sum_{i=0}^{3} P_i B_i^3(t) \quad , \qquad P_0 = (0,2)\ P_1 = (0,1)\ P_3 = (1,0)\ P_4 = (2,0)$$

Draw a sketch of the curve (there is no need to compute a large number of points, and draw exactly). What is the value of the curve at the point $t = \frac{1}{2}$?

b) (10 pts.) Convert the curve to Hermite form.

c) (5 pts.) For the following curves, say if it is possible to represent them as a **cubic** Bezier curve, and if so explain how the control points should be placed to get such a curve.
   1. A point.
   2. A straight line.
   3. A semicircle.

d) (5 pts.) A Computer Graphics student started working at Adobe, and was asked to code a framework that allows the user to draw curves. From the curve families we have learned in class, which are the curves that are best fitted for this task? Why?

Question 4 (25 pts.)

a) (5 pts.) Explain, or show with an example, why it is not possible to interpolate between orientations by linearly interpolating between the coefficients of the corresponding rotation matrices.

b) (5 pts.) Let α, β, γ be three Euler angles, and R the corresponding 3D rotation matrix. Is it always possible to uniquely compute the three Euler angles from R? Explain.

c) (10 pts.) Let K be a sub-group of the quaternions. For each of the following cases, prove or show a counter example: if $q_1, q_2 \in K$ then $q_1 \cdot q_2 \in K$.
   1. K contains the quaternions of the form $[c,(0,0,0)]$.
   2. K contains the quaternions of the form $[x,(y,0,0)]$.
   3. K contains the quaternions of the form $[0,(x,y,z)]$.

d) (5 pts.) Let q_1, q_2 be two quaternions that represent 3D orientations. Is it possible to generate an animation between the two orientations using the quaternion $q(t) = (1-t)q_1 + tq_2$? If it is possible, show how, and if not, explain why not.