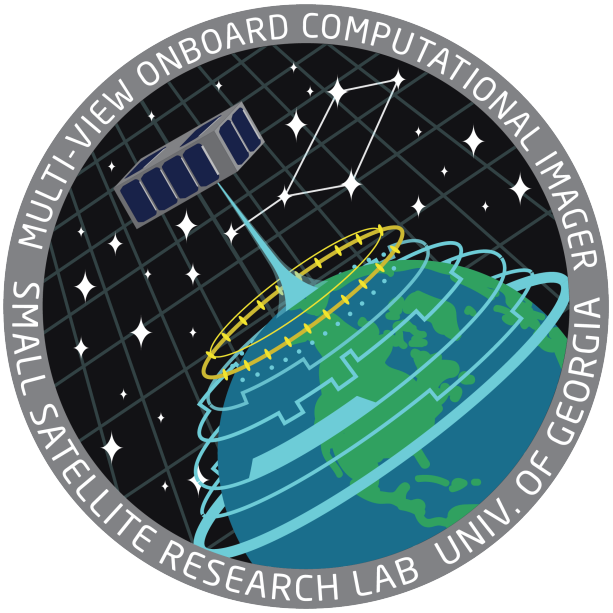

Multiview Onboard Computaional Imager

Algorithm Theoretical Basis Document

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Contents

Chapter 1

Introduction

1.1 File Formats

1.2 Feature Detection

1.3 Feature Discription

1.4 Feature Matching

1.5 2 View Reprojection

1.6 N View Reprojection

1.7 Surface Reconstruction

Chapter 2

Feature Detection

Chapter 3

Feature Extraction

Chapter 4

Feature Matching

4.1 Overview

4.2 2 View Matching

4.3 N View Matching

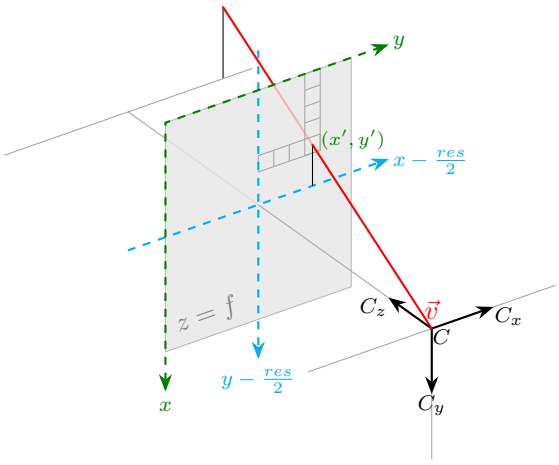
Chapter 5

Two View Reprojection

5.1 Overview

The goal with the 2-view reprojection is to take the pairs of matched points from the SIFT steps and move them from $\mathbb{R}^2$ into $\mathbb{R}^3$ so that equations of lines can be generated from the focal point of the camera into each matched point. Then, we want to find the minimum distance between those lines and choose the midpoint of that line segment at our reprojected point.

5.2 Getting Equations of Lines



The goal here is to generate a parametric equation of a line given camera position and orientation coordinates C, the camera focal length f, and the position of a coordinate in $\mathbb{R}^2$ on the image plane. We wish to generate vector v that can be used to make the parametric equation. The 2-view reprojection takes the matched points between 2 images and places them into $\mathbb{R}^3$. To place each set of points into $\mathbb{R}^3$ some trigonometry and and matrix transformations need to take place. The first step to moving a keypoint into $\mathbb{R}^3$ is to place it onto a plane in $\mathbb{R}^2$. the coordinates (x', y') in $\mathbb{R}^2$ require the size of a pixel $dpix$, the location of the keypoint (x, y), and the resolution of the image $(xres, yres)$ to yield:

$$x' = dpix(x - \frac{xres}{2}) \qquad y' = dpix(y - \frac{yres}{2})$$

This is repeated for the other matching keypoint. The coordinate (x', y', z') in $\mathbb{R}^3$ of the keypoint (x', y') in $\mathbb{R}^2$ is given by three rotation matrices and one translation matrix. First we treat (x', y') in $\mathbb{R}^2$ as a homogenous vector in $\mathbb{R}^3$ to yield $(x', y', 1)$. Given a unit vector representing the camera, in our case the spacecraft's camera's, orientation (r_x, r_y, r_z) we find the angle to rotate in each axis $(\theta_x, \theta_y, \theta_z)$. in our simple case we find the angle in the xy plane with:

$$\theta_z = \cos^{-1} \frac{(\begin{bmatrix}1 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}r_x & r_y & r_z\end{bmatrix})}{\sqrt{\begin{bmatrix}r_x & r_y & r_z\end{bmatrix} \cdot \begin{bmatrix}r_x & r_y & r_z\end{bmatrix}}}$$

Future software will generate rotations for all planes in an identical way. Now, given a rotation in each plane $(\theta_x, \theta_y, \theta_z)$ we calculate the homogeneous coordinate $(r_x, r_y, r_z, 1)$ in $\mathbb{R}^4$ using linear transformations. The values (T_x, T_y, T_z) represent

a translation in $\mathbb{R}^3$ and use camera position coordinates (C_x, C_y, C_z), the camera unit vectors representing orientation (u_x, u_y, u_z), and focal length f:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) & 0 \\ 0 & \sin(\theta_x) & \cos(\theta_x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} C_x - (x_r + f * u_x) \\ C_y - (y_r + f * u_y) \\ C_z - (z_r + f * u_z) \\ 1 \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} = \begin{bmatrix} x'_r \\ y'_r \\ z'_r \\ 1 \end{bmatrix}$$

The above process should happen for both points that have been matched. This should result in 2 homogeneous points that we will call $\begin{bmatrix} x_0 & y_0 & z_0 & 1 \end{bmatrix}^T$ and $\begin{bmatrix} x_1 & y_1 & z_1 & 1 \end{bmatrix}^T$. Each point has a corresponding camera vector, which is already known thanks to the camera coordinates, $\begin{bmatrix} C_{x0} & C_{y0} & C_{z0} & 1 \end{bmatrix}^T$ and $\begin{bmatrix} C_{x1} & C_{y1} & C_{z1} & 1 \end{bmatrix}^T$. From this we can make parametic lines L_0 and L_1 with the parametic variables t_0 and t_1:

$$L_0 = \begin{bmatrix} x_0 - C_{x0} \\ y_0 - C_{y0} \\ z_0 - C_{z0} \end{bmatrix} \begin{bmatrix} t_0 \\ t_0 \\ t_0 \end{bmatrix} + \begin{bmatrix} C_{x0} \\ C_{y0} \\ C_{z0} \end{bmatrix}$$

$$L_1 = \begin{bmatrix} x_1 - C_{x1} \\ y_1 - C_{y1} \\ z_1 - C_{z1} \end{bmatrix} \begin{bmatrix} t_1 \\ t_1 \\ t_1 \end{bmatrix} + \begin{bmatrix} C_{x1} \\ C_{y1} \\ C_{z1} \end{bmatrix}$$

The Host functions are simple and will not be addressed here, refer to standard CUDA memory management. For the Kernel, we must insure that we have a few global variables about the camera data/parameters availible to the GPU. These variables are the focal length foc, the feild of view fov, and the resolution of the imager res. This assumes that the cameras are in the xy plane, which can only be assumed for this special 2-view case. It may be helpful to read the section on file formats, which is after the main introduction.

In the future, only the first 3 components, the position coordinates, of C_0 and C_1 are used. Vectors v_0 and v_1 are used to parateterizeds lines.

Algorithm 1 Device Kernel

Let: $dpix = foc * \tan \frac{fov/2}{res/2}$
Let: $i = blockIdx.x * blockDim.x + threadIdx.x$
procedure GENERATELINES(R2POINTS, R3CAMERAS)
    $C_0[6] = (R3cameras[0], R3cameras[1], R3cameras[2], R3cameras[3], R3cameras[4], R3cameras[5])$
    $C_1[6] = (R3cameras[6], R3cameras[7], R3cameras[8], R3cameras[9], R3cameras[10], R3cameras[11])$
    $x_0 = (R2points[i] - res/2.0)$
    $y_0 = (-R2points[i+1]) + res/2.0)$
    $x_1 = (R2points[i+2] - res/2.0)$
    $y_1 = (-R2points[i+3]) + res/2.0)$
    $kp_0 = [x_0, y_0, 0.0]$
    $kp_1 = [x_1, y_1, 0.0]$
    $\theta_{x0} = \cos^{-1}(\frac{kp0 \cdot \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T}{\sqrt{\|kp_0\|}})$
    $\theta_{x1} = \cos^{-1}(\frac{kp1 \cdot \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T}{\sqrt{\|kp_1\|}})$
    $kp_0 = kp_0 \begin{bmatrix}1 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ 0 & \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2})\end{bmatrix} \begin{bmatrix}\cos(\theta_{x0}+\frac{\pi}{2}) & -\sin(\theta_{x0}+\frac{\pi}{2}) & 0 \\ \sin(\theta_{x0}+\frac{\pi}{2}) & \cos(\theta_{x0}+\frac{\pi}{2}) & 0 \\ 0 & 0 & 1\end{bmatrix}$
    $kp_1 = kp_1 \begin{bmatrix}1 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ 0 & \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2})\end{bmatrix} \begin{bmatrix}\cos(\theta_{x1}+\frac{\pi}{2}) & -\sin(\theta_{x1}+\frac{\pi}{2}) & 0 \\ \sin(\theta_{x1}+\frac{\pi}{2}) & \cos(\theta_{x1}+\frac{\pi}{2}) & 0 \\ 0 & 0 & 1\end{bmatrix}$
    $kp_0[0] = C_0[0] - (kp_0[0](C_0[3] * foc))$
    $kp_0[1] = C_0[1] - (kp_0[1](C_0[4] * foc))$ ▷ kp0 now represents the keypoint location in $\mathbb{R}^3$
    $kp_1[0] = C_1[0] - (kp_1[0](C_1[3] * foc))$
    $kp_1[1] = C_1[1] - (kp_1[1](C_1[4] * foc))$ ▷ kp1 now represents the keypoint location in $\mathbb{R}^3$
    $v_0 = kp_0 - C_0$ ▷ line L_0's vector
    $v_1 = kp_1 - C_1$ ▷ line L_1's vector

5.3 Minimum Distance Between Skew Lines

Now that we have lines L_0 and L_1, the challenge is to find the points s_0 and s_1 of closest approach. First, we must test the assumption that our lines are skew, meaining they are not parrallel and do not intersect. To start to think of this we take the forms of L_0 and L_1 and simplify them by thinking of them as parametic vectors where C_0 and C_1, represent the camera position vectors and v_0 and v_1 represent the vector was previously calculated from the subtraction of match coordinates with the camera vector. We make the simple equations:

$$L_0 = v_0 t_0 + C_0 \qquad L_1 = v_1 t_1 + C_1$$

To make sure that the lines are not parallel, which is unlikely, we must verify that their cross product is not zero. if $v_0 \times v_1 = 0$ then we have a degenerate case with infinitely many solutions. As long as we know this is not the case we can proceed. We know that the cross product of the two vectors $c = v_0 \times v_1$ is perpendicular to the lines L_0 and L_1. We know that the plane P, formed by the translation of $L1$ along c, contains C_1. We also know that the point C_1 is perpendicular to the vector $n_0 = v_1 \times (v_0 \times v_1)$. Thus, the intersection of L_0 with P is also the point, s_0, that is nearest to L_1, given by the equation:

$$s_0 = C_0 + \frac{(C_1 - C_0) \cdot n_0}{v_0 \cdot n_0} \cdot v_0$$

This also holds for the second line L_1, the point s_1, and vector $n_1 = v_0 \times (v_1 \times v_0)$. with the equation:

$$s_1 = C_1 + \frac{(C_0 - C_1) \cdot n_1}{v_1 \cdot n_1} \cdot v_1$$

Now, given to points that represent the closest points of approach, we simply find the midpoint m:

$$m = \begin{bmatrix}(s_0[x] + s_1[x])/2 \\ (s_0[y] + s_1[y])/2 \\ (s_0[z] + s_1[z])/2\end{bmatrix}$$

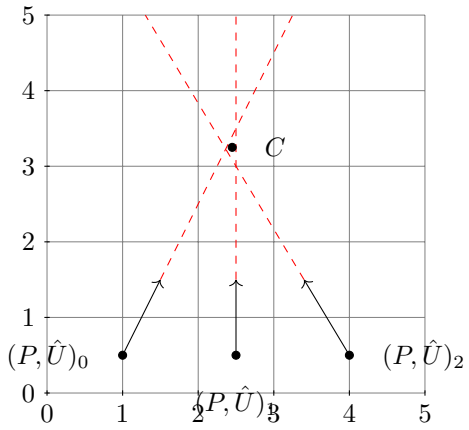
5.4 Least Square Approximation for Two Lines

Chapter 6

N/Multiview Reprojection

6.1 Overview

At this stage we are exclusivley in $\mathbb{R}^3$. For the purpose of our software, we expect points in the form $P = (p_x, p_y, p_z)$ and their corresponding orientation as a unit vector $\hat{U} = (\hat{u}_x, \hat{u}_y, \hat{u}_z)$, we expect these together in a tuple $((p_x, p_y, p_z), (\hat{u}_x, \hat{u}_y, \hat{u}_z))_n$. This tuple comes with several (n many) tuples which all uniquely correspond with a matched set. So we have a $\mathbb{R}^3$ match set $M = \{(P, \hat{U})_0, (P, \hat{U})_1, ..., (P, \hat{U})_n\}$ where n is the numnber of tuple pairs and has a one-to-one correspondence with the number of views in which the $\mathbb{R}^2$ match was found. We all also get a set of $\mathbb{R}^3$ matches, M_i, resulting in a set which has a one-to-one correspondence with the total number of points we should have after reprojection.



I know I said we're in $\mathbb{R}^3$ (and we are), just imagine that the diagram above illustrates the values within a particular Z plane. Note that the diagram could be similar if any arbitary plane is chosen.

In the above diagram, assume that point C is the correct real world point and has no orientation. We're trying to make a best guess at was C is given our imperfect information.

6.2 3 by 3 Inversion Method

Caleb (me the person writing this) made up the name to this method, so don't search it because you probably won't find anything about it.

First, remember the identity:

$$(m \times n) \cdot (m \times n) = ||m||^2 ||n||^2 - (m \cdot n)^2$$

Then note, we have the distance function, measuring how much a given $((p_x, p_y, p_z), (\hat{u}_x, \hat{u}_y, \hat{u}_z))_n$ tuple (which represents a line) misses the real world targe point C. Not this function is calculated for each tuple.

$$D_n = \frac{||(C - P_n) \times \hat{U}_n||}{||\hat{U}_n||}$$

We will want to use the square of the distance (as is common in many optimization problems) to insure convex optimization and positive distance values. We also use the identity mentioned above to get our primary distance equation. Then, taking the first derivative of the distance function will, to find a 0 solution, will give us a local minimum value.

$$
\begin{aligned}
D_n &= \frac{||(C - P_n) \times \hat{U}_n||}{||\hat{U}_n||} \\
D_n^2 &= \Big(\frac{||(C - P_n) \times \hat{U}_n||}{||\hat{U}_n||}\Big)^2 \\
D_n^2 &= \frac{||(C - P_n) \times \hat{U}_n||^2}{||\hat{U}_n||^2} \\
D_n^2 &= \frac{||C - P_n||^2||\hat{U}_n||^2 - ||(C - P_n) \cdot \hat{U}_n||^2}{||\hat{U}_n||^2} \\
D_n^2 &= ||C - P_n||^2 - \frac{||(C - P_n) \cdot \hat{U}_n||^2}{||\hat{U}_n||^2} \\
\frac{dD_n^2}{dC} &= 2(C - P_n) - 2\hat{U}_n \frac{(C - P_n) \cdot \hat{U}_n}{||\hat{U}_n||^2}
\end{aligned}
$$

So, we need to find a zero for the following (note that we are dealing with a vector in $\mathbb{R}^3$, so $0 = [0, 0, 0]^T$). the value m is the total number of $\mathbb{R}^3$ match points:

$$
\begin{aligned}
0 &= \sum_{n=0}^{m} C - P_n - \hat{U}_n \frac{(C - P_n) \cdot \hat{U}_n}{||\hat{U}_n||^2} \\
0 &= \sum_{n=0}^{m} C - P_n - \frac{\hat{U}_n(C \cdot \hat{U}_n)}{||\hat{U}_n||^2} + \frac{\hat{U}_n(P_n \cdot \hat{U}_n)}{||\hat{U}_n||^2} \\
0 &= \sum_{n=0}^{m} C - P_n - \frac{\hat{U}_n\hat{U}_n^T C}{||\hat{U}_n||^2} + \frac{\hat{U}_n\hat{U}_n^T P_n}{||\hat{U}_n||^2} \\
0 &= \sum_{n=0}^{m} \Big(I - \frac{\hat{U}_n\hat{U}_n^T}{||\hat{U}_n||^2}\Big)C - \Big(P_n - \frac{\hat{U}_n\hat{U}_n^T P_n}{||\hat{U}_n||^2}\Big)
\end{aligned}
$$

Notice this is of the form $Ax = b$ because we now have $0 = Ax - b$. The next thing to note is we can remove the summations and get a system that results in taking an inverse of a 3 by 3 matrix.

Notice the possible expansion:

$$
\begin{aligned}
0 &= \sum_{n=0}^{m} \Big(I - \frac{\hat{U}_n\hat{U}_n^T}{||\hat{U}_n||^2}\Big)C - \Big(P_n - \frac{\hat{U}_n\hat{U}_n^T P_n}{||\hat{U}_n||^2}\Big) \\
0 &= \left(\Big(I - \frac{\hat{U}_0\hat{U}_0^T}{||\hat{U}_0||^2}\Big) + \Big(I - \frac{\hat{U}_1\hat{U}_1^T}{||\hat{U}_1||^2}\Big) + \Big(I - \frac{\hat{U}_2\hat{U}_2^T}{||\hat{U}_2||^2}\Big) + ...\right)C - \left(\Big(P_0 - \frac{\hat{U}_0\hat{U}_0^T P_0}{||\hat{U}_0||^2}\Big) + \Big(P_1 - \frac{\hat{U}_1\hat{U}_1^T P_1}{||\hat{U}_1||^2}\Big) + ...\right) \\
0 &= (A)C - (b) \\
AC &= b \\
C &= A^{-1}b
\end{aligned}
$$

So, the meat of this method is to calculate the A matrix's inverse and muplyply it by vector b to find the estimated point C. succinctly, these are calcualted:

$$
\begin{aligned}
A &= \sum_{n=0}^{m} \Big(I - \frac{\hat{U}_n\hat{U}_n^T}{||\hat{U}_n||^2}\Big) \\
b &= \sum_{n=0}^{m} \Big(P_n - \frac{\hat{U}_n\hat{U}_n^T P_n}{||\hat{U}_n||^2}\Big)
\end{aligned}
$$

Chapter 7

Reconstruction

7.1 Overview

7.2 Orienting the Point Cloud

Chapter 8

Bundle Adjustment

8.1 Introduction

Bundle adjustment is defined as the problem of refining a visual reconstruction of a scene geometry to produce jointly optimal 3D point cloud representation of the scene and viewing parameter (camera pose and calibration) estimates.

8.2 Notation

Here we list some notation used in this paper.

I	$\{I_1, ..., I_N\}$. Input set of N images
C_i	camera center location corresponding to I_i (world coordinates)
c_{foc}	camera focal length
c_{fov}	camera field of view
K	intrinsic camera matrix
R_i	i-th camera rotation matrix
M_i	i-th camera projection matrix
PC	$\{P_1, ..., P_L\}$. Point cloud set
P_j	$\{x_j, y_j, z_j\}$. i-th computed 3D feature point (world coordinates)
$p_{(i,j)}$	observed 2D image point on image i corresponding to P_j (pixel coordinates)
s	state vector
δ	step vector
$f(s)$	cost/error function

8.3 Problem Formulation

We are given a set of N images along with the feature points for each image. As a prerequisite, a set of 3D points, PC, are estimated for each pixel point match set. It is convenient to define a "track" for each $P_j \in PC$ as

$$T_j = \{P_j, Q(P_j)\}$$

where $Q(P_j) = \{p_{(1,j)}, p_{(2,j)}, ..., p(L, j)\}$ is the set of observed image points that were previously used to predict P_j, and where $p_{(i,j)} = null$ whenever no point on I_i contributed to estimation of P_j (i.e., when P_j is not visible on I_i).
[[reword this after writing earlier parts of pipeline to refer to]]
Then as we previously defined, for a particular image point $p_{(i,j)}$, the projection error, or distance between the observed location and the projected image point location $\bar{p}_{(i,j)} = M_i P_j$, is denoted

$$d(p_{(i,j)}, \bar{p}_{(i,j)})$$

This distance function depends on all entries in the projection matrix M_i, namely R_i, C_i, c_{foc}, and c_{fov}, as well as the 3D coordinates of each feature point $P_j = (x_j, y_j, z_j)$. Now we organize all of these variables into one state vector s:

$$s = \{ \bigcup_i R_i, \bigcup_i C_i, \ c_{foc}, \ c_{fov}, \bigcup_j P_j \ \}$$

with $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, L$. Then our goal is to globally minimize the amount of projection error with respect to s. We define the cost function

$$f(s) = \sum_{i=i}^{N} \sum_{j=1}^{L} d(p_{(i,j)},\ \bar{p}_{(i,j)}) = \sum_{i=i}^{N} \sum_{j=1}^{L} d(p_{(i,j)},\ M_i P_j) \tag{8.1}$$

The goal of bundle adjustment then is to compute the optimal state vector parameters to minimize the cost function:

$$\min_{s} f(s) \tag{8.2}$$

8.4 BA Algorithm

This section explains the bundle adjustment algorithm in detail. First we explain the mathematical solution to the problem expressed in section 3. After that we describe the methods used to reduce computational cost so that the algorithm becomes feasible.

8.4.1 Numerical Optimization Estimate

We need to minimize the cost function $f(s)$ over parameters s starting from the given initial estimate. Real valued cost functions are too complicated to minimize in closed form, and the parameter space is non linear, so instead we approximate $f(s)$ by a local linear model - in our case a quadratic Taylor series expansion of f at the current state s. For a small step δ,

$$f(s + \delta) \approx f(s) + g(s)^T \delta + \frac{1}{2} \delta^T H(s) \delta \tag{8.3}$$

where $g \equiv \frac{df}{ds}(s)$ and $H \equiv \frac{d^2 f}{ds^2}(s)$ are the gradient vector and Hessian matrix of f, respectively. Assuming that H is positive definite, the local model is a simple quadratic with a unique global minimum, which can be found explicitly. To do this, we use the Gauss-Newton method to solve for step δ in the following linear system.

$$H(s)\delta = -g(s) \tag{8.4}$$

8.4.2 Schur Complement

Note that calculating the second derivatives H and then inverting the matrix to solve (4) by

$$\delta = -H^{-1} g$$

is extremely computationally expensive for complex cost functions such as this f, so bundle adjustment methods typically estimate the hessian using clever techniques such as exploiting the sparse nature of H.

8.5 Pseudocode

8.6 Appendix