Physics Laws for Spaghetti Bridge PINN

Beam Theory

Euler-Bernoulli Beam Equation

$EI(d^4w/dx^4) = q(x)$

Beam Curvature Equation

$d^2w/dx^2 = M/EI$

Moment-Curvature Relationship

$M = EI(d^2w/dx^2)$

Shear Force Equation

$V = dM/dx$

Loading Relationship

$dV/dx = -q(x)$

Axial and Torsional Effects

Axial Stress

$\sigma = F/A$

Axial Strain

$\varepsilon = \Delta L/L$

Axial Deformation

$\Delta L = FL/(AE)$

Torsional Shear Stress

$\tau = Tr/J$

Angle of Twist

$\theta = TL/GJ$

Stability Analysis

Euler Critical Load (Buckling)

$P_{cr} = (\pi^2 EI)/(KL)^2$

Lateral-Torsional Buckling Critical Moment

$M_{cr} = (\pi/L)\sqrt{(EI_y \cdot GJ)}$

Material Behavior

Hooke's Law

$\sigma = E\varepsilon$

Shear Stress-Strain Relationship

$\tau = G\gamma$

Poisson's Effect

$\varepsilon_{lateral} = -\nu\varepsilon_{axial}$

Shear Modulus Relationship

$G = E/(2(1+\nu))$

Principal Stresses

$\sigma_{1,2} = (\sigma_x+\sigma_y)/2 \pm \sqrt{[(\sigma_x-\sigma_y)^2/4 + \tau_{xy}^2]}$

Equilibrium Conditions

Force Equilibrium

$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$

Moment Equilibrium

$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$

Failure Criteria

Maximum Normal Stress Criterion

$\sigma_{max} \geq \sigma_{ultimate}$ (failure)

Maximum Shear Stress (Tresca)

$\tau_{max} = (\sigma_1 - \sigma_3)/2 \leq \tau_{yield}$

Von Mises Yield Criterion

$\sqrt{[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]/2} \leq \sigma_{yield}$

Stress Intensity Factor

$K_I = \sigma\sqrt{(\pi a)}$

Fracture Criterion

$K_I \geq K_{Ic}$ (failure)

Energy Method

Strain Energy Density

$u = \sigma\varepsilon/2 = \sigma^2/(2E)$

Virtual Work Principle

$\delta W_{ext} = \delta W_{int}$

Castigliano's Theorem

$\delta_i = \partial U/\partial P_i$