

UNIVERSITY OF TWENTE.



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UNIVERSITY OF TWENTE, NETHERLANDS



# Overview

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- My background
- VerCors
  - Separation Logic
  - PVL (Prototypal Verification Language)
  - SYCL

# Formal Methods & Tools

EEMCS Faculty at the University of Twente


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- <https://www.utwente.nl/en/eemcs/fmt/>




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- Deductive verifier of concurrent and parallel software
- Static analysis of source code
- Supports multiple input languages
- Verification is automated, given user-provided annotations
- Annotations are written as pre/post-condition style contracts
  - Logic based on permission-based separation logic
- There are two tool papers on VerCors:
  - The VerCors tool set: verification of parallel and concurrent software. Blom, S., et al. (IFM 2017)
  - The VerCors verifier: a progress report. Amborst, L., et al. (CAV 2024)

Java/C/C++  
OpenCL/CUDA  
PVL  


VerCors

  
Viper

↓  


- The explanations use PVL
  - Easiest to explain concepts
  - Language that is best supported
  - Java-like syntax

```
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```

```
requires 0 <= r && r < 256;
```

```
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```



```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
  
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == (r + g + b) / 3;  
  
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == (r + g + b) / 3;  
ensures 0 <= \result && \result < 256;  
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == (r + g + b) / 3;  
ensures 0 <= \result && \result < 256;  
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```



```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == (r + g + b) / 3;  
ensures 0 <= \result && \result < 128;  
int meanRGB(int r, int g, int b){  
    return (g + b + r) / 3;  
}
```



```
requires 0 <= r && r < 256;
requires 0 <= g && g < 256;
requires 0 <= b && b < 256;
ensures \result == add(g,b,r) / 3;
ensures 0 <= \result && \result < 256;
int meanRGB(int r, int g, int b){
    return add(r,b,g) / 3;
}
```

```
requires 0 <= r && r < 256;
requires 0 <= g && g < 256;
requires 0 <= b && b < 256;
ensures \result == r+g+b;
int add(int r, int g, int b) {
    return (r+g+b);
}
```

`add` and `meanRGB` are verified separately.

```
requires 0 <= r && r < 256;
requires 0 <= g && g < 256;
requires 0 <= b && b < 256;
ensures \result == add(g,b,r) / 3;
ensures 0 <= \result && \result < 256;
int meanRGB(int r, int g, int b){
    return add(r,b,g) / 3;
}
```

```
requires 0 <= r && r < 256;
requires 0 <= g && g < 256;
requires 0 <= b && b < 256;
ensures \result == r+g+b;
int add(int r, int g, int b) {
    return (r+g+b);
}
```

`add` and `meanRGB` are  
verified separately.

Assume the preconditions  
Prove the postconditions

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == add(g,b,r) / 3;  
ensures 0 <= \result && \result < 256;  
int meanRGB(int r, int g, int b){  
    return add(r,b,g) / 3;  
}
```

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == r+g+b;  
int add(int r, int g, int b) {  
    return (r+g+b);  
}
```



`add` and `meanRGB` are  
verified separately.

Prove the preconditions  
Get the postconditions

Assume the preconditions  
Prove the postconditions

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == add(g,b,r) / 3;  
ensures 0 <= \result && \result < 256;  
int meanRGB(int r, int g, int b) {  
    return add(r,b,g) / 3;  
}
```

```
requires 0 <= r && r < 256;  
requires 0 <= g && g < 256;  
requires 0 <= b && b < 256;  
ensures \result == r+g+b;  
int add(int r, int g, int b) {  
    return (r+g+b);  
}
```

- What do we prove?
  - Functional-correctness properties
  - Data-race freedom and memory safety (by default)
  - Partial correctness (no proof for termination)

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```



- Permissions to specify ownership
  - Tokens to specify access mode (read/write)
- Specify permissions for every shared location
- Permissions are fractions between 0 and 1
- $0 < p < 1$  is a read permission
- 1 is a write permission
- For any shared location, at most 1 permission in the whole system

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r, write) ** Perm(g, write) ** Perm(b, write);  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r,      )  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```



```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r, 1\1)  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r, write)  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r, write) ** Perm(g, write) ** Perm(b, write);  
    ensures r == 0 && g == 0 && b == 0;  
    void clear() {  
        r = 0;  
        g = 0;  
        b = 0;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r, 1\2);  
    ensures \result == r;  
    int getR() {  
        return r;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    context Perm(r, read) ;  
    ensures \result == r;  
    int getR() {  
        return r;  
    }  
}
```

```
class RGB {  
    int r;  
    int g;  
    int b;  
  
    requires Perm(r, 3\4);  
    ensures  Perm(r, 1\4);  
    int getR() {  
        return r;  
    }  
}  
  
void main() {  
    RGB rgb = new RGB();  
  
    int r = rgb.getR();  
    r = rgb.getR();  
    assert r == 0;  
}
```

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# Verification of Matrix Multiplication in

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# Overview

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- Matrix Multiplication
- Verification Concepts/Techniques
- Matrix Multiplication in SYCL



# Matrix Multiplication

---

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \mathbf{A} \times \mathbf{B} = \mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{pmatrix}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^n a_{ik}b_{kj}$$

# Matrix Multiplication

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \mathbf{A} \times \mathbf{B} = \mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{pmatrix}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^n a_{ik}b_{kj}$$

# Matrix Multiplication

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \mathbf{A} \mathbf{B} = \mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{pmatrix}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^n a_{ik}b_{kj}$$

# Matrix Multiplication

---

- Kernel 1: Initialize matrix a
- Kernel 2: Initialize matrix b
- Kernel 3: Calculate  $c = a \times b$

# Matrix Multiplication

Ghost state

---

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication

Ghost state

Program  
state

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

Ghost  
state

$$\mathbf{A} =$$

$$\mathbf{B} =$$

# Matrix Multiplication

Ghost state

Program  
state

int\*

$\mathbf{A} =$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$\mathbf{B} =$

$$\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

Ghost  
state

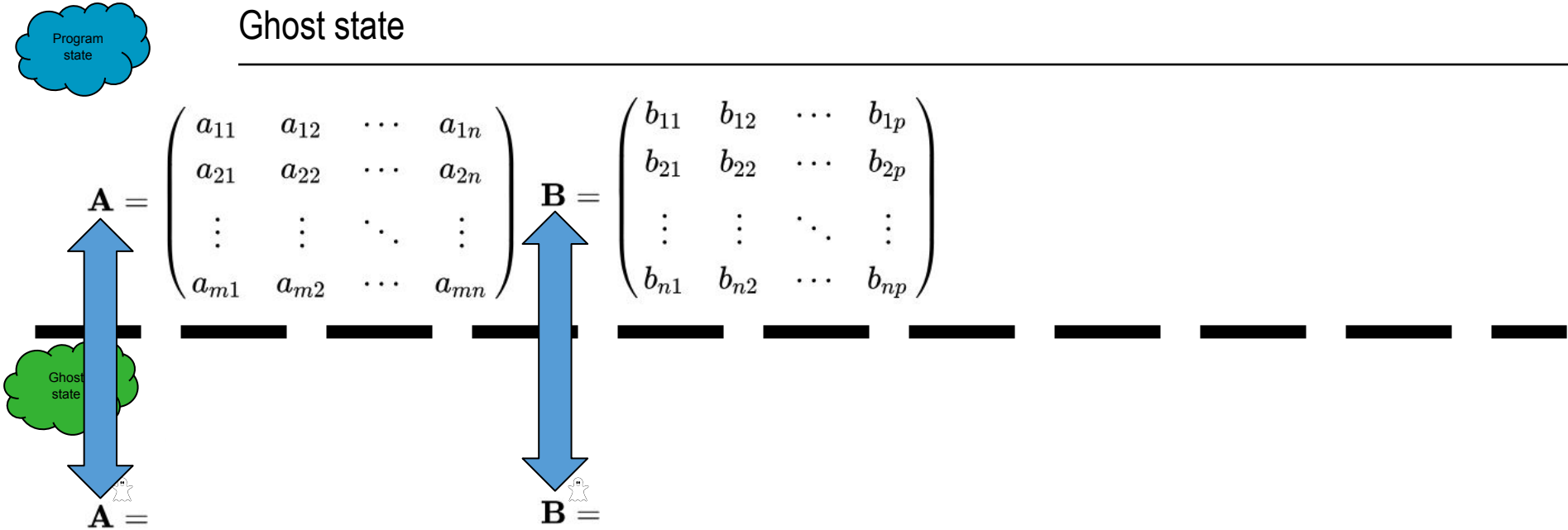
seq<int>

$\mathbf{A} =$

$\mathbf{B} =$

# Matrix Multiplication

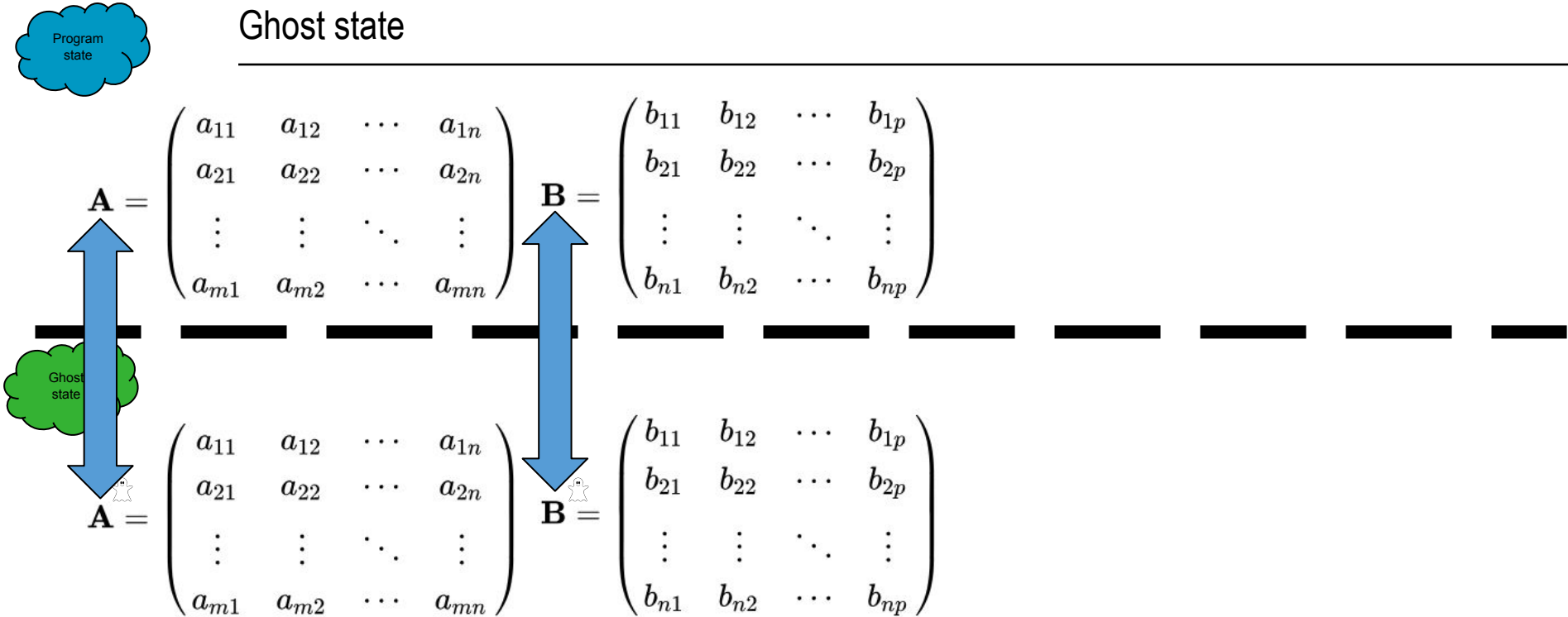
Ghost state





# Matrix Multiplication

Ghost state



# Matrix Multiplication

Ghost state

Program  
state

int\*

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

Ghost  
state

seq<int>

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication

Ghost state

Program  
state

int\*

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{pmatrix}$$

Ghost  
state

seq<int>

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication

Loop invariant

Program  
state

int\*

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \text{sum} = 0$$

Ghost  
state

seq<int>

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication

Loop invariant

Program  
state

int\*

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \text{sum} = a_{11} * b_{11}$$

Ghost  
state

seq<int>

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication

Loop invariant

Program  
state

int\*

$\mathbf{A} =$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$\mathbf{B} =$

$$\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

$$\text{sum} = a_{11} * b_{11} + a_{12} * b_{21}$$

Ghost  
state

seq<int>

$\mathbf{A} =$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$\mathbf{B} =$

$$\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication

Loop invariant

Program  
state

int\*

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix} \quad \text{sum} = a_{11} * b_{11} + a_{12} * b_{21} \dots a_{1n} * b_{n1}$$

Ghost  
state

seq<int>

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

# Matrix Multiplication in SYCL

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VSCode



# Matrix Multiplication in SYCL

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- Current improvement points of SYCL support
  - Currently only buffers/accessors are supported
    - USM support is a possible followup
  - The proof takes ~4 minutes
    - Also proven the concrete case
    - Parsing C++ takes significant time, ~1 minute