PEVA Cheat Sheet 1

Lecture 1 General

Amdahl's law: $T(n) = (1-\alpha) \cdot t + \alpha \cdot \frac{t}{n}$, $lim_{n\to\infty}\ S(n) = \frac{1}{1-\alpha}$
Kendall notation for queues:
arrivals | service | servers | buffersize | population | scheduling

`arrival`	Distribution of interarrival time
`service`	Distribution of service time
`servers`	Number of servers.
`buffer size`	Maximum number of customers in queueing station (including servers).
`population`	Number of customers in and outside the queueing station.
`scheduling`	Employed scheduling strategy.

Lecture 2 DTMCs

Limiting distribution: $\underline{v}(P-I) = \underline{0}$ and $\sum_j v_j = 1$

$$(P-I) = \begin{pmatrix} -0.1 & 0.1 \\ 0.4 & -0.4 \end{pmatrix} \implies \begin{matrix} -0.1v_0 + 0.4v_1 = 0 \\ 0.1v_0 - 0.4v_1 = 0 \end{matrix}$$

`recurrent`	A state is recurrent if we return to it with probability 1.
`transient`	A state is transient (or nonrecurrent) if there is a positive probability of not returning to this state.
`positive recurrent`	A state is positive recurrent (or recurrent non-null) if its mean recurrence time is finite.
`null recurrent`	A state is null recurrent if its mean recurrence time is infinite.
`absorbing`	A state i is absorbing if and only if pi,i = 1.
`period`	The period d_i of state i is the greatest common divisor of all the values n for which $p_{i,i}(n) > 0$.
`irreducible`	A DTMC is called irreducible if every state can be reached from every other state in a finite number of steps. In an irreducible DTMC, all states have the same period.
`Markov property`	Future evolution (next state) only depends on the current state, not on the past history!
`time-homogeneous`	DTMCs are time-homogeneous: the matrix **P** does not change over time.

Lecture 3 CTMCs

For each state i, introduce rate q_i for an exponentially distributed residence time; mean residence time is $1/q_i$.

Compute steady-state of CTMC by: **1.** GBEs or "fluxin, fluxout", **2.** Determine steady-state of embedded DTMC and renormalize $\underline{v}$ with $p_i = (\frac{v_i}{q_i})\ div\ (\sum_j \frac{v_j}{q_j})$, **3.** Generator Matrix **Q** with $q_{i,j} = q_i \cdot p_{i,j},\ i \neq j$ and $-q_i,\ i = j$. With **Q**, solve $\underline{p} \cdot \mathbf{Q} = \underline{0}$.

Lecture 4 M|M|1 queues

`FCFS`	First come, first served.
`RR`	Round robin.
`PS`	Processor sharing.
`SJN`	Shortest job next.
`LCFS`	Last come, first served.
`IS`	Infinite server.
`PRIO`	priority scheduling.
`PASTA`	Poisson Arrivals See Time Averages.

What is the expected number of jobs in the system in steady state?
1. compute steady-state probabilities using GBEs or something else,
2. use steady-state probabilities to compute expectation.
$E[N] = \sum_{i=0}^{n} i \cdot p_i$.
If server is infinite (like with a M|M|1 queue), the expected number of customer:
In system, $E[N] = \frac{\rho}{1-\rho}$.
In queue, $E[N_q] = \frac{\rho^2}{1-\rho}$.
In server, $E[N_s] = \rho$.
Little's Law helps to go from **system-oriented** measures tot **user-oriented** measures.

Little's law for:

`Full station`	$E[N] = \lambda \cdot E[R]$, with $E[R]$ as the expected response time, average time each customer spends in system.
`Queue only`	$E[N_q] = \lambda \cdot E[W]$, with $E[W]$ as average waiting time.
`Server only`	$\rho = \lambda \cdot E[S]$, with $E[S]$ as average service time.

For finite stations with one server:

`Little's law`	$E[N] = X \cdot E[R]$, with $X = \mu$ if overloaded
`Loss prob p_{m}`	probability that an arriving job has to leave because the buffer is full (PASTA)
`Throughput`	$X = \lambda \cdot (1-p_m) = \mu \cdot (1-p_0)$, number of jobs served per time unit
`Utilization`	$X \cdot E[S] = 1 - p_0$

For infinite stations with m server:

`Utilization`	For each individual server $\rho = \frac{\lambda}{m \cdot \mu}$
`Expected busy servers`	Number of busy servers $m \cdot \rho = \frac{\lambda}{\mu}$

Lecture 5 Simulation

Monte Carlo method: X_i and Y_i random variables, uniform on [0,1] = random points in a unit square. Define J_i if $Y_i \leq X_i$ then 1 else 0. Estimate $\widetilde{A} = \frac{1}{N}\sum_{i=1}^{N} J_i \approx \int_0^1 x^2 dx$.

The estimate $\widetilde{a}$ is a realization of the random variable $\widetilde{A}$. Random variable $\widetilde{A}$ is called an estimator of a. $\widetilde{A}$ should be unbiased, so $E[\widetilde{A}] = a$ and $\widetilde{A}$ shoudl be consistent, so $lim_{n\to\infty} var[\widetilde{A}] = 0$
Different ways to classify simulation methods: **1.** Stochastic vs. deterministic: usage of random numbers, **2.** Discrete-event vs. continuous-event, **3.** Steady-state vs. transient and **4.** Time-based vs. event-based.
Time-based simulation: Also called synchronous simulation, Time proceeds in steps of size Δt, In each iteration all events are processed that happen in the interval $[t, t+\Delta t]$, System state is changed accordingly, Assumption: ordering of events in an interval is not important, events are independent and Δt has to be small.
Event-based simulation: Also called asynchronous simulation, Time 'jumps' from event to event, In each iteration: determine the next event, set simulation time to its occurrence time, process the event and generate new events.
User-oriented measure: Estimate of average response time from n jobs = $\widetilde{r} = \frac{1}{n}\sum_{i=1}^{n}(t_i^{(d)} - t_i^{(a)})$, with $t_i^{(a)}$ arrival time of ith job and $t_i^{(d)}$ departure time of ith job.

Lecture 6 M|G|1 queues

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.