PEVA Cheat Sheet 1

Lecture 1 General

Amdahl's law: $T(n) = (1-\alpha)\cdot t + \alpha \cdot \frac{t}{n}$, $lim_{n\to\infty}\ S(n) = \frac{1}{1-\alpha}$
Kendall notation for queues:
arrivals | service | servers | buffersize | population | scheduling

`arrival`	Distribution of interarrival time
`service`	Distribution of service time
`servers`	Number of servers.
`buffer size`	Maximum number of customers in queueing station (including servers).
`population`	Number of customers in and outside the queueing station.
`scheduling`	Employed scheduling strategy.

Lecture 2 DTMCs

Limiting distribution: $\underline{v}(P-I) = \underline{0}$ and $\sum_j v_j = 1$

$$(P-I) = \begin{pmatrix} -0.1 & 0.1 \\ 0.4 & -0.4 \end{pmatrix} \implies \begin{matrix} -0.1v_0 + 0.4v_1 = 0 \\ 0.1v_0 - 0.4v_1 = 0 \end{matrix}$$

`recurrent`	A state is recurrent if we return to it with probability 1.
`transient`	A state is transient (or nonrecurrent) if there is a positive probability of not returning to this state.
`positive recurrent`	A state is positive recurrent (or recurrent nonnull) if its mean recurrence time is finite.
`null recurrent`	A state is null recurrent if its mean recurrence time is infinite.
`absorbing`	A state i is absorbing if and only if pi,i = 1.
`period`	The period d_i of state i is the greatest common divisor of all the values n for which $p_{i,i}(n) > 0$.
`irreducible`	A DTMC is called irreducible if every state can be reached from every other state in a finite number of steps. In an irreducible DTMC, all states have the same period.
`Markov property`	Future evolution (next state) only depends on the current state, not on the past history!
`time-homogeneous`	DTMCs are time-homogeneous: the matrix **P** does not change over time.

Lecture 3 CTMCs

For each state i, introduce rate q_i for an exponentially distributed residence time; mean residence time is $1/q_i$.

Compute steady-state of CTMC by: **1.** GBEs or "fluxin, fluxout", **2.** Determine steady-state of embedded DTMC and renormalize $\underline{v}$ with $p_i = (\frac{v_i}{q_i})\ div\ (\sum_j \frac{v_j}{q_j})$, **3.** Generator Matrix **Q** with $q_{i,j} = q_i \cdot p_{i,j},\ i \neq j$ and $-q_i,\ i = j$. With **Q**, solve $\underline{p}\cdot\mathbf{Q} = \underline{0}$.

Lecture 4 M|M|1 queues

`FCFS`	First come, first served.
`RR`	Round robin.
`PS`	Processor sharing.
`SJN`	Shortest job next.
`LCFS`	Last come, first served.
`IS`	Infinite server.
`PRIO`	priority scheduling.
`PASTA`	Poisson Arrivals See Time Averages.

What is the expected number of jobs in the system in steady state?
1. compute steady-state probabilities using GBEs or something else,
2. use steady-state probabilities to compute expectation.
$E[N] = \sum_{i=0}^{n} i \cdot p_i$.
If server is infinite (like with a M|M|1 queue), the expected number of customer:
In system, $E[N] = \frac{\rho}{1-\rho}$.
In queue, $E[N_q] = \frac{\rho^2}{1-\rho}$.
In server, $E[N_s] = \rho$.
Little's Law helps to go from **system-oriented** measures to **user-oriented** measures.

Little's law for:

`Full station`	$E[N] = \lambda \cdot E[R]$, with $E[R]$ as the expected response time, average time each customer spends in system.
`Queue only`	$E[N_q] = \lambda \cdot E[W]$, with $E[W]$ as average waiting time.
`Server only`	$\rho = \lambda \cdot E[S]$, with $E[S]$ as average service time.

For finite stations with one server:

`Little's law`	$E[N] = X \cdot E[R]$, with $X = \mu$ if overloaded
`Loss prob p_{m}`	probability that an arriving job has to leave because the buffer is full (PASTA)
`Throughput`	$X = \lambda \cdot (1-p_m) = \mu \cdot (1-p_0)$, number of jobs served per time unit
`Utilization`	$X \cdot E[S] = 1 - p_0$

For infinite stations with m server:

`Utilization`	For each individual server $\rho = \frac{\lambda}{m\cdot\mu}$
`Expected busy servers`	Number of busy servers $m \cdot \rho = \frac{\lambda}{\mu}$

Lecture 5 Simulation

Monte Carlo method: X_i and Y_i random variables, uniform on [0,1] = random points in a unit square. Define J_i if $Y_i \leq X_i^2$ then 1 else 0. Estimate $\widetilde{A} = \frac{1}{N}\sum_{i=1}^{N} J_i \approx \int_0^1 x^2 dx$.

The estimate $\widetilde{a}$ is a realization of the random variable $\widetilde{A}$. Random variable $\widetilde{A}$ is called an estimator of a. $\widetilde{A}$ should be unbiased, so $E[\widetilde{A}] = a$ and $\widetilde{A}$ should be consistent, so $lim_{n\to\infty} var[\widetilde{A}] = 0$
Different ways to classify simulation methods: **1.** Stochastic vs. deterministic: usage of random numbers, **2.** Discrete-event vs. continuous-event, **3.** Steady-state vs. transient and **4.** Time-based vs. event-based.
Time-based simulation: Also called synchronous simulation, Time proceeds in steps of size Δt, In each iteration all events are processed that happen in the interval $[t, t+\Delta t]$, System state is changed accordingly, Assumption: ordering of events in an interval is not important, events are independent and Δt has to be small.
Event-based simulation: Also called asynchronous simulation, Time 'jumps' from event to event, In each iteration: determine the next event, set simulation time to its occurrence time, process the event and generate new events.
User-oriented measure: Estimate of average response time from n jobs = $\widetilde{r} = \frac{1}{n}\sum_{i=1}^{n}(t_i^{(d)} - t_i^{(a)})$, with $t_i^{(a)}$ arrival time of ith job and $t_i^{(d)}$ departure time of ith job.
Mean values: We want to determine an approximation of $E[X]$ of a random variable X (for example, response time). Simulation is used to generate n samples, each of which is a realization of a random variable X_i. All X_i have the same distribution as X. The X_i are (should be) independent of each other. Random variable $\widetilde{X}$ is an estimator of $E[X]$: $\widetilde{X} = \frac{1}{n}\cdot\sum_{i=1}^{n} X_i$ and hopefully $\widetilde{X}$ is unbiased $(E[\widetilde{X}] = E[X])$ and consistent $(\lim_{n\to\infty} var[\widetilde{X}] = 0)$.
Confidence intervals: The X_i are (hopefully) independent and identically distributed, with mean $E[X]$ and some (unknown) variance $\sigma^2 = var[X]$. Central limit theorem says: $\widetilde{X}$ is approximately normal distributed with mean $E[N]$ and variance σ^2/n since it is the sum of independant vairables X_i. σ^2 is not known either, but can be estimated by: $\widetilde{\sigma}^2 = \frac{n}{n-1}\cdot(\frac{\sum_{i=1}^{n} X_i^2}{n} - \widetilde{X}^2)$ which is the mean of the squares minus square of the mean. And $var[\widetilde{X}] = \frac{\sigma^2}{n} \approx \frac{\widetilde{\sigma}^2}{n}$.

`Std deviations`	Probability that a normally-distributed random variable deviates more than 1.645 standard deviations from the mean, is 10%.
$\widetilde{X}$	$\in [E[X] - 1.645\cdot\frac{\sigma}{\sqrt{n}}, E[X] + 1.645\cdot\frac{\sigma}{\sqrt{n}}]$
`Random interval`	$[\widetilde{X} - 1.645\cdot\frac{\widetilde{\sigma}}{\sqrt{n}}, \widetilde{X} + 1.645\cdot\frac{\widetilde{\sigma}}{\sqrt{n}}]$ which with (approx.) 90% probability contains the true mean E[X]. This is called the 90% confidence interval.
`68%`	Use σ instead of 1.645
`95%`	Use $2\cdot\sigma$instead of 1.645
`99%`	Use $3\cdot\sigma$instead of 1.645

Lecture 6 M|G|1 queues

$\sigma^2 = var[X] = E[X^2] - E[X]^2$ and normalized variance w.r.t. mean value $C_X^2 = \frac{var[X]}{E[X]^2}$

`Deterministic`	$X_{Det} \sim Det(d)$, $Pr\{X = d\} = 1$, $E[X_{Det}] = d$, $E[X_{Det}^2] = d^2$, $var[X_{Det}] = 0$ and $C_{X_{Det}}^2 = 0$
`Uniform`	$X_{Unif} \sim Unif(a,b)$, $E[X_{Unif}] = \frac{a+b}{2}$, $E[X_{Unif}^2] = \frac{b^3-a^3}{3(b-a)}$, $var[X_{Unif}] = \frac{(b-a)^2}{12}$ and $C_{X_{Unif}}^2 = \frac{(b-a)^2}{3(a+b)^2}$
`Exponential`	$X_{Exp} \sim Exp(\lambda)$, $E[X_{Exp}] = 1/\lambda$, $E[X_{Exp}^2] = 2/\lambda^2$, $var[X_{Exp}] = 1/\lambda^2$ and $C_{X_{Exp}}^2 = 1$

Expected residual time: $E[R]$ **of an arriving job: 1.** $E[R_1] = \rho\cdot\frac{E[S^2]}{2E[S]}$ expected residual service time of the job in service (if any), **2.** $E[R_2] = E[N_q]\cdot E[S]$ expected service time of the job(s) already waiting and **3.** $E[R_3] = E[S]$ expected service time of the arriving job itself. Then $E[R] = E[S] + \frac{\lambda\cdot E[S^2]}{2(1-\rho)}$, $E[N] = \lambda E[R] = \lambda E[S] + \frac{\lambda^2 E[S^2]}{2(1-\rho)}$, $E[W] = \frac{\lambda E[S^2]}{2(1-\rho)}$ and $E[N_q] = \frac{\lambda^2 E[S^2]}{2(1-\rho)}$