PEVA Cheat Sheet 2

Lecture 6 M|G|1 queues

Comparison of different queues

`M|M|1`	$E[W_M] = \frac{\rho E[S]}{1-\rho}$
`M|D|1`	$E[W_D] = \frac{1}{2}E[W_M]$
`M|E_2|1`	$E[W_{E_2}] = \frac{3}{4}E[W_M]$

Batch arrivals: $E[H] = \sum_{k=0}^{\infty} k \cdot h_k$, second

`Arrival`	Poisson arrival process with rate λ.
`Packets`	An arrival consists of H mini-packets.
`E[H]`	$E[H] = \sum_{k=0}^{\infty} k \cdot h_k$
`2nd factorial moment`	$E[H^2] - E[H] = E[H^2 - H] = E[H(H-1)] = \sum_{k=0}^{\infty} k \cdot (k-1) \cdot h_k$
`Service time`	service time distribution for a mini-packet with $E[S]$ and $E[S^2]$
`Utilisation`	$\rho = \lambda E[H] \cdot E[S]$
`E[W]`	$E[W] = \frac{\lambda E[H] \cdot E[S^2]}{2(1-\rho)} + \frac{E[H(H-1)]E[S]}{2E[H](1-\rho)}$
`E[W_unsplit]`	$E[W_{unsplit}] = \frac{\lambda E[S^2]}{2(1-\rho)} = \frac{\lambda 2 \cdot E[S]^2}{2(1-\rho)}$

Priority PRIO: With PRIO every class has an λ_r and $E[S_r]$.

`Non-Preemtive`	a job in service is always first finished before a new job is put into service
`remaining service time`	$E[T_P] = \frac{1}{2}\sum_{k=1}^{P} \lambda_k E[S_k^2]$
`Cobham`	$E[W_r] = \frac{E[T_P]}{(1-\sigma_r)(1-\sigma_{r-1})}$, with $\sigma_0 = 0$
`Sigma r`	$\sigma_r = \sum_{k=1}^{r} \rho_k$
`Conservation law`	$\rho E[W] = \sum_{r=1}^{P} \rho_r E[W_r]$
`Preemtive`	a job in service is stopped being served as soon as a higher priority job arrives.
`Scheduling`	preemtive resume (PRS), preemtive repeat identical (PRI) and preemtive repeat different (PRD).
`For class 1 job`	$E[W_1] = \frac{\lambda_1 E[S_1^2]}{2(1-\sigma_r)}$.
`E[W_r]`	$E[W_r] = \frac{\sigma_{r-1}E[S_r]}{1-\sigma_{r-1}} + \frac{E[T_r]}{(1-\sigma_r)(1-\sigma_{r-1})}$

Shortest Job next (SJN):

`Utilisation`	$\rho_t = \lambda_t t = \lambda s f_s(s)dt \cdot t$
`Cumulative utilisation`	$\beta_t = \int_0^t \lambda s f_s(s)ds$
`Total utilisation`	$\beta_\infty = \lambda \int_0^\infty s f_s(s)ds = \lambda E[S] = \rho$
`Remaining service time`	$E[T_\infty] = \frac{1}{2}\lambda E[S^2]$
`Waiting time`	$E[W_t] = \frac{\lambda E[S^2]}{2(1-\beta_t)^2}$
`Waiting time`	$E[W_{SJN}] = \int_0^\infty E[W_t]f_s(t)dt$

Round Robin (RR): an arriving job needs an integer number of service rounds, each of constant length Q (quantum). The number of quanta G an arriving job needs follows a (discrete) geometric distribution. For an arriving job consisting of 1 quantum only, the mean response time is simple: $E[R_{G=1}] = \rho\frac{Q}{2} + E[N_q]Q + Q$

RR to the limit (processor Sharing (PS)) $Q \to 0$: with response time $E[R_{PS}] = \frac{t}{1-\rho}$, stretch factor $(1-\rho)^{-1}$ and virtual average waiting time for jobs of length t $E[W_{PS}(t)] = E[R_{PS}] - t = \frac{\rho t}{1-\rho}$. Unconditional virtual average waiting time $E[W_{PS}] = \frac{\rho E[S]}{1-\rho}$

Lecture 7 Open queueing networks

Open queueing networks: Infinite source/sink (node 0), arrivals from source, departures to sink, number of customers in QN is not constant, stability if number of arrivals equals number of departures.

Feed-Forward QNs: Nodes can be ordered in such a way that whenever customers flow from node i to node j, we have $i < j$. Acyclic customer/packet/job flow. FF QN must be open.

TODO Burke's Theorem See slide 15

Consequences of Burke's Theorem: $E[N_i] = \frac{\rho_i}{1-\rho_i}$ with $\rho_i = \frac{\lambda_i}{\mu_i}$.

Steady-state probs: $Pr\{\underline{N} = \underline{n}\} = \frac{1}{G}\prod_{i=1}^{M} \rho_i^{n_i}$ with $G = (\prod_{i=1}^{M}(1-\rho_i))^{-1}$.

Feedback Jackson QNs: Steady-state probs $Pr\{\underline{N} = \underline{n}\} = \frac{1}{G}\prod_{i=1}^{M} \rho_i^{n_i}$ with $G = (\prod_{i=1}^{M}(1-\rho_i))^{-1}$. $E[R]$? Calulate $E[N]$ which is the sum of $E[N_i] = \frac{\rho_i}{1-\rho_i}$ for all queues and use Litlle's Law $E[R] = \frac{E[N]}{\lambda}$

Lecture 8 Closed queueing networks

Gordon Newell QNs: always stable.

`Visit count`	$V_j = \sum_{i=1}^{M} V_i r_{i,j}$, Solavble if one of the Vs is given
`Service Demand`	$D_i = V_i \cdot E[S_i]$. Highest service demand is bottleneck

If we increase K, then $X(K) \to \frac{1}{D_b}$ and $\rho_b \to 1$. For non-bottleneck stations, $\rho_i \to \frac{D_i}{D_b} < 1$

Gordon And Newell product-form: $I(M,K)$ is every possible combination of $\underline{n} = (n_1, n_2 ... n_M)$. $P(\underline{N} = \underline{n}) = \frac{1}{G(M,K)}\prod_{i=1}^{M} D_i^{n_i}$ with normalisation constant $G(M,K) = \sum_{n \in I(M,K)}\prod_{i=1}^{M} D_i^{n_i}$. Size of $I(M,K) = \binom{M+K-1}{M-1}$

MVA:

$E[\hat{R}_i(k)]$	$(E[N_i(k-1)]+1)D_i$
$E[\hat{R}(k)]$	$\sum E[\hat{R}_i(K)]$
$X(K)$	$\frac{K}{E[\hat{R}(k)]}$
$E[N_i(k)]$	$X(k)E[\hat{R}_i(k)]$
`For IS node`	$E[R_i](K) = E[S_i]$ and $E[\hat{R}_i](K) = D_i$

k	0	1	2	3
$E[\hat{R}_1(k)]$	–			
$E[\hat{R}_2(k)]$	–			
$E[\hat{R}_3(k)]$	–			
$X(k)$	–			
$E[N_1(k)]$	0			
$E[N_2(k)]$	0			
$E[N_3(k)]$	0			

Asumptotic bounds:

`X(K) upperbound`	$X(K) \le \min\{\frac{K}{E[Z]+D_\Sigma}, \frac{1}{D_+}\}$
`E[R] lower bound`	$E[\hat{R}(K)] \ge \max\{E[Z]+D_\Sigma, KD_+\}$
`Saturation point`	$K^* = \frac{D_\Sigma + E[Z]}{D_+}$. The integer part of K^* can be interpreted as the maximum number of customers that could be accommodated without any queueing when all the service times are of deterministic length. Stated differently, if the number of customers is larger than K* we are sure that queueing effects in the network contribute to the response times.

Bard-Schweiter Approximation: MVA with $E[\hat{R}_i(K)] \approx (\frac{K-1}{K}E[N_i(K)]+1)D_i$ and as a first guess $E[N_i](k) = \frac{K}{M}$.

Balanced Queueing Networks: Assume all station ahve the same service demands and $E[N_i(K)] = \frac{K}{M}$.

$E[\hat{R}_i(K)]$	$\frac{D(K+M-1)}{M}$
$E[\hat{R}_(K)]$	$D(K+M-1)$
$X(K)$	$\frac{K}{D(K+M-1)}$
$\rho_i(K)$	$\frac{K}{K+M-1}$
`Simple bounds`	$\frac{K}{D_+(K+M-1)} \le X(K) \le \frac{K}{D_-(K+M-1)}$ $D_-(K+M-1) \le E[\hat{R}(K)] \le D_+(K+M-1)$
`Tighter bounds`	
`Performance`	best if $D = \bar{D} = \frac{D_\Sigma}{M}$
`Throughput`	$X(K) \le \min\{\frac{K}{\bar{D}(K+M-1)}, \frac{1}{D_+}\}$ $\max\{\bar{D}(K+M-1), KD_+\} \le E[\hat{R}(K)] \le D_+(K-1) + D_\Sigma$

Lecture 9 Random number generation

Inversion method: Given some $F_y(y)$, solve $z = F_y(y)$. Here $Z \sim U(0,1)$.
If density function $f_y(y)$ is given, calculate $F_y(y)$. For example, if $f_y(y) = \frac{2}{3}x^2$ for $-1 \le x \le 1$ and zero for everything else, then $F_y(y) = \int_{-\infty}^{x} f_y(u)du = \int_{-1}^{x}\frac{2}{3}u^2du = [\frac{1}{2}x^3]_{-1}^{x} = \frac{1}{2}x^3 + \frac{1}{2}$.
Rejection Method: Given the desity function with finite domain $[a,b]$ and finite image $[0,c]$, you can get approximate random numbers. Generate two uniformly distributed numbers u_1 and u_2 on $[0,1]$. Derive random numbers $x = a + (b-a)u_1$ and $y = cu_2$. If $(y < f_X(x))$, then accept x, else generate new u_1 and u_2. This is fairly efficient when area under $f_X(x)$ is close to $c(b-a)$.

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