PEVA Cheat Sheet 1

Lecture 1 General

Amdahl's law: $T(n) = (1 - \alpha) \cdot t + \alpha \cdot \frac{t}{n}$, $\lim_{n \to \infty} S(n) = \frac{1}{1 - \alpha}$

Kendall notation for queues:

 $arrivals \mid service \mid servers \mid buffersize \mid population \mid scheduling$

arrival Distribution of interarrival time Distribution of service time service

servers Number of servers.

buffer size Maximum number of customers in queueing station (in-

cluding servers).

population Number of customers in and outside the queueing sta-

tion.

scheduling Employed scheduling strategy.

Lecture 2 DTMCs

Limiting distribution: $\underline{v}(P-I) = \underline{0}$ and $\sum_i v_i = 1$

$$(P-I) = \begin{pmatrix} -0.1 & 0.1 \\ 0.4 & -0.4 \end{pmatrix} \implies \begin{matrix} -0.1v_0 + 0.4v_1 = 0 \\ 0.1v_0 - 0.4v_1 = 0 \end{matrix}$$

A state is recurrent if we return to it with probrecurrent

transient A state is transient (or nonrecurrent) if there

is a positive probability of not returning to this

positive recurrent A state is positive recurrent (or recurrent non-

null) if its mean recurrence time is finite.

A state is null recurrent if its mean recurrence null recurrent

time is infinite.

absorbing A state i is absorbing if and only if pi, i = 1. The period d_i of state i is the greatest common period

divisor of all the values n for which $p_{i,i}(n) > 0$. A DTMC is called irreducible if every state can

be reached from every other state in a finite number of steps. In an irreducible DTMC, all

states have the same period.

Future evolution (next state) only depends on Markov property

the current state, not on the past history! DTMCs are time-homogeneous: the matrix P time-homogeneous

does not change over time.

Lecture 3 CTMCs

irreducible

For each state i, introduce rate q_i for an exponentially distributed residence time; mean residence time is $1/q_i$.

Compute steady-state of CTMC by: 1. GBEs or "fluxin, fluxout". 2. Determine steady-state of embedded DTMC and renormalize vwith $p_i = \left(\frac{v_i}{q_i}\right) div \left(\sum_j \frac{v_j}{q_j}\right)$, **3.** Generator Matrix **Q** with $q_{i,j} = q_i \cdot p_{i,j}, \ i \neq j \text{ and } -q_i, \ i = j.$ With **Q**, solve $p \cdot \mathbf{Q} = \underline{0}$.

Lecture 4

FCFS First come, first served. RR Round robin. PS Processor sharing. SJNShortest job next. LCFS Last come, first served. IS Infinite server. PRIO priority scheduling. PASTA Poisson Arrivals See Time Averages.

What is the expected number of jobs in the system in steady state? 1. compute steady-state probabilities using GBEs or something else. 2. use steady-state probabilities to compute expectation.

 $E[N] = \sum_{i=0}^{n} i \cdot p_i.$

If server is infinite (like with a M|M|1 queue), the expected number of customer:

In system, $E[N] = \frac{\rho}{1-\rho}$.

In queue, $E[N_q] = \frac{\rho^2}{1-\rho}$. In server, $E[N_s] = \rho$.

Little's Law helps to go from system-oriented measures tot user-oriented measures.

Little's law for:

 $E[N] = \lambda \cdot E[R]$, with E[R] as the expected re-Full station sponse time, average time each customer spends in system. $E[N_a] = \lambda \cdot E[W]$, with E[W] as average waiting Queue only $\rho = \lambda \cdot E[S]$, with E[S] as average service time.

Server only

For finite stations with one server: $E[N] = X \cdot E[R]$, with $X = \mu$ if overloaded Little's law Loss prob p_{m} probability that an arriving job has to leave be-

cause the buer is full (PASTA)

 $X = \lambda \cdot (1 - p_m) = \mu \cdot (1 - p_0)$, number of jobs Throughput served per time unit

Utilization $X \cdot E[S] = 1 - p_0$

For infinite stations with m server:

For each individual server $\rho = \frac{\lambda}{m \cdot \mu}$ Utilization Expected busy servers For each individual server $m \cdot \rho = \frac{\lambda}{2}$

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam. luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut

elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor, Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed

Lecture 5 Lecture 6