Lecture Notes week 5

Ömer Şakar

December 13, 2016

Contents

1 Lecture 1

Example: Coordination Game
40% upscaled and 60% low-priced.
Competitor $= 4 \cdot you$.

American Footbal Game
$P_2(p, run) = p \cdot 0 + (1-p) \cdot (-10) = 10p - 10$
$P_2(p, pass) = p \cdot (-5) + (1-p) \cdot 0 = -5p$

If $p > \frac{2}{3} \implies 10p - 10 > -5p \implies 2^{nd}$ player: run.
If $p < \frac{2}{3} \implies 10p - 10 < -5p \implies 2^{nd}$ player: pass.

$P_1(p, q) = \sum_i p_i(S_i) \cdot \sum_j q_j \cdot P_1(S_i, S_j)$
$P_1(S_i, q) = \sum_j q_j \cdot P_1(S_i, S_j)$
$P_1(p, q) = P_1(S_i, q)\ \forall S_i$
$\implies 1 \cdot P_1 p, q \geq \sum_i p'_i \cdot P_1(S_i.q) = P_1(p', q)$

2 Lecture 2

Questions from slides
Answer Question 1:
STUB
Answer Question 2:
Yes, for network routing games there is always a **pure** Nash equilibrium.

Player i switches from $P_i$to P_i'.
He saves $\sum_{l \in P_i \setminus P_i'} l_e(n_e(z))$.
He pays: $\sum_{l \in P_i' \setminus P_i} l_e(n_e(z)+1)$ The sum of these are $< 0 \implies$ potential change.

1) $p(Z) = \sum_e l_e(1) + \cdots + l_e(n_e) \leq \sum_e n_e \cdot l_e(n_e) = tc(Z)$

2)$P(Z) = \sum_e (a_e \cdot 1 + b_e) + \cdots + (a_e \cdot n_e + b_e) = \sum_e a_e \frac{n_e \cdot (n_e+1)}{2} + n_e \cdot b_e \geq \sum_e \frac{1}{2} n_e \cdot (n_e \cdot a_e + b_e) = \frac{1}{2} \sum_e n_e \cdot l_e(n_e) = \frac{1}{2} \cdot tc(Z)$, where $l_e(n_e) = (n_e \cdot a_e + b_e)$