Lecture Notes week 1

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1 Lecture 1

1.1 Bayes' Rule

Given events A and B:

Definition 1. *The Conditional Probability* $P(A|B) = \frac{P(A \cap B)}{P(B)}$, *with* $P(B) > 0$.

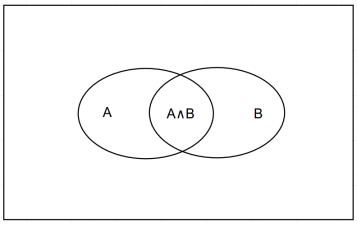

Figure 1: Visual representation of Bayes' Rule

Example: There are 40 Math students of which 15 are girls and there are 50 Computer Science students of which 10 are girls.
Let A = *randomly chosen student is a girl* and B = *randomly chosen student is a math student*.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{15}{90}}{\frac{40}{90}} = \frac{3}{8}$

And from Bayes' Rule follows $\Rightarrow P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A), P(A) > 0$.
Thus we can rewrite it as:

Definition 2. *The Conditional Probability* $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$.

1.1.1 Full Probability Formula

Definition 3. $P(A) = P(B) \cdot P(A|B) + P(\bar{B}) \cdot P(A|\bar{B})$

Example: $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)} = \frac{\frac{40}{90} \cdot \frac{3}{8}}{\frac{25}{90}} = \frac{3}{8}$

1.2 A Herding Experiment

Envelope 1 contains 8 red and 4 blue domino pieces (R) and envelope 1 contains 4 red and 8 blue domino pieces (B).
$P(R) = P(B) = \frac{1}{2}$
The first person that draws either a red or blue piece.
$P(R|(saw)blue) = \frac{P(blue|R) \cdot P(R)}{P(blue)}$.
$P(blue) = P(R) \cdot P(blue|R) + P(B) \cdot P(blue|B) = \frac{1}{2} \cdot \frac{4}{12} + \frac{1}{2} \cdot \frac{8}{12} = \frac{1}{2}$
$P(R|blue) = \frac{\frac{4}{12} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3} < \frac{1}{2}$ *and* $P(B|blue) = \frac{\frac{8}{12} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{2}{3} > \frac{1}{2}$
Now lets look at when a second person draws.

Definition 4. $D_i = \{blue\}$ *or* $\{red\}$ $-$ *what person* i *says they saw*

Definition 5. $E_i = \{blue\}$ *or* $\{red\}$ $-$ *what person* i *saw*

Lets say that the first person says what he sees ($D_1 = E_1$)
$P(B|blue, blue) = \frac{P(B) \cdot P(blue, blue|B)}{P(blue, blue)}$
$P(blue, blue) = \frac{1}{2} \cdot (\frac{2}{3})^2 + \frac{1}{2} \cdot (\frac{2}{3})^2 = \frac{5}{18}$
Thus $P(B|blue, blue) = \frac{\frac{1}{2} \cdot (\frac{2}{3})^2}{\frac{5}{18}} = \frac{4}{5} > \frac{1}{2}$
Conclusion: If $D_1 = \{blue\}$ and $E_2 = \{blue\} \implies D_2 = \{blue\}$

2 Lecture 2