Lecture Notes week 1

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1 Lecture 1

1.1 Bayes' Rule

Given events A and B:

Definition 1. *The Conditional Probability* $P(A|B) = \frac{P(A \cap B)}{P(B)}$, *with* $P(B) > 0$.

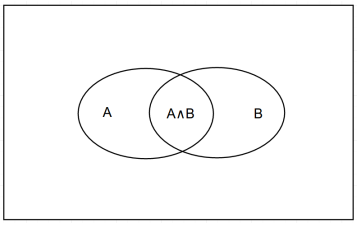

Figure 1: Visual representation of Bayes' Rule

Example: There are 40 Math students of which 15 are girls and there are 50 Computer Science students of which 10 are girls.
Let A = *randomly chosen student is a girl* and B = *randomly chosen student is a math student*.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{15}{90}}{\frac{40}{90}} = \frac{3}{8}$$

And from Bayes' Rule follows $\Rightarrow P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A), P(A) > 0$.
Thus we can rewrite it as:

Definition 2. *The Conditional Probability* $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$.

1.1.1 Full Probability Formula

Definition 3. $P(A) = P(B) \cdot P(A|B) + P(\bar{B}) \cdot P(A|\bar{B})$

Example: $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)} = \frac{\frac{40}{90} \cdot \frac{3}{8}}{\frac{25}{90}} = \frac{3}{8}$

1.2 A Herding Experiment

Envelope 1 contains 8 red and 4 blue domino pieces (R) and envelope 1 contains 4 red and 8 blue domino pieces (B).
$P(R) = P(B) = \frac{1}{2}$
The first person that draws either a red or blue piece.
$P(R|(saw)blue) = \frac{P(blue|R) \cdot P(R)}{P(blue)}$.
$P(blue) = P(R) \cdot P(blue|R) + P(B) \cdot P(blue|B) = \frac{1}{2} \cdot \frac{4}{12} + \frac{1}{2} \cdot \frac{8}{12} = \frac{1}{2}$
$P(R|blue) = \frac{\frac{4}{12} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3} < \frac{1}{2}$ *and* $P(B|blue) = \frac{\frac{8}{12} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{2}{3} > \frac{1}{2}$
Now lets look at when a second person draws.

Definition 4. $D_i = \{blue\}$ *or* $\{red\}$ $-$ *what person i says they saw*

Definition 5. $E_i = \{blue\}$ *or* $\{red\}$ $-$ *what person i saw*

Lets say that the first person says what he sees ($D_1 = E_1$)
$P(B|blue, blue) = \frac{P(B) \cdot P(blue,blue|B)}{P(blue,blue)}$
$P(blue, blue) = \frac{1}{2} \cdot (\frac{2}{3})^2 + \frac{1}{2} \cdot (\frac{2}{3})^2 = \frac{5}{18}$
Thus $P(B|blue, blue) = \frac{\frac{1}{2} \cdot (\frac{2}{3})^2}{\frac{5}{18}} = \frac{4}{5} > \frac{1}{2}$
Conclusion: If $D_1 = \{blue\}$ and $E_2 = \{blue\} \implies D_2 = \{blue\}$

$P(R) = P(B) = \frac{1}{2}$
$P(B|blue, red) = \frac{P(B) \cdot P(blue,red|B)}{P(blue,red)} = \frac{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2}} = \frac{1}{2}$
$D_1 = \{blue\}, E_2 = \{red\} \implies D_2 = \{red\}$ If your opinions differ then
$P(Bcascade|R) = \frac{1}{3} \cdot \frac{1}{3} + (\frac{1}{3} \cdot \frac{2}{3}) \cdot 2 \cdot \frac{1}{3} \cdot \frac{1}{3} + (\frac{1}{3} \cdot \frac{2}{3} \cdot 2)^2 \cdot \frac{1}{3} \cdot \frac{1}{3} + \cdots = \frac{1}{9} \cdot \frac{1}{1-\frac{4}{9}} = \frac{1}{5} \equiv 20\%$

1.3 Genral Cascade Model

Example: Adopting a product
1) Let B be bad and G be good. We estimate $P(B) = 1 - p$ and $P(G) = p$.
2) And let v_g be if we guess G right and v_b if we guess B right.
$v_g \cdot p + v_b \cdot (1 - p) = 0$ - Expected Reward. Rewritten it looks like $v_b + (v_g - v_b) \cdot p$.
It is accepted if $P(G|signal) > p$ and rejected if $P(G|signal) < p$.
3) low(L) and high(H)

	B	G
L	q	1-q
H	1-q	q

Table 1: With q > $\frac{1}{2}$

$P(L|B) = q > \frac{1}{2},\ P(H|B) = 1 - q.\ P(L|G) = 1 - q,\ P(H|G) = q > \frac{1}{2}.$

First person $P(G|H) = \frac{P(H|G) \cdot P(G)}{P(H)} = \frac{p \cdot q}{p \cdot q + (1-p) \cdot (1-q)} > \frac{p \cdot q}{p \cdot q + (1-p) \cdot q} = \frac{p \cdot q}{q} = p$
Thus $E_1 = \{H\} \implies D_1 = \{G\}$

Let $a + b$ - person and S - signal a times H and b times L.
$P(G|S) = \frac{P(S|G) \cdot P(G)}{P(S)} = \frac{p \cdot q^a \cdot (1-q)^b}{p \cdot q^a \cdot (1-q)^b + (1-p) \cdot (1-q)^a \cdot q^b} > \frac{p \cdot q^a \cdot (1-q)^b}{p \cdot q^a \cdot (1-q)^b + (1-p) \cdot q^a \cdot (1-q)^b} = p$
This is when $a < b \iff (1 - q)^a \cdot q^b > q^a \cdot (1 - q)^b$ (because $q > 1 - q$).
Thus when $a < b, D = \{B\}$
If $a > b \iff (1-q)^a \cdot q^b < q^a \cdot (1-q)^b$. This would result in $P(G|S) > p \implies D = \{G\}$
//TODO insert picture of graph

2 Lecture 2