ADVANCED TIME SERIES ANALYSIS FINAL PROJECT

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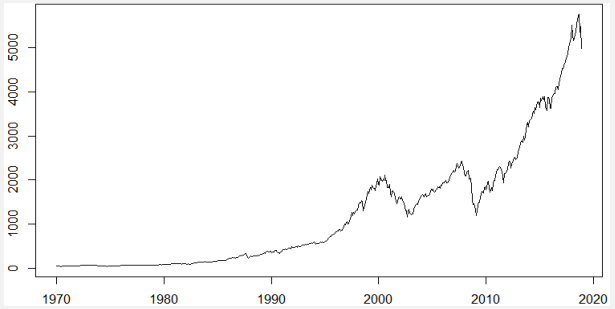
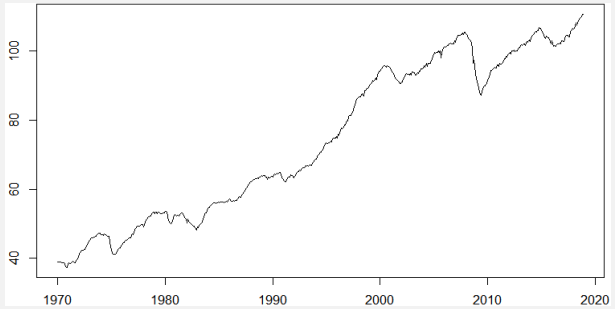
THE DATA

- The data consists of two monthly time series in USA, from January 1970 to December 2018.
  1. ***trsp500:*** S&P 500 Total Return; Monthly Dividend Reinvest (EOP – Equity Office Properties)
  2. ***indpro:*** Industrial Production Index: 2012 = 100 (SA – Seasonally Adjusted)

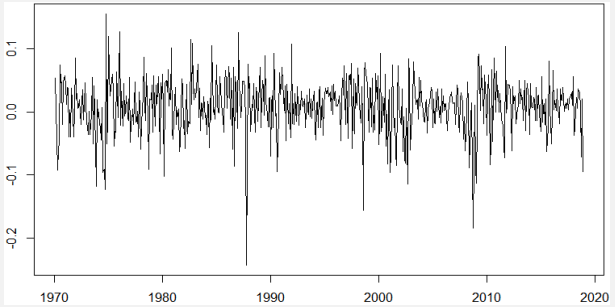


trsp500

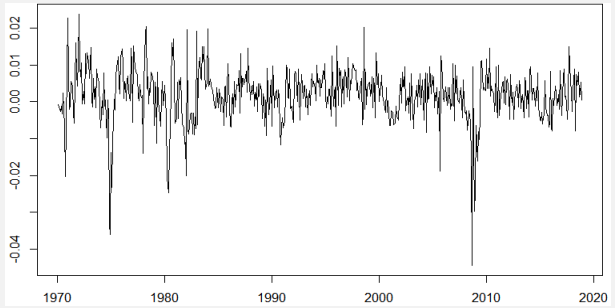


indpro

- There are high autocorrelation and no seasonality effect for both series – checked with acf and monthplot. Series are persistent.
- Log-difference transformation is applied to obtain stationary time series. Augmented Dickey Fuller test is used to ensure that with both «drift» and «trend» options.



log-differenced trsp500 (investment)

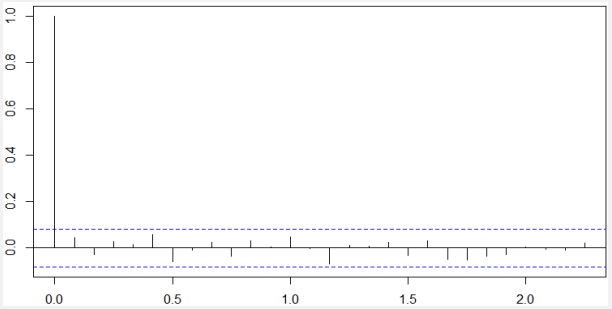


log-differenced indpro (production)

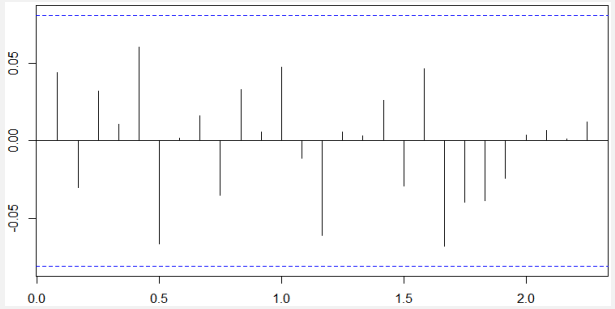
Sources
http://www.economagic.com/em-cgi/data.exe/fedstl/trsp500
http://www.economagic.com/em-cgi/data.exe/fedstl/indpro

UNIVARIATE ANALYSIS – MODEL SELECTION

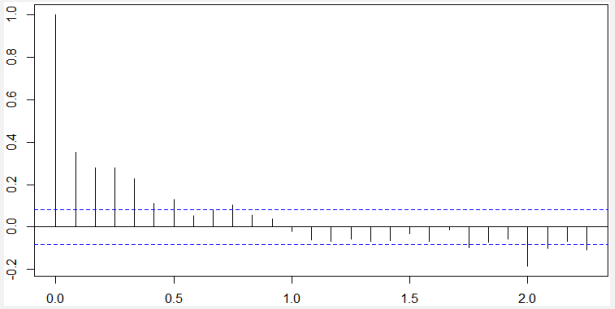
- In order to select a suitable model for the time series, correlograms of the transformed series are examined.
  - *trsp500*: No significant autocorrelation or partial autocorrelation occur at any lags.
  - *indpro*: Seasonal difference might be needed at lag 2 (24 months)



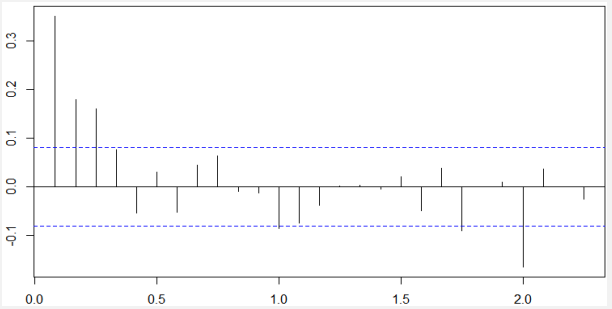
ACF of log-differenced *trsp500*



PACF of log-differenced *trsp500*



ACF of log-differenced *indpro*



PACF of log-differenced *indpro*

- After that, 2 parsimonious models are created for each time series; one of which is found with auto.arima function.
- All models are validated with Box-Ljung test on residuals. Also, correlograms of model residuals show no autocorrelation.

		Model	Estimated σ2	AIC	RMSE	Ljung-Box p-value
Series	*trsp500*	ARMA(1,1)	0.001911	-2000.82	0.0437162	0.8918
		SARMA(0,0)(1,0)[12]	0.001923	-2001.09	0.0437800	0.8652
	indpro	SARMA(1,1)(2,1)[12]	0.04239	-4229.33	0.006510382	0.363
		SARMA(1,1)(2,0)[12]	0.04286	-4230.01	0.006518542	0.292

- For both series, different models are better in different measures. In order to decide which model is significantly better, Diebold-Mariano test (with squared forecast error) is used.

DIEBOLD-MARIANO TEST

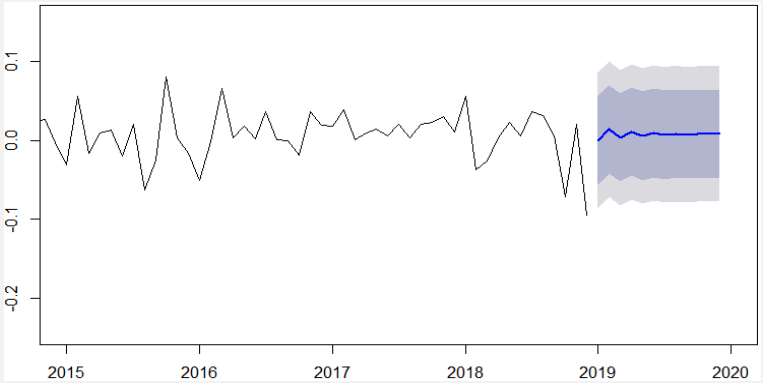
- Diebold-Mariano test compares the h-step ahead forecast errors of two models. With this model, it is possible to test which of the two models perform better.
- H0: E[h-step ahead forecast error of Model A] = E[h-step ahead forecast error of Model B]
- Paired t-test is applied. However, Newey-West correction is used on the standard error.

		Model	Error Mean	DM Test Statistics	p-value
Series	*trsp500*	ARMA(1,1)	0.001797509	-0.25936	0.7957
		SARMA(0,0)(1,0)[12]	0.001801771		
	indpro	SARMA(1,1)(2,1)[12]	0.00004614633	-1.7149	0.08849
		SARMA(1,1)(2,0)[12]	0.00004630990		

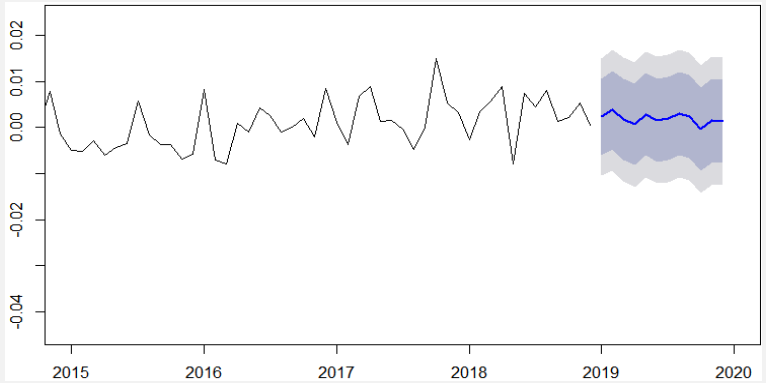
- According to the Diebold-Mariano tests, there are no significant differences between the models selected ($\alpha = 0.05$).
- For parsimony, the models with lower error means are selected for further analyses. They are:
  - trsp500 → ARMA(1,1)
  - indpro → SARMA(1,1)(2,1)[12]

FORECASTS

- With the selected models, the values of the next 12 months (from January 2019 to December 2019) are predicted.

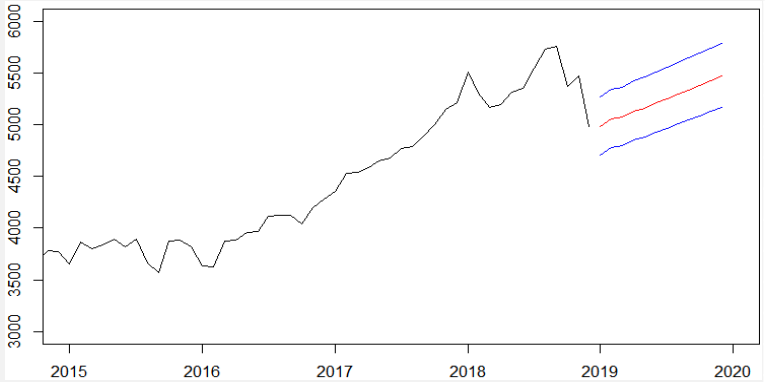


Predictions of trsp500 (log differenced)



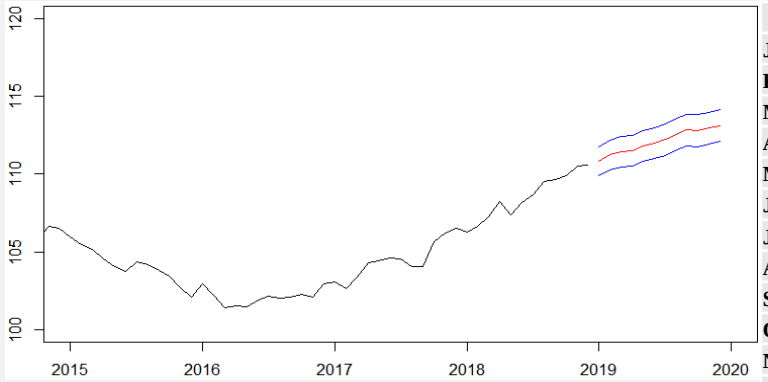
Predictions of indpro (log differenced)

- However, since the predictions are for the transformed values, back-transformation is applied to get real forecasted values.



Predictions of trsp500 in 2019

	2019
January	4983.183
February	5054.429
March	5074.265
April	5131.685
May	5162.658
June	5213.257
July	5250.336
August	5297.748
September	5338.336
October	5384.449
November	5427.206
December	5473.003



Predictions of indpro in 2019

	2019
January	110.8097
February	111.2286
March	111.4215
April	111.4996
May	111.8193
June	111.9984
July	112.2195
August	112.5602
September	112.8350
October	112.7953
November	112.9608
December	113.1146

MULTIVARIATE ANALYSIS – COINTEGRATION

- Cointegration between two time series should be checked.
- If cointegration exists, long run equilibrium relationship between the two variables can be estimated by OLS.
- Engle-Granger approach is used to detect cointegration. Variables are regressed on each other
  - For raw data, linear models have significantly low p-values, but they are not valid according to the Box-Ljung test. (Spurious regression)
  - For differenced data, linear models are not adequate, and they are not valid according to the Box-Ljung test.
  - For log-differenced data, linear models are not adequate, but one model is valid according to the Box-Ljung test.
    - Cointegrated Augmented Dickey-Fuller test (CADF) is applied on the residuals of that model. Significant result is achieved.

- ECM (Error Correcting Model) is applied to describe short-run dynamics between two time series.
  - $Y_t \rightarrow$ log-differenced indpro
  - $X_t \rightarrow$ log-differenced trsp500
  - ECT → error correcting term: lagged residuals of the model

- ECM is also inadequate – with high p-value and low R2 value.
- We cannot confidently conclude that cointegration occurs.

DYNAMIC MODELS & GRANGER CAUSALITY

- Using dynamic models, one stationary time series is regressed on the other one.
- Lagged variables are needed, because previous models were not adequate to explain the relationship between two time series.
- Distributed lag models (DLM) and autoregressive distributed lag models (ADLM) are tried with different orders.
  - DL(1) and DL(2) are tried. Although the residuals are homoscedastic and uncorrelated, the models are inadequate to explain the variability.
  - ADL(1) and ADL(2) are tried. The conclusions are the same as the distributed lag models.

	Model p-value	Adjusted R-square	Ljung-Box test p-value
DL(1)	0.1282	0.003617	0.7613
DL(2)	0.2277	0.002298	0.7623
ADL(1)	0.6008	-0.001944	0.7956
ADL(2)	0.6008	-0.001944	0.7956

- Although DL models seem to perform better than ADL models, the results are inconclusive.
- Also, DLM and ADLM are not very suitable for predictions.

- The condition of Granger-Causality is tested with lag one, since it seems to be the most adequate model tested.
- H_0: There is no Granger-Causality.

Model 1: ts_trsp500_dlog.0 ~ ts_trsp500_dlog.1 + ts_indpro_dlog.1
Model 2: ts_trsp500_dlog.0 ~ ts_trsp500_dlog.1

	Res. Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	582	1.1177				
2	583	1.1234	-1	-0.0057042	2.9704	0.08533

- Granger-Causality does not occur ($\alpha = 0.05$).

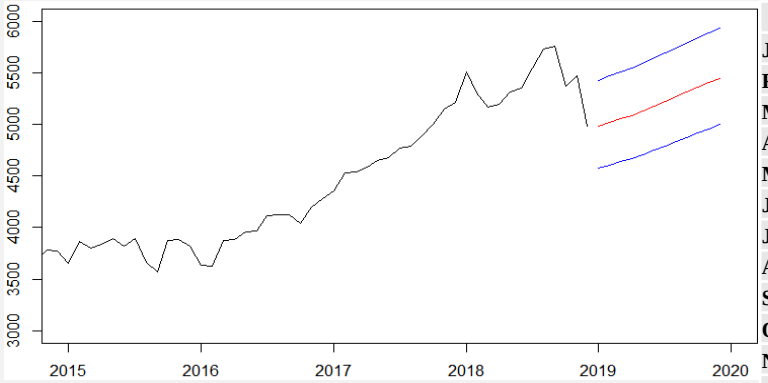
VAR MODEL

- Two time series are combined, and a 2-dimensional VAR model is created.
- The most suitable model is found to be VAR(3).

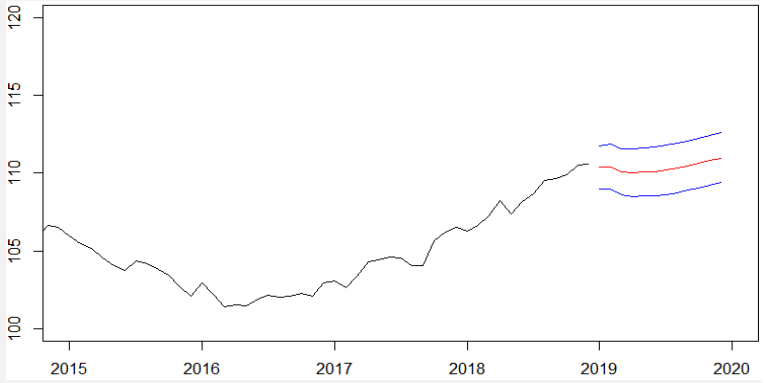
```
Estimation results for equation investment:
===========================================
investment = investment.l1 + production.l1 + investment.l2 + production.l2 + investment.l3 + production.l3 + const

               Estimate Std. Error t value Pr(>|t|)
investment.l1  0.049554   0.041638   1.190   0.2345
production.l1  0.552571   0.271194   2.038   0.0421 *
investment.l2 -0.028880   0.041697  -0.693   0.4888
production.l2 -0.121430   0.277747  -0.437   0.6621
investment.l3  0.022616   0.041870   0.540   0.5893
production.l3 -0.344388   0.270577  -1.273   0.2036
const          0.007816   0.001981   3.946 8.93e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.0437 on 577 degrees of freedom
Multiple R-Squared: 0.01236,    Adjusted R-squared: 0.002091
F-statistic: 1.204 on 6 and 577 DF,  p-value: 0.3026
```

```
Estimation results for equation production:
===========================================
production = investment.l1 + production.l1 + investment.l2 + production.l2 + investment.l3 + production.l3 + const

               Estimate Std. Error t value Pr(>|t|)
investment.l1 0.0085672  0.0061320   1.397  0.16291
production.l1 0.2315208  0.0399386   5.797 1.11e-08 ***
investment.l2 0.0127295  0.0061406   2.073  0.03862 *
production.l2 0.1256533  0.0409036   3.072  0.00223 **
investment.l3 0.0368411  0.0061661   5.975 4.03e-09 ***
production.l3 0.1690897  0.0398476   4.243 2.56e-05 ***
const         0.0003622  0.0002917   1.242  0.21480
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.006435 on 577 degrees of freedom
Multiple R-Squared: 0.2299,     Adjusted R-squared: 0.2219
F-statistic: 28.71 on 6 and 577 DF,  p-value: < 2.2e-16
```

- Estimations for production index are more accurate than those for dividend reinvestment, due to lower p-value and higher R2.
- Again, real predictions are obtained by back-transforming the log-differenced predictions.



2019	
January	4983.020
February	5023.956
March	5056.262
April	5090.254
May	5131.787
June	5180.790
July	5224.756
August	5269.017
September	5314.767
October	5360.071
November	5405.396
December	5451.064



2019	
January	110.3681
February	110.4201
March	110.0797
April	110.0330
May	110.0766
June	110.1062
July	110.1981
August	110.3248
September	110.4693
October	110.6289
November	110.8003
December	110.9802

COMPARING PREDICTIONS

- Two sets of predictions are obtained for two time series: One from univariate analysis, one from multivariate analysis.
- Luckily, real values of the series for 2019 exist until November – which may not always be the case.
- Predictions are compared, based on absolute errors and squared errors.

	trsp500						
	Real values	**Univariate predictions**	**Multivariate predictions**	**Abs. error (Univariate)**	**Abs. error (Multivariate)**	**Sq. error (Univariate)**	**Sq. Error (Multivariate)**
January	5383,63	4983,183	4983,02	400,447	400,61	160357,7998	160488,3721
February	5556,49	5054,429	5023,956	502,061	532,534	252065,2477	283592,4612
March	5664,46	5074,265	5056,262	590,195	608,198	348330,138	369904,8072
April	5893,81	5131,685	5090,254	762,125	803,556	580834,5156	645702,2451
May	5519,27	5162,658	5131,787	356,612	387,483	127172,1185	150143,0753
June	5908,25	5213,257	5180,79	694,993	727,46	483015,27	529198,0516
July	5993,17	5250,336	5224,756	742,834	768,414	551802,3516	590460,0754
August	5898,23	5297,748	5269,017	600,482	629,213	360578,6323	395908,9994
September	6008,59	5338,336	5314,767	670,254	693,823	449240,4245	481390,3553
October	6138,73	5384,449	5360,071	754,281	778,659	568939,827	606309,8383
November	-	5427,206	5405,396	-	-	-	-
December	-	5473,003	5451,064	-	-	-	-

	indpro						
	Real values	**Univariate predictions**	**Multivariate predictions**	**Abs. error (Univariate)**	**Abs. error (Multivariate)**	**Sq. error (Univariate)**	**Sq. Error (Multivariate)**
January	110,1185	110,8097	110,3681	0,6912	0,2496	0,47775744	0,06230016
February	109,5631	111,2286	110,4201	1,6655	0,857	2,77389025	0,734449
March	109,6811	111,4215	110,0797	1,7404	0,3986	3,02899216	0,15888196
April	108,9888	111,4996	110,033	2,5108	1,0442	6,30411664	1,09035364
May	109,2264	111,8193	110,0766	2,5929	0,8502	6,72313041	0,72284004
June	109,2678	111,9984	110,1062	2,7306	0,8384	7,45617636	0,70291456
July	109,1183	112,2195	110,1981	3,1012	1,0798	9,61744144	1,16596804
August	109,9273	112,5602	110,3248	2,6329	0,3975	6,93216241	0,15800625
September	109,594	112,8350	110,4693	3,241	0,8753	10,504081	0,76615009
October	108,6714	112,7953	110,6289	4,1239	1,9575	17,00655121	3,83180625
November	-	112,9608	110,8003	-	-	-	-
December	-	113,1146	110,9802	-	-	-	-

	trsp500	**indpro**
MAD (Univariate)	607,4284	2,50304
MAD (Multivariate)	632,9950	0,85481
MSQ (Univariate)	388233,6325	7,08243
MSQ (Multivariate)	421309,8281	0,93937

- Univariate analysis give better results for trsp500.
- Multivariate analysis give better results for indpro.

CONCLUSIONS & COMMENTS

- Both time series have an increasing trend. Trend for trsp500 seems to be exponential, whereas the trend for indpro seems to be linear.
- Sharp declines in both time series (for instance, in 2009) coincides with the economic crisis in USA.
- In order to achieve more accurate results without violating the assumptions, time series should be stationary. Stationartiy is satisfied with log-difference transformation.
- In order to avoid computationally exhaustive analysis, the number of models tried are kept low.
- Although the plots of the time series indicate similar patterns, cointegration is not observed.
- Even though they are valid according to Box-Ljung test, error correcting model and dynamic models are inadequate to explain the variability.
- There may be other external regressors that affects both time series, like housing prices, interest rates, GDP per capita etc. However, a causal relationship between two time series cannot be proven. Granger causality also did not occur.
- Since the predictions from multivariate analysis perform better than those from univariate analysis for indpro, it can be said that industrial production is affected by external factors more than dividend reinvestment is.