Access to HE Diploma: Mathematics
YHS984
Vectors and Matrices
MOCK
Task 2 of 2

Task 2 is for you to demonstrate your understanding of the module and how well you can apply these skills. The questions in this assessment cover a variety of attainment levels; the more that you answer correctly, the higher the chance you will receive the highest grade.

- working out MUST be shown in all questions
- You have 2 hours to complete this assessment

1. a) A projectile has been launched at 3 m/s in the x- direction and 5 m/s in the y-direction, express the velocity of the projectile in vector-form.

Assessment Criteria 1.1

b) Another projectile is launched in the OPPOSITE x-direction at 4 m/s and 6 m/s in the y-direction. Express its velocity as a single vector

c) Express the difference in velocity as a single vector

d) Find the relative speed between the two projectiles

Assessment Criteria 1.2

2. A line passes through the point $\begin{pmatrix}1\\3\end{pmatrix}$ in the $\begin{pmatrix}-2\\1\end{pmatrix}$ direction.

   a) State the vector equation of the line.

   b) Prove the point $\begin{pmatrix}-9\\8\end{pmatrix}$ lies on the line.

Assessment Criteria 1.3

Prove that the line

$$\ell : \vec{r} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

is perpendicular to the line in $2a$)

Assessment Criteria 1.4

3. A plane is described by

$$\mathbf{P} : \vec{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + s \begin{pmatrix} 7 \\ -5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

a) Find the normal vector $\vec{n}$

Assessment Criteria 2.2

b) Another plane has normal vector

$$\vec{n} = \begin{pmatrix} 3 \\ -5 \\ 6 \end{pmatrix}$$

find the angle θ in **degrees** between the two planes.

b) Give the vector equation of this plane.

Assessment Criteria 2.4

4. A 2 day festival sells **beer, water, milk, and vodka**.

   On Saturday they sell 1350 beers, 2 bottles of water, 1630 glasses of milk, and 1630 shots of vodka.

   On Sunday they sell 2 beers, 1350 bottles of water, 245 glasses of milk, and 245 shots of vodka.

a)

i) Express this as a matrix A_1

Assessment Criteria 2.4

ii) What are the dimensions of this matrix

Assessment Criteria 2.3

b) Next year their sales look like this, where the columns represent beer, water, milk, and vodka sold while the rows represent Saturday and Sunday:

$$A_2 = \begin{pmatrix} 4000 & 23 & 3021 & 453 \\ 32 & 4120 & 1467 & 1732 \end{pmatrix}$$

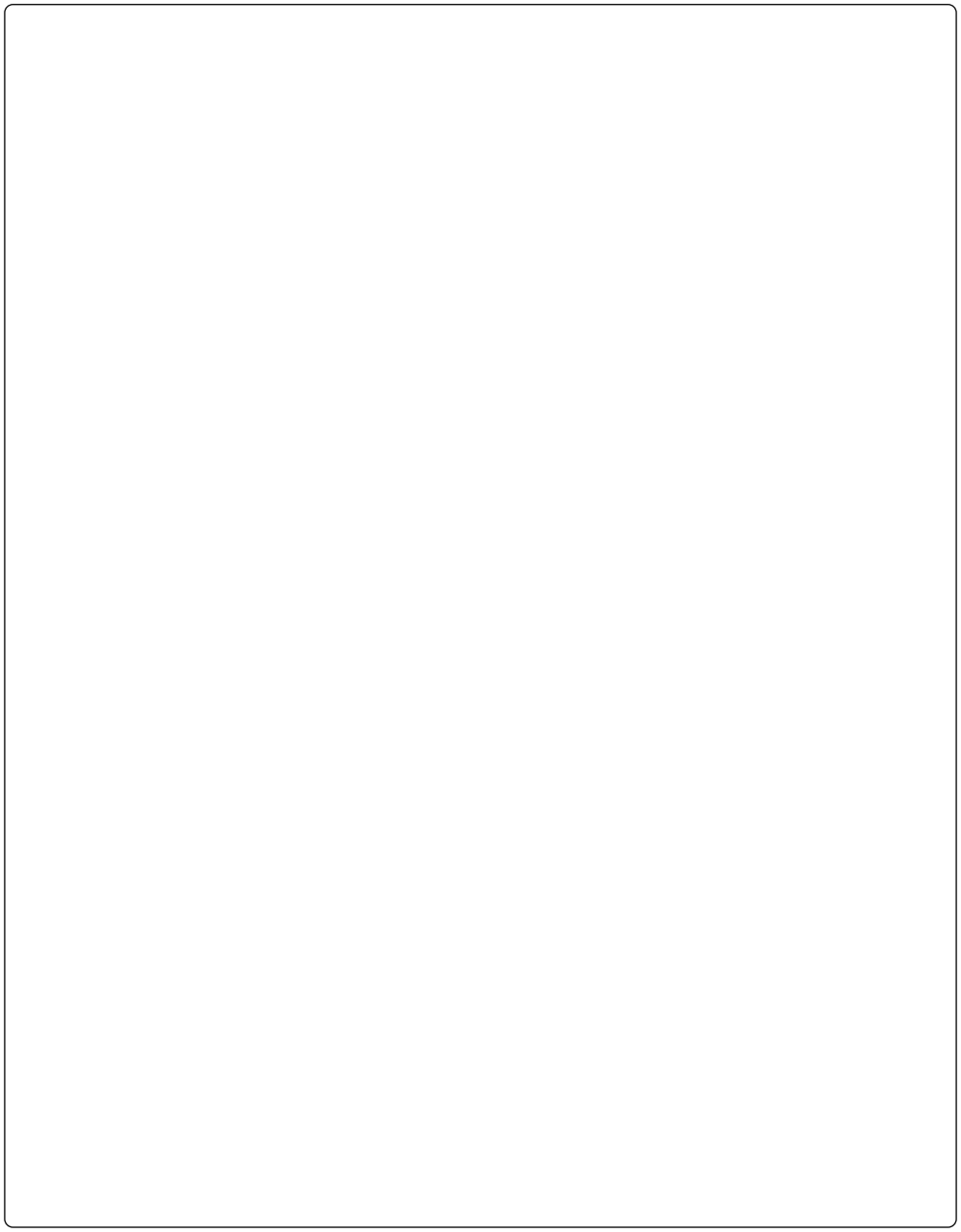
Find the matrix A_{TOT} that expresses the total sales across the two years.

c) Explain why the two matrices cannot be multiplied i.e. why is $A_1 A_2$ not a valid operation.

Assessment Criteria 3.3

5. a) Find the determinant of the matrix

$$A = \begin{pmatrix} 4 & 1 & 6 \\ 3 & 2 & 5 \\ -2 & 7 & 4 \end{pmatrix} \tag{1}$$



b) To find the inverse of A we first need to compute the Adjugate (or Adjoint) Find adj(A) using the following steps...

i) Compute the Minor Matrix;

(Each element of the minor matrix M_{ij} is calculated by taking the element A_{ij} and crossing the row i and column j then calculating the 2×2 determinant of the remaining 4 elements in A.)

ii) Apply the Cofactor signs

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \tag{2}$$

iii) Transpose the matrix

$$C_{ij} \rightarrow C_{ji} \tag{3}$$

c) Using your adj(A) , express the inverse using the formula

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) \tag{4}$$

d) Use the inverse matrix method to solve the following simultaneous equations.

$$\begin{aligned} 4x + y + 6z &= 3 \\ 3x + 2y + 5z &= 7 \\ -2x + 7y + 4z &= -5 \end{aligned} \tag{5}$$

Assessment Criteria 3.3

END OF TASK 2