IE 313
SUPPLY CHAIN MANAGEMENT

Spring 2023

ASSIGNMENT 1

GROUP 2
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Introduction

Q&H is one of the leading detergent manufacturers in Turkey. There is a demand forecast for a particular product of Q&H. By considering constraints like the facility capacity, inventory holding cost, production cost, and the labor cost, the LP model decides on the amount of production and inventory mainly in an effort to maximize the profit. In addition to these, since the promotions affect the demand for the product of the company, the LP model also takes these parameters into account. As a result, different amounts of profits are obtained in different scenarios.

The LP Model

1. **Defining the indices.**
   t : $index\ of\ months\ t\ =\ 1,\ldots,12$

2. **Defining the decision variables.**
   W_t : $manhours\ of\ regular\ labor\ used\ in\ period\ t$
   O_t : $manhours\ of\ overtime\ labor\ used\ in\ period\ t$
   I_t : $units\ of\ product\ to\ be\ left\ over\ as\ inventory\ in\ period\ t$
   X_t : $units\ of\ product\ to\ be\ produced\ in\ period\ t$
   SB_t : $units\ of\ product\ to\ be\ subcontracted\ in\ period\ t$

3. **Defining the parameters.**
   v_t : $unit\ production\ cost\ in\ month\ t$
   c_t : $inventory\ carrying\ cost\ in\ month\ t$
   w_t : $cost\ per\ manhour\ of\ regular\ labor\ in\ month\ t$
   o_t : $cost\ per\ manhour\ overtime\ labor\ in\ month\ t$
   sb_t : $subcontracting\ cost\ in\ month\ t$
   rm_t : $total\ manhours\ of\ regular\ labor\ available\ in\ period\ t$
   om_t : $total\ manhours\ of\ overtime\ labor\ available\ in\ period\ t$
   d_t : $forecast\ demand\ for\ the\ product\ in\ month\ t$
   sp_t : $sale\ price\ of\ the\ product\ in\ month\ t$

4. **Defining The Constraints.**

$Balance\ Equation$	$X_t + I_{t-1} = d_t + I_t$
$Work\ hours\ Availability$	$k \times X_t \leq W_t + O_t$
$Regular\ Work\ Hour\ Constraint$	$W_t = rm_t$
$Overtime\ Work\ Hour\ Constraint$	$O_t = ow_t$
$Safety\ Stock$	$I_t \geq 100$

5. **Defining The Objective Function.**

$$max \sum_{t} \quad (sp_t \times d_t - sb_t \times SB_t - v_t \times X_t - c_t \times I_t - w_t \times W_t - o_t \times O_t)$$

$$SB_t, X_t, I_t, W_t, O_t \geq 0 \qquad \forall t$$

Overview of the Algorithm Code

1. Importing the PuLP library and defining the parameters of the objective function.

```
In [1]: from pulp import*

In [2]: #Define the model.
        model = LpProblem(name="IE13_Assignment1", sense = LpMaximize)
```

Image 1. Code part 1

2. Setting the index.

```
In [3]: #Define the sets.
        period = list(range(13))
```

Image 2. Code part 2

3. Defining the decision variables. The decision variable called SB is added for the subcontracting situation.

```
In [4]: #Define the decision variables.
        X = LpVariable.dicts("Amount of Production" , (period), lowBound = 0)
        I = LpVariable.dicts("Inventory Level" , (period), lowBound = 0)
        W = LpVariable.dicts("Regular Time Workhours Used" , (period), lowBound = 0)
        O = LpVariable.dicts("Overtime Workhours Used" , (period), lowBound = 0)
        SB = LpVariable.dicts("Subcontracting Amount" , (period), lowBound = 0)
```

Image 3. Code part 3

4. Defining the parameters. Since the demand and the sale price are changing according to the promotions, these set of parameters are different in every notebook. In addition to this, the beginning and ending inventory are set as 150.

```
In [5]: #Define the parameters.

#demand:
dt = {
    0:0,
    1:280,
    2:301,
    3:277,
    4:310,
    5:285,
    6:278,
    7:291,
    8:277,
    9:304,
    10:295,
    11:302,
    12:297,
    13:0
}
I[0] = 150
I[12] = 150

#Constant Workforce Production
#Number of employees:
ne = 100
#Manhours required to produce one unit of the product:
k = 100
#The cost per manhour of regular labor in month t:
wt = 10
#The cost per manhour overtime labor in month t:
ot = 15
vt = 1000
#The unit inventory cost in month t:
ct = 100
#The subcontracting cost
sb = 2300
#The sale price
spt = {
    0:0,
    1:2600,
    2:2600,
    3:2600,
    4:2600,
    5:2600,
    6:2600,
    7:2600,
    8:2600,
    9:2600,
    10:2600,
    11:2600,
    12:2600,
    13:0
}

#The total manhours of regular labor available in month t:
rmt = 20 * 2 * 8 * ne
#The total manhours of overtime labor available in month t:
owt = 20 * 2 * ne #multplied by 2 because of 2 shifts
```

Image 4. Code part 4

5. Defining the constraints and the objective function. The balance equation, the work hours availability, the regular & over time work hours, and the safety stock constraints are added to the model. Then, the objective function is defined as the difference between sales and costs.

```
In [6]: #Define the constraints.
        for t in period[1:]:
            model += (SB[t] + X[t] + I[t-1] == dt[t] + I[t] , "Balance Equation %s"%t)
            model += (k*X[t] <= W[t] + O[t] , "Workhours Availability %s"%t)
            model += (W[t] == rmt , "Regular Workhour Constraint %s"%t)
            model += (O[t] <= owt , "Overtime Workhours Constraint %s"%t)
            model += (I[t] >= 100 , "Safety Stock Constraint %s"%t)
        #Define the objective function.
        model += lpSum(spt[t] * dt[t] - sb * SB[t] - vt * X[t] - ct * I[t] - wt* W[t] - ot * O[t] for t in period[1:])
```

Image 5. Code part 5

6. Checking for optimality & finding the optimal objective function value.

```
In [7]: model

Out[7]: IE13_Assignment1:
MAXIMIZE
-1000*Amount_of_Production_1 + -1000*Amount_of_Production_10 + -1000*Amount_of_Production_11 + -1000*Amount_of_Production_12
+ -1000*Amount_of_Production_2 + -1000*Amount_of_Production_3 + -1000*Amount_of_Production_4 + -1000*Amount_of_Production_5 +
-1000*Amount_of_Production_6 + -1000*Amount_of_Production_7 + -1000*Amount_of_Production_8 + -1000*Amount_of_Production_9 + -
100*Inventory_Level_1 + -100*Inventory_Level_10 + -100*Inventory_Level_11 + -100*Inventory_Level_2 + -100*Inventory_Level_3 +
-100*Inventory_Level_4 + -100*Inventory_Level_5 + -100*Inventory_Level_6 + -100*Inventory_Level_7 + -100*Inventory_Level_8 +
-100*Inventory_Level_9 + -15*Overtime_Workhours_Used_1 + -15*Overtime_Workhours_Used_10 + -15*Overtime_Workhours_Used_11 + -1
5*Overtime_Workhours_Used_12 + -15*Overtime_Workhours_Used_2 + -15*Overtime_Workhours_Used_3 + -15*Overtime_Workhours_Used_4
+ -15*Overtime_Workhours_Used_5 + -15*Overtime_Workhours_Used_6 + -15*Overtime_Workhours_Used_7 + -15*Overtime_Workhours_Used
_8 + -15*Overtime_Workhours_Used_9 + -10*Regular_Time_Workhours_Used_1 + -10*Regular_Time_Workhours_Used_10 + -10*Regular_Tim
e_Workhours_Used_11 + -10*Regular_Time_Workhours_Used_12 + -10*Regular_Time_Workhours_Used_2 + -10*Regular_Time_Workhours_Use
d_3 + -10*Regular_Time_Workhours_Used_4 + -10*Regular_Time_Workhours_Used_5 + -10*Regular_Time_Workhours_Used_6 + -10*Regular
_Time_Workhours_Used_7 + -10*Regular_Time_Workhours_Used_8 + -10*Regular_Time_Workhours_Used_9 + 9077200
SUBJECT TO
Balance_Equation_1: Amount_of_Production_1 - Inventory_Level_1 = 130

Workhours_Availability_1: 100 Amount_of_Production_1
 - Overtime_Workhours_Used_1 - Regular_Time_Workhours_Used_1 <= 0

In [8]: model.solve()
        LpStatus[model.status]

Out[8]: 'Optimal'

In [9]: print("Optimal objective function value = ", value(model.objective))

        Optimal objective function value =  1626600.0
```

Image 6. Code part 6

7. Printing the optimal values of the decision variables.

```
In [10]: for v in model.variables():
             print(v.name, "=", v.varValue)

Amount_of_Production_1 = 230.0
Amount_of_Production_10 = 304.0
Amount_of_Production_11 = 320.0
Amount_of_Production_12 = 320.0
Amount_of_Production_2 = 301.0
Amount_of_Production_3 = 277.0
Amount_of_Production_4 = 310.0
Amount_of_Production_5 = 285.0
Amount_of_Production_6 = 278.0
Amount_of_Production_7 = 291.0
Amount_of_Production_8 = 277.0
Amount_of_Production_9 = 304.0
Inventory_Level_1 = 100.0
Inventory_Level_10 = 109.0
Inventory_Level_11 = 127.0
Inventory_Level_2 = 100.0
Inventory_Level_3 = 100.0
Inventory_Level_4 = 100.0
Inventory_Level_5 = 100.0
```

Image 7. Code part 7

8. Creating a dataframe with the decision variables to store them and make data visualizations.

```
In [12]: import pandas as pd
production_index = [0,4,5,6,7,8,9,10,11,1,2,3]
inventory_index = [12,15,16,17,18,19,20,21,22,13,14]
overtime_index = [23,27,28,29,30,31,32,33,34,24,25,26]
regular_index = [35,39,40,41,42,43,44,45,46,36,37,38]
demand=[]
production = []
inventory = []
regular_workhours = []
overtime_workhours = []
working_utilization = []
for i in range(1,13):
    demand.append(list(dt.values())[i])
for i in inventory_index:
    inventory.append(model.variables()[i].varValue)
inventory.append(150)
for i in overtime_index:
    overtime_workhours.append(model.variables()[i].varValue)
for i in regular_index:
    regular_workhours.append(model.variables()[i].varValue)
for i in production_index:
    production.append(model.variables()[i].varValue)
for i in production_index:
    working_utilization.append(32000-model.variables()[i].varValue*100)

d = {
    'Months' : ["January", "February", "March", "April" , "May", "June",
                "July" , "August" , "September" , "October" , "November" , "December"],
    'Amount of Production' : production,
    'Inventory Level At The End Of The Month' : inventory,
    'Regular Time Workhours' : regular_workhours,
    'Overtime Workhours' : overtime_workhours,
    'Amount of Excess Hours' : working_utilization
}
df = pd.DataFrame(data=d)
df
```

Image 8. Code part 8

9. Deciding on the KPI's to analyze the results and visualizing the data.

```
In [13]: import matplotlib.pyplot as plt
import seaborn as sns
plt.figure(figsize=(12,4))
sns.barplot(data=df, x = "Months",y = "Inventory Level At The End Of The Month")
plt.title("Inventory Levels")
plt.show()
plt.figure(figsize=(12,4))
sns.barplot(data=df, x = "Months",y = "Amount of Excess Hours")
plt.title("Unused Capacity of The Facility In Terms Of Working Hours")
plt.show()
plt.figure(figsize=(12,6))
plt.bar(df["Months"],df["Amount of Production"], color='purple')
plt.bar(df["Months"],df["Inventory Level At The End Of The Month"], bottom=df["Amount of Production"], color='pink')
plt.plot(demand, color = 'red')
plt.legend(['Demand','Amount of Production', 'Inventory  Level'])
plt.title("Production vs. Inventory Level")
plt.show()
```

Image 9. Code part 9

Answers of the Questions with Necessary Interpretations

Answer 1:

The optimal production plan is the given above with the minimum cost of 7.393.300$. In some months, the facility is not producing at its capacity because of the low demand and carrying cost of the inventory. Thus, the optimal model prefers to increase the production in the last months of the year to keep the inventory level at 150 at the end of the year as can be seen from the bar chart below. The model simply prefers to minimize the inventory cost because it is less expensive than making overtime in December to approach the ending inventory target.

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours
0	January	130.0	0.0	32000.0	0.0
1	February	301.0	0.0	32000.0	0.0
2	March	277.0	0.0	32000.0	0.0
3	April	310.0	0.0	32000.0	0.0
4	May	285.0	0.0	32000.0	0.0
5	June	278.0	0.0	32000.0	0.0
6	July	316.0	25.0	32000.0	0.0
7	August	320.0	68.0	32000.0	0.0
8	September	320.0	84.0	32000.0	0.0
9	October	320.0	109.0	32000.0	0.0
10	November	320.0	127.0	32000.0	0.0
11	December	320.0	150.0	32000.0	0.0

Table 1. Data Table 1

In the first half of the year, the production catches the demand as the demand is lower than the facility production capacity. In that situation, holding the employees that do not produce anything is costly. The facility could have been designed by considering the demand, the market volume, and the market share of the company. In addition to this, the demand should be forecasted accurately for not holding any safety stock for 6 months for not being in short.

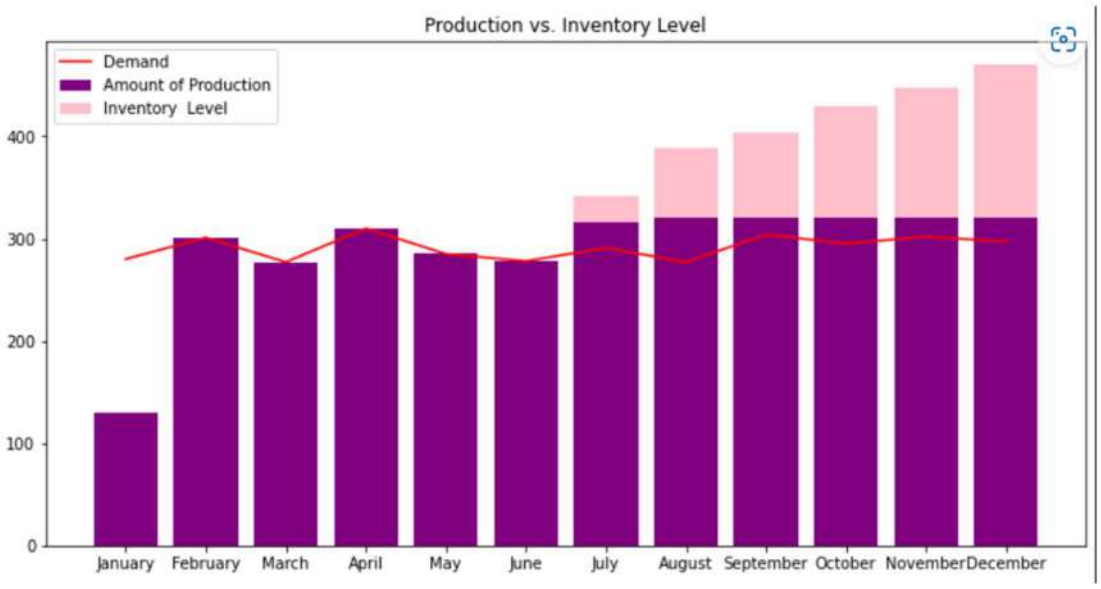

Table 2. Production vs. Inventory 1

Answer 2:

The objective function is maximizing profit. Profit is equal to the difference of the revenue of what is sold and the cost of what is produced. Thus, the objective function is being changed by multiplying with -1 since it is minimizing before and adding Sale Price * Demand of The Month. Because the objective of the model was changed, the value of it should not be compared to the value in the first question. They should be interpreted separately. ***In the optimal situation, the maximum profit is 1.626.600$.***

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours
0	January	230.0	100.0	32000.0	0.0
1	February	301.0	100.0	32000.0	0.0
2	March	277.0	100.0	32000.0	0.0
3	April	310.0	100.0	32000.0	0.0
4	May	285.0	100.0	32000.0	0.0
5	June	278.0	100.0	32000.0	0.0
6	July	291.0	100.0	32000.0	0.0
7	August	277.0	100.0	32000.0	0.0
8	September	304.0	100.0	32000.0	0.0
9	October	304.0	109.0	32000.0	0.0
10	November	320.0	127.0	32000.0	0.0
11	December	320.0	150.0	32000.0	0.0

Table 3. Data Table 2

The cost of holding inventory is 100, whereas the cost of working an hour more is 15 per an employee. In the optimal production plan, since the objective function is minimizing the cost, the model prefers to hold inventory to make over time production. As it can be seen from the table, the facility is not working over time.

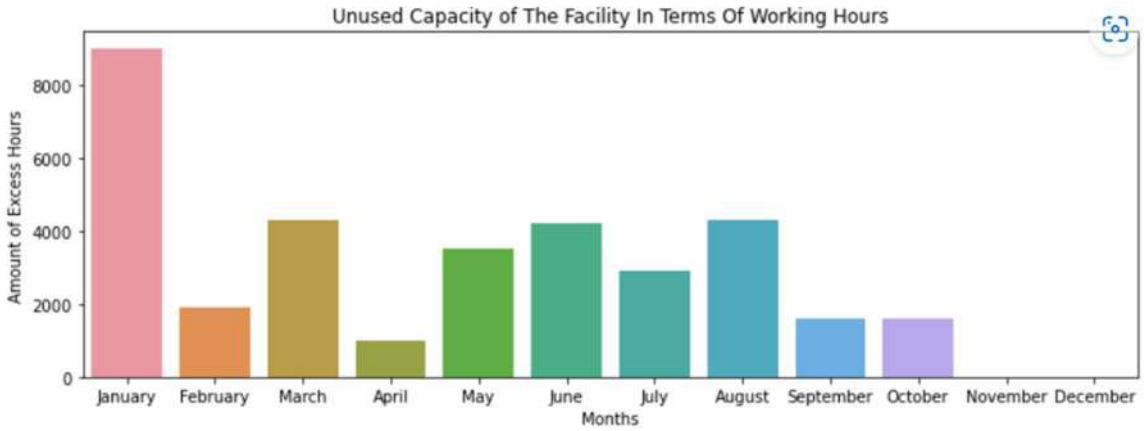

Table 4. Unused Capacities 1

Answer 3:

In the third question, the situation of promotion in April was examined. ***In the optimal situation, the maximum profit is 1.521.804$.*** As a result of the promotion and decrease in the sale price, the demand for the company's products is increasing in April. But, this increase results in forward buying from the next two months. In that situation, the demand of the promotion month is very important since it increases by 50% and may result in holding inventory or overtime working according to the costs. ***For example, the model prefers to overwork in April and satisfy the demand in that month instead of producing full capacity in the previous months and keeping to inventory.*** It is obvious that the cost of overtime working in April is smaller than keeping inventory more in February and March.

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours
0	January	320.0	190.0	32000.0	0.0
1	February	345.6	234.6	32000.0	2560.0
2	March	360.0	317.6	32000.0	4000.0
3	April	360.0	100.0	32000.0	4000.0
4	May	228.0	100.0	32000.0	0.0
5	June	222.4	100.0	32000.0	0.0
6	July	291.0	100.0	32000.0	0.0
7	August	277.0	100.0	32000.0	0.0
8	September	304.0	100.0	32000.0	0.0
9	October	304.0	109.0	32000.0	0.0
10	November	320.0	127.0	32000.0	0.0
11	December	320.0	150.0	32000.0	0.0

Table 5. Data Table 3

In addition to these, the decrease in the demand of the following 2 months of the promotion and the increase in the demand of the promotion month result in idle months after the months with overtime. ***It may result in an increase in employee motivation***. They are working really hard in some term of the year and then it is more like resting and less hustle and bustle.

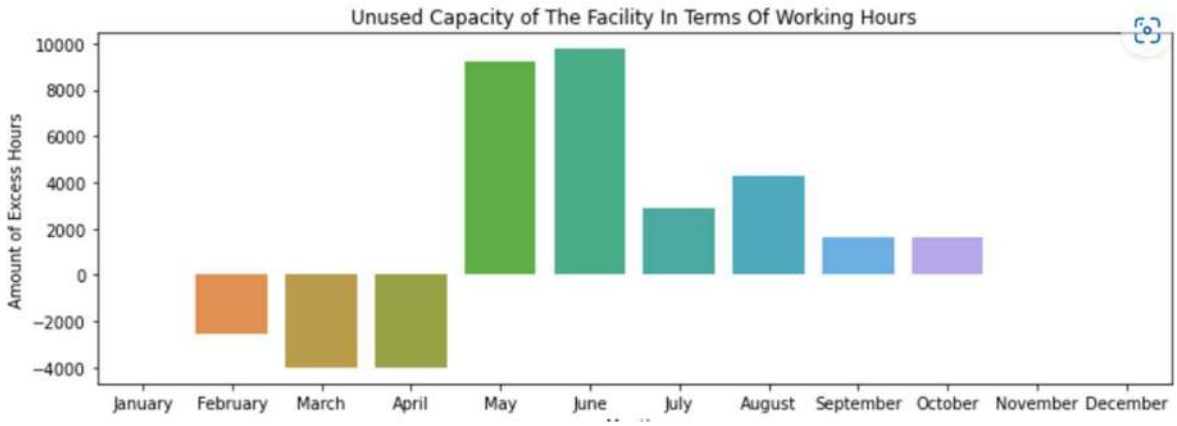

Table 6. Unused Capacities 2

Answer 4:

In the fourth question, the company prefers to make a promotion in June. Since the promotion is in the middle of the year, there is a time for producing slowly and keeping inventory in small amounts instead of making overtime work hours in around the promotion month. As can be seen from the table above, the facility is making over working only in the promotion month unlike the situation in the third question. ***Thus, it can be said that if there is time for production, producing full capacity, keeping the goods to the inventory, and only making over time in the promotion month are more profitable.***

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours
0	January	320.0	190.0	32000.0	0.0
1	February	320.0	209.0	32000.0	0.0
2	March	320.0	252.0	32000.0	0.0
3	April	320.0	262.0	32000.0	0.0
4	May	320.0	297.0	32000.0	0.0
5	June	333.6	100.0	32000.0	1360.0
6	July	232.8	100.0	32000.0	0.0
7	August	221.6	100.0	32000.0	0.0
8	September	304.0	100.0	32000.0	0.0
9	October	304.0	109.0	32000.0	0.0
10	November	320.0	127.0	32000.0	0.0
11	December	320.0	150.0	32000.0	0.0

Table 7. Data Table 4

As it can be seen from the graph, the inventory is gradually increasing till June while the facility is producing with full capacity by not making overtime. ***The maximum profit that can be earned under these constraints is 1.619.644$.***

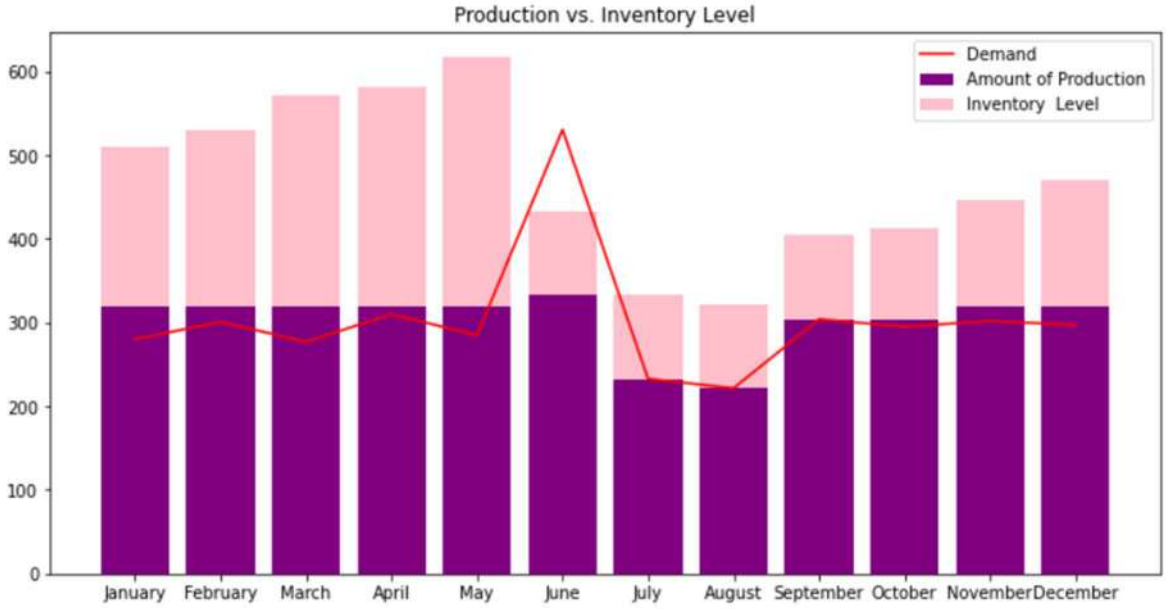

Table 8. Production vs. Inventory 1

Answer 5:

The maximum profit that the model gives under the situation given in the fifth question is 1.378.600$. It is the case where the company is not promoting while the competitor does. This decision affects the demand of the company to drop by 50 percent in the promotion month. ***The profit is less compared to other options, due to the drop in the demand in April, the facility becomes idle and an unused capacity occurs.***

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours
0	January	230.0	100.0	32000.0	0.0
1	February	301.0	100.0	32000.0	0.0
2	March	277.0	100.0	32000.0	0.0
3	April	155.0	100.0	32000.0	0.0
4	May	285.0	100.0	32000.0	0.0
5	June	278.0	100.0	32000.0	0.0
6	July	291.0	100.0	32000.0	0.0
7	August	277.0	100.0	32000.0	0.0
8	September	304.0	100.0	32000.0	0.0
9	October	304.0	109.0	32000.0	0.0
10	November	320.0	127.0	32000.0	0.0
11	December	320.0	150.0	32000.0	0.0

Table 9. Data Table 5

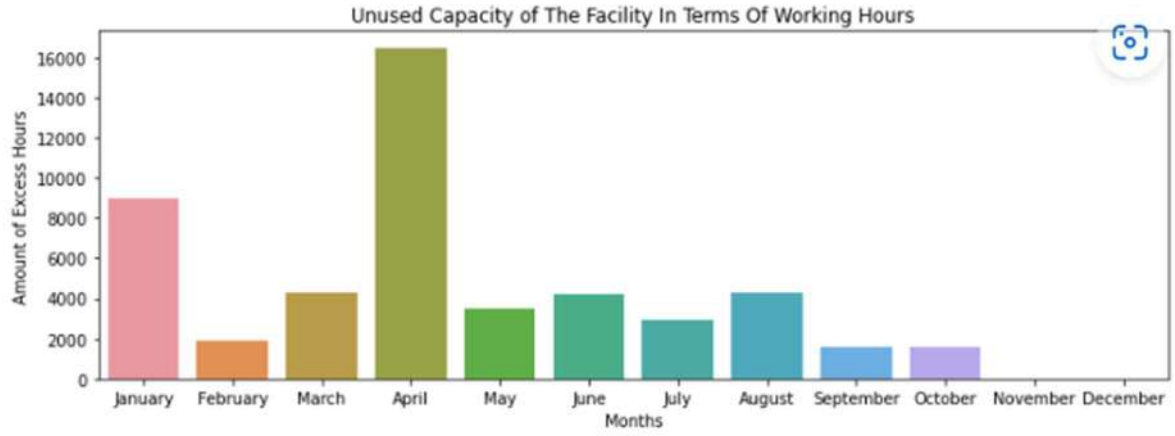

Table 10. Unused Capacities 3

Since the demand decreases due to not promoting while the competitor does, the facility stays idle. The facility does not make over time production, it even does not make a production. ***Thus, there is no demand or amount of production that can be considered to satisfy by subcontracting.*** The optimal production plan is the same as before and ***the maximum profit is the same also which is 1.378.600$.***

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours	Amount of Subcontacting
0	January	230.0	100.0	32000.0	0.0	0.0
1	February	301.0	100.0	32000.0	0.0	0.0
2	March	277.0	100.0	32000.0	0.0	0.0
3	April	155.0	100.0	32000.0	0.0	0.0
4	May	285.0	100.0	32000.0	0.0	0.0
5	June	278.0	100.0	32000.0	0.0	0.0
6	July	291.0	100.0	32000.0	0.0	0.0
7	August	277.0	100.0	32000.0	0.0	0.0
8	September	304.0	100.0	32000.0	0.0	0.0
9	October	304.0	109.0	32000.0	0.0	0.0
10	November	320.0	127.0	32000.0	0.0	0.0
11	December	320.0	150.0	32000.0	0.0	0.0

Table 11. Data Table 5 w/Subcontracting

Answer 6:

In this scenario, Q&H promotes in April, Unilock does no promotion throughout the year. ***The profit of the company in this scenario is estimated as 1.521.804$.***

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours	Amount of Excess Hours
0	January	320.0	190.0	32000.0	0.0	0.0
1	February	345.6	234.6	32000.0	2560.0	-2560.0
2	March	360.0	317.6	32000.0	4000.0	-4000.0
3	April	360.0	100.0	32000.0	4000.0	-4000.0
4	May	228.0	100.0	32000.0	0.0	9200.0
5	June	222.4	100.0	32000.0	0.0	9760.0
6	July	291.0	100.0	32000.0	0.0	2900.0
7	August	277.0	100.0	32000.0	0.0	4300.0
8	September	304.0	100.0	32000.0	0.0	1600.0
9	October	304.0	109.0	32000.0	0.0	1600.0
10	November	320.0	127.0	32000.0	0.0	0.0
11	December	320.0	150.0	32000.0	0.0	0.0

Table 12. Data Table 6

There is an inventory accumulation in the first 3 months due to the promotion and demand increase in April. To accumulate this inventory, the factory works overtime in February, March, and April.

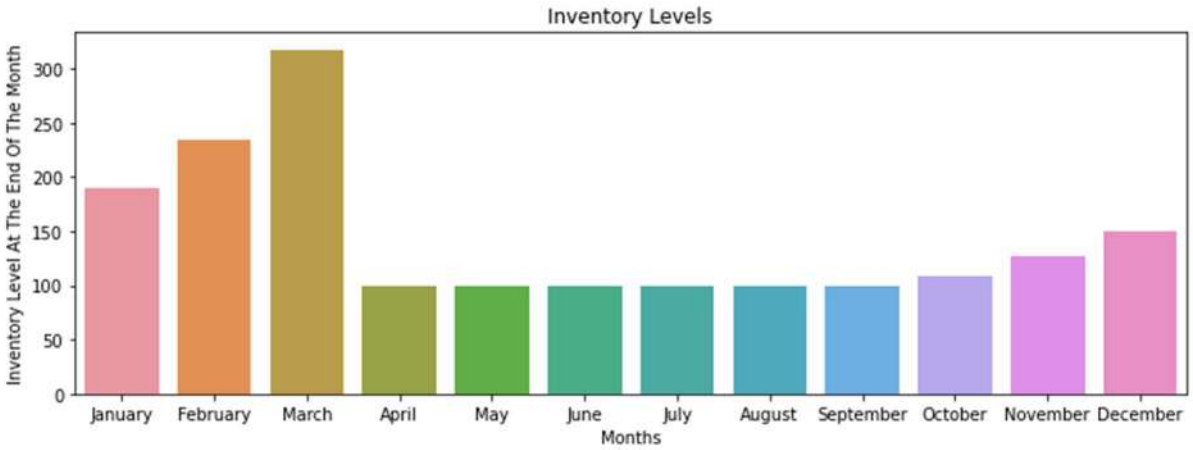

Table 13. Inventory Levels 1

After the promotion, there is significant unused capacity in the following 2 months which can be seen in **Table 14.**, which is followed by smaller unused capacity for the next 4 months after.

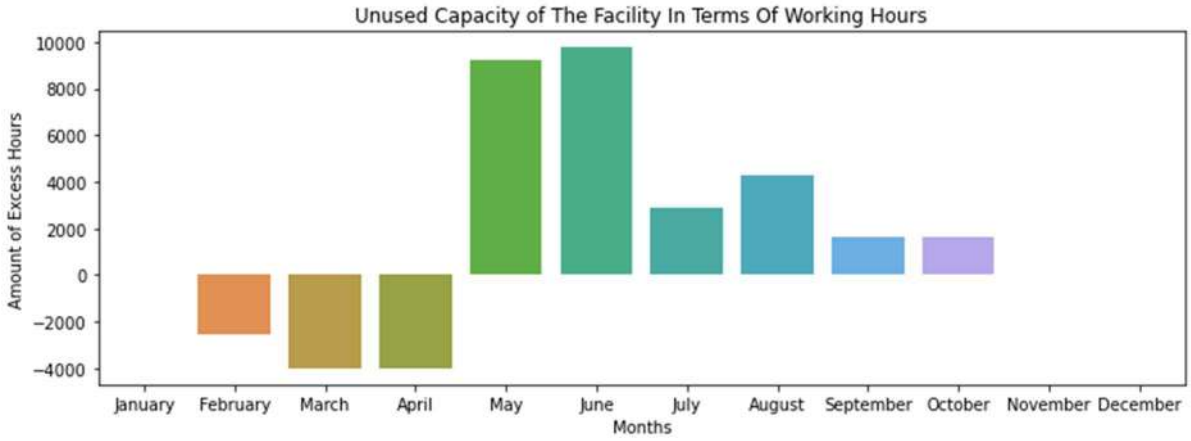

Table 14. Unused Capacities 4

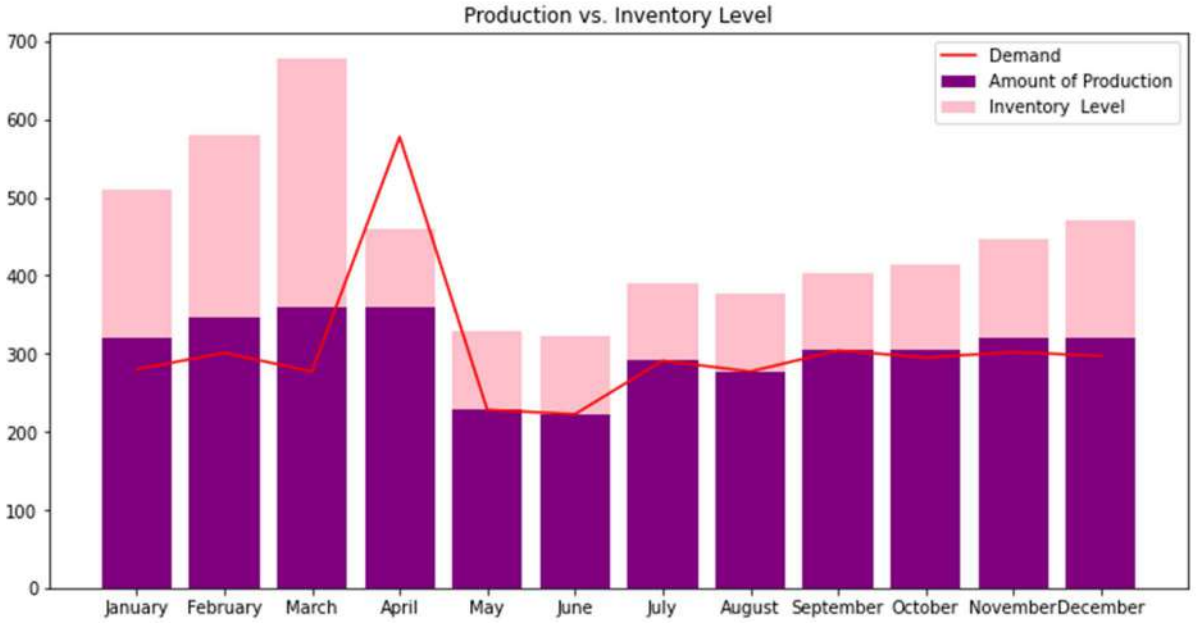

Table 15. Production vs. Inventory 2

If the subcontracting option was present, the company would subcontract instead of working overtime in February, March, and April, which can be observed in the table below. Although there is a subcontracting option still an inventory accumulation before April can be observed. This is the result of having lower inventory holding costs compared to subcontracting all the deficit in April.

To add more, while building up the inventory, the model uses no overtime since it will not only be more costly but also can increase the inventory holding cost which would exceed the subcontracting cost. As a result, the model found a solution in between. This would result

in **30.240$** of increase in the overall profit of the company which raises the total profits to **1.552.044$**.

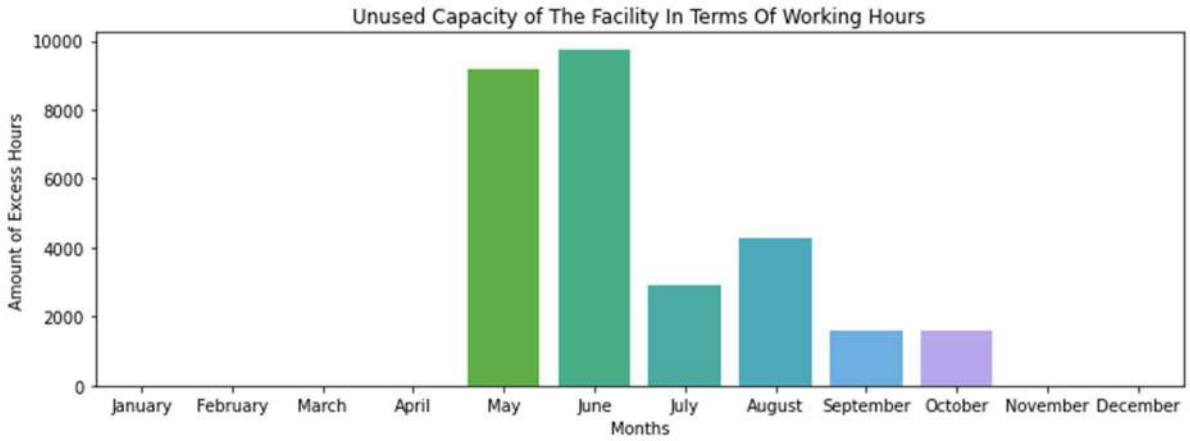

Table 14. Unused Capacities 6 w/Subcontracting

Answer 7:

In this scenario, both firms decide to do their promotions in April. As a result of this, forward buying from the next two months increases the overall demand in April thus decreasing the demand for the next two months. To satisfy this demand an inventory accumulation in the first 3 months can be observed from the inventory levels graph. Inventory accumulation for 3 months can also be interpreted as working only regular hours and holding inventory is less costly than holding less inventory and working overtime in a short period before the promotion begins.

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours	Amount of Excess Hours
0	January	298.75	168.75	32000.0	0.0	2125.0
1	February	320.00	187.75	32000.0	0.0	0.0
2	March	320.00	230.75	32000.0	0.0	0.0
3	April	320.00	100.00	32000.0	0.0	0.0
4	May	213.75	100.00	32000.0	0.0	10625.0
5	June	208.50	100.00	32000.0	0.0	11150.0
6	July	291.00	100.00	32000.0	0.0	2900.0
7	August	277.00	100.00	32000.0	0.0	4300.0
8	September	304.00	100.00	32000.0	0.0	1600.0
9	October	304.00	109.00	32000.0	0.0	1600.0
10	November	320.00	127.00	32000.0	0.0	0.0
11	December	320.00	150.00	32000.0	0.0	0.0

Table 15. Data Table 7

After the promotion inventory levels stay in minimum safety stock level for a long time and the demand is satisfied by the monthly production only for the next 6 months following April.

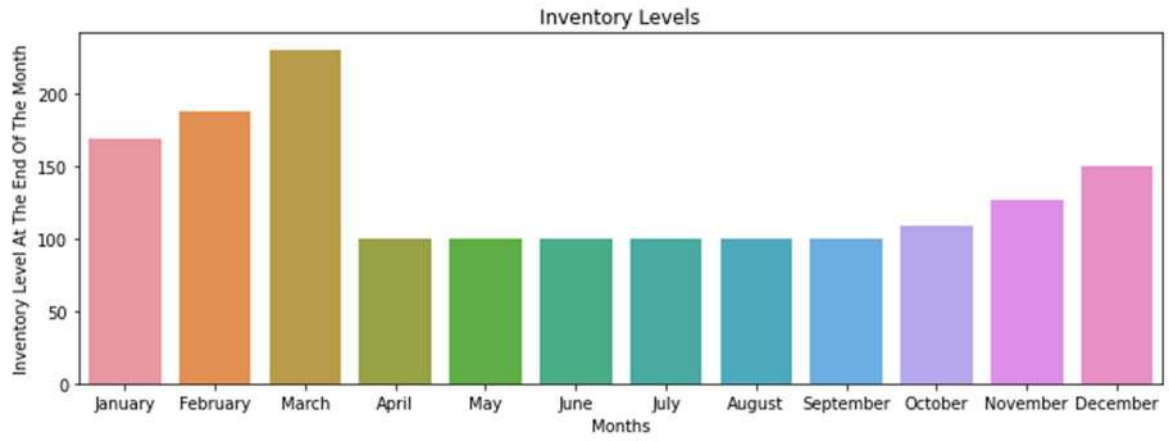

Table 16. Inventory Levels 2

In these 6 months there is a sharp increase in unused capacity of working hours, which reach a peak level in June. The model prefers not using this capacity instead of accumulating inventory for the upcoming demand forecasts, since it is more profitable. ***As a result of this scenario the profit of the firm is $1.480.680.***

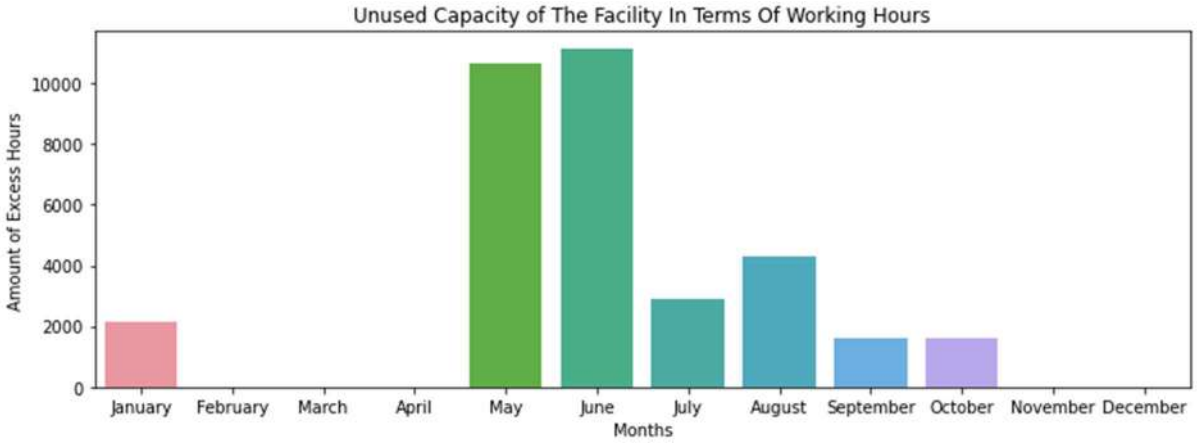

Table 17. Unused Capacities 7

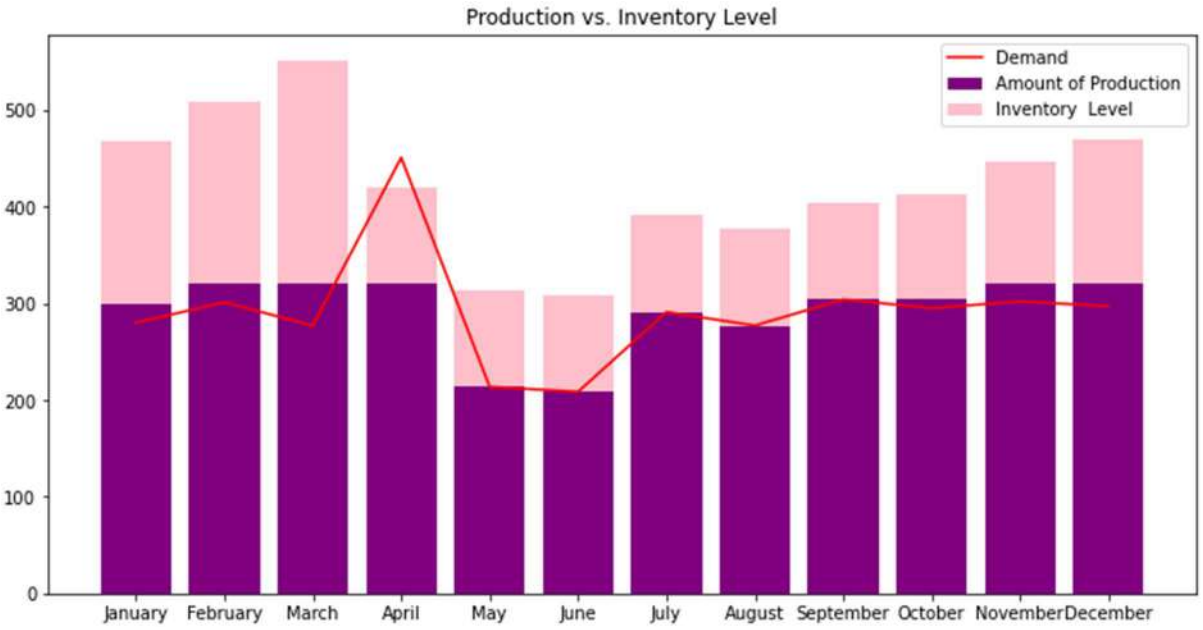

Table 18. Production vs. Inventory 3

If a third-party subcontractor is available for this scenario, the profit of the firm stays the same since the model does not prefer subcontracting in any month over the 1-year horizon.

Answer 8:

In this scenario, both firms decide to do their promotions in June. As a result of this, forward buying from the next two months increases the overall demand in June thus decreasing the demand for the next two months.

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours	Amount of Excess Hours
0	January	230.00	100.0	32000.0	0.0	9000.0
1	February	313.00	112.0	32000.0	0.0	700.0
2	March	320.00	155.0	32000.0	0.0	0.0
3	April	320.00	165.0	32000.0	0.0	0.0
4	May	320.00	200.0	32000.0	0.0	0.0
5	June	320.00	100.0	32000.0	0.0	0.0
6	July	218.25	100.0	32000.0	0.0	10175.0
7	August	207.75	100.0	32000.0	0.0	11225.0
8	September	304.00	100.0	32000.0	0.0	1600.0
9	October	304.00	109.0	32000.0	0.0	1600.0
10	November	320.00	127.0	32000.0	0.0	0.0
11	December	320.00	150.0	32000.0	0.0	0.0

Table 19. Data Table 8

To satisfy this demand an inventory accumulation starting from February until the end of May can be observed from **Table 20.** Inventory accumulation for 4 months can also be interpreted

as working only regular hours and holding inventory is less costly than holding less inventory and working overtime in a short period before the promotion begins.

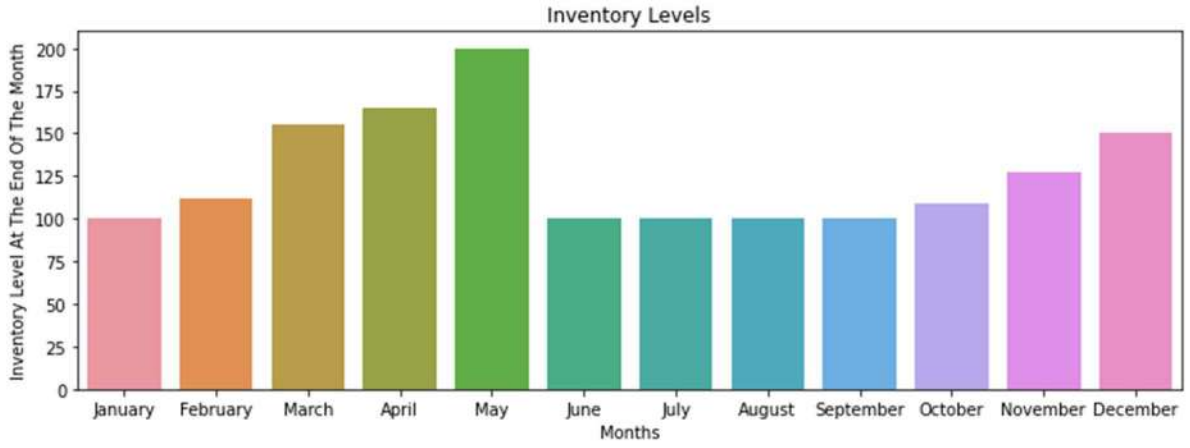

Table 20. Inventory Levels 3

There are 3 months that leave a significant amount of capacity unused; January, July, August. There is unused capacity in February, September, And October which is very low. Since the promotion is done in June, unused capacity in July and August can be explained as a result of the demand decrease in these months following a forward buying which happened in the promotion month June. Model lowers the production in the following months to escape from high inventory holding costs. On the other side, unused capacity in January and February can be explained by high inventory holding costs. Model prefers using existing inventory to lower the inventory levels.

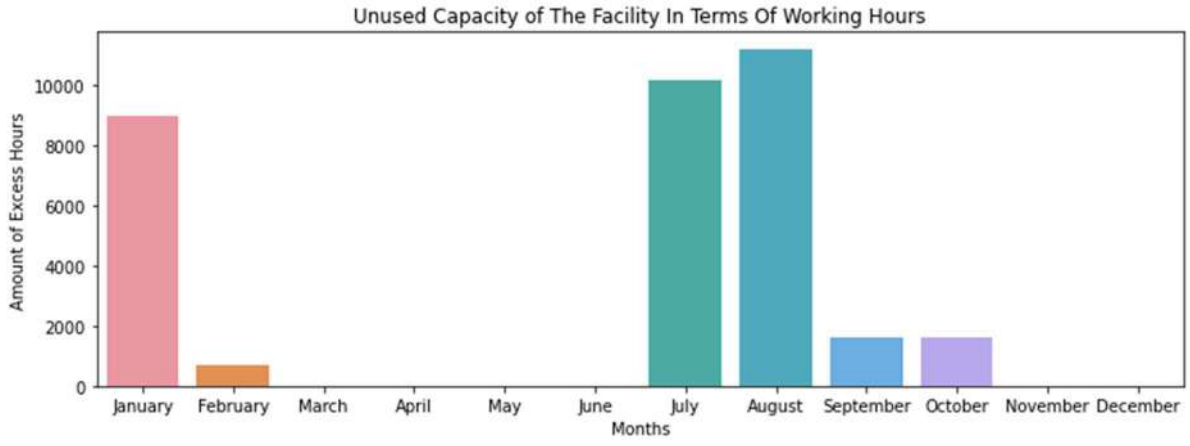

Table 21. Unused Capacities 8

As can be seen from **Table 22.**, the inventory is used only in January in June (promotion month).

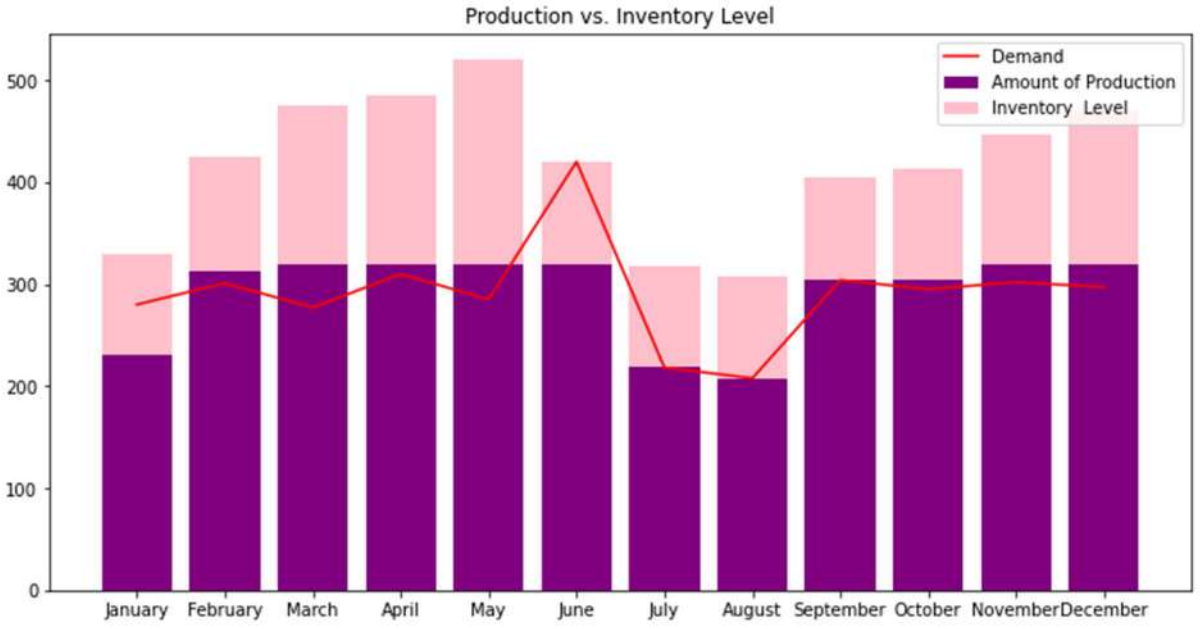

Table 22. Production vs. Inventory 4

Considering a third-party subcontractor for this scenario, the model chooses not to use any subcontracting under these circumstances.

Answer 9:

In this scenario, Q&H accumulates inventory in the first 4 months to meet the demand in the promotion month April. Except the factory works overtime despite its higher cost compared to regular hours, which the model prefers since it makes the overall cost lower. ***The profit of the firm in this scenario is 1.343.884$.***

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours	Amount of Excess Hours
0	January	320.0	190.0	32000.0	0.0	0.0
1	February	345.6	234.6	32000.0	2560.0	-2560.0
2	March	360.0	317.6	32000.0	4000.0	-4000.0
3	April	360.0	100.0	32000.0	4000.0	-4000.0
4	May	228.0	100.0	32000.0	0.0	9200.0
5	June	111.2	100.0	32000.0	0.0	20880.0
6	July	291.0	100.0	32000.0	0.0	2900.0
7	August	277.0	100.0	32000.0	0.0	4300.0
8	September	304.0	100.0	32000.0	0.0	1600.0
9	October	304.0	109.0	32000.0	0.0	1600.0
10	November	320.0	127.0	32000.0	0.0	0.0
11	December	320.0	150.0	32000.0	0.0	0.0

Table 23. Data Table 9

There is a sharp decrease in demand following the promotion month since both Q&H's promotion in April and UNILOCK's promotion in June caused a decrease in demand.

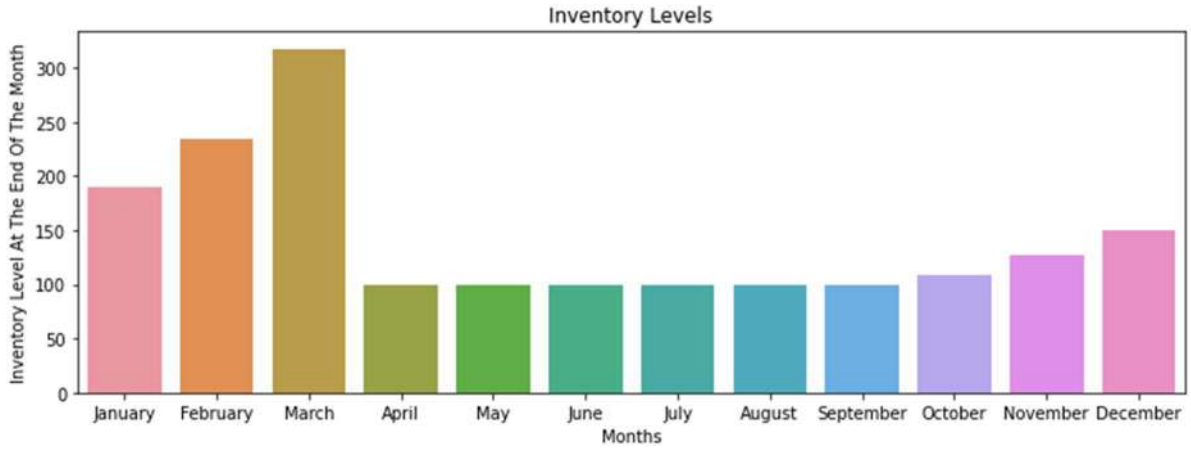

Table 23. Inventory Levels 4

In the 6 consecutive months following the promotion in April there is unused capacity which makes a peak in June. The model prefers not using the capacity despite the firm having to pay the workers full time salary. The reason for this **model escapes from production costs and accumulated inventory holding costs**.

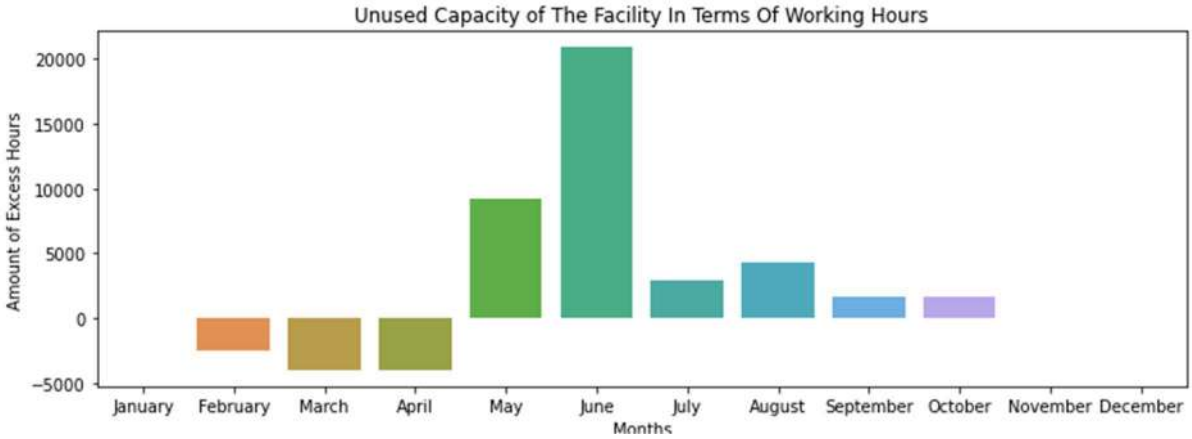

Table 24. Unused Capacities 9

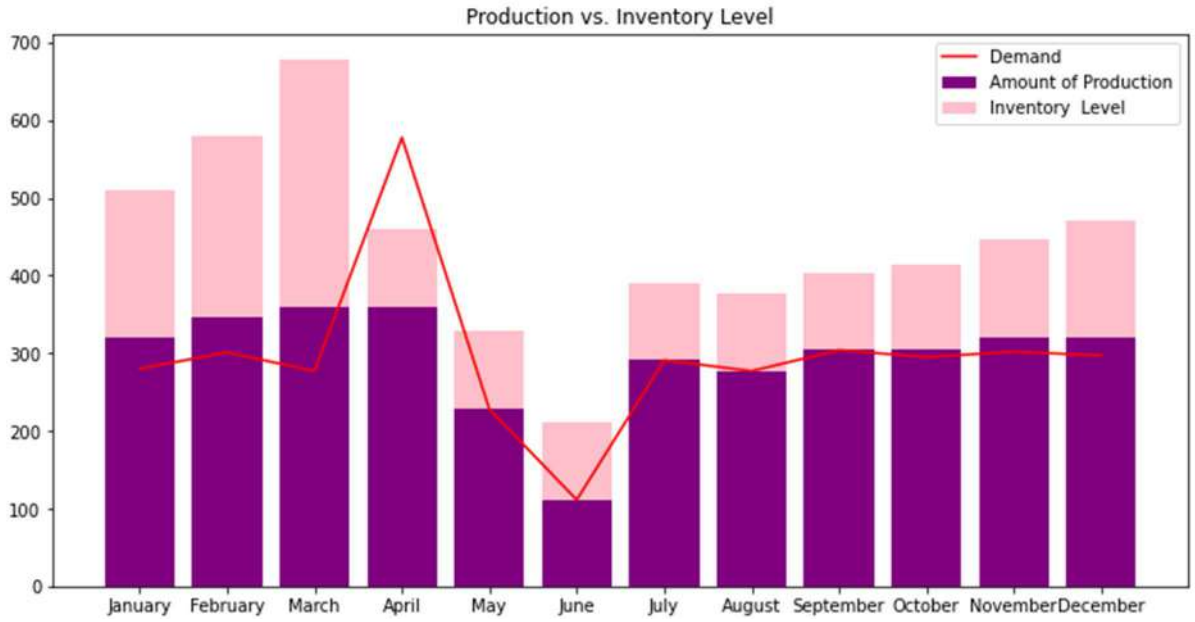

Table 25. Production vs. Inventory 5

If subcontracting was available, the firm would use subcontracting in April to meet the demand instead of working overtime (can be seen in the below **Table 26.**) in February, March, and April. As a result of this strategy, the costs would be lower and the profits would get 30.240$ higher to 1.374.124$.

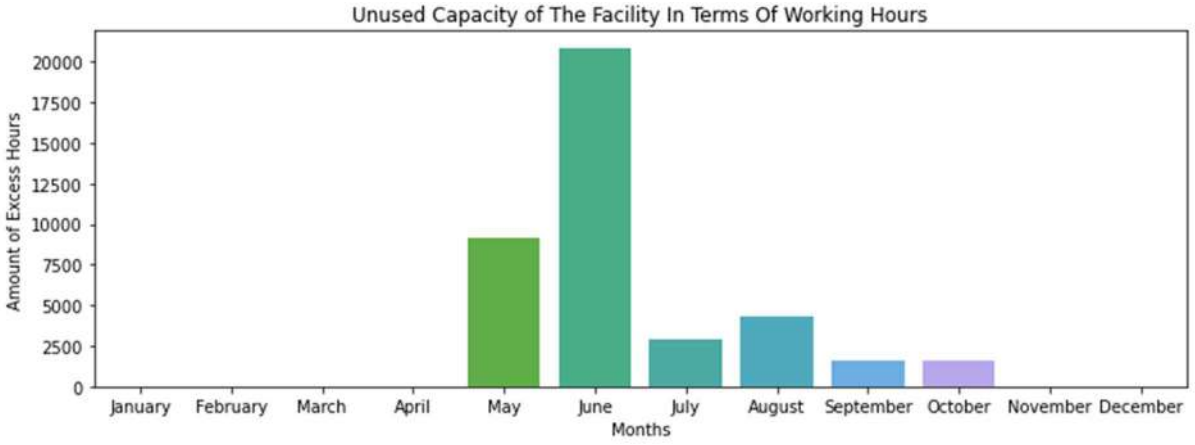

Table 26. Unused Capacities 9 w/Subcontracting

Answer 10:

In this scenario, Q&H promotes in June whereas the competitor UNILOCK makes a promotion in April.

	Months	Amount of Production	Inventory Level At The End Of The Month	Regular Time Workhours	Overtime Workhours	Amount of Excess Hours
0	January	230.0	100.0	32000.0	0.0	9000.0
1	February	301.0	100.0	32000.0	0.0	1900.0
2	March	287.6	110.6	32000.0	0.0	3240.0
3	April	320.0	275.6	32000.0	0.0	0.0
4	May	320.0	310.6	32000.0	0.0	0.0
5	June	320.0	100.0	32000.0	0.0	0.0
6	July	232.8	100.0	32000.0	0.0	8720.0
7	August	221.6	100.0	32000.0	0.0	9840.0
8	September	304.0	100.0	32000.0	0.0	1600.0
9	October	304.0	109.0	32000.0	0.0	1600.0
10	November	320.0	127.0	32000.0	0.0	0.0
11	December	320.0	150.0	32000.0	0.0	0.0

Table 27. Data Table 10

This results in a demand decrease in April for Q&H, however, to satisfy the demand in June, the inventory accumulation begins in April and continues until the promotion is made.

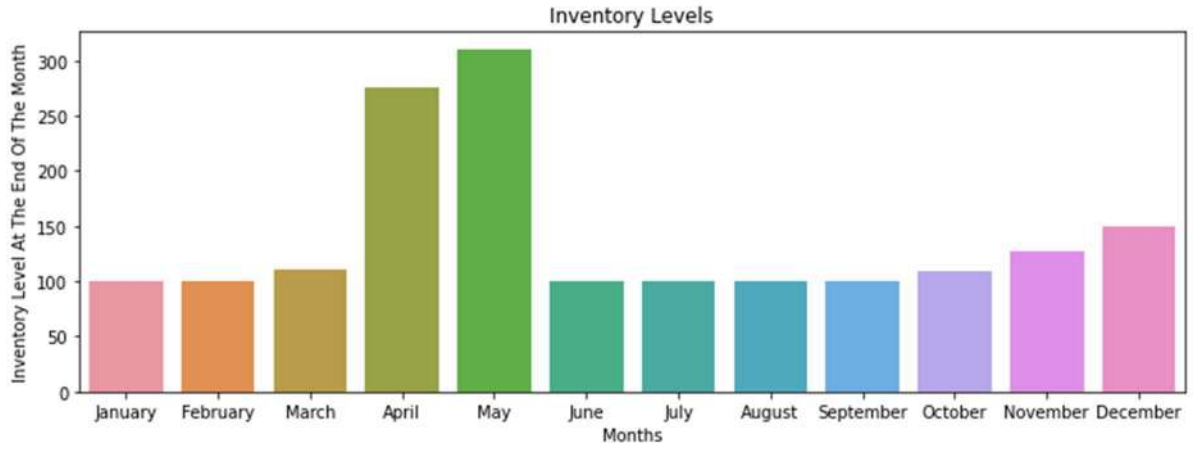

Table 28. Inventory Levels 5

As a result of promotion, forward buying happens in June, which lowers the demand of July and August by 20%, which makes the model leave unused capacity in those months under the present cost circumstances. On one hand; April, May, and June are the busiest months for the firm, on the other hand there is significant unused capacity in January, July, and August.

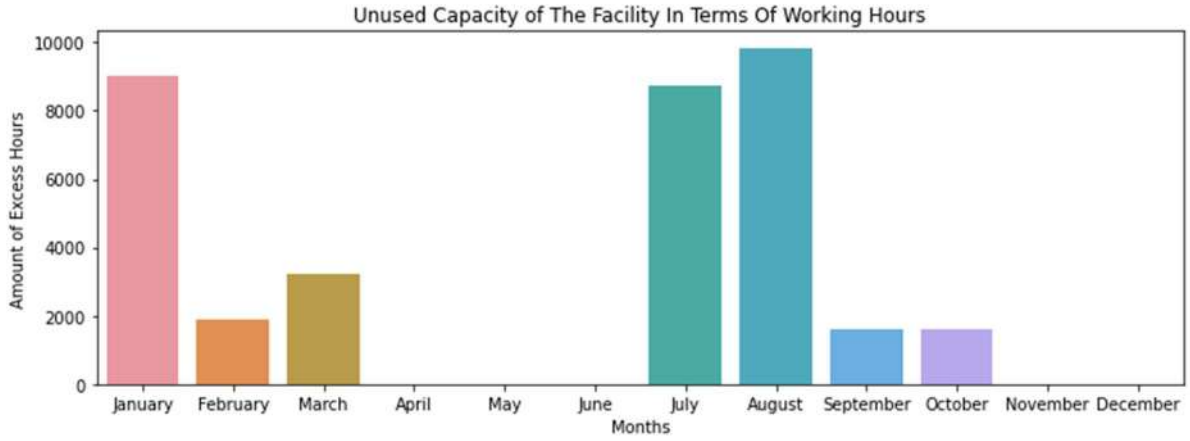

Table 29. Unused Capacities 10

As can be seen in the below table, the increase in demand in the promotion month is satisfied both by the monthly production that particular month and the inventory accumulated in the previous months.

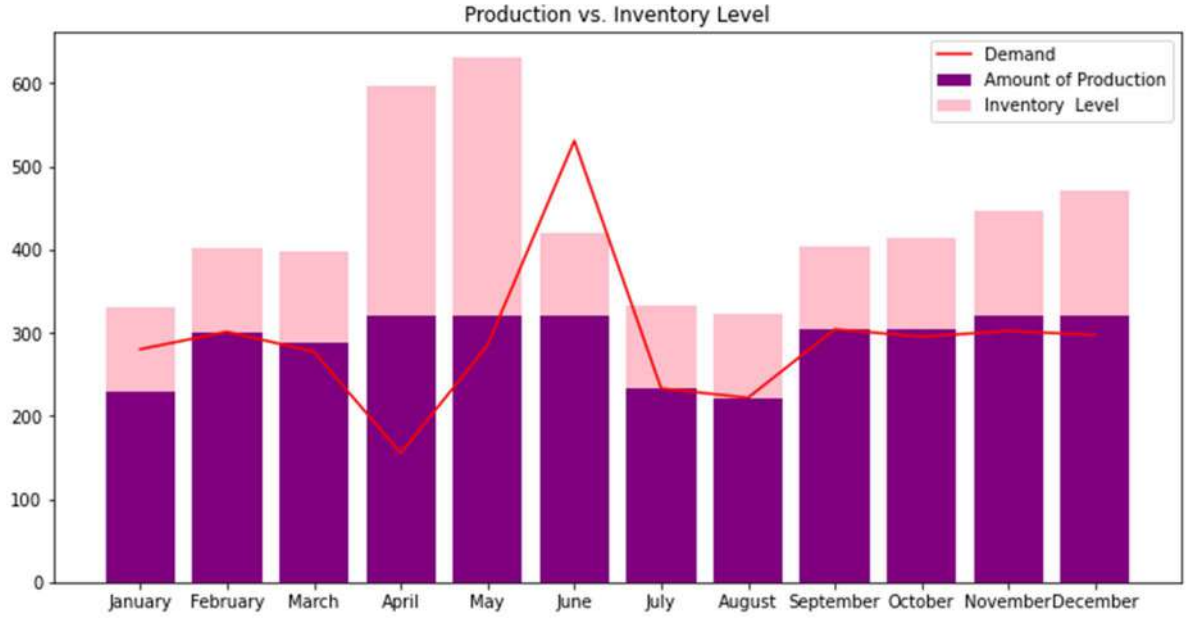

Table 30. Production vs. Inventory Level 6

As subcontracting is made available for this scenario, the model chooses not to use it anywhere, since the cost of the alternatives is less than satisfying the demand by subcontracting.

Answer 11:

According to **Table 31.**, Q&H maximizes profits in the scenario neither Q&H nor UNILOCK makes a promotion. For instance, if we compare the scenarios in which Q&H does not make a promotion, promotes in June, and promotes in April respectively while UNILOCK never promotes, it is obvious that **no promotion** scenario is the best with its closest follower

promotion made in June. **This can be explained by the capacity and the costs, which are not suitable and profitable for a promotion in given circumstances.** To satisfy the demand in the promotion months Q&H makes costly maneuvers that eventually lowers the profit.

Q&H	UNILOCK	Profit w\o Subcontracting	Profit w/ Subcontracting	Difference
-	-	$1.626.600	$1.626.600	$0
June	-	$1.619.644	$1.622.364	$2.720
April	-	$1.521.804	$1.552.044	$30.240
-	April	$1.378.600	$1.378.600	$0
April	April	$1.480.680	$1.480.680	$0
June	June	$1.494.200	$1.494.200	$0
April	June	$1.343.884	$1.374.124	$30.240
June	April	$1.423.364	$1.423.364	$0
-	June	$1.404.200	$1.404.200	$0

Table 31. Comparison Table with Overall Profits

Answer 12:

The best decision for Q&H no matter what its competitor UNILOCK does can be found by a comparison with the following method. Average profits and minimum profits for Q&H promotion scenarios are calculated for distinct UNILOCK promotions. For instance, average profit for promoting in June regardless of UNILOCK's decision is calculated as **[(June, -) + (June, April) + (June, June)] / 3 = Average profit of promoting in June**. After these calculations are done for all the scenarios which can be seen in **Table 32.**. To conclude, regardless of UNILOCK's decision it is best for Q&H to prepare a production plan aiming to **make their promotion in June**, which gives the highest minimum profits.

Q&H	UNILOCK	Average Profit	Minimum Profit
-	All Scenarios	$1.469.800	$1.378.600
April	All Scenarios	$1.448.789	$1.343.884
June	All Scenarios	$1.512.403	$1.423.364

Table 32. Decision Comparison

Answer 13:

The scenario which enables a third-party subcontractor is discussed in the answers and interpretations of each question.