IE203 – HW2

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Introduction

In this assignment, we are asked to calculate the steady state probability of an ergodic markov chain, first by creating a transition probability matrix then using this matrix in two different methods, matrix multiplication method and monte carlo simulation method, with three different N values, 5-25-50. I used python programming language and related libraries for this assignment

Results For Matrix Multiplication Method

In this method, I multiplied the matrix with itself until the rows converge. Without an absorbing state, I observed a steady state distribution for all N values. However, when an absorbing state added (or one of the states is converted into an absorbing state) we cannot observe a steady state distribution since one state absorbs all the probability distribution.

Results For Monte Carlo Simulation Method

In this method, I used M=200000 to observe a sequence this long. I randomly picked an initial state, I randomly picked a value from an interval and decided on my next state. Finally I calculated the number of observations at each state. I observed similar result to matrix multiplication method. Without an absorbing state there is a steady state distribution, however, when there is one absorbing state it absorbs the probability distribution.

Generating Transition Probability Matrix

```
import numpy as np
N=50 #number of states
TPM=np.identity(N) #TPM=transition probability matrix
TPM = TPM + np.random.uniform(low=0, high=1, size=(N, N)) #I assign random numbers between 0-1
TPM = TPM / TPM.sum(axis=1, keepdims=1) #then I divide them to sum, as a result I generate my TPM
print(TPM)
```

W/o Absorbing States

For n=5

```
[[0.32678891 0.10950343 0.20340509 0.31315522 0.04714734]
 [0.00261485 0.48451434 0.31500118 0.14273364 0.055136  ]
 [0.19063385 0.06670372 0.50349057 0.14327372 0.09589816]
 [0.22988232 0.01122448 0.23924758 0.45039098 0.06925464]
 [0.19028037 0.16362397 0.16225406 0.12081108 0.36303053]]
```

For n=25

```
[[1.05139205e-01 2.70653899e-02 3.52725749e-02 2.29316658e-02
  1.09485079e-02 1.08943616e-03 4.47914611e-02 4.19929689e-03
  6.61757345e-02 4.08598938e-02 7.32572509e-02 7.84667239e-02
  1.16835394e-02 2.61669300e-02 9.36375197e-04 3.30362596e-02
  6.68610246e-02 7.89211961e-02 6.37024094e-02 5.51229849e-02
  2.83753954e-02 1.65037330e-02 5.59889741e-02 3.44717551e-02
  1.80322823e-02]
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  6.26790901e-02 4.25658105e-03 6.26967455e-02 5.32474555e-02
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  4.15784089e-02]
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 7.57881929e-02 1.68979736e-02 2.16122263e-02 2.94306479e-02
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```

For n=50

```
[[0.03610441 0.03090751 0.02329107 ... 0.00432984 0.00492102 0.01794566]
 [0.00759838 0.04706426 0.01517043 ... 0.02701856 0.01214291 0.01319289]
 [0.01104791 0.00307493 0.042244   ... 0.00492918 0.02188494 0.00238549]
 ...
 [0.00879804 0.00407466 0.02745029 ... 0.05743711 0.01348208 0.00594593]
 [0.01365519 0.03160348 0.01021212 ... 0.01463398 0.07106463 0.04067589]
 [0.00825341 0.0083719  0.00841524 ... 0.01417751 0.00399198 0.07832597]]
```

W/ One Absorbing State

For n = 5

```
[[0.43154594 0.17185313 0.15452269 0.05459752 0.18748073]
 [0.1247785  0.43591708 0.24164246 0.03483906 0.16282289]
 [0.16742889 0.00398551 0.45450145 0.16145904 0.21262511]
 [0.11117763 0.26609991 0.02899044 0.47638518 0.11734684]
 [0.         0.         0.         0.         1.        ]]
```

For n=25

```
[[1.23960083e-01 4.64253281e-02 7.46983827e-02 1.46536389e-02
  6.05924717e-02 6.15271332e-02 5.76709054e-02 3.14674587e-02
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  4.58502052e-02 5.72494855e-02 7.46042633e-03 4.03783668e-03
  9.65330685e-03 5.06412076e-02 1.02003622e-02 6.47672845e-02
  2.71562081e-02]
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1.14193050e-02 3.21616634e-03 5.54896007e-02 6.75482037e-02
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4.37256293e-02]
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1.54559101e-02 1.43845940e-02 6.16141497e-02 6.29290997e-04

4.27506054e-02 5.83549667e-02 4.61501367e-02 2.07391522e-02
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7.83398826e-05 5.71425542e-02 2.73387066e-02 5.74729108e-02
4.00152582e-02 2.42506969e-02 1.26464751e-02 7.11383989e-02
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5.24473012e-02]
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2.31562936e-02 2.48654839e-02 4.74198333e-03 3.80094276e-02
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4.47746738e-02 3.73734310e-02 4.33750443e-02 9.13378619e-03
5.32330755e-02 1.23267957e-01 5.41495682e-02 6.43462719e-02
5.66339627e-02]
[1.20558711e-02 7.28028387e-02 2.71279388e-02 6.62609173e-02
1.04944587e-02 5.56783917e-02 2.27335620e-02 2.73039825e-02
6.87999987e-02 3.23230496e-02 1.73981976e-02 3.32309511e-02
7.38127678e-02 9.91759148e-03 8.45603315e-02 2.18501381e-02
1.33458421e-02 4.10521826e-02 2.28712153e-02 9.77042994e-03
6.47195033e-02 3.25006617e-02 1.20883209e-01 2.52614022e-02
3.32445674e-02]
[6.29444606e-02 1.40935876e-02 6.94991814e-03 3.68623277e-02
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2.46484186e-02 1.64639805e-02 6.40931741e-02 4.20510344e-02
4.37814407e-02 6.95887047e-02 3.41044245e-02 3.06760456e-02
1.90081147e-02 2.95011416e-02 2.75780664e-02 3.41519551e-02
3.99145463e-02 5.50564661e-02 5.14130474e-02 8.62208339e-02
1.10794931e-02]
[0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
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0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
1.00000000e+00]]

For n=50

[[5.37563485e-02 1.34084037e-02 2.68995314e-02 ... 2.79283599e-02

```
 3.04200058e-02 2.41846857e-02]
[6.49408805e-03 7.34180980e-02 1.02756077e-02 ... 1.52203004e-02
 4.17617786e-02 2.42152170e-02]
[2.10334491e-02 9.38476030e-03 6.85190246e-02 ... 8.12442055e-03
 1.88538445e-02 2.68432133e-02]
...
[2.38463273e-02 9.92747319e-04 6.61470440e-03 ... 5.33174750e-02
 2.70976919e-02 3.58526868e-03]
[1.42481574e-02 3.64779082e-02 2.88601822e-02 ... 2.43768714e-02
 7.55190483e-02 1.29425251e-02]
[0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
 0.00000000e+00 1.00000000e+00]]
```

Matrix Multiplication Method

```
import collections
import random
import numpy as np

e=0.0005
i=0
MMTPM=TPM

while(1):
    ++i
    MMTPM=np.dot(MMTPM,MMTPM)
    Average=np.mean(MMTPM, axis=0)
    rdnode=random.randint(0,N-1)
    sub=Average-MMTPM[rdnode]
    sub=np.absolute(sub)


    if(np.mean(sub)<e):
        break

print(MMTPM)
```

W/o Absorbing States

For n=5

```
[[0.20570281 0.12210768 0.31707524 0.25399556 0.10111871]
 [0.20489157 0.12323393 0.31766175 0.25298014 0.10123261]
 [0.20553431 0.12235672 0.31717043 0.25377972 0.10115883]
 [0.20590515 0.12183455 0.31691459 0.2542461  0.10109961]
 [0.20512776 0.1229235  0.31740785 0.25333761 0.10120328]]
```

For n=25

```
[[0.03785859 0.03900315 0.04352345 0.03915799 0.03740575 0.03858831
  0.03927221 0.03267949 0.04178363 0.04346462 0.04905545 0.03520522
  0.04143118 0.04161392 0.04669278 0.040413   0.03814362 0.03954621
  0.04487803 0.04438127 0.0347447  0.04227989 0.0307213  0.0417275
  0.03642875]
 [0.03786066 0.03889272 0.04354965 0.03918291 0.03744657 0.03862108
  0.03928467 0.03271503 0.04181524 0.04347796 0.04907693 0.03515154
  0.0414267  0.04159486 0.04676422 0.0403391  0.03790708 0.03956382
  0.04496852 0.04438196 0.03489641 0.04224171 0.03065269 0.04164474
  0.03654326]
 [0.03776623 0.03871907 0.04355    0.03921669 0.0374949  0.03876111
  0.03931104 0.03260309 0.04174848 0.04366998 0.04895234 0.03502947
  0.0415849  0.04162482 0.04675728 0.04033738 0.03771138 0.0395486
  0.04492632 0.04436014 0.03523121 0.04209998 0.03074646 0.04148254
  0.03676662]
 [0.03779587 0.03886083 0.04343041 0.03954783 0.03746084 0.0387398
  0.03923217 0.03263988 0.04167094 0.04323547 0.04896249 0.03487425
  0.04180038 0.04143907 0.04684554 0.04013414 0.03771658 0.03982401
  0.04494978 0.04468878 0.03522706 0.04182403 0.03060297 0.04170255
  0.03679431]
 [0.03780893 0.03872084 0.0436766  0.03902347 0.03750854 0.03871197
  0.03922651 0.03268711 0.04184843 0.04374308 0.04902212 0.03503748
  0.04163933 0.04166995 0.04677982 0.04030929 0.03785125 0.03952117
  0.04485428 0.04429502 0.03488714 0.04238919 0.03085456 0.04148453
```

0.0364494]
[0.03769711 0.0386926 0.04350756 0.03940431 0.03756317 0.03884427
0.03927157 0.03238875 0.0417206 0.04357122 0.0488388 0.03487578
0.04167783 0.04160883 0.04663954 0.04033761 0.03777361 0.03961211
0.04494305 0.0444299 0.03539832 0.04194651 0.03084507 0.0415446
0.03686729]
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0.03924271 0.03262613 0.04178264 0.04366666 0.04901958 0.03505412
0.04150529 0.0416932 0.04667405 0.04038782 0.03790251 0.03947606
0.04492327 0.04427819 0.03505794 0.04223996 0.03080589 0.04155837
0.03663702]
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0.03923001 0.03262171 0.0418692 0.04410361 0.04895111 0.03505489
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0.03658458]
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0.04142509 0.04166923 0.04669751 0.04034888 0.03791293 0.03948301
0.0450282 0.0443453 0.03501694 0.04223767 0.03064014 0.04162513
0.03662581]
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0.04192719 0.04175315 0.04676819 0.04017037 0.03760141 0.03970174
0.04480969 0.04421306 0.03523105 0.04200786 0.03081174 0.0414257
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0.04487575 0.04421122 0.03513136 0.04224998 0.03087857 0.04129717
0.03662055]
[0.03777498 0.03872393 0.04361328 0.03918303 0.03754754 0.03872677
0.03920089 0.0325809 0.04180283 0.04367413 0.04898137 0.03496696
0.0415788 0.0416849 0.04668096 0.04035204 0.03788241 0.03949782
0.04494213 0.04433441 0.0350632 0.04225608 0.03081823 0.04150298
0.03662945]
[0.03779572 0.03885791 0.0436058 0.03917382 0.03740654 0.03855759
0.03920458 0.03261303 0.04173928 0.04367375 0.04911463 0.03504931
0.04151757 0.04172944 0.04672888 0.04040038 0.03800836 0.03950438
0.04490369 0.04437877 0.0348676 0.0422908 0.03062624 0.04167547
0.03657647]
[0.0377294 0.03862851 0.04347795 0.03947579 0.03747318 0.03882492
0.03924536 0.03251001 0.04160751 0.04362872 0.04895085 0.03498205
0.04163222 0.04161372 0.04667364 0.04024107 0.03778073 0.03954465
0.04504928 0.04436828 0.03545369 0.04183952 0.03074304 0.04156709
0.03695885]]

For n=50

[[0.01936413 0.01969083 0.01678717 ... 0.01795871 0.01619883 0.01955685]
[0.01936843 0.01970334 0.01679634 ... 0.01794272 0.01621626 0.0195553]
[0.01935621 0.01970284 0.01680199 ... 0.01798813 0.01621209 0.01956044]
...
[0.01937855 0.01969859 0.01679344 ... 0.01796178 0.01618585 0.01952525]
[0.01935059 0.01969106 0.01679679 ... 0.01795693 0.01618944 0.01958814]
[0.01936554 0.01969479 0.01678679 ... 0.01800031 0.01617599 0.01955524]]

W/ One Absorbing State

For n=5

```
[[5.38175712e-04 5.26228267e-04 5.99267380e-04 4.13848259e-04
  9.97922480e-01]
 [5.56538269e-04 5.44183177e-04 6.19714386e-04 4.27968763e-04
  9.97851595e-01]
 [5.27141444e-04 5.15438959e-04 5.86980545e-04 4.05363089e-04
  9.97965076e-01]
 [6.39390985e-04 6.25196571e-04 7.11972228e-04 4.91681137e-04
  9.97531759e-01]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  1.00000000e+00]]
```

For n=25

```
[[0.00236919 0.00193294 0.00195834 0.00248489 0.00201274 0.00261426
  0.00227455 0.00216603 0.00229841 0.00233129 0.00268766 0.00253085
  0.00238295 0.00294383 0.00231567 0.0021711  0.00190433 0.00254232
  0.0024816  0.00171462 0.00227878 0.00247771 0.00258009 0.00233745
  0.9442084 ]
 [0.0023742  0.00193702 0.00196248 0.00249014 0.002017   0.00261978
  0.00227936 0.00217061 0.00230326 0.00233622 0.00269334 0.0025362
  0.00238799 0.00295005 0.00232056 0.00217569 0.00190836 0.00254769
  0.00248685 0.00171824 0.0022836  0.00248295 0.00258554 0.00234239
  0.9440905 ]
 [0.00244756 0.00199687 0.00202311 0.00256708 0.00207932 0.00270073
  0.00234979 0.00223768 0.00237443 0.0024084  0.00277656 0.00261456
  0.00246177 0.0030412  0.00239226 0.00224291 0.00196732 0.00262641
  0.00256369 0.00177133 0.00235416 0.00255967 0.00266543 0.00241476
  0.94236302]
 [0.00253281 0.00206643 0.00209359 0.0026565  0.00215175 0.0027948
  0.00243164 0.00231562 0.00245714 0.0024923  0.00287327 0.00270564
  0.00254753 0.00314714 0.00247559 0.00232104 0.00203585 0.00271789
  0.00265299 0.00183303 0.00243616 0.00264883 0.00275827 0.00249888
  0.94035533]
 [0.00255483 0.00208439 0.00211178 0.00267959 0.00217045 0.0028191
  0.00245278 0.00233575 0.0024785  0.00251396 0.00289825 0.00272915
  0.00256967 0.00317449 0.00249711 0.00234122 0.00205354 0.00274152
  0.00267605 0.00184897 0.00245734 0.00267186 0.00278225 0.0025206
  0.93983685]
 [0.00241032 0.00196649 0.00199233 0.00252802 0.00204768 0.00265964
  0.00231404 0.00220363 0.00233831 0.00237176 0.00273431 0.00257478
  0.00242432 0.00299493 0.00235587 0.00220879 0.00193739 0.00258645
  0.00252468 0.00174438 0.00231834 0.00252072 0.00262487 0.00237802
  0.94323992]
 [0.00250562 0.00204425 0.00207111 0.00262798 0.00212865 0.0027648
  0.00240554 0.00229077 0.00243076 0.00246554 0.00284243 0.00267659
  0.00252018 0.00311336 0.00244902 0.00229613 0.00201399 0.00268872
  0.00262451 0.00181336 0.00241001 0.0026204  0.00272867 0.00247205
  0.94099556]
 [0.00245905 0.00200625 0.00203261 0.00257913 0.00208908 0.00271341
  0.00236082 0.00224818 0.00238558 0.00241971 0.00278959 0.00262684
  0.00247333 0.00305548 0.0024035  0.00225344 0.00197656 0.00263874
  0.00257573 0.00177965 0.00236521 0.00257169 0.00267794 0.0024261
  0.94209237]
 [0.00229704 0.00187407 0.0018987  0.00240921 0.00195144 0.00253464
  0.00220528 0.00210007 0.00222841 0.00226029 0.00260581 0.00245377
  0.00231038 0.00285418 0.00224515 0.00210498 0.00184634 0.00246489
  0.00240603 0.0016624  0.00220938 0.00240226 0.00250151 0.00226626
  0.94590751]
 [0.00244455 0.00199442 0.00202063 0.00256393 0.00207677 0.00269741
```

0.00234691 0.00223493 0.00237152 0.00240545 0.00277315 0.00261135
0.00245875 0.00303747 0.00238933 0.00224016 0.00196491 0.00262318
0.00256054 0.00176916 0.00235127 0.00255653 0.00266216 0.0024118
0.94243371]
[0.00245731 0.00200483 0.00203118 0.00257731 0.00208761 0.00271149
0.00235916 0.0022466 0.0023839 0.00241801 0.00278762 0.00262498
0.00247159 0.00305333 0.0024018 0.00225185 0.00197516 0.00263688
0.00257391 0.00177839 0.00236354 0.00256987 0.00267605 0.00242439
0.94213323]
[0.00256347 0.00209144 0.00211893 0.00268865 0.00217779 0.00282863
0.00246107 0.00234365 0.00248688 0.00252246 0.00290805 0.00273838
0.00257836 0.00318523 0.00250556 0.00234913 0.00206049 0.00275079
0.0026851 0.00185522 0.00246564 0.00268089 0.00279166 0.00252912
0.93963341]
[0.00257595 0.00210162 0.00212924 0.00270174 0.00218839 0.0028424
0.00247305 0.00235506 0.00249899 0.00253474 0.00292221 0.00275171
0.00259091 0.00320073 0.00251775 0.00236057 0.00207052 0.00276418
0.00269817 0.00186425 0.00247765 0.00269394 0.00280525 0.00254144
0.93933954]
[0.00247557 0.00201973 0.00204627 0.00259646 0.00210312 0.00273164
0.00237668 0.00226329 0.00240161 0.00243597 0.00280834 0.00264449
0.00248995 0.00307601 0.00241965 0.00226859 0.00198984 0.00265647
0.00259303 0.00179161 0.0023811 0.00258897 0.00269594 0.0024424
0.94170326]
[0.00249061 0.00203199 0.0020587 0.00261223 0.00211589 0.00274823
0.00239112 0.00227704 0.0024162 0.00245077 0.00282539 0.00266055
0.00250508 0.0030947 0.00243434 0.00228236 0.00200192 0.0026726
0.00260878 0.00180249 0.00239556 0.00260469 0.00271231 0.00245724
0.9413492]
[0.00246707 0.00201279 0.00203925 0.00258755 0.0020959 0.00272226
0.00236853 0.00225552 0.00239336 0.00242761 0.0027987 0.00263541
0.0024814 0.00306545 0.00241134 0.0022608 0.00198301 0.00264735
0.00258413 0.00178546 0.00237293 0.00258008 0.00268668 0.00243402
0.9419034]
[0.00239619 0.00195496 0.00198065 0.0025132 0.00203568 0.00264405
0.00230047 0.00219071 0.0023246 0.00235786 0.00271828 0.00255969
0.00241011 0.00297738 0.00234206 0.00219584 0.00192603 0.00257128
0.00250988 0.00173416 0.00230475 0.00250595 0.00260949 0.00236408
0.94357266]
[0.0023979 0.00195636 0.00198207 0.002515   0.00203714 0.00264594
0.00230212 0.00219228 0.00232626 0.00235955 0.00272023 0.00256152
0.00241183 0.00297951 0.00234373 0.00219741 0.00192741 0.00257313
0.00251168 0.0017354 0.0023064 0.00250774 0.00261136 0.00236578
0.94353223]
[0.00239012 0.00195001 0.00197564 0.00250684 0.00203052 0.00263735
0.00229464 0.00218516 0.00231871 0.00235188 0.0027114 0.0025532
0.002404   0.00296983 0.00233612 0.00219028 0.00192115 0.00256477
0.00250352 0.00172976 0.00229891 0.0024996 0.00260288 0.0023581
0.94371561]
[0.00246262 0.00200916 0.00203557 0.00258288 0.00209212 0.00271735
0.00236425 0.00225145 0.00238905 0.00242323 0.00279365 0.00263066
0.00247693 0.00305992 0.00240699 0.00225672 0.00197943 0.00264257
0.00257947 0.00178224 0.00236865 0.00257542 0.00268183 0.00242963
0.94200822]
[0.00243729 0.0019885 0.00201463 0.00255632 0.0020706 0.0026894
0.00233994 0.00222829 0.00236448 0.00239831 0.00276491 0.0026036
0.00245145 0.00302845 0.00238223 0.00223351 0.00195907 0.00261539
0.00255294 0.00176391 0.00234429 0.00254894 0.00265425 0.00240464
0.94260467]
[0.00242667 0.00197983 0.00200585 0.00254517 0.00206157 0.00267768
0.00232974 0.00221858 0.00235417 0.00238785 0.00275286 0.00259225
0.00244077 0.00301525 0.00237185 0.00222377 0.00195053 0.00260399
0.00254181 0.00175622 0.00233407 0.00253783 0.00264268 0.00239416
0.94285483]

```
 [0.00248369 0.00202635 0.00205298 0.00260498 0.00211001 0.0027406
  0.00238448 0.00227071 0.00240948 0.00244396 0.00281755 0.00265316
  0.00249812 0.0030861  0.00242758 0.00227602 0.00199636 0.00266518
  0.00260154 0.00179748 0.00238891 0.00259746 0.00270478 0.00245041
  0.94151211]
 [0.00255751 0.00208658 0.002114   0.00268241 0.00217273 0.00282206
  0.00245535 0.00233821 0.0024811  0.0025166  0.00290129 0.00273202
  0.00257237 0.00317783 0.00249974 0.00234368 0.0020557  0.0027444
  0.00267886 0.00185091 0.00245992 0.00267466 0.00278517 0.00252325
  0.93977362]
 [0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.         0.
  1.        ]]
```

For n=50

```
[[0.0169796  0.01712207 0.01803379 ... 0.01720547 0.01617218 0.14214764]
 [0.0170065  0.01714921 0.01806236 ... 0.01723274 0.01619782 0.14078818]
 [0.01695326 0.01709551 0.01800581 ... 0.01717878 0.0161471  0.14347846]
 ...
 [0.01737971 0.01752553 0.01845874 ... 0.0176109  0.01655326 0.12193329]
 [0.0172347  0.01737933 0.01830473 ... 0.01746398 0.01641517 0.1292589 ]
 [0.         0.         0.         ... 0.         0.         1.        ]]
```

Monte Carlo Simulation Method

```
import collections
import random

M=200000
sqArray = np.zeros(N)

rdNode = random.randint(0,N-1)

for i in range (M):
    rand=random.random()
    sumrand=0.0
    for j in range(N):
        if(rand >=sumrand) & (rand <= sumrand + TPM[rdNode][j]):
            rdNode=j
            sqArray[j] = sqArray[j] + 1
            break
        sumrand = sumrand+TPM[rdNode][j]

Pi = np.zeros(N)
Pi = sqArray/M
print(Pi)
```

W/o Absorbing States

n=5

```
[0.206165 0.121755 0.318315 0.253265 0.1005   ]
```

n=25

[0.037775 0.038225 0.043625 0.038575 0.03684 0.03875 0.039205 0.033085
 0.04197 0.04347 0.04851 0.03578 0.041075 0.042345 0.047285 0.03982
 0.037915 0.039935 0.04495 0.043995 0.03466 0.04128 0.03082 0.042945
 0.037165]

n=50

[0.019415 0.01965 0.016265 0.021715 0.019505 0.0215   0.021005 0.018865
 0.020955 0.01897 0.018425 0.02128 0.022205 0.01996 0.0162   0.01986
 0.019295 0.020625 0.021965 0.02206 0.02016 0.01749 0.02019 0.02061
 0.01728 0.018095 0.01998 0.02002 0.02082 0.0249   0.02478 0.01777
 0.019265 0.02204 0.018875 0.021535 0.020505 0.018665 0.022335 0.016795
 0.02126 0.01972 0.020335 0.02075 0.021775 0.020555 0.019975 0.01824
 0.01608 0.01948]

W/ One Absorbing State

n=5
[0. 0. 0. 0. 1.]

n=25
[2.50000e-05 0.00000e+00 1.00000e-05 2.00000e-05 0.00000e+00 1.00000e-05
2.00000e-05 1.00000e-05 1.00000e-05 5.00000e-06 1.00000e-05 2.00000e-05
1.00000e-05 1.00000e-05 5.00000e-06 5.00000e-06 1.00000e-05 5.00000e-06
5.00000e-06 5.00000e-06 5.00000e-06 0.00000e+00 2.00000e-05 5.00000e-06
9.99775e-01]

n=50
[0.0000e+00 5.0000e-06 0.0000e+00 1.0000e-05 0.0000e+00 0.0000e+00
1.0000e-05 0.0000e+00 5.0000e-06 1.0000e-05 2.0000e-05 1.5000e-05
1.5000e-05 0.0000e+00 1.0000e-05 5.0000e-06 0.0000e+00 1.0000e-05
1.0000e-05 5.0000e-06 1.5000e-05 0.0000e+00 5.0000e-06 5.0000e-06
0.0000e+00 5.0000e-06 1.0000e-05 5.0000e-06 5.0000e-06 1.0000e-05
5.0000e-06 5.0000e-06 0.0000e+00 5.0000e-06 2.0000e-05 0.0000e+00
0.0000e+00 5.0000e-06 0.0000e+00 1.0000e-05 0.0000e+00 0.0000e+00
0.0000e+00 0.0000e+00 5.0000e-06 0.0000e+00 0.0000e+00 1.5000e-05
0.0000e+00 9.9974e-01]

References

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