

## QuickSort

Tested in

```
g++ version: 4.9.2 (windows 10)
g++ version: 4.8.5 - ITU SSH
```

Running Commands

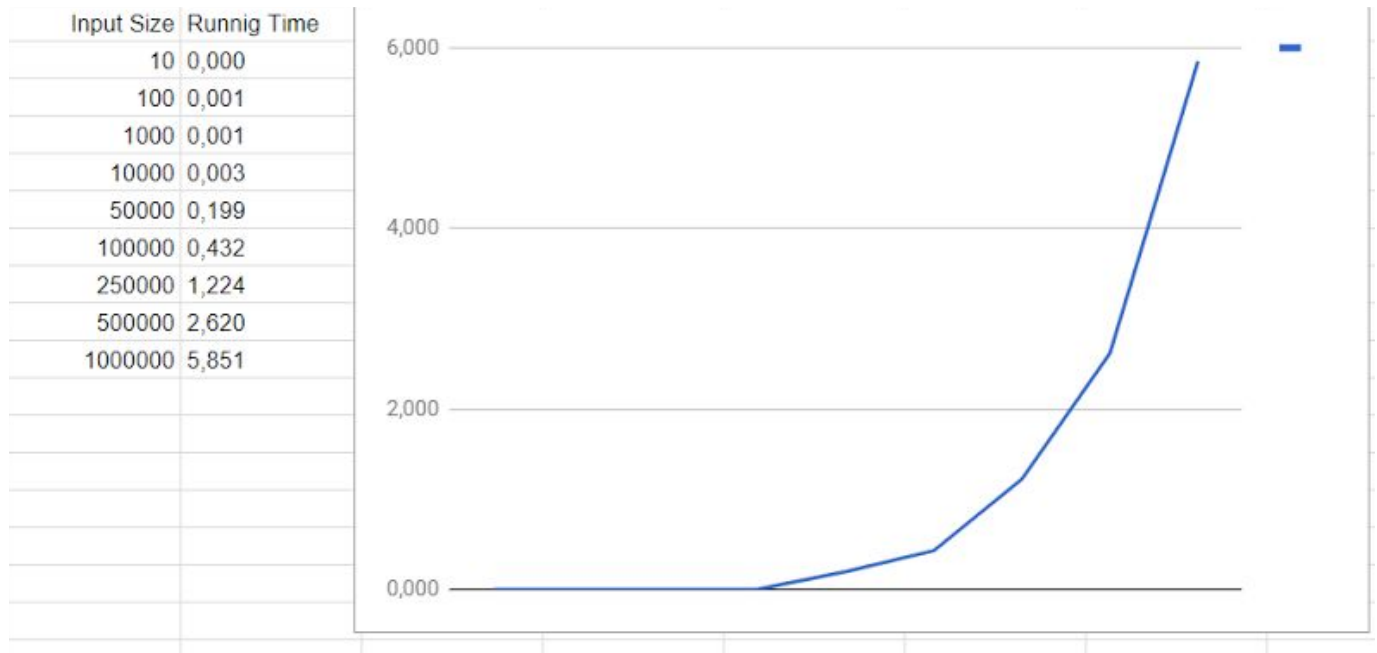
```
g++ main.cpp -o main.exe
./main.exe input_size
```

a)  $T(N)$  = the worst case running time for  $N$  elements.

The worst case in quicksort occurs when the dataset is already sorted. Because we select our pivot as left-most or right-most element and if dataset is already sorted, our pivot value will be smallest or largest element in the dataset. As a result of this situation, imbalanced partitioning will happen in every recursion and one of the our subarrays will be empty every time.

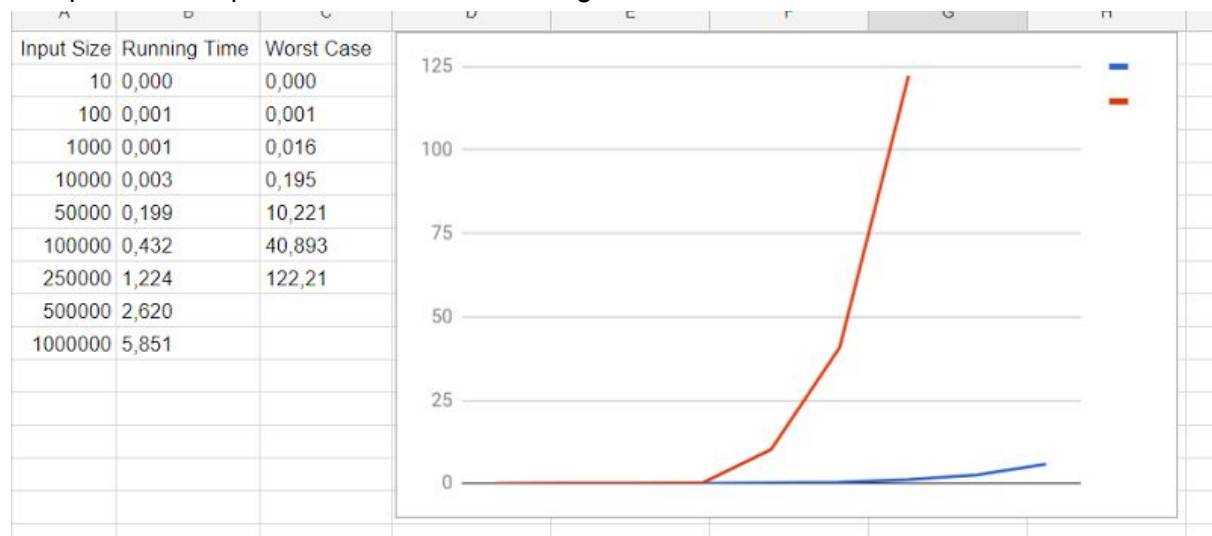
$T(N) = T(0) + T(N-1) + \theta(N)$ $= \theta(1) + T(N-1) + \theta(N)$ $= T(N-1) + \theta(N)$ $= \theta(N^2)$ <p>If we sum up all of the arithmetic series  <math>cn + c(n-1) + c(n-2) + \dots + 2c</math>  <math>= c(n + (n-1) + (n-2) + \dots + 2)</math>  <math>= c((n+1)(n/2) - 1)</math>  Then we will ignore low order terms and constants when using big-<math>\theta</math> notation.</p> <p>Worst runnig time is <math>\theta(N^2)</math></p>	<table> <tr> <th>Subproblem Sizes</th><th>Total Partitioning Time</th></tr> <tr> <td>n</td><td>cn</td></tr> <tr> <td> </td><td></td></tr> <tr> <td>n-1</td><td>c(n-1)</td></tr> <tr> <td> </td><td></td></tr> <tr> <td>n-2</td><td>c(n-2)</td></tr> <tr> <td> </td><td></td></tr> <tr> <td>n-3</td><td>c(n-3)</td></tr> <tr> <td> </td><td></td></tr> <tr> <td>n-4</td><td>c(n-4)</td></tr> <tr> <td> </td><td></td></tr> <tr> <td>n-5</td><td>c(n-5)</td></tr> <tr> <td> </td><td></td></tr> <tr> <td>n-6</td><td>c(n-6)</td></tr> <tr> <td>⋮</td><td></td></tr> <tr> <td>1</td><td>0</td></tr> </table>	Subproblem Sizes	Total Partitioning Time	n	cn			n-1	c(n-1)			n-2	c(n-2)			n-3	c(n-3)			n-4	c(n-4)			n-5	c(n-5)			n-6	c(n-6)	⋮		1	0
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b) A balanced partitioning will be best case for us. Which means left and right subarrays have  $N/2$  where  $N$  is the number of elements in original array. Both these quicksort calls in best case partitioning will take time  $T(N/2)$ . So  $T(N)$  for best case will be  $T(n) = 2T(N/2) + aN$ . If we apply the master method  $a=2, b=2$ . case2:  $T(N)=\theta(n\log n)$



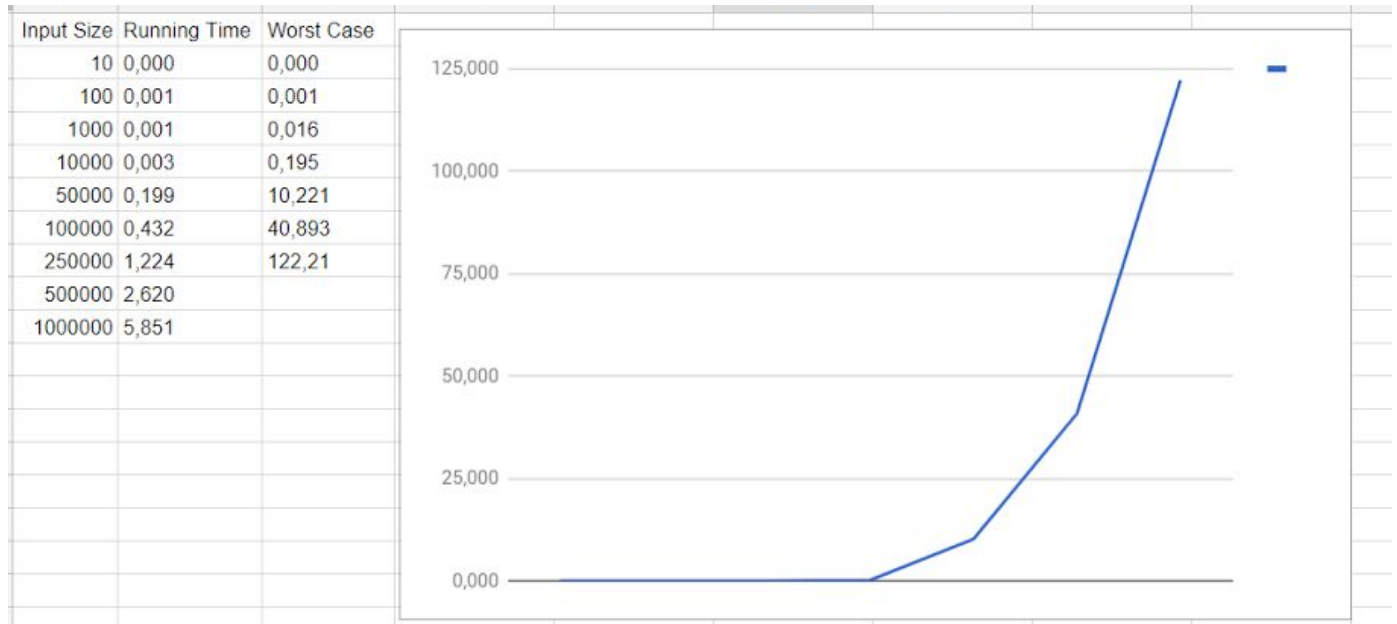
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Comparative Graphs for worst case - average case.



c) The worst case in quicksort occurs when the dataset is already sorted in ascending or descending order. Because we select our pivot as left-most or right-most element and if data set is already sorted, our pivot value will be smallest or largest element in the dataset. As a result of this situation, imbalanced partitioning will happen in every recursion and one of the our subarrays will be empty every time.

To construct worst case, I have exported all dataset in ascending order and applied quicksort algorithm to exported .csv file. (I commented out exporting function calls in source code.)



Time complexity for quicksort is  $O(n \log n)$  in best or average case and  $O(n^2)$  in worst case. But worst case can be avoided by using randomized version of quicksort.

Other possible solutions to avoid the worst cases

1. Choosing the middle index of the partition as a pivot.
2. Choosing the median of the first, middle and last element of the partition for the pivot.

d) Quicksort is not a stable sorting algorithm. In a stable sorting algorithm, the relative order of records with equal key is preserved.

For example: assume that we want to sort a list of points in cartesian plane. Each records in the list here is a pair of integers, first integer is X-coordinate and second integer is Y-coordinate. If we want for sort in increasing order of X-coordinates. We have two records with equal X-coordinate. If we will use a stable sorting algorithm then (4,5) that is before point (4,3) in the original list will be before (4,3) in sorted arrangement also. But this is not guaranteed if we will use an unstable algorithm like quicksort. In the second arrangement elements are sorted in increasing order of X-coordinate but (4,3) is coming before (4,5).

Points in cartesian plane:  
 (1,2) (4,5) (2,3) (4,3) (5,2)  
 (1,2) (2,3) (4,5) (4,3) (5,2)  
 or  
 (1,2) (2,3) (4,3) (4,5) (5,2)

Partitioning logic does not ensure stability in quicksort algorithm.

For my quicksort implementation: These two datas printed out in different orders in executions.

10 , 70 , 74 , female , 74565 , 86000000US74565  
 10 , 25 , 29 , female , 97843 , 86000000US97843  
 10 , 5 , 9 , male , 99778 , 86000000US99778  
 11 , 0 , 4 , female , 4675 , 86000000US04675  
 11 , 18 , 19 , male , 5669 , 86000000US05669  
 11 , 75 , 79 , male , 12412 , 86000000US12412  
 11 , 20 , 20 , female , 12865 , 86000000US12865  
 11 , 10 , 14 , male , 13162 , 86000000US13162  
 11 , 35 , 39 , male , 17934 , 86000000US17934  
 11 , 62 , 64 , female , 17934 , 86000000US17934  
 11 , 25 , 29 , male , 24139 , 86000000US24139  
 11 , 80 , 84 , male , 29584 , 86000000US29584  
 11 , 18 , 19 , male , 42236 , 86000000US42236  
 11 , 10 , 14 , female , 45032 , 86000000US45032  
 11 , 40 , 44 , female , 52225 , 86000000US52225  
 11 , 15 , 17 , male , 59241 , 86000000US59241  
 11 , 75 , 79 , female , 59925 , 86000000US59925  
 11 , 65 , 66 , male , 60928 , 86000000US60928  
 11 , 85 , 0 , female , 62421 , 86000000US62421  
 11 , 65 , 66 , male , 72351 , 86000000US72351  
 11 , 25 , 29 , male , 76827 , 86000000US76827  
 12 , 55 , 59 , female , 12416 , 86000000US12416  
 12 , 18 , 19 , male , 14720 , 86000000US14720