MA1522 Tutorial 8

AY 24/25 Sem 1 — github/omgeta

Q1. (a) Orthonormal basis $= \left\{ \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \frac{1}{2\sqrt{11}}\begin{pmatrix}3\\-5\\3\\-1\end{pmatrix}, \frac{1}{\sqrt{110}}\begin{pmatrix}7\\3\\-4\\-6\end{pmatrix}, \frac{1}{\sqrt{10}}\begin{pmatrix}1\\-1\\-2\\2\end{pmatrix} \right\}$ ■

(b) Orthonormal set $= \left\{ \frac{1}{\sqrt{10}}\begin{pmatrix}1\\2\\2\\1\end{pmatrix}, \frac{1}{\sqrt{110}}\begin{pmatrix}3\\6\\-4\\-7\end{pmatrix}, \frac{1}{\sqrt{33}}\begin{pmatrix}4\\-3\\2\\-2\end{pmatrix} \right\}$ which is not a basis since it only contains 3 vectors ■

Q2. (a) Reduce the augmented matrix for $Ax = b$:

$$[A \mid b] = \left(\begin{array}{cccc|c}0&1&1&0&6\\1&-1&1&-1&3\\1&0&1&0&-1\\1&1&1&1&1\end{array}\right) \xrightarrow{RREF} \left(\begin{array}{cccc|c}1&0&0&1&0\\0&1&0&1&0\\0&0&1&-1&0\\0&0&0&0&1\end{array}\right)$$

which is inconsistent ■

(b) Reduce the augmented matrix for $A^TAx = A^Tb$:

$$[A^TA \mid A^Tb] = \left(\begin{array}{cccc|c}3&0&3&0&3\\0&3&1&2&4\\3&1&4&0&9\\0&2&0&2&-2\end{array}\right) \xrightarrow{RREF} \left(\begin{array}{cccc|c}1&0&0&1&-6\\0&1&0&1&-1\\0&0&1&-1&7\\0&0&0&0&0\end{array}\right)$$

so a least squares solution is $\vec{u} = \begin{pmatrix}-6\\-1\\7\\0\end{pmatrix}$ ■

(c) Projection is $A\vec{u} = \begin{pmatrix}6\\2\\1\\0\end{pmatrix}$ ■

Q3. (a) Solve the augmented matrix for $N\vec{x} = \vec{y}$:

$$[N\vec{x} \mid \vec{y}] = \begin{pmatrix}1&\frac{1}{100}&\frac{11}{4}\\1&\frac{3}{2500}&\frac{331}{100}\\1&\frac{3}{200}&\frac{98}{25}\\1&\frac{1}{50}&\frac{99}{20}\end{pmatrix}$$

which is inconsistent ■

(b) Solve $N^TN\vec{x} = N^T\vec{b} : \vec{x} = \begin{pmatrix}0.0001\\0.0216\end{pmatrix}$ ■

Q4. (a) $A = QR = \begin{pmatrix}\frac{1}{3}&0&-\frac{1}{\sqrt{6}}\\\frac{1}{3}&0&-\frac{1}{\sqrt{6}}\\\frac{1}{3}&0&\frac{2}{\sqrt{6}}\\0&1&0\end{pmatrix}\begin{pmatrix}\sqrt{3}&\sqrt{3}&\frac{1}{\sqrt{3}}\\0&1&1\\0&0&\frac{\sqrt{2}}{\sqrt{3}}\end{pmatrix}$ ■

(b) Solve $A^TA\vec{x} = A^T\vec{b}$

$$
\begin{aligned}
A^TA\vec{x} &= A^T\vec{b} \\
(QR)^TQR\vec{x} &= (QR)^T\vec{b} \\
R^TQ^TQR\vec{x} &= R^TQ^T\vec{b} \\
R^TR\vec{x} &= R^TQ^T\vec{b} \\
R\vec{x} &= Q^T\vec{b} \\
\therefore \vec{x} &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \blacksquare
\end{aligned}
$$