MA1522 Tutorial 1

AY 24/25 Sem 1 — github/omgeta

Q1. (a) Substitute $y = t$ into $x = 1 + 2t$:

$$\begin{aligned} x &= 1 + 2y \\ x - 2y &= 1 \quad \blacksquare \end{aligned}$$

(b) Suppose $x = t$:

$$\begin{aligned} t - 2y &= 1 \\ -2y &= 1 - t \\ y &= \frac{1}{2}t - \frac{1}{2} \quad \blacksquare \end{aligned}$$

Q2. Substitute $y = s$ and $z = t$ into $x = 3 - 4s + t$:

$$\begin{aligned} x &= 3 - 4y + z \\ x + 4y - z &= 3 \quad \blacksquare \end{aligned}$$

Q3. (a) Reduce the corresponding augmented matrix:

$$\begin{bmatrix} 3 & 2 & -4 & 3 \\ 2 & 3 & 3 & 15 \\ 5 & -3 & 1 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Therefore, the unique solution is:

$$x = 3, y = 1, z = 2 \quad \blacksquare$$

(b) Reduce the corresponding augmented matrix:

$$\begin{bmatrix} 1 & 1 & -1 & -2 & 0 \\ 2 & 1 & -1 & 1 & -2 \\ -1 & 1 & -3 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3 & -2 \\ 0 & 1 & 0 & -\frac{19}{2} & 2 \\ 0 & 0 & 1 & -\frac{9}{2} & 0 \end{bmatrix}$$

Therefore, the general solution is:

$$\begin{cases} a & = -2 - 3d \\ b & = 2 + \frac{19}{2}d \\ c & = \frac{9}{2}d \\ d & \text{ is free} \end{cases} \quad \blacksquare$$

(c) Reduce the corresponding augmented matrix:

$$\begin{bmatrix} 1 & -4 & 2 & -2 \\ 1 & 2 & -2 & -3 \\ 1 & -1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

System of equations is inconsistent, therefore, there is no solution. $\blacksquare$

Q4. Reduce the augmented matrix:

$$\begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix} \sim \begin{bmatrix} a & 0 & b & 2 \\ 0 & a & 4-b & 2 \\ 0 & 0 & b-2 & 0 \end{bmatrix}$$

(a) There are no solutions when:

$$a = 0, b \neq 2 \quad \blacksquare$$

(b) There is a unique solution when:

$$a \neq 0, b \neq 2 \quad \blacksquare$$

(c) There are infinite solutions with one free parameter when:

$$a \neq 0, b = 2 \quad \blacksquare$$

(d) There are infinite solutions with two free parameters when:

$$a = 0, b = 2 \quad \blacksquare$$

Q5. (a) Yes $\blacksquare$

$$\begin{aligned} x + y + z &= 0 \\ x + y + z &= 1 \end{aligned}$$

(b) Yes $\blacksquare$

$$\begin{aligned} x &= 2 \\ y &= 3 \\ x + y &= 5 \end{aligned}$$

(c) No $\blacksquare$
For a unique solution, there must be no free variables.
However, there must be more unknowns than equations, and each row can have at most 1 pivot.
$\therefore$ there must be some column with no pivot.
$\therefore$ there must be a non-pivot column.
$\therefore$ cannot have a unique solution.

(d) Yes $\blacksquare$

$$\begin{aligned} x - y &= 0 \\ 2x - 2y &= 0 \\ 3x - 3y &= 0 \end{aligned}$$

Q6. Reduce the corresponding augmented matrix:

$$\begin{bmatrix} 1 & -1 & 2 & 6 \\ 2 & 2 & -5 & 3 \\ 2 & 5 & 1 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Therefore the solution is:

$$\begin{aligned} &x^2 = 4, y^2 = 0, z^2 = 1 \\ &x = \pm 2, y = 0, z = \pm 1 \quad \blacksquare \end{aligned}$$

Q7. We have the linear equations

$$\begin{aligned} x_1 + x_3 &= 800 \\ x_1 - x_2 + x_4 &= 200 \\ x_2 - x_5 &= 500 \\ x_3 + x_6 &= 750 \\ x_4 + x_6 - x_7 &= 600 \\ x_5 - x_7 &= -50 \end{aligned}$$

Reduce the corresponding augmented matrix:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 450 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, the general solution is:

$$\begin{cases} x_1 & = 50 + x_6 \\ x_2 & = 450 + x_7 \\ x_3 & = 750 - x_6 \\ x_4 & = 600 - x_6 + x_7 \\ x_5 & = -50 + x_7 \\ x_6 & \text{ is free} \\ x_7 & \text{ is free} \end{cases}$$

(a) No, there are an infinite number of solutions ■

(b) $x_1 = 100, x_2 = 550, x_3 = 700, x_4 = 650, x_5 = 50$ ■

(c) Suppose $x_1 = 0$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -1 & 450 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 650 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This solution is not possible as it would result in the impossible with negative traffic ■