GEA1000 Tutorial 4

AY 24/25 Sem 2 — github/omgeta

Q1. (a.) (i.) T

(ii.) H

(iii.) H — 9. {H, T, H, H, H}

(iv.) H — 26. {T, T, H, H, T}

(b.) $P(T|HHHH) = 0.5$

(c.) Disagree; the probability of the result of each toss is independent of previous events.

(d.) (i.) $\frac{5}{32}$

(ii.) No, because $P(X \geq 4) = \frac{6}{32} = 18.75\% > 5\%$

Q2. (a.) $P(\text{infected}|\text{positive}) = \frac{2400}{2470} \approx 0.9717$

	Infected (3000)	**Not Infected (7000)**
Positive (2470)	2400 (True Positives)	70 (False Positives)
Negative (7530)	600 (False Negatives)	6930 (True Negatives)

(b.) No; base rate fallacy assumes the same prevalence as in Country X, which may not hold in Country Y.

Q3. (a.) Sample proportion, $p^* = 0.525$

(b.) (i.) Margin of error, $e = 1.96 \cdot \sqrt{\frac{0.495(1-0.495)}{200}} \approx 0.069$

(ii.) Yes; Confidence interval $= 0.495 \pm 0.069$ which includes 0.506

(c.) (i.) Larger; higher sample size reduces error margin

(ii.) $n \geq \frac{1.96^2 \cdot 0.525(1-0.525)}{0.03^2} = 1064.\cdots \implies n = 1065$

(d.) (i.) Sample size n is directly proportional to $p(1-p)$

(ii.) $n \geq \frac{1.96^2 \cdot 0.5 \cdot 0.5}{0.03^2} \approx 1067.11 \implies n = 1068$

(e.) Null hypothesis, H_0: $\mu = 28$
Alternative hypothesis, H_1: $\mu < 28$
Level of significance, $\alpha = 0.05$
Test statistic $= \dfrac{27.038 - 28}{5.206/\sqrt{1000}} = -5.843$
Since $p < 0.001 < \alpha = 0.05$, therefore we reject the null hypothesis to conclude that mean age is less than 28.