CS2109S Tutorial 4

AY 25/26 Sem 1 — github/omgeta

A. 1. (A, B, A^2, B^2) (ellipse features except AB which is only needed for tilt)

2. (A)

3. (a.) (AB) with boundary $AB = 0$

   (b.) (A, B, AB) with boundary $(A - sA)(B - sB) = 0$

B. 1. Let $z = w^T x$ then $p = \sigma(z) = \frac{1}{1+e^{-w^T x}}$:

$$\begin{aligned}\frac{\partial}{\partial w_j} \log p &= \frac{\sigma'(z)}{\sigma(z)} \cdot \frac{\partial z}{\partial w_j} \\ &= (1 - \sigma(z))x_j \\ &= (1 - p)x_j\end{aligned}$$

2. Let $1 - p = 1 - \sigma(z)$:

$$\begin{aligned}\frac{\partial}{\partial w_j} \log(1 - p) &= \frac{-\sigma'(z)}{1 - \sigma(z)} \cdot \frac{\partial z}{\partial w_j} \\ &= -\frac{p(1 - p)}{1 - p} x_j \\ &= -px_j\end{aligned}$$

3. Given $J_{BCE}(w) = -y \log h_w(x) - (1 - y) \log(1 - h_w(x))$:

$$\begin{aligned}\frac{\partial J_{BCE}}{\partial w_j} &= -y(1 - p)x_j - (1 - y)(-px_j) \\ &= (p - y)x_j\end{aligned}$$

C. 1. $(4.2kg, 0.4m) \implies p_{cat} \approx 0.9839, p_{horse} \approx 0.00177, p_{elephant} \approx 0 \implies$ cat
   $(720kg, 2.4m) \implies p_{cat} \approx 0.0360, p_{horse} \approx 0.99834, p_{elephant} \approx 0 \implies$ horse
   $(2350kg, 5.5m) \implies p_{cat} \approx 0, p_{horse} \approx 0, p_{elephant} \approx 1 \implies$ elephant

2. Only as long as it does not interfere with existing decision boundaries.

3. Pairwise boundaries occur where two scores are equal: $(w_i - w_j)^T x = 0$

4. Without scaling, the loss functions would disproportionately be impacted by changes in the larger scale feature.