ST2334 Tutorial 9

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. (a), (b), (d)

Q2. $c = \frac{1038-1000}{146/\sqrt{64}} = 2.08$

Q3. (b); p-value < 0.05 suggests we must reject null hypothesis so $2 \notin$ CI

Long Form Questions

Q1. Let $H_0 : \mu = 14.0, H_1 : \mu \neq 14.0$ at $\alpha = 0.05$
Test statistic: $T = \frac{\overline{X}-\mu}{s/\sqrt{n}} \sim t_4$ so $t = \frac{14.4-14.0}{0.158/\sqrt{5}} = 5.66$
Critical region: $t < -t_{4;0.025} = -2.776$ or $t > t_{4;0.025} = 2.776$
Since $t = 5.66 > 2.776$, we reject null hypothesis in favour of the alternative hypothesis $\mu \neq 14.0$ at 0.05 level of significance

Q2. Let $H_0 : \hat{x} - \hat{y} = 0, H_1 : \hat{x} - \hat{y} \neq 0$ at $\alpha = 0.05$
Test statistic: $Z = \frac{\overline{X}-\overline{Y}}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \sim N(0,1)$ so $z = \frac{115.1-114.6}{\sqrt{\frac{0.47^2}{33}+\frac{0.38^2}{31}}} = 4.69$
p-value: $2P(Z > 4.69) \approx 0$ Since p-value < 0.05 , we reject null hypothesis at 0.05 level of significance

Q3. Assuming equal variance since $\frac{s_1}{s_2} = 1.73$, pooled estimator $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = 5$
CI: $(\overline{x} - \overline{y}) \pm t_{8;0.025} \cdot s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 3 \pm 2.306 \cdot \sqrt{5} \cdot \sqrt{\frac{2}{5}} = 3 \pm 3.26 = (-0.26, 6.26)$

Q4. (a.) Note $\frac{s_1}{s_2} = 0.67$ so variance is assumed equal. Pooled estimator $s_p^2 = 5.85685$.
Let $H_0 : \mu_1 - \mu_2 = 0$ vs $H_1 : \mu_1 - \mu_2 < 0$
$\alpha = 0.05$
Sample statistic: $T = \frac{\overline{X}-\overline{Y}}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{18}$
Critical region: $t < t_{18;0.05} = -1.734$
Test statistic: $t = \frac{7.1-9.4}{2.41\sqrt{\frac{2}{10}}} = -2.13$
Since $t < -1.734$, we reject null hypothesis at level of signifance $\alpha = 0.05$ in support of the conclusion that instituting a coffee break reduces number of mean errors.

(b.) Assume two populations follow normal distributions

(c.) $P(t_{18} < -2.13) \approx 0.023$

Q5. Let $H_0 : \mu_D = 0$ vs $H_1 : \mu_D \neq 0$
Test statistic: $t = \frac{-0.101}{\frac{0.11367}{\sqrt{10}}} = -2.8098$
P-value: $2P(t_9 > 2.8098) = 0.0204$
Since p-value < 0.05, we will reject H_0 and conclude there is a difference in mean results.