MA1521 Homework 8

AY 24/25 Sem 1 — github/omgeta

Q1. (a) $f(x,y) = \dfrac{x^2}{y^2+2} - \dfrac{y^2}{x^2+y}$

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{(y^2+2)(2x)-(x^2)(0)}{(y^2+2)^2} - \frac{(x^2+y)(0)-(y^2)(2x)}{(x^2+y)^2} \\ &= \frac{2x}{y^2+2} + \frac{2xy^2}{(x^2+y)^2} \quad \blacksquare \\ \frac{\partial f}{\partial y} &= \frac{(y^2+2)(0)-(x^2)(2y)}{(y^2+2)^2} - \frac{(x^2+y)(2y)-(y^2)(1)}{(x^2+y)^2} \\ &= \frac{-2yx^2}{(y^2+2)^2} - \frac{2yx^2+y^2}{(x^2+y)^2} \quad \blacksquare\end{aligned}$$

(b) $g(x,y,z) = xy^2z^3 + 3yz + \ln(z^2+1)$

$$\begin{aligned}\frac{\partial g}{\partial x} &= y^2z^3 \quad \blacksquare \\ \frac{\partial g}{\partial y} &= 2xyz^3 + 3z \quad \blacksquare \\ \frac{\partial g}{\partial z} &= 3xy^2z^2 + 3y + \frac{2z}{z^2+1} \quad \blacksquare\end{aligned}$$

Q2. $f(x,y) = e^x y^2 + x\sin xy$

$$\begin{aligned}f_x &= e^x y^2 + \sin xy + xy\cos xy \quad \blacksquare \\ f_y &= 2e^x y + x^2\cos xy \quad \blacksquare \\ f_{xy} &= \frac{\partial}{\partial y} f_x \\ &= 2e^x y + x\cos xy + x\cos xy - x^2 y\sin xy \\ &= 2e^x y + 2x\cos xy - x^2 y\sin xy \quad \blacksquare\end{aligned}$$

Q3. $f(x,y,z) = e^{xy^2} + x^2 + \sin^{-1} y$

$$\begin{aligned}f_x &= y^2 e^{xy^2} + 2x \\ f_{xx} &= y^4 e^{xy^2} + 2 \\ f_{xxy} &= 2xy^5 e^{xy^2} + 4y^3 e^{xy^2} \\ &= 2y^3 e^{xy^2}(xy^2+2) \quad \blacksquare\end{aligned}$$

Q4. $x^3 + 2y^3 + 6xz^3 = 61z$

Differentiate w.r.t. x:

$$\begin{aligned}3x^2 + 6z^3 + 18xz^2\frac{\partial z}{\partial x} &= 61\frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial x} &= \frac{3x^2+6z^3}{61-18xz^2} \quad \blacksquare\end{aligned}$$

Differentiate w.r.t. y:

$$\begin{aligned}6y^2 + 18xz^2\frac{\partial z}{\partial y} &= 61\frac{\partial z}{\partial y} \\ \frac{\partial z}{\partial y} &= \frac{6y^2}{61-18xz^2} \quad \blacksquare\end{aligned}$$

Q5. (a) $I = 7V \implies \frac{dI}{dV} = 7$ (dollars per day per thousand views) ■

(b) When $300 = V \times R \implies R = \frac{300}{V}$:

$$\frac{dR}{dV} = -\frac{300}{V^2}$$

$$\frac{dR}{dV}|_{V=50} = -0.12 \text{ (dollars per thousand views per thousands of view per day)} \quad ■$$

(c) Differentiate w.r.t. t:

$$\frac{dI}{dt} = \frac{dV}{dt}R + \frac{dR}{dt}V$$

Substitute $R = 5, V = 70, \frac{dR}{dt} = 0.02, \frac{dI}{dt} = 8$:

$$8 = 5\frac{dV}{dt} + (0.02)(70)$$

$$\frac{dV}{dt} = 1.32 \text{ (thousands of views per day per day)} \quad ■$$

Q6. (a) At $(-1, 1, -\frac{1}{6})$:

$$\frac{\partial z}{\partial x} = -\frac{7}{6}$$

$$\frac{\partial z}{\partial y} = -\frac{35}{36}$$

$$\text{Tangent: } z + \frac{1}{6} = -\frac{7}{6}(x+1) - \frac{35}{36}(y-1)$$

$$z = -\frac{7}{6}x - \frac{35}{36}y - \frac{13}{36} \quad ■$$

(b) At $(3, 2, 1)$:

$$\frac{\partial z}{\partial x} = \frac{33}{7}$$

$$\frac{\partial z}{\partial y} = \frac{24}{7}$$

$$\text{Tangent: } z - 1 = \frac{33}{7}(x-3) + \frac{24}{7}(y-2)$$

$$z = \frac{33}{7}x + \frac{24}{7}y - 20 \quad ■$$