FDP2021 Special Physics Class 1, 2

AY 24/25 Sem 2 - 25/26 Sem 1 — github/omgeta

1. Vector Algebra

Differential Calculus

Gradient of scalar function f, ∇f, is a vector rate of change of f with maximum increase in the direction ∇f:

i. $\nabla(fg) = f(\nabla g) + g(\nabla f)$
ii. $\nabla(\vec{a}\cdot\vec{b}) = \vec{a}\times(\nabla\times\vec{b}) + \vec{b}\times(\nabla\times\vec{a}) + (\vec{a}\cdot\nabla)\vec{b} + (\vec{b}\cdot\nabla)\vec{a}$

Divergence of vector function $\vec{v}$, $\nabla\cdot\vec{v}$, is a scalar of how much $\vec{v}$ spreads out:

i. $\nabla\cdot(f\vec{a}) = f(\nabla\cdot\vec{a}) + \vec{a}\cdot(\nabla f)$
ii. $\nabla\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot(\nabla\times\vec{a}) - \vec{a}\cdot(\nabla\times\vec{b})$

Curl of vector function $\vec{v}$, $\nabla\cdot\vec{v}$, is a vector of how much $\vec{v}$ curls around:

i. $\nabla\times(f\vec{a}) = f(\nabla\times\vec{a}) - \vec{a}\times(\nabla f)$
ii. $\nabla\times(\vec{a}\times\vec{b}) = (\vec{b}\cdot\nabla)\vec{a} - (\vec{a}\cdot\nabla)\vec{b} + \vec{a}(\nabla\cdot\vec{b}) - \vec{b}(\nabla\cdot\vec{a})$

Laplacian of scalar function f, $\nabla^2 f = \nabla\cdot\nabla f$, is a scalar. Other second derivatives are:

i. $\nabla\cdot(\nabla\times\vec{a}) = 0$
ii. $\nabla\times(\nabla f) = \vec{0}$
iii. $\nabla\times(\nabla\times\vec{a}) = \nabla(\nabla\cdot\vec{a}) - \nabla^2\vec{a}$

Integral Calculus

Line Integral: $\int_a^b \vec{v}\cdot d\vec{\ell}$

Surface Integral: $\int_{\mathcal{S}} \vec{v}\cdot d\vec{S}$

Volume Integral: $\int_{\mathcal{V}} f\,d\tau$

Fundamental Theorems:

i. $\int_a^b (\nabla f)\cdot d\vec{\ell} = f(\vec{b}) - f(\vec{a})$ (Gradient)
ii. $\int (\nabla\cdot\vec{a})\,d\tau = \oint \vec{a}\cdot d\vec{S}$ (Divergence)
iii. $\int (\nabla\times\vec{a})\cdot d\vec{S} = \oint \vec{a}\cdot d\vec{\ell}$ (Curl)

Coordinate Systems

Cartesian (x, y, z):

$$d\vec{\ell} = \hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz, \quad d\tau = dx\,dy\,dz$$
$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$
$$\nabla\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$
$$\nabla\times\vec{v} = \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical (r, θ, ϕ) – origin radius r, z-angle θ, xy-angle ϕ:

$$d\vec{\ell} = \hat{r}\,dr + \hat{\theta}\,(r\,d\theta) + \hat{\phi}\,(r\sin\theta\,d\phi), \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$$
$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\phi}$$
$$\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$
$$\nabla\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\,v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$
$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Cylindrical (s, ϕ, z) – z-radius s, xy-angle ϕ, height z:

$$d\vec{\ell} = \hat{s}\,ds + \hat{\phi}\,(s\,d\phi) + \hat{z}\,dz, \quad d\tau = s\,ds\,d\phi\,dz$$
$$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial\phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$
$$\nabla\cdot\vec{v} = \frac{1}{s}\frac{\partial}{\partial s}(s\,v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$
$$\nabla\times\vec{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \left[\frac{1}{s}\frac{\partial}{\partial s}(s\,v_\phi) - \frac{1}{s}\frac{\partial v_s}{\partial\phi}\right]\hat{z}$$
$$\nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

2. Electrostatics

3. Potentials

4. Electric Fields

5. Magnetostatics

6. Magnetic Fields

7. Electrodynamics