CS3230 Tutorial 11

AY 25/26 Sem 1 — github/omgeta

Q1). $\Theta(n)$; given subroutine f which calculates Euclidean distance from origin in $O(1)$, this problem reduces to Quickselect for $k = \sqrt{n}$ smallest element, which runs in $\Theta(n)$

Q2). Yes; choose a good median (e.g. with median-of-medians)

Q3). $O(n)$ for part 1, and $O(n^2)$ for part 2

Q4).

a. $E[X_k] = Pr[X_k = 1] = \frac{1}{n}$

b. Since exactly one $X_k = 1$, $T(n) = \sum_{k=0}^{n-1} X_k(T(\max(k, n-1-k))) + \Theta(n)$, then $E[T(n)] = \sum_{k=0}^{n-1} E[X_k T(\max(k, n-1-k))] + \Theta(n)$.

Since X_k is independent, then $E[T(n)] = \frac{1}{n}\sum_{k=0}^{n-1} E[T(\max(k, n-1-k))] + \Theta(n) \leq \frac{2}{n}\sum_{k=\lfloor \frac{n}{2} \rfloor}^{n-1} E[T(k)] + \Theta(n)$ (by symmetry)

c. Induction Hypothesis: $E[T(n)] \leq cn$
Base Case: pick c large enough so $E[T(n)] \leq cn$ holds for small n
Inductive Step: Assume IH holds for all $k < n$, then
$E[T(n)] \leq \frac{2}{n}\sum_{k=0}^{n-1} ck + an \leq \frac{2c}{n} \cdot \frac{3}{8}n^2 + an = (\frac{3c}{4} + a)n$. Choose $c \geq 4a$, then $\frac{3c}{4} + a \leq c$ so $E[T(n)] \leq cn$
Therefore $E[T(n)] \in O(n)$