

CS2100 Computer Organisation

AY 24/25 Sem 2 — github/omgeta

1. Number Systems

Weighted-positional number systems with base- R , has each position weighted in powers of R .

- N positions give R^N values
- M values need $\lceil \log_R M \rceil$ positions

Conversion

Base- $R \rightarrow$ Decimal: repeated multiply by R^{n-1}

Decimal \rightarrow Binary (Generalised for Base- R):

- Whole numbers: repeated division by 2, $LSB \rightarrow MSB$
- Fractional: repeated multiplication by 2, $MSB \rightarrow LSB$

Binary \rightarrow Octal: partition in groups of 3

Octal \rightarrow Binary: convert each digit to 3 bits

Binary \rightarrow Hexadecimal: partition in groups of 4

Hexadecimal \rightarrow Binary: convert each digit to 4 bits

C Programming

ASCII Chars are represented with 7 bits and 1 parity bit

`char` is 1 byte, `int` + `float` are 4 bytes, `long` + `double` are 8 bytes.

Functions only modify `struct` if passed as pointer or in array with: `(*object_p).a = 1;`, `object_p->a = 1;`

Arrays in `structs` are deep-copied (but not pointers to arrays).

Negative Numbers

Sign-and-Magnitude:

- MSB is the sign bit (0 is positive)
- Range: $-(2^{N-1} - 1)$ to $2^{N-1} - 1$
- Negation: Flip MSB

1s Complement (useful for arithmetic):

- Negated x , $-x = 2^N - x - 1$
- Range: $-(2^{N-1} - 1)$ to $2^{N-1} - 1$
- Negation: Invert bits
- Overflow: add carry to result, wrap around
- $(b-1)s$: $-x = b^N - x - 1$

2s Complement (useful for arithmetic):

- Negated x , $-x = 2^N - x$
- Range: -2^{N-1} to $2^{N-1} - 1$
- Negation: Invert bits, then add 1
- Overflow: truncate
- bs : $-x = b^N - x$

Excess- M (useful for comparisons):

- Start at $-M = -2^{N-1}$, for N bits.
- Range: -2^{N-1} to $2^{N-1} - 1$
- Express x in Excess- N : $x + N$

Real Numbers

Fixed-Point (limited range):

- Reserve bits for whole numbers and for fraction, converting with 1s or 2s complement

IEEE 754 Floating-Point (more complex):

- Sign 1-bit (0 is positive)
- Exponent 8-bits (excess-127)
- Mantissa 23-bits (normalised to $1.m \times 2^{exp}$)

sign	exponent	mantissa
------	----------	----------

single-precision: 1-bit sign / 8-bit exponent / 23-bit mantissa

2. ISA

Instruction Set Architecture (ISA) defines instructions for how software controls the hardware.

von Neumann Architecture:

- Processor: Perform computations.
- Memory: Stores code and data (stored memory).
- Bus: Bridge between processor and memory.

Storage Architectures:

- Stack: Operands are implicitly on top of the stack.
- Accumulator: One operand is implicitly in the accumulator.
- Memory-Memory: All operands in memory.
- Register-Register: All operands in registers.

Endianness (ordering of bytes in a word):

- Big Endian: MSB in lowest address
- Little Endian: LSB in lowest address

Instruction Length:

- Fixed-Length: easy fetch and decode, simplified pipelining, instruction bits are scarce
- Variable-Length: more flexible

Instruction Encoding, under fixed-length instructions involves extending opcode for instruction types with unused bits:

- Minimise: maximise opcode range of instruction type with least opcode bits
- Maximise: minimise opcode range of instruction type with least opcode bits

Shortcut for Maximise given $A > B > C$ instruction types, where $|X|$ is the minimum number of instruction X
 $= 2^A - |B|2^{A-B} - |C|2^{A-C} + |B| + |C|$

3. MIPS

MIPS Assembly Language:

- Mainly Register-Register, Fixed-Length
- 32 registers, each 32-bits (4-byte) long
- 2^{30} words contains 32-bits (4-byte) long
- Memory addresses are 32-bits long

Arithmetic Instructions:

- $C16, C5$ are 16, 5 bit patterns
- $C16_{2s}$ is a sign-extended 2's complement
- NOT: `nor` with `$zero`, or `xor` with all 1s
- Large constants: `lui` (31:16 bits) + `ori` (15:0 bits)

Operation	Opcode in MIPS	Meaning
Addition	<code>add \$rd, \$rs, \$rt</code>	$\$rd = \$rs + \$rt$
	<code>addi \$rt, \$rs, C16_{2s}</code>	$\$rt = \$rs + C16_{2s}$
Subtraction	<code>sub \$rd, \$rs, \$rt</code>	$\$rd = \$rs - \$rt$
Shift left logical	<code>sll \$rd, \$rt, C5</code>	$\$rd = \$rt \ll C5$
Shift right logical	<code>srl \$rd, \$rt, C5</code>	$\$rd = \$rt \gg C5$
AND bitwise	<code>and \$rd, \$rs, \$rt</code>	$\$rd = \$rs \& \$rt$
	<code>andi \$rt, \$rs, C16</code>	$\$rt = \$rs \& C16$
OR bitwise	<code>or \$rd, \$rs, \$rt</code>	$\$rd = \$rs \$rt$
	<code>ori \$rt, \$rs, C16</code>	$\$rt = \$rs C16$
NOR bitwise	<code>nor \$rd, \$rs, \$rt</code>	$\$rd = \$rs \downarrow \$rt$
XOR bitwise	<code>xor \$rd, \$rs, \$rt</code>	$\$rd = \$rs \wedge \$rt$
	<code>xori \$rt, \$rs, C16</code>	$\$rt = \$rs \wedge C16$

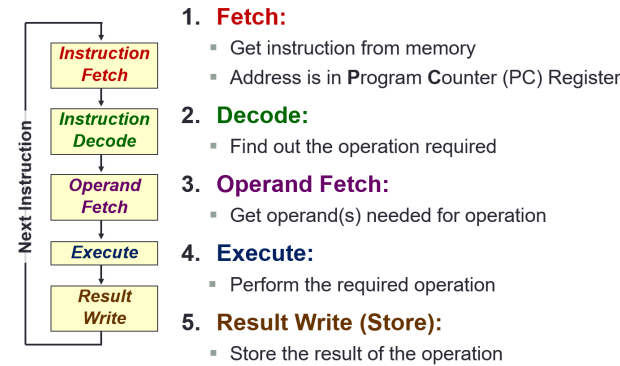
Memory Instructions:

- `lw $rt, offset($rs)`: load contents to `$rt`
- `sw $rt, offset($rs)`: store contents to `$rs`
- Use `lb`, `sb` for loading bytes (like chars)

Control Instructions:

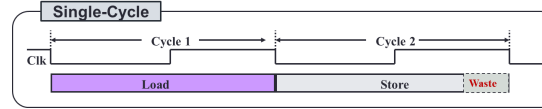
- `beq/bne $rs, $rt, label/imm`: jump if \neq .
- Branch: adds $\text{imm} \times 4$ to $\$PC = \$PC + 4$ on branch. Range: $\pm 2^{15}$ instructions
- `j label/imm`: jump unconditionally.
- Pseudo-direct: takes 4 MSBs from $\$PC + 4$, and 2 LSB omitted since instructions are word-aligned. Range: 256-byte boundary of $\$PC + 4$ 4 MSBs
- `slt $rd, $rs, $rt`: set `$rd` to 1 if $\$rs < \rt

Datapath



Clock:

- Single-Cycle: Read + Compute are done within clock period. PC Write on rising clock edge. Time taken depends on longest instruction.



- Multicycle: Break up execution into multiple steps (e.g. fetch, decode, execute, memory, writeback), with each step taking one cycle. Time taken depends on slowest step.



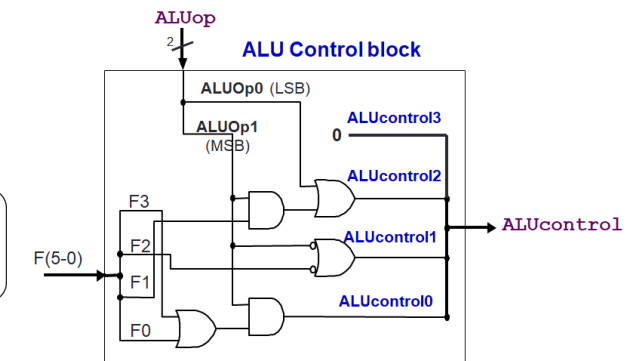
Critical Paths:

- R-type/Immediate:
Inst Mem \rightarrow RegFile \rightarrow MUX (ALUSrc) \rightarrow ALU \rightarrow MUX (MemToReg) \rightarrow RegFile
- `lw/sw`:
Inst Mem \rightarrow RegFile \rightarrow ALU \rightarrow DataMem \rightarrow MUX (MemToReg) \rightarrow RegFile
- `beq`:
Inst Mem \rightarrow RegFile \rightarrow MUX (ALUSrc) \rightarrow ALU \rightarrow AND \rightarrow MUX (PCSrc)

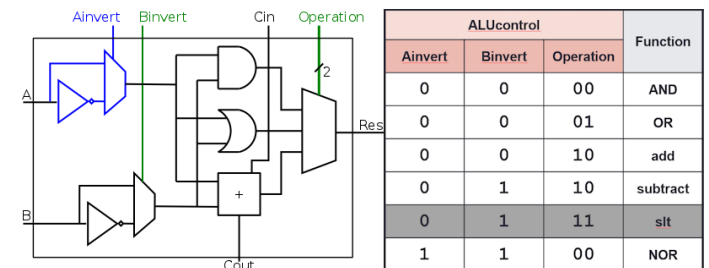
Control

Control Signals:

- RegDst @Decode:** Selects destination register.
(0) \rightarrow `$rt` (1) \rightarrow `$rd`
- RegWrite @Decode:** Enable register write.
(0) \rightarrow no write (1) \rightarrow WD written to WR
- ALUSrc @ALU:** Second 2nd ALU operand.
(0) \rightarrow RD2 (1) \rightarrow SignExt(Imm)
- ALUcontrol @ALU:** Second ALU operation.
2-bit ALUop from opcode + funct for 4-bits
- MemRead @Memory:** Enable memory read.
(0) \rightarrow no read (1) \rightarrow read Addr. into Read Data
- MemWrite @Memory:** Enable memory write.
(0) \rightarrow no write (1) \rightarrow write Write Data into Addr.
- MemToReg @Writeback:** Select writeback result.
(0) \rightarrow ALU result (1) \rightarrow Memory Read data
- Branch @Memory:** Select next $\$PC$.
(0) \rightarrow $\$PC + 4$ (1) \rightarrow $(\$PC + 4) + (\text{imm} \times 4)$

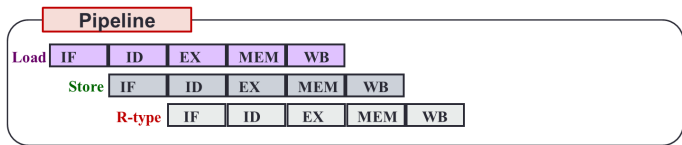


1-bit ALU



Pipelining

Pipelining improves throughput of workload by running instructions in parallel, with each pipeline stage taking one cycle with no overlapping of same stage.



Pipelining Stages:

- i. **IF** (Fetch): Fetch instruction, PC + 4
- ii. **ID** (Decode): Opcode, register file read
- iii. **EX** (Execute): ALU execution or branch calc
- iv. **MEM** (Memory Access): **lw/sw** access memory
- v. **WB** (Writeback): Write result to register file

Pipeline Registers:

- i. **IF/ID**: Instruction, PC+4
- ii. **ID/EX**: RD1/2, Imm, PC+4, WriteReg
- iii. **EX/MEM**: ALU result, (PC+4)+(imm×4), RD2, WriteReg
- iv. **MEM/WB**: ALU result, Mem Read Data, WriteReg

Performance

Time for I instructions, for T_k stage time, under N stages:

- i. Single-Cycle:
 - $CT_{seq} = \max(\sum_k^N = 1T_k)$
 - $Time_{seq} = I \times CT_{seq}$
- ii. Multi-Cycle:
 - $CT_{multi} = \max(T_k)$
 - $Time_{multi} = I \times AverageCPI \times CT_{multi}$
- iii. Pipeline:
 - $CT_{pipeline} = \max(T_k) + t_d$, with overhead t_d
 - $Time_{pipeline} = Cycles \times CT_{pipeline}$
 - Ideal Cycles = $I + N - 1$

Ideally, speedup over single-cycle is $\frac{Time_{seq}}{Time_{pipeline}} \approx N$

Hazards

Structural: Conflict in shared resource (e.g. MEM)

Data: Data dependencies (e.g. read-after-write; RAW)

- i. Forwarding: EX/MEM or MEM/WB → EX
- ii. Stalling: For **lw** delay

Control: Branch taken not known till EX stage, worst case 3-cycle stall

- i. Early branch: Branch check at ID for 1-cycle stall
- ii. Prediction: Assume all branches not taken, flush pipeline if wrong
- iii. Delayed branch: Move instructions executed regardless of branch outcome into branch delay slots (1 per stall)

Forwarding Paths

RAW:

- i. EX/MEM→EX
- ii. EX/MEM→MEM (**xx**→**sw**)

Memory:

- i. MEM→MEM (**lw**→**sw**)

Early Branch:

- i. EX/MEM→ID

Common Delays

Instruction	Delay
RAW	+2
Load Word	+2
Branch at MEM	+3
Branch at ID	+1

Instruction	Delay
RAW	+0
Load Word	+1
Branch at MEM	+3
Branch at ID	+1
RAW + Branch at ID	+1
Load + Branch at ID	+2

4. Boolean Algebra

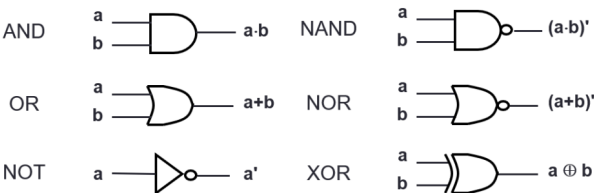
Standard forms:

- i. Literal: boolean variable or it's complement
- ii. Product term: logical product (AND) of literals
- iii. Sum term: logical sum (OR) of literals
- iv. Sum-of-Products: logical sum of product terms
- v. Product-of-Sums: logical product of sum terms

Canonical forms:

- i. Minterm (of n variables): prod. term of n literals. Denoted m_0 to $m[2^n - 1]$; e.g. $m_3 = x' \cdot y \cdot z$
- ii. Maxterm (of n variables): sum term of n literals; Denoted M_0 to $M[2^n - 1]$; e.g. $M_3 = X + Y' + Z'$
- iii. Sum-of-Minterms: canonical SOP expression e.g. $\sum m(0, 1, 2, 3) = m_0 + m_1 + m_2 + m_3$
- iv. Product-of-Maxterms: canonical POS expression e.g. $\prod M(4, 5, 6, 7) = M_4 \cdot M_5 \cdot M_6 \cdot M_7$

Logic Gates



Complete sets of logic are sufficient for any boolean function: {AND, OR, NOT}, {NAND}, {NOR}

SOP Circuit: 2-level AND-OR, or 2-level NAND.
POS Circuit: 2-level OR-AND, or 2-level NOR.

Karnaugh Maps

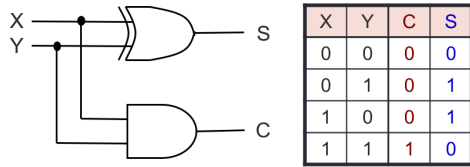
Implicants are product terms covering minterms:

- 1. Prime implicant (PI): implicant of maximum size
- 2. Essential prime implicant (EPI): PI including minterm not covered by any other PI

5. Combinational Circuits

Combinational circuit outputs depend only on inputs.

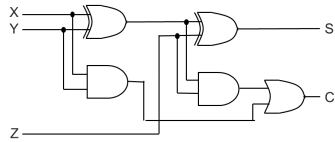
Half Adder



$$C = X \cdot Y$$

$$S = X' \cdot Y + X \cdot Y' = X \oplus Y$$

Full Adder



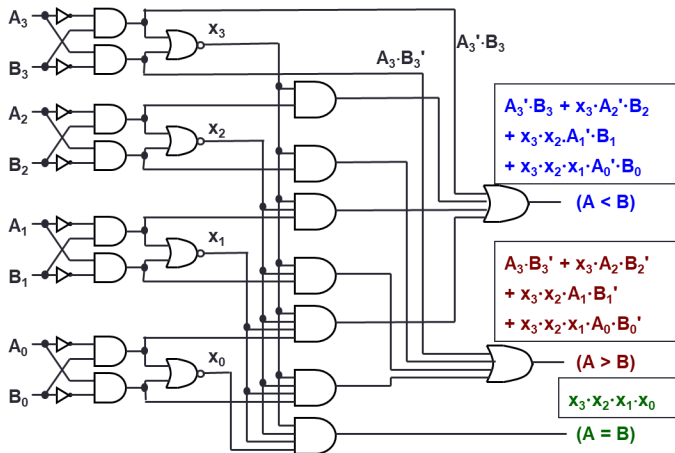
$$C = X \cdot Y + X \cdot Z + Y \cdot Z = X \cdot Y + (X \oplus Y) \cdot Z$$

$$S = X \oplus Y \oplus Z$$

$$= X' \cdot Y \cdot Z + X \cdot Y' \cdot Z + X \cdot Y \cdot Z' + X \cdot Y \cdot Z$$

Magnitude Comparator

Let $A = A_3A_2A_1A_0$, $B = B_3B_2B_1B_0$; $x_i = A_i \cdot B_i + A'_i \cdot B'_i$



Circuit Delays

Inputs with delays t_1, \dots, t_n into gate with delay t will have delay $\max(t_1, \dots, t_n) + t$.

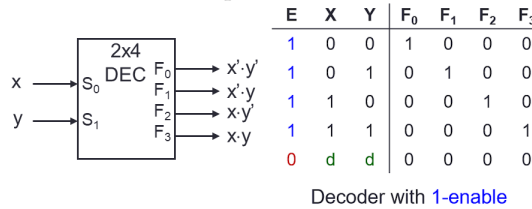
Delay of n -bit parallel adder, t is delay of full adders:

$$S_n = ((n-1)2 + 2)t, \quad C_{n+1} = ((n-1)2 + 3)t$$

MSI Components

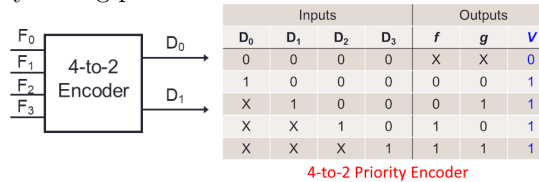
Decoders (n to 2^n)

Output line are minterms, and circuits can be represented with sum of decoder outputs.

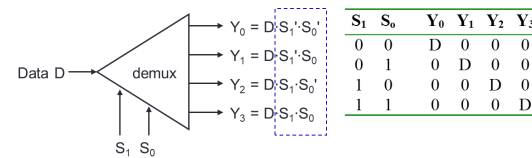


Encoders (2^n to n)

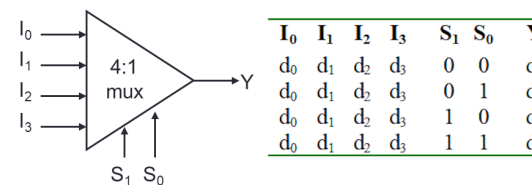
Priority encoders allow multiple inputs on, with highest priority taking precedence.



Demultiplexer (data decoders)



Multiplexer (data encoders)

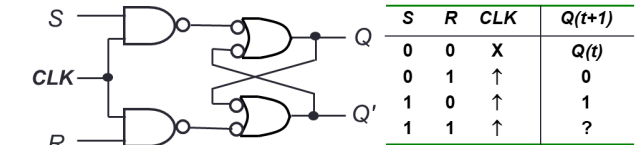


6. Sequential Circuits

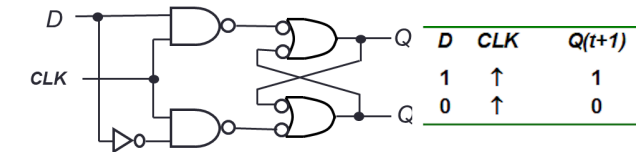
Sequential circuit outputs depend on inputs and state:

- Latches pulse-triggered; flip-flops edge-triggered
- Gated circuits are enable-input
- Self-correcting if unused states \rightarrow used states

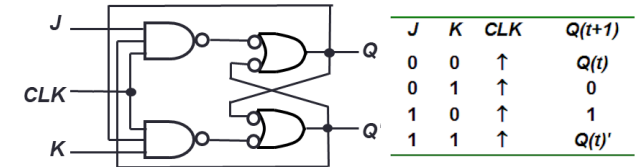
SR Flip-Flop



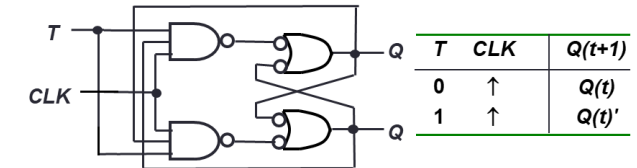
D Flip-Flop



JK Flip-Flop



T Flip-Flop



Excitation

Q	Q'	J	K	Q	Q'	S	R	Q	Q'	D	Q	Q'	T
0	0	0	X	0	0	0	X	0	0	0	0	0	0
0	1	1	X	0	1	1	0	0	1	1	0	1	1
1	0	X	1	1	0	0	1	1	0	0	1	0	1
1	1	X	0	1	1	X	0	1	1	1	1	1	0

JK Flip-flop SR Flip-flop D Flip-flop T Flip-flop

7. Cache

Caches reduce frequency of slow direct memory access in DRAM (slow & cheap) by using SRAM (fast & expensive).

- i. Hit: data in cache
 - Rate: fraction of accesses that are hits
 - Time: time to access cache data
- ii. Miss: data not in cache
 - Penalty: time to replace cache block + hit time

Average Access Time

$= \text{Hit rate} \times \text{Hit Time} + (1 - \text{Hit rate}) \times \text{Miss penalty}$

Principle of Locality:

- i. Temporal: Same data likely re-accessed soon.
- ii. Spatial: Nearby data likely be assessed soon.

Types of Misses:

- i. Cold: Due to never being previously allocated.
- ii. Conflict: In direct mapped/set associative cache; due to previous eviction by different block.
- iii. Capacity: In fully associative cache; due to previous eviction as a result of small cache size.

Policies

Read Miss Policy:

- i. Load data into cache, then from cache to register

Write Policy:

- i. Write-through: Write to both cache and memory.
- ii. Write-back: Write to cache and set dirty bit; write to memory on block eviction and reset dirty bit

Write Miss Policy (data to write not in cache):

- i. Write Allocate: Cache data and follow write policy.
- ii. Write Around: Write directly to memory only.

Block Replacement Policy:

- i. Least Recently Used (LRU; most common)
- ii. First In First Out (FIFO)
- iii. Random Replacement (RR)
- iv. Least Frequently Used (LFU)

Addresses can be grouped by block number:

- i. 2^N bytes in block $\rightarrow N$ offsets



Direct Mapped Cache

Blocks mapped to location index in cache:

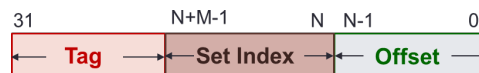
- i. 2^M cache blocks $\rightarrow M$ indexes
- ii. Block stored with identification tag and valid bit



k-way Associative Cache

Blocks mapped to k locations at set index in cache:

- i. 2^M cache sets (blocks / k) $\rightarrow M$ set indexes
- ii. Block stored with identification tag and valid bit



Fully Associative Cache

Blocks placed anywhere but need to search all blocks



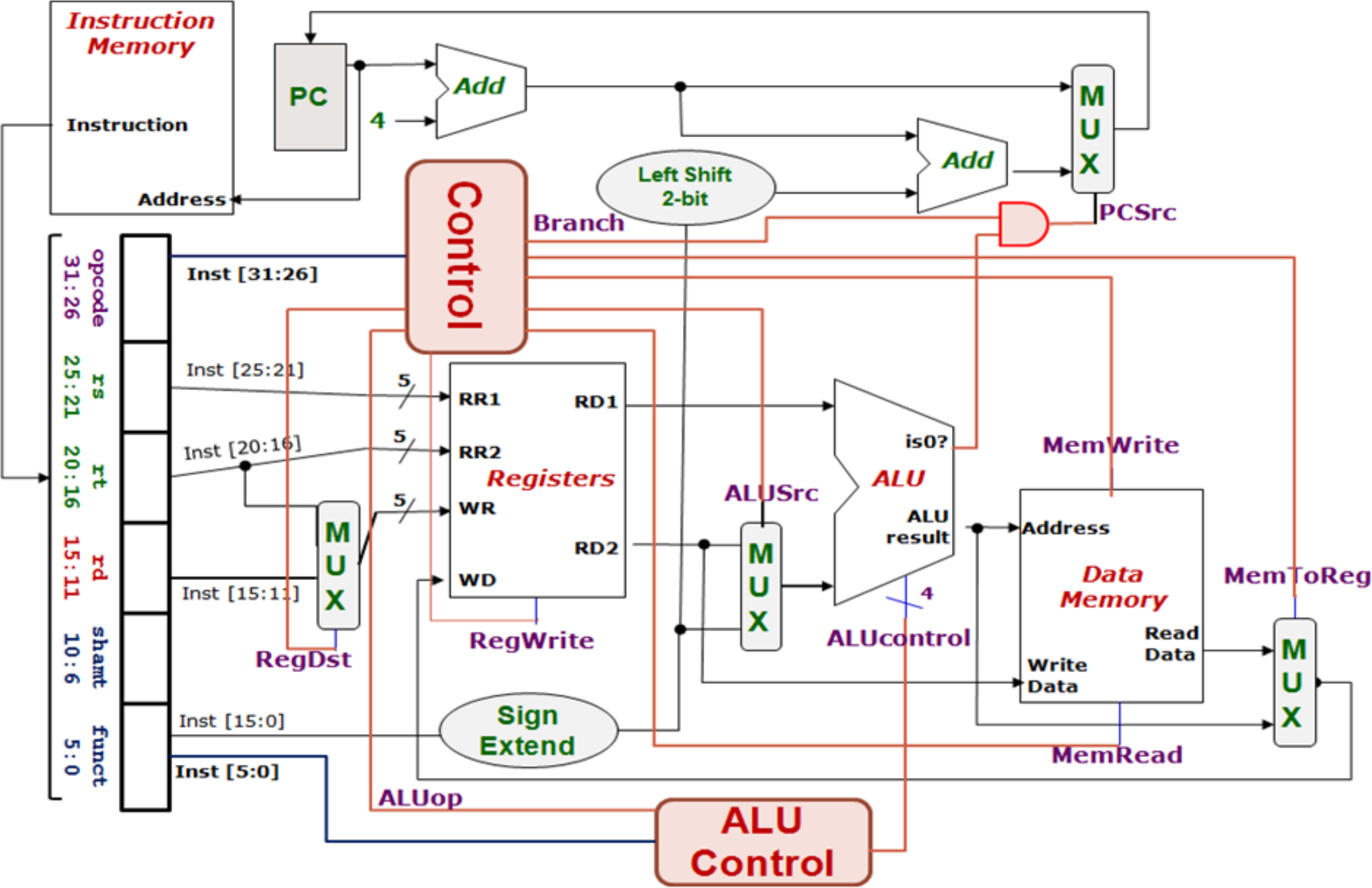
Performance

Direct mapped cache of size N has almost same miss rate as 2-way associative cache of size $N/2$.

Miss Efficiencies:

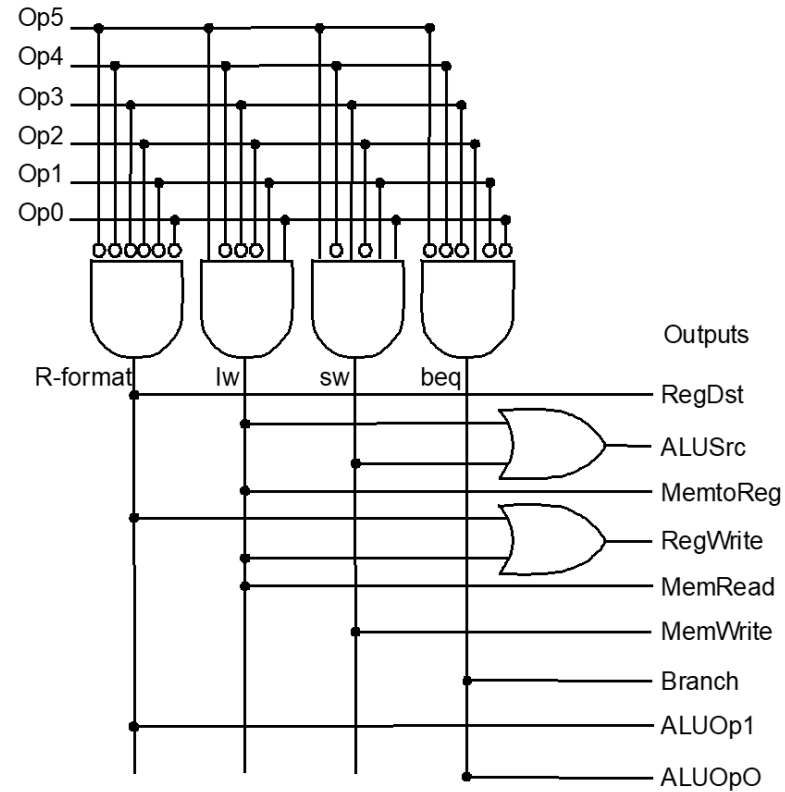
- i. Cold miss does not depend on size/associativity
- ii. Conflict miss decreases with increasing associativity
- iii. Capacity miss does not depend on associativity but decreases with increasing size

Reference Sheet



	Opcode					
	Op5	Op4	Op3	Op2	Op1	Op0
R-type	0	0	0	0	0	0
lw	1	0	0	0	1	1
sw	1	0	1	0	1	1
beq	0	0	0	1	0	0

Inputs



	RegDst	ALUSrc	MemTo Reg	Reg Write	Mem Read	Mem Write	Branch	ALUOp	
								op1	op0
R-type	1	0	0	1	0	0	0	1	0
lw	0	1	1	1	1	0	0	0	0
sw	X	1	X	0	0	1	0	0	0
beq	X	0	X	0	0	0	1	0	1

	EX Stage				MEM Stage			WB Stage	
	RegDst	ALUSrc	ALUOp		Mem Read	Mem Write	Branch	MemTo Reg	Reg Write
			op1	op0					
R-type	1	0	1	0	0	0	0	0	1
lw	0	1	0	0	1	0	0	1	1
sw	X	1	0	0	0	1	0	X	0
beq	X	0	0	1	0	0	1	X	0

Opcode	ALUOp	Instruction Operation	Func field	ALU action	ALU control
lw	00	load word	xxxxxx	add	0010
sw	00	store word	xxxxxx	add	0010
beq	01	branch equal	xxxxxx	subtract	0110
R-type	10	add	10 0000	add	0010
R-type	10	subtract	10 0010	subtract	0110
R-type	10	AND	10 0100	AND	0000
R-type	10	OR	10 0101	OR	0001
R-type	10	set on less than	10 1010	set on less than	0111

Instruction Type	ALUOp
lw / sw	00
beq	01
R-type	10

ALUcontrol	Function
0000	AND
0001	OR
0010	add
0110	subtract
0111	slt
1100	NOR

ASCII Table

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	\$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(72	48	110	H	104	68	150	h
9	9	11		41	29	51)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	
29	1D	35		61	3D	75	=	93	5D	135]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

Boolean Algebra Laws		
Identity	$A + 0 = 0 + A = A$	$A \cdot 1 = 1 \cdot A = A$
Complement	$A + A' = 1$	$A \cdot A' = 0$
Commutative	$A + B = B + A$	$A \cdot B = B \cdot A$
Associative	$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Distributive	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$
Idempotency	$X + X = X$	$X \cdot X = X$
One/ Zero Element	$X + 1 = 1$	$X \cdot 0 = 0$
Involution	$(X')' = X$	
Absorption 1	$X + X \cdot Y = X$	$X \cdot (X + Y) = X$
Absorption 2	$X + X' \cdot Y = X + Y$	$X \cdot (X' + Y) = X \cdot Y$
De Morgan's	$(X + Y)' = X' \cdot Y'$	$(X \cdot Y)' = X' + Y'$
Consensus	$X + Y \cdot Z + Y' \cdot Z = X + Y' \cdot Z$	$(X + Y) \cdot (X' + Z) \cdot (Y + Z) = (X + Y) \cdot (X' + Z)$

C Precedence		
Postfix Unary	<code>++, --</code>	Left to right
Prefix Unary	<code>+, -, ++, --, &, *, (type), !</code>	Right to left
Multiplicative Binary	<code>*, /, %</code>	Left to right
Additive Binary	<code>+, -</code>	Left to right
Relational	<code><=, >=, <, ></code>	Left to right
Equality	<code>==, !=</code>	Left to right
Conjunction	<code>&&</code>	Left to right
Disjunction	<code> </code>	Left to right
Assignment	<code>=, +=, -=, *=, /=, %=</code>	Right to left

Prefix Multipliers			
Nano	n	10^{-9}	2^{-30}
Micro	μ	10^{-6}	2^{-20}
Milli	m	10^{-3}	2^{-10}
Kilo	k	10^3	2^{10}
Mega	M	10^6	2^{20}
Giga	G	10^9	2^{30}
Tera	T	10^{12}	2^{40}