CS2040S Recitation 4

AY 24/25 Sem 2 — github/omgeta

Q1. (a.) Yes; rule 1 and rule 3

(b.) Minimum height: $O(\log_a n) + 1$. Maximum height: $O(\log_b n)$

(c.) We can use an array of (key, subtree) pairs.

(d.) $O(\log_a n \cdot \log b) = O(\log_a n) = O(\log n)$

(e.) Internal node: $\{5, 6, 7, 8\}$ splits at median 6 which is moved to the parent
Root node: $\{10, 20, 30\}$ splits at median 20 which is the new root

(f.) Nodes split proactively at $b - 1$ keys so each split child meets $a - 1$ minimum.
After splitting, the leaf has $< b - 1$ keys allowing safe insertion This preserves leaf depth and key ranges.

(g.) $O(\log n)$ as splits can propagate

(h.) Merge: $\{5\}, \{7, 8\}$ split by 6 becomes $\{5, 6, 7, 8\}$
Share: $\{5, 6\}, \{8\}$ split by 7 becomes $\{5, 6, 7, 8\}$ and splits back into $\{5\}, \{7, 8\}$ split by 6
$O(\log n)$ as merges and shares propagate up.

Q2. (a.) $\frac{n}{B}$

(b.) $\log(\frac{n}{B})$

(c.) All $O(1)$

(d.) $O(\log_B n)$