ST2334 Tutorial 8

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. (a); $E(U) = E(X)/n = \frac{np}{n} = p$
(c); Note $E(X^2) = V(X) + [E(X)]^2 = np(1-p) + (np)^2 = np + (n^2 - n)p^2$, so
$E(W) = \frac{E(X^2) - E(X)}{n(n-1)} = \frac{np + n(n-1)p^2 - np}{n(n-1)} = p^2$

Q2. (a); CI $= (\overline{x}_1 - \overline{x}_2) \pm z_{0.03}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n^2}} = (80 - 75) \pm (1.88)\sqrt{\frac{5^2}{25} + \frac{3^2}{36}} = 5 \pm 2.102 = (2.898, 7.102)$

Q3. $E = z_{0.025} \cdot \frac{\sigma}{\sqrt{n}}$ which requires sample size factor of 4 to double

Long Form Questions

Q1. $P(\overline{X} \geq 24) = P(\frac{\overline{X} - 20}{4.1/\sqrt{9}} > \frac{24 - 20}{4.1/\sqrt{9}}) = P(t_8 > 2.9268) \approx 0.00955$ so we can reject the null hypothesis $\mu = 20$ in favour of the alternate hypothesis $\mu > 20$

Q2. (a.) CI: $\overline{y} \pm z_{0.025}\frac{0.75}{\sqrt{20}} = 4.85 \pm 0.3287 = (4.5213, 5.1787)$

(b.) Length $= 2z_{0.025}\frac{0.75}{\sqrt{n}} = 0.4 \implies n = 54$

(c.) CI: $\overline{y} \pm t_{19;0.025}\frac{0.75}{\sqrt{20}} = 4.85 \pm 2.093\frac{0.75}{\sqrt{20}} = 4.85 \pm 0.351 = (4.499, 5.201)$

Q3. CI: $48.5 \pm t_{11;0.05}\frac{1.5}{\sqrt{12}} = 48.5 \pm 0.7777 = (47.722, 49.278)$

Q4. CI: $(\overline{x}_1 - \overline{x}_2) \pm z_{0.01}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (12.2 - 9.1) \pm 2.33\sqrt{\frac{1.1^2}{100} + \frac{0.9^2}{200}} = 3.1 \pm 0.296 = (2.804, 3.396)$. Since it does not cover 0 and is positive, the treatments appears to reduce the mean amount of metal removed

Q5. (a.) CI: $\overline{x} \pm E = 87 \pm 10 = (77, 97)$

(b.) $E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ so to reduce error by factor of 4, we must increase sample size by 16, giving $n = 36 \cdot 16 = 576$

(c.) Since the 95% confidence CI contains 80, we do not reject the null hypothesis.

Q6. (a.) CI: $4.5 \pm z_{0.025}\frac{0.75}{\sqrt{49}} = 4.5 \pm 0.21 = (4.29, 4.71)$

(b.) No, the CI contains 4.3, so we cannot reject the null hypothesis.