ST2334 Tutorial 6

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. (b); Success probability, $p = P((HHH, TTT)') = 1 - (\frac{1}{8} + \frac{1}{8}) = \frac{3}{4}$
Let X be the number of experiments required to get such a success. $X \sim Geom(\frac{3}{4})$ so $P(X \leq x) = 1 - (\frac{1}{4})^x$

Q2. (a); $P(\min X_i > t) = P(X_1 > t, X_2 > t, \cdots, X_n > t) = \Pi_{i=1}^{n} e^{-\lambda t} = e^{-n\lambda t} \implies Exp(n\lambda)$

Q3. $P(X_1 = x \mid X_1 + X_2 = 10) = \frac{P(X_1 = X, X_1 + X_2 = 10)}{P(X_1 + X_2 = 10)} = \frac{P(X_1 = x, X_2 = 10 - x)}{P(X_1 + X_2 = 10)} = \frac{(e^{-2}2^x/x!)(e^{-3}3^{10-x}/(10-x)!)}{e^{-5}5^10/10!} = \frac{10!}{x!(10-x)!} \cdot \frac{2^x 3^{10-x}}{5^10} = \binom{10}{x}(\frac{2}{5})^x(1 - \frac{2}{5})^{10-x}$ which is the $Bin(10, 2/5)$ probability function.
Therefore, $E(X_1 \mid X_1 + X_2 = 10) = 10(\frac{2}{5}) = 4$

Q4. $P(T \geq 10 \mid T > 9) = P(T \geq 1) = e^{-0.5}$

Long Form Questions

Q1. Let $N \sim Poisson(5)$

(i) $P(N = 0) = e^{-5} \approx 0.0067$

(ii) $P(N > 10) = 1 - e^{-5} \sum_{k=0}^{10} \frac{5^k}{k!} \approx 0.0137$

(iii) $N' \sim Poisson(15) \implies P(N' > 20) = 1 - e^{15} \sum_{k=0}^{20} \frac{15^k}{k!}$

Q2. Let $X \sim Bin(10000, 0.0005)$

(i) $E(X) = 10000 \cdot 0.0005 = 5$ and $Var(X) = np(1-p) = 5 \cdot (0.9995) \approx 4.9975$

(ii) Approximating $X \sim Poisson(\lambda = 5)$, then $P(X \geq 10) \approx 1 - P(X \leq 9) \approx 0.0318$

(iii) Similarly, $P(X = 0) \approx e^{-5}$

Q3. (i) $\frac{2}{3}$

(ii) $\frac{5}{15} = \frac{1}{3}$

Q4. Let $X \sim Exp(\frac{1}{4})$

(i) $P(X > 3) = e^{-\frac{3}{4}} \approx 0.4724$

(ii) $P(X < 3) = 1 - e^{-\frac{3}{4}} \approx 0.5276$

(iii) Let $Y \sim Bin(6, 0.5276)$ then $P(Y \geq 4) = \sum_{k=4}^{6} \binom{6}{k} p^k (1-p)^{6-k} \approx 0.3968$

Q5. Let $X \sim Exp(\frac{1}{25000})$

(i) $P(X \geq 20000) = e^{-\frac{20000}{25000}} \approx 0.4493$ and $P(X \leq 30000) = 1 - e^{-\frac{30000}{25000}} \approx 0.6988$ so $P(20000 \leq X \leq 30000) = 0.1481$

(ii) $P(X > 75000) = e^{-\frac{75000}{25000}} \approx 0.0498$