

CS3230 Tutorial 8

AY 25/26 Sem 1 — github/omgeta

- Q1). (b); Only total bank balance needs to be non-negative over Q operations
- Q2). Charge $\$3 \in O(1)$ for every insertion, where $\$1$ is the cost of insertion, and $\$2$ is stored in the bank. Every time we double in size, we use $\$1$ for the item to copy itself over and $\$1$ for another item, so the bank is non-negative. Since actual cost \leq am. cost, then actual cost is $O(1)$
- Q3). **ENQUEUE(x)**: charge $\$2 \in O(1)$ where $\$1$ is cost of addition, and $\$1$ is stored in the bank for later
DEQUEUE(): charge $\$0 \in O(1)$ by using the stored $\$1$ in the bank for the item to remove
DELETE(k): charge $k\$0 = 0 \in O(1)$ by using the stored $\$1$ s in the bank for each item removal
ADD(A): enqueue $|A|$ items for charge $\$2|A| \in O(|A|)$ cost
- Q4). Let $\Phi = |Q|$,
ENQUEUE(x): actual cost = 1, $\Delta\Phi = +1 \implies$ am. cost $2 \in O(1)$
DEQUEUE(): actual cost = 1, $\Delta\Phi = -1 \implies$ am. cost $0 \in O(1)$
DELETE(k): actual cost = k , $\Delta\Phi = -k \implies$ am. cost $0 \in O(1)$
ADD(A): actual cost = $|A|$, $\Delta\Phi = +|A| \implies$ am. cost $2|A| \in O(|A|)$
- Q5). Let $\Phi = \text{size}(T) - \text{items}(T) \geq 0$,
Case 1 (no shrinkage): actual cost = 1, $\Delta\Phi = +1 \implies$ am. cost $2 \in O(1)$
Case 2 (shrinkage): actual cost = $1 + n$, $\Delta\Phi = (n - n) - (2n - (n - 1)) = 1 - n \implies$ am. cost $2 \in O(1)$