CS1231S Tutorial 1

AY 24/25 Sem 1 — github/omgeta

Q1. Let p be "I use the umbrella", q be "it rains"

(a) "I use the umbrella if it rains" $\equiv q \rightarrow p$
"I use the umbrella only if it rains" $\equiv p \rightarrow q$

(b) In "I use the umbrella if it rains":
"it rains" is sufficient for "I use the umbrella"
"I use the umbrella" is necessary for "it rains"

(c) "I use the umbrella if and only if it rains"$\equiv (q \rightarrow p) \wedge (p \rightarrow q) \equiv p \leftrightarrow q$

(d) In "I use the umbrella if and only if it rains":
"I use the umbrella" is a necessary and sufficient condition for "it rains"

Q2. The students do not preserve the brackets after applying De Morgan's law which makes the logical statement ambiguous as $\wedge$ and $\vee$ have equal precedence

(a) $a \wedge {\sim}(b \wedge c) \equiv a \wedge ({\sim}b \vee {\sim}c)$

(b) ${\sim}(x \vee y) \vee z \equiv ({\sim}x \wedge {\sim}y) \vee z$

Q3. (a) ${\sim}a \wedge ({\sim}a \rightarrow (b \wedge a))$
$\equiv {\sim}a \wedge ({\sim}({\sim}a) \vee (b \wedge a))$ (Implication law)
$\equiv {\sim}a \wedge (a \vee (b \wedge a))$ (Double negation law)
$\equiv {\sim}a \wedge (a \vee (a \wedge b))$ (Commutative law)
$\equiv {\sim}a \wedge (a)$ (Absorption law)
$\equiv$ false (Negation law)

(b) $(p \vee {\sim}q) \rightarrow q$
$\equiv {\sim}(p \vee {\sim}q) \vee q$ (Implication law)
$\equiv ({\sim}p \wedge q) \vee q$ (DeMorgan's law)
$\equiv q \vee (q \wedge {\sim}p)$ (Commutative law)
$\equiv q$ (Absorption law)

(c) ${\sim}(p \vee {\sim}q) \vee ({\sim}p \wedge {\sim}q)$
$\equiv ({\sim}p \wedge q) \vee ({\sim}p \wedge {\sim}q)$ (DeMorgan's law)
$\equiv {\sim}p \wedge (q \vee {\sim}q)$ (Distributive law)
$\equiv {\sim}p \wedge true$ (Negation law)
$\equiv {\sim}p$ (Identity law)

(d) $(p \rightarrow q) \rightarrow r$
$\equiv ({\sim}p \vee q) \rightarrow r$ (Implication law)
$\equiv ({\sim}({\sim}p \vee q)) \vee r$ (Implication law)
$\equiv (p \wedge {\sim}q) \vee r$ (DeMorgan's law)

Q4. Since the truth tables do not match, $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
F	T	T	T	T	T	T
T	F	F	F	T	T	T
F	T	F	T	F	**F**	**T**
F	F	T	T	T	T	T
F	F	F	T	T	**F**	**T**

Q5. Let p be $12x - 7 = 29$, and q be $x = 3$
Original: $p \to q$
Negation: $p \wedge {\sim}q$
Contrapositive: ${\sim}q \to {\sim}p$
Converse: $q \to p$
Inverse: ${\sim}p \to {\sim}q$

Suppose $12x - 7 = 29$, then $12x = 36$ and $x = 3$, which matches the conclusion. Therefore, the conditional statement is true.

Suppose $x = 3$, then indeed $12x - 7 = 29$, which matches the conclusion. Therefore, the converse statement is also true.

No, the converse and the inverse are logically equivalent because they are contrapositive of each other.

p	q	$(q \to p)$	$({\sim}p \to {\sim}q)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

Q6. Alternative 1, it is evident that the transitive rule of inference does not hold

p	q	r	$p \to_a q$	$q \to_a r$	$p \to_a r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
F	T	T	F	T	F
T	F	F	F	F	F
F	F	T	F	F	F
F	T	F	F	F	F
F	F	F	F	F	F

Alternative 2, it is evident that the transitive rule of inference does not hold

p	q	r	$p \to_b q$	$q \to_b r$	$p \to_b r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
F	T	T	T	T	T
T	F	F	F	F	F
F	F	T	F	T	T
F	T	F	T	F	F
F	F	F	F	F	F

Alternative 3, it is evident that the transitive rule of inference does not hold

p	q	r	$p \to_c q$	$q \to_c r$	$p \to_c r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
F	T	T	F	T	F
T	F	F	F	T	F
F	F	T	T	F	F
F	T	F	F	F	T
F	F	F	T	T	T

Q7. (a) Let p be "Sandra knows Java", q be "Sandra knows C++"
$p \wedge q$
$\therefore q$ (By specialization)

(b) Let p be "at least one of these two numbers is divisible by 6", q be "product of these two numbers is divisible by 6"
$p \to q$
$\sim p$
$\therefore \sim q$ (Inverse error)

(c) Let p be "there are as many rational numbers as there are irrational numbers", q be "the set of all irrational numbers is infinite"
$p \to q$
q
$\therefore p$ (Converse error)

(d) Let p be "I get a Christmas bonus", q be "I sell my motorcycle", r be "I'll buy a stereo"
$p \to r$
$q \to r$
$\therefore (p \vee q) \to r$ (By construction)

Q8. (a) $\sim p \implies p =$ false
Since $p \vee (q \wedge q) =$ true and $p =$ false, $(q \wedge r)$ must be true $\implies q = r =$ true
Therefore, the conclusion is also true, and the argument is valid.

(b) Let $p =$ true, $q =$ false, $r =$ false
Premise 1: $p \vee (q \wedge r)$is true
Premise 2: $\sim(p \wedge q)$ is true
Conclusion: r is false Which shows that the argument is not valid

(c) Let p be "I go to the beach", q be "I will take my shades", r be "I will take my sunscreen"
$p \to (q \vee r)$
q
$\sim r$
$\therefore p$ (Converse error, therefore the argument is invalid)

(d) Let p be "I will buy a new goat", q be "I will buy a used Yugo", r be "I will need a loan"
$p \vee q$
$(p \wedge q) \to r$
$q \wedge \sim r$
$\therefore \sim p$
The argument is valid

Q9. (a) **Proof (by contradiction).**

1. If A is a knight, then:

   1.1 What A says is true. (by definition of knight)
   1.2 $\therefore B$ is a knight too. (that's what A says)
   1.3 $\therefore$ What B says is true. (by definition of knight)
   1.4 $\therefore A$ is a knave. (that's what B says)
   1.5 $\therefore A$ is not a knight.
   1.6 $\therefore$ Contradiction to 1.

2. $\therefore A$ is not a knight.
3. $\therefore A$ is a knave. (since A is either a knight or a knave, but not both)
4. $\therefore$ What B says is true.
5. $\therefore B$ cannot be a knave. (as B has said something true)
6. $\therefore B$ is a knight. (one is a knight or a knave)

(b) **Proof (by exhaustion).**

1. If C is a knight, then:

   1.1 What C says is true. (by definition of knight)
   1.2 $\therefore$ D is a knave. (that's what C says)
   1.3 $\therefore$ What D says is false. (by definition of knave)
   1.4 $\therefore$ C is not a knave. (that's what D says)
   1.5 $\therefore$ C is a knight.
   1.6 $\therefore$ there is no contradiction.
   1.7 $\therefore$ there is 1 knight and 1 knave.

2. If C is a knave, then:

   1.1 What C says is false. (by definition of knave)
   1.2 $\therefore$ D is not a knave. (that's what C says)
   1.3 $\therefore$ D is a knight.
   1.4 $\therefore$ What D says is true. (by definition of knight)
   1.5 $\therefore$ C is a knave. (that's what D says)
   1.6 $\therefore$ there is no contradiction.
   1.7 $\therefore$ there is 1 knight and 1 knave.

3. $\therefore$ there is always 1 knight and 1 knave. (in both cases)

Q10. Let $x = 2n + 1, y = 2m + 1$ be two odd numbers,

$$\begin{aligned} x \cdot y &= (2n+1) \cdot (2m+1) \\ &= 4mn + 2n + 2m + 1 \\ &= 2(2mn + m + n) + 1 \\ &= 2k + 1, \text{ where } k = 2mn + m + n \in \mathbb{Z} \\ &\text{is odd, by definition of odd numbers} \end{aligned}$$