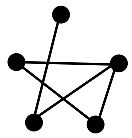
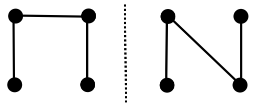
CS1231S Tutorial 11

AY 24/25 Sem 1 — github/omgeta

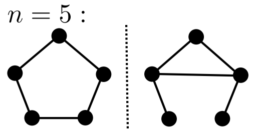
Q1. (a)



(b) $n = 4$:



$n = 5$:



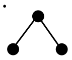
For $n = 3, 6$, K_n has odd edges and cannot be divided into two halves ■

Q2. $4 \times 3 = 12$ ■

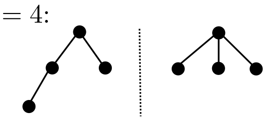
Q3. (a) $n = 1$:

$n = 2$:

$n = 3$:



$n = 4$:

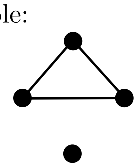


(b) $n = 1$ has 1, $n = 2$ has 1, $n = 3$ has $\frac{3!}{2} = 3$, $n = 4$ has $\frac{4!}{2} + 4 = 12 + 4 = 16$ ■

Q4. (a)

1. Suppose $G = (V, E)$ is a connected, simple, undirected graph
2. There is spanning tree $T = (V, E')$, where $E' \subseteq E$ (Proposition 10.7.1)
3. Since T is a tree, $|E'| = |V| - 1$ (Theorem 10.5.2)
4. Therefore, $|E| \geq |V| - 1$ ■

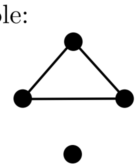
(b) No; Counterexample:



Q5. (a)

1. Suppose $G = (V, E)$ is an acyclic, simple, undirected graph
2. Take all complete subgraphs of G, $H_1(V_1, E_1), \cdots, H_n(V_n, E_n)$ which also form trees
3. Then, $|E_1| = |V_1 - 1|, \cdots, |E_n| = |V_n| - 1$ (Theorem 10.5.2)
4. $|E| = |E_1| + \cdots + |E_n| = |V| - n$
5. Therefore, $|E| \leq |V| - 1$ ■

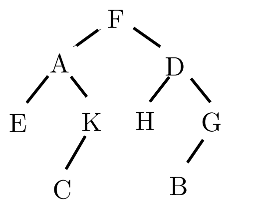
(b) No; Counterexample:



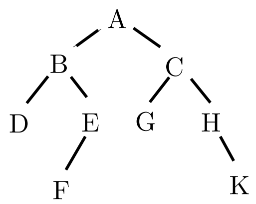
Q6.
1. Prove G is tree $\rightarrow$ there is exactly one path between every pair of vertices:
   - 1.1. Suppose $G = (V, E)$ is a tree
   - 1.2. G is connected and acyclic (Definition of tree)
   - 1.3. Any two vertices have a path between them (Definition of connected)
   - 1.4. Suppose there are vertices with two or more paths connecting them:
     - 1.4.1. Then, G is cyclic (Lemma 10.5.5)
     - 1.4.2. This contradicts 1.2
   - 1.5. Hence, the supposition is false, and there is exactly one path between every pair of vectors
2. Prove there is exactly one path between every pair of vertices $\rightarrow$ G is a tree:
   - 2.1. Suppose $G = (V, E)$ is a graph with exactly one path between every pair of vertices
   - 2.2. G is connected (Definition of connected)
   - 2.3. Suppose G is cyclic:
     - 2.3.1. There is a cycle C in G (Definition of cyclic)
     - 2.3.2. Any two vertices in C have two paths connecting them
     - 2.3.3. This contradicts 2.1
   - 2.4. Hence, the supposition is false, and G is acyclic
   - 2.5. G is a tree (Definition of tree)
3. $\therefore$ G is a tree $\leftrightarrow$ there is exactly one path between every pair of vectors $\blacksquare$

Q7.
1. Suppose $G = (V, E)$ is a graph where each complete subgraph is a group of stones
2. Initially, we have K_n which has $\frac{n(n-1)}{2}$ edges
3. With each splitting into complete subgraphs of k_1, k_2 vertices, we remove $k_1 \times k_2$ edges
4. Finally, we stop when there are exactly n subgraphs, each with 1 vertex with no edges
5. At this point, we will have removed all $\frac{n(n-1)}{2}$ edges, which is also the maximum earnt $\blacksquare$

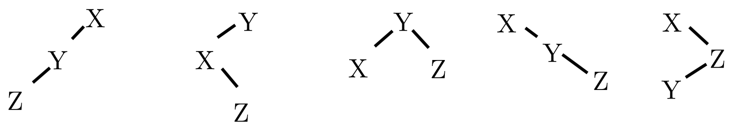
Q8.
(a) Post-order: E C K A H B G D F



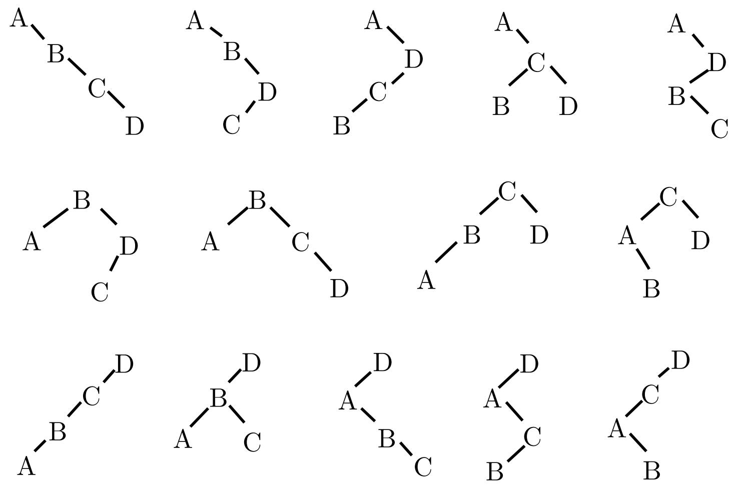
(b) Pre-order: A B D E F C G H K



Q9. (a)



(b)



Q10.