MA1522 Tutorial 11

AY 24/25 Sem 1 — github/omgeta

Q1. (a) Yes; Standard matrix $= \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

Basis for range $= \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$; Basis for kernel $= \phi$ ■

(b) Not a linear transformation. ■

(c) Yes; Standard matrix $= \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

Basis for range $= \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$; Basis for kernel $= \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$ ■

(d) Not a linear transformation. ■

(e) Yes; Standard matrix $= \begin{pmatrix} 0 & 0 & 1 & 2 & -1 \end{pmatrix}$

Basis for range $= \{1\}$; Basis for kernel $= \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ ■

(f) Not a linear transformation. ■

Q2. (a) $A_F = \begin{pmatrix} 1 & -2 & 0 \\ 1 & 1 & -3 \\ 0 & 5 & -1 \end{pmatrix}$ and $B_G = \begin{pmatrix} -1 & 0 & 1 \\ 5 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ ■

(b) Yes, with the standard matrix $\begin{pmatrix} 0 & -2 & 1 \\ 6 & 2 & -3 \\ 1 & 6 & 0 \end{pmatrix}$ ■

(c) Standard matrix is $\begin{pmatrix} -11 & -2 & 1 \\ 1 & -2 & -2 \\ 24 & 4 & -1 \end{pmatrix}$ ■

(d) H is the transformation with standard matrix $B_G^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{5}{3} & -\frac{2}{3} & \frac{5}{3} \\ \frac{4}{3} & \frac{1}{3} & -\frac{1}{3} \end{pmatrix}$ ■

Q3. (a) Yes; standard matrix is $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 2 & 4 \\ 0 & -1 & 1 \\ 1 & 4 & 6 \end{pmatrix}$ $\blacksquare$

(b) Yes; since $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2}(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix})$ then standard matrix is $\begin{pmatrix} \frac{1}{2}T\begin{pmatrix} 2 \\ 0 \end{pmatrix} & \frac{1}{2}(T\begin{pmatrix} 1 \\ 1 \end{pmatrix} - T\begin{pmatrix} 1 \\ -1 \end{pmatrix}) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ $\blacksquare$

(c) No; given input vectors are linearly dependent and do not span $\mathbb{R}_{63}$

Q4. (a) nullity$(T) = 4 - 4 = 0$, so T is one-to-one but not onto $\blacksquare$

(b) rank$(T) = 6 - 2 = 4$, so T is onto but not one-to-one $\blacksquare$

(c) rank$(T) = 3$, nullity$(T) = 4 - 3 = 1$, so T is neither one-to-one or onto $\blacksquare$

(d) nullity$(T) = 0$, rank$(T) = 3 - 0 = 3$ so T is one-to-one and onto $\blacksquare$