MA1521 Homework 10

AY 24/25 Sem 1 — github/omgeta

Q1. (a)

$$\begin{aligned}
\int\int_D \frac{2y}{x^2+16} dA &= \int_0^3 \int_0^{\sqrt{x}} \frac{2y}{x^2+16} dydx \\
&= \int_0^3 [\frac{y^2}{x^2+16}]_0^{\sqrt{x}} dx \\
&= \int_0^3 \frac{x}{x^2+16} dx \\
&= \frac{1}{2}[\ln|x^2+16|]_0^3 \\
&= \frac{1}{2}(\ln(25+\ln 16) \\
&= \ln\frac{5}{4} \quad \blacksquare
\end{aligned}$$

(b)

$$\begin{aligned}
\int\int_D x\cos y dA &= \int_0^1 \int_0^{x^2} x\cos y dydx \\
&= \int_0^1 [x\sin y]_0^{x^2} dx \\
&= \int_0^1 x\sin x^2 dx \\
&= \frac{1}{2}\int_0^1 \sin u du & (u = x^2,\ du = 2xdx) \\
&= \frac{1}{2}[-\cos u]_0^1 \\
&= \frac{1}{2}(1-\cos 1) \quad \blacksquare
\end{aligned}$$

Q2. (a)

$$\begin{aligned}
\int_0^1 \int_{x-1}^{1-x} xdydx &= \int_0^1 \int_0^{1-y} xdxdy + \int_{-1}^0 \int_0^{y+1} xdxdy \\
&= \int_0^1 [\frac{1}{2}x^2]_0^{1-y} dy + \int_{-1}^0 [\frac{1}{2}x^2]_0^{y+1} dy \\
&= \frac{1}{2}\int_0^1 (1-y)^2 dy + \frac{1}{2}\int_{-1}^0 (y+1)^2 dy \\
&= \frac{1}{2}[y - y^2 + \frac{1}{3}y^3]_0^1 + \frac{1}{2}[\frac{1}{3}y^3 + y^2 + y]_{-1}^0 \\
&= \frac{1}{6} + \frac{1}{6} \\
&= \frac{1}{3} \quad \blacksquare
\end{aligned}$$

(b)

$$\begin{aligned}
\int_0^8 \int_{\sqrt[3]{y}}^2 (\frac{1}{16} + e^{x^4}) dxdy &= \int_0^2 \int_0^{x^3} (\frac{1}{16} + e^{x^4}) dydx \\
&= \int_0^2 [\frac{1}{16}y + ye^{x^4}]_0^{x^3} dx \\
&= \int_0^2 x^3 (\frac{1}{16} + e^{x^4}) dx \\
&= \frac{1}{4}\int_0^{16} (\frac{1}{16} + e^u) du \qquad (u = x^4,\ du = 4x^3 dx) \\
&= \frac{1}{4}[\frac{1}{16}u + e^u]_0^{16} \\
&= \frac{1}{4}(1 + e^{16} - e^0) \\
&= \frac{e^{16}}{4} \quad \blacksquare
\end{aligned}$$

Q3. At intersection:

$$\begin{aligned}4 - x^2 &= 3x \\ x^2 + 3x - 4 &= 0 \\ (x-1)(x+4) &= 0 \\ x &= -4, 1\end{aligned}$$

Then, find the volume:

$$\begin{aligned}\int_{-4}^{1}\int_{3x}^{4-x^2}(x+4)dydx &= \int_{-4}^{1}[y(x+4)]_{3x}^{4-x^2}dx \\ &= \int_{-4}^{1}(x+4)(4-x^2)-(3x)(x+4)dx \\ &= \int_{-4}^{1}(-x^3-7x^2-8x+16)dx \\ &= [-\frac{1}{4}x^4 - \frac{7}{3}x^3 - 4x^2 + 16x]_{-4}^{1} \\ &= \frac{625}{12} \quad \blacksquare\end{aligned}$$

Q4.

$$\begin{aligned}\int\int_R xy^2 dA &= \int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} xy^2 dxdy \\ &= \int_0^a [\frac{1}{2}x^2y^2]_{a-y}^{\sqrt{a^2-y^2}} dy \\ &= \frac{1}{2}\int_0^a (a^2-y^2)y^2 - (a-y)^2y^2 dy \\ &= \frac{1}{2}\int_0^a y^2(a-y)(a+y-a+y)dy \\ &= \frac{1}{2}\int_0^a 2y^3(a-y)dy \\ &= \int_0^a ay^3 - y^4 dy \\ &= [\frac{a}{4}y^4 - \frac{1}{5}y^5]_0^a \\ &= \frac{a^5}{4} - \frac{a^5}{5} \\ &= \frac{a^5}{20} \quad \blacksquare\end{aligned}$$

Q5.

$$
\begin{aligned}
\int\int_D 2xdA &= \int_0^2 \int_{y^2}^{\frac{3y}{2}+1} 2xdxdy + \int_{-3}^0 \int_{y^2}^{1-\frac{8y}{3}} 2xdxdy \\
&= \int_0^2 [x^2]_{y^2}^{\frac{3y}{2}+1} dy + \int_{-3}^0 [x^2]_{y^2}^{1-\frac{8y}{3}} dy \\
&= \int_0^2 \frac{(3y+2)^2}{4} - y^4 dy + \int_{-3}^0 \frac{(3-8x)^2}{9} - y^4 dy \\
&= \int_0^2 (\frac{9}{4}y^2 + 3y + 1 - y^4)dy + \int_{-3}^0 (1 - \frac{16}{3}y + \frac{64}{9}y^2 - y^4)dy \\
&= [\frac{3}{4}y^3 + \frac{3}{2}y^2 + y - \frac{1}{5}y^5]_0^2 + [y - \frac{8}{3}y^2 + \frac{64}{27}y^3 - \frac{1}{5}y^5]_{-3}^0 \\
&= \frac{38}{5} + \frac{212}{5} \\
&= 50 \quad \blacksquare
\end{aligned}
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