ST2334 Tutorial 1

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. $((A \cup B) \cap C)' = (A \cup B)' \cup C' = (A' \cap B') \cup C' = (A' \cup C') \cap (B' \cup C')$ which is (c)

Q2. $\binom{5}{3}\binom{21}{0} + \binom{5}{2}\binom{21}{1} + \binom{5}{1}\binom{21}{2} = 1270$

Q3. $4! \cdot \binom{5}{3} \cdot 3! = 1440$

Q4. $9! = 362880$

Long Form Questions

Q1. (i) $S = \{3, 4, 5, 13, 14, 15, 23, 24, 25, 123, 124, 125, 213, 214, 215\}$

(ii) $A = \{3, 4, 5\}$

(iii) $B = \{5, 15, 25, 125, 215\}$

(iv) $C = \{3, 4, 5, 23, 24, 25\}$

(v) $A \cap B = \{5\} \neq \phi$ so they are not mutually exclusive $A \cup B = \{3, 4, 5, 15, 25, 125, 215\}$
$A \cap B \cap C = \{5\}$

Q2. (i) $\binom{6}{3} \cdot 3! = 120$

(ii) Fix 9 as last digit, $\binom{5}{2} \cdot 2! = 20$

(iii) Fixing 6 or 8 as first digit, $\binom{3}{1} + \binom{4}{1} = 7$

Q3. (i) $\binom{7}{5} = 21$

(ii) $\binom{5}{3} = 10$

(iii) $\binom{5}{3} + \binom{2}{1}\binom{5}{4} = 20$

(iv) $\binom{3}{2}\binom{4}{3} = 12$

Q4. (i) Exactly $13 + 8 = 21$ steps required, so choose where to place 13 east steps: $\binom{21}{13} = 203490$

(ii) Paths through Y: $\binom{16}{10}\binom{5}{2} = 80080$, so paths avoiding Y: $203490 - 80080 = 123410$

(iii) Paths through X avoiding Y: $\binom{4}{2}\binom{17}{6} - \binom{4}{2}\binom{12}{8}\binom{5}{2} = 44556$