ST2334 Tutorial 4

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. $P(Y = 3 \mid X = 2) = \frac{0.1}{0.2+0.1+0.05} = \frac{4}{7}$

Q2. $E(X \mid Y = 2) = 1(\frac{0.05}{0.5}) + 2(\frac{0.1}{0.5}) + 3(\frac{0.35}{0.5}) = 2.6$

Q3. $E(3X + 2Y) = 3E(X) + 2E(Y) =$
$3[0(0.25 + 0.23) + 1(0.35 + 0.17)] + 2[0(0.25 + 0.35) + 1(0.23 + 0.17)] = 2.36$

Q4. $f_X(x) = \int f(x, y)dy = \int_0^1 (x + y)dy = x + \frac{1}{2}$ $(0 \le x \le 1)$

$\implies f_{Y|X}(y \mid x = 0.2) = \frac{f(x,y)}{f_X(x)} = \frac{x+y}{x+\frac{1}{2}}$

$\therefore E(Y \mid X = 0.2) = \int y \cdot f_{Y|X}(y \mid X = 0.2)\, dy = \int_0^1 y\frac{0.2+y}{0.7}\, dy = \frac{13}{21}$

Q5. (d)

Long Form Questions

Q1. (i) $E(X) = 2(0.01) + 3(0.25) + 4(0.4) + 5(0.3) + 6(0.04) = 4.11$
$E(X^2) = 2^2(0.01) + 3^2(0.25) + 4^2(0.4) + 5^2(0.3) + 6^2(0.04) = 17.63$
$V(X) = E(X^2) = [E(X)]^2 = 0.739$

Q2. (i) $P(0.6 \ge X \ge 1.2) = \int_{0.6}^1 x\, dx + \int_1^{1.2}(2 - x)dx = 0.5$

(ii) $E(X) = 1, V(X) = \frac{1}{6}$

Q3. (i) $P(X = x, Y = y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}$ $(x = 0, 1, 2, 3, y = 0, 1, 2, 1 \le x + y \le 4)$

(ii) $P(X = 1, Y = 1) = \frac{9}{35}$

(iii) $P(X + Y \le 2) = P(X = 0, Y = 1) + P(X = 0, Y = 2) + P(X = 1, Y = 0) + P(X = 2, Y = 0) + P(X = 1, Y = 1) = \frac{1}{2}$

(iv) $f_X(x) = \frac{\binom{3}{x}\binom{5}{4-x}}{\binom{8}{4}}$

(v) $f_{Y|X}(y \mid x = 2) = \frac{P(Y=y|X=2)}{P(X=2)} = \frac{\binom{2}{y}\binom{3}{2-y}}{\binom{5}{2}}$ so $P(Y = 0 \mid X = 2) = 0.3$

Q4. $f_X(x) = \int_1^2 \frac{12}{13}x(x + y)dy = \frac{12}{13}x(x + \frac{3}{2})$ so $f_{Y|X}(y \mid x = 0.5) = \frac{f(0.5,y)}{f_X(0.5)} = 0.25 + 0.5y$
$\implies P(Y \le 1.5 \mid X = 0.5) = \int_1^{1.5}(0.25 + 0.5y)dy = \frac{7}{16}$ and
$E(Y \mid X = 0.5) = \int_1^2 y(0.25 + 0.5y)dy = \frac{37}{24}$