ST2334 Tutorial 3

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. (c); $\sum_{R_X} f(x) = 1$

Q2. (b); continuous and monotonically increasing

Q3. (c); $P(X = 3) = 0.1 \neq 0.15$

Q4. $\sum_{x=0}^{3} c(x^2 + 4) = 1 \implies 30c = 1 \implies c = \frac{1}{30}$

Q5. (a)

Long Form Questions

Q1. (i) $P(A \cap B) = P(B \mid A) \cdot P(A)$ so
$P(A \cap B \cap C) = P(C \mid A \cap B) \cdot P(A \cap B) = 0.8 \cdot 0.9 \cdot 0.75 = 0.54$

(ii) $P(B) = 0.75 \cdot 0.9 + 0.25 \cdot 0.8 = 0.875$

(iii) $P(A \mid B) = \frac{0.75 \cdot 0.9}{0.875} = 0.7714$

(iv) $P(B \cap C) = 0.54 + (0.25 \cdot 0.8 \cdot 0.7) = 0.68$

(v) $P(A \mid B \cap C) = \frac{0.54}{0.68} = 0.7941$

Q2. (i) $P(T) = 0.3, P(S) = 0.6$

(ii) $P(T \cap S) = 0.2 \neq P(T)P(S) = 0.18 \implies$ dependent

(iii) $P(T \cap S) = 0.18 \implies$ independent

Q3. $P(A_2 \mid N) = \dfrac{0.3 \cdot 0.08}{0.5(0.05) + 0.3(0.08) + 0.2(0.1)} = 0.3478$

Q4. $P(Y = y) = \dfrac{\binom{y-1}{0}\binom{5-y}{1}}{\binom{5}{2}} = \dfrac{5-y}{10}$ so $P(Y = 1, 2, 3, 4) = (0.4, 0.3, 0.2, 0.1)$

Q5. (i) $\int_0^1 k\sqrt{x}dx = \frac{2}{3}k = 1 \implies k = \frac{3}{2}$

(ii) $F(x) = \begin{cases} 0, & x \leq 0 \\ x^{\frac{3}{2}}, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$ so $P(0.3 < X < 0.6) = 0.6^{\frac{3}{2}} - 0.3^{\frac{3}{2}} \approx 0.3004$

Q6. (i) $P(X \leq 0.2) = 1 - e^{-1.6} \approx 0.7981$

(ii) $f(x) = F'(x) = 8e^{-8x}, x \geq 0$