ST2334 Tutorial 7

AY 25/26 Sem 1 — github/omgeta

Short Form Questions

Q1. (b); Let $X \sim Bin(400, 0.5)$, then $E(X) = 200, V(X) = 10^2$
so approximately $X \sim N(200, 10^2)$ and $Z = \frac{X-200}{10}$ $Z(0, 1)$.
$P(185 \leq X \leq 210) = P(184.5 < X < 210.5) = P(\frac{184.5-200}{10} < Z < \frac{210.5-200}{10}) = P(-1.55 < Z < 1.05) = \Phi(1.05) - \Phi(-1.55)$

Q2. True

Q3. True

Q4. $E(X_i) = 1 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3} = \frac{4}{3}$,
$E(X_i^2) = 1 \cdot \frac{1}{3} + 3^2 \cdot \frac{1}{3} = \frac{10}{3} \implies V(X_i) = E(X_i^2) - [E(X_i)]^2 = \frac{14}{9} \implies V(\overline{X}) = \frac{V(X_i)}{10} \approx 0.156$

Long Form Questions

Q1. Let $X \sim N(200, 15^2)$ so $Z = \frac{X-200}{15} \sim N(0, 1)$

(a.) $P(X > 224) = P(Z > \frac{224-200}{15}) = P(Z > 1.6) = 1 - \Phi(1.6) \approx 0.548$

(b.) $P(191 \leq X \leq 209) = P(-0.6 < Z < 0.6) = \Phi(0.6) - \Phi(-0.6) = 0.4514$

(c.) Note $P(X > 230) = P(Z > 2.0) = 1 - \Phi(2.0) \approx 0.02275$.
Let $Y \sim Bin(10000, 0.02275)$, $E(Y) = 10000(0.02275) \approx 23$

(d.) $P(Z < z) = 0.25 \implies z = z_{0.75} = -0.6745$. So this occurs at
$Z < z_{0.75} \leftrightarrow \frac{X-\mu}{\sigma} < z_{0.75} \leftrightarrow X < \mu + \sigma z_{0.75} = 189.88$ml

Q2. Let $X \sim N(24, 3.8^2)$

(a.) $P(X > 15) = P(Z > -2.368) = 1 - \Phi(-2.37) = 0.9911$

(b.) Note $P(X > 30) = P(Z > 1.579) = 1 - \Phi(1.579) = 0.05717$
Let $Y \sim Bin(3, 0.05717)$, $P(Y = 2) = \binom{3}{2}(0.05717)^2(1 - 0.05717)^1 = 0.00922$

Q3. $E(\overline{X}) = E(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n} \mu_X = \mu_X$
$V(\overline{X}) = V(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n^2}\sum_{i=1}^{n} V(X_i) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_X^2 = \frac{\sigma_X^2}{n}$

Q4. (a.) $\mu = 4(0.2) + 5(0.4) + 6(0.3) + 7(0.1) = 5.3$,
$E(X^2) = 4^2(0.2) + 5^2(0.4) + 6^2(0.3) + 7^2(0.1) = 28.9$
$sigma^2 = E(X^2) - [E(X)]^2 = 0.81$

(b.) $\mu_{\overline{X}} = 5.3$, $\sigma_{\overline{X}}^2 = \frac{\sigma_X^2}{36} = 0.0225$

(c.) Apply CLT, $\overline{X} \sim N(5.3, 0.0255)$ so $P(\overline{X} < 5.5) = P(Z < 1.33) = 0.9082$

Q5. (a.) Let $Z = \frac{\overline{X}_b - \overline{X}_a}{\sigma_{\overline{X}_a - \overline{X}_b}} \sim N(0, 1)$.
Note $V(\overline{X}_B - \overline{X}_A) = V(\overline{X}_B) + V(\overline{X}_A) = \frac{2}{36}$ so
$P(\overline{X}_B - \overline{X}_A > 0.2) = P(Z > \frac{0.2}{\sqrt{2/36}} = 0.1981$

(b.) No, since p-value ≈ 0.2 is quite large we cannot reject the null hypothesis of $\mu_A = \mu_B$.