CS1231S Tutorial 10

AY 24/25 Sem 1 — github/omgeta

Q1.

$$\begin{aligned}(2x^2+\frac{1}{x})^9 &= (2x^2)^9 + \binom{9}{1}(2x^2)^8(\frac{1}{x}) + \cdots + \binom{9}{6}(2x^2)^3(\frac{1}{x})^6 + \cdots \\ &= \cdots + (84)(8x^6)(\frac{1}{x^6}) + \cdots \\ &= \cdots + 672 + \cdots\end{aligned}$$

Therefore, term independent of x is 672 ■

Q2. $\binom{n+1}{2} = \frac{(n+1)!}{(n-1!)2!} = \frac{n(n+1)}{2}$ ■

Q3. (a) $P(6) = \frac{2}{9}$ ■

(b) $P(1)(1) + P(2)(2) + P(3)(3) + P(4)(4) + P(5)(5) + P(6)(6) =$ $(\frac{1}{81})(1) + \frac{3}{81}(2) + \frac{5}{81}(3) + \frac{16}{81}(4) + \frac{24}{81}(5) + \frac{32}{81}(6) = \frac{398}{81}$ ■

Q4. Let X, Y denote two ball selections

$$\begin{aligned}E(X+Y) &= E(X) + E(Y) && \text{(Linearly of expectation)} \\ &= 2(\frac{1}{5}(1) + \frac{2}{5}(2) + \frac{2}{5}(8)) \\ &= 8.4 \quad \blacksquare\end{aligned}$$

Q5. (a)

$$\begin{aligned}P(\text{infected}|+) &= \frac{P(+|\text{infected}) \cdot P(\text{infected})}{P(+)} \\ &= \frac{0.85 \cdot 0.001}{0.1} \\ &= 0.00850 \quad \blacksquare\end{aligned}$$

(b)

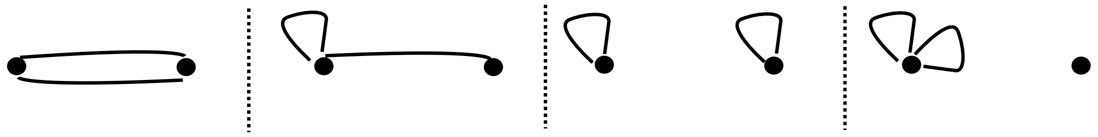
$$\begin{aligned}P(+|\overline{\text{infected}}) &= \frac{P(\overline{\text{infected}}|+) \cdot P(+)}{P(\overline{\text{infected}})} \\ &= \frac{(1-0.00850)\cdot 0.1}{0.999} \\ &= 0.0992 \quad \blacksquare\end{aligned}$$

Q6. (a) $\frac{1}{16}; \frac{2^{n^2-n}}{2^{n^2}}$ ■

(b) $\frac{1}{64}; \frac{2^{\frac{n(n+1)}{2}}}{2^{n^2}}$ ■

Q7. Eulerian but not Hamiltonian ■

Q8.



Q9. (a) $A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$ ■

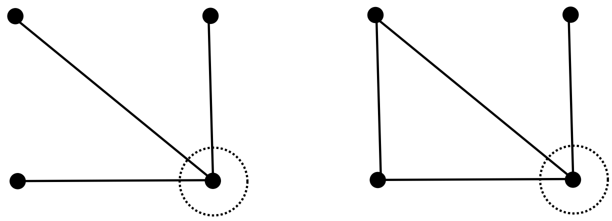
(b) $A^0 = I_4, A^2 = \begin{pmatrix} 3 & 3 & 1 & 1 \\ 3 & 5 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 3 & 2 \end{pmatrix}, A^3 = \begin{pmatrix} 5 & 3 & 9 & 6 \\ 3 & 0 & 13 & 8 \\ 9 & 13 & 1 & 1 \\ 6 & 8 & 1 & 1 \end{pmatrix}$ ■

(c) $\text{Walks}_{a\to b}$ of length $2 = A^2_{ab} = 3$, which are ae_1de_3b, ae_3ce_5b, ae_3ce_6b ■
$\text{Walks}_{c\to c}$ of length $2 = A^2_{cc} = 5$, which are ce_3ae_3c, ce_5be_5c, ce_6be_6c, ce_5be_6c, ce_6be_5c ■

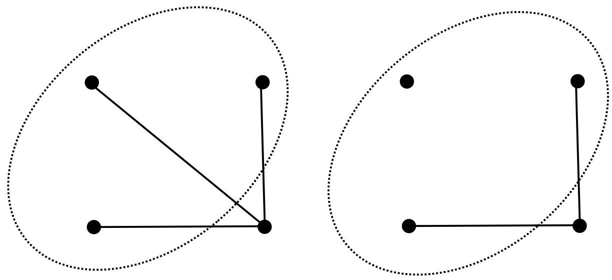
(d) $\text{Walks}_{a\to c}$ of length $3 = A^3_{ac} = 9$, which are $ae_2ae_2ae_3c$, $ae_1de_1ae_3c$, $ae_1de_4be_5c$, $ae_1de_4be_6c$, $ae_3ce_3ae_3c$, $ae_3ce_5be_5c$, $ae_3ce_6be_6c$, $ae_3ce_5be_6c$, $e_3cee_6be_5c$ ■

Q10.
1. Suppose P is party attendees, $|P| = n$, and H is number of possible handshakes.
2. Since every person shook atleast the hosts hand, $H = \{1, \cdots, n-1\}$
3. Since $\frac{|P|}{|H|} = \frac{n-1}{n-2} > 1$, $\exists$ handshakes $h \in H$ shared by atleast 2 $p \in P$ ■ (Gen. PHP)

Q11. (a)



(b)



Q12.
1. Suppose graph G with 6 vertices and its complement $\overline{G}$
2. $\forall v$, $deg(v)$ w.r.t one of $G, \overline{G}$ is atleast 3 (Gen. PHP)
3. Choose some v, and let A be the graph with $deg(v) \geq 3$ with adjacent vertices $\{w_1, w_2, w_3\}$
4. Case 1 ($\exists w_i, w_j$ $(\{w_i, w_j\} \in E(A))$):
   4.1. $\{v, w_i, w_j\}$ is a triangle in A
5. Case 2 ($\sim(\exists w_i, w_j$ $(\{w_i, w_j\} \in E(A)))$):
   5.1. $\forall w_i, w_j$ $(\{w_i, w_j \notin E(A)\})$
   5.2. $\forall w_i, w_j$ $(\{w_i, w_j\} \in E(\overline{A}))$ (Definition of graph complement)
   5.3. $\{w_1, w_2, w_3\}$ is a triangle in $\overline{A}$
6. In both cases, there is a triangle in either G or $\overline{G}$ ■