MA1521 Calculus for Computing

AY 24/25 Sem 1 — github/omgeta

1. Limits

Function $f(x)$ is continuous at $x = c$ if and only if it is differentiable at c or $\lim_{x\to c} f(x)$ exists and $\lim_{x\to c} f(x) = f(c)$.

Laws of Limits:

i. $\lim_{x\to c}(f(x) \pm g(x)) = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$
ii. $\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$
iii. $\lim_{x\to c}(f(x)g(x)) = (\lim_{x\to c} f(x))(\lim_{x\to c} g(x))$
iv. $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$
v. g is continuous at $x = b \wedge \lim_{x\to c} f(x) = b$
$\implies \lim_{x\to c} g(f(x)) = g(b) = g(\lim_{x\to c} f(x))$

Squeeze Theorem:

$$g(x) \le f(x) \le h(x) \wedge \lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$$
$$\implies \lim_{x\to c} f(x) = L$$

Intermediate Value Theorem:

f is continuous on $[a, b] \wedge k$ is between $f(a)$ and $f(b)$
$\implies f(c) = k$ for some $c \in [a, b]$

Trignometric Identities:

$$\lim_{x\to c} g(x) = 0$$
$$\implies \lim_{x\to c} \frac{g(x)}{\sin(g(x))} = \lim_{x\to c} \frac{\sin(g(x))}{g(x)} = 1$$
$$\implies \lim_{x\to c} \frac{g(x)}{\tan(g(x))} = \lim_{x\to c} \frac{\tan(g(x))}{g(x)} = 1$$

Exponential Trick

$$\lim_{x\to c} \ln f(x) = L \implies \lim_{x\to c} f(x) = e^L$$

L'Hôpital's Rule

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty} \implies \lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

2. Differentiation

Derivative of a function f is given by:

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Derivative of a parametric function in t is given by:

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}, \quad \frac{dy^2}{dx^2} = \frac{d}{dt}(\frac{dy}{dx}) \div \frac{dx}{dt}$$

Critical points at $x = c$ of function f are non-endpoints where $f'(c)$ is 0 or does not exist.

First Derivative Test:

i. $f'(c^-) > 0 \wedge f'(c^+) < 0$ (Local maximum)
ii. $f'(c^-) < 0 \wedge f'(c^+) > 0$ (Local minimum)
iii. Otherwise (Point of inflection)

Second Derivative Test:

i. $f''(c) < 0$ (Local maximum)
ii. $f''(c) > 0$ (Local minimum)

Rolle's Theorem:

f continuous on $[a, b]$, differentiable on $(a, b) \wedge f(a) = f(b)$
$\implies f'(c) = 0$ for some $c \in [a, b]$

Mean Value Theorem:

f is continuous on $[a, b] \wedge f$ is differentiable on (a, b)
$\implies f'(c) = 0$ for some $c \in [a, b]$

Standard Derivatives

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan(g(x))$	$g'(x)\sec^2(g(x))$
$\sec(g(x))$	$g'(x)\sec(g(x))\tan(g(x))$
$\operatorname{cosec}(g(x))$	$-g'(x)\operatorname{cosec}(g(x))\cot(g(x))$
$\cot(g(x))$	$-g'(x)\operatorname{cosec}^2(g(x))$
$\sin^{-1}(g(x))$	$\frac{g'(x)}{\sqrt{1-g(x)^2}}$
$\cos^{-1}(g(x))$	$-\frac{g'(x)}{\sqrt{1-g(x)^2}}$
$\tan^{-1}(g(x))$	$\frac{g'(x)}{1+g(x)^2}$
$\cot^{-1}(g(x))$	$-\frac{g'(x)}{1+g(x)^2}$
$\sec^{-1}(g(x))$	$\frac{g'(x)}{\lvert g(x)\rvert\sqrt{g(x)^2-1}}, \lvert g(x)\rvert > 1$
$\operatorname{cosec}^{-1}(g(x))$	$-\frac{g'(x)}{\lvert g(x)\rvert\sqrt{g(x)^2-1}}, \lvert g(x)\rvert > 1$
a^x	$a^x \ln(a)$

3. Integration

Definite integrals of function f have Riemann Sum:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \Sigma_{i=1}^n \frac{b-a}{n} f\left(a + (b-a)\frac{i}{n}\right)$$

Integration by substitution involves choosing $u = g(x)$ and replacing all original variables, limits and dx.

Integration by parts for $\int f(x)g(x)dx$ involves choosing u and $\frac{dv}{dx}$ (u by LIATE) so $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

Volume of revolution about same axis, in a disk:

$$V = \pi\int_a^b [f(x)]^2 dx, \quad V = \pi\int_c^d [g(y)]^2 dy$$

Volume of revolution about diff. axis, in a cylindrical shell:

$$V = 2\pi\int_a^b x\lvert f(x)\rvert dx, \quad V = 2\pi\int_c^d y\lvert g(y)\rvert dy$$

Arc length of a curve measured along x or y:

$$l = \int_a^b \sqrt{1 + [f'(x)]^2}dx, \quad l = \int_c^d \sqrt{1 + [g'(y)]^2}dy$$

Standard Integrals

$\mathbf{f(x)}$	$\mathbf{F(x) - C}$
$[f(x)]^n, n \neq -1$	$\frac{[f(x)]^{n+1}}{(n+1)f'(x)}$
$\tan(f(x))$	$\frac{1}{f'(x)}\ln\lvert\sec(f(x))\rvert$
$\sec(f(x))$	$\frac{1}{f'(x)}\ln\lvert\sec(f(x)) + \tan(f(x))\rvert$
$\operatorname{cosec}(f(x))$	$-\frac{1}{f'(x)}\ln\lvert\operatorname{cosec}(f(x)) + \cot(f(x))\rvert$
$\cot(f(x))$	$-\frac{1}{f'(x)}\ln\lvert\operatorname{cosec}(f(x))\rvert$
$\sec^2(f(x))$	$\frac{1}{f'(x)}\tan(f(x))$
$\operatorname{cosec}^2(f(x))$	$-\frac{1}{f'(x)}\cot(f(x))$
$\sec(f(x))\tan(f(x))$	$\frac{1}{f'(x)}\sec(f(x))$
$\operatorname{cosec}(f(x))\cot(f(x))$	$-\frac{1}{f'(x)}\operatorname{cosec}(f(x))$
$\frac{1}{a^2+[f(x)]^2}$	$\frac{1}{af'(x)}\tan^{-1}(\frac{f(x)}{a})$
$\frac{1}{\sqrt{a^2-[f(x)]^2}}$	$\frac{1}{f'(x)}\sin^{-1}(\frac{f(x)}{a})$
$-\frac{1}{\sqrt{a^2-[f(x)]^2}}$	$\frac{1}{f'(x)}\cos^{-1}(\frac{f(x)}{a})$
$\frac{1}{a^2-[f(x)]^2}$	$\frac{1}{2af'(x)}\ln\lvert\frac{f(x)+a}{f(x)-a}\rvert$
$\frac{1}{[f(x)]^2-a^2}$	$\frac{1}{2af'(x)}\ln\lvert\frac{f(x)-a}{f(x)+a}\rvert$
$\frac{1}{\sqrt{[f(x)]^2+a^2}}$	$\frac{1}{f'(x)}\ln\lvert f(x) + \sqrt{[f(x)]^2 + a^2}\rvert$
$\frac{1}{\sqrt{[f(x)]^2-a^2}}$	$\frac{1}{f'(x)}\ln\lvert f(x) + \sqrt{[f(x)]^2 - a^2}\rvert$
$\sqrt{a^2 - x^2}$	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}(\frac{x}{a})$
$\sqrt{x^2 - a^2}$	$\frac{x}{2}\sqrt{x^2 - a^2} + \frac{a^2}{2}\ln\lvert x + \sqrt{x^2 - a^2}\rvert$

4. Sequences and Series

n^{th} **Term:** $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges

Integral Test for $a_n = f(n)$, where f is continuous, positive, decreasing for $x \geq 1$:

$$\int_1^{\infty} f(x) \text{ converges} \iff \sum_{n=1}^{\infty} a_n \text{ converges}$$

p**-series:** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges $\iff p > 1$

Comparison Test for $0 \leq a_n \leq b_n$:

$$\sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

$$\sum_{n=1}^{\infty} a_n \text{ diverges} \implies \sum_{n=1}^{\infty} b_n \text{ diverges}$$

Absolute Convergence:

$$\sum_{n=0}^{\infty} |a_n| \text{ converges} \implies \sum_{n=0}^{\infty} a_n \text{ converges}$$

Ratio/Root Test:

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L \text{ or } \lim_{n\to\infty} \sqrt[n]{|a_n|} = L$$

i. $0 \leq L < 1$ (Absolute Convergence)

ii. $L > 1$ (Divergence)

iii. $L = 1$ (Inconclusive)

Alternating Series Test for terms $a_n = (-1)^n b_n$ or $a_n = (-1)^{n-1} b_n$, where b_n is decreasing:

$$\lim_{n\to\infty} b_n = 0 \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

Radius of Convergence $R = \frac{1}{L}$ about $x = a$ for power series $b_n = c_n(x-a)^n$ is interval for absolute convergence:

$$\lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = L \text{ or } \lim_{n\to\infty} \sqrt[n]{|c_n|} = L$$

Power series represented functions for $R > 0$ have:

i. $f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$

ii. $\int f(x)dx = \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$

Taylor Series for a function with power series representation is:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

with MacLaurin Series at $x = 0$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$$

Common Series

$\mathbf{a_n}$	$\lim_{\mathbf{n\to\infty}} \sum_{\mathbf{r=1}}^{\mathbf{n}} \mathbf{a_r}$
$ar^{n-1}, \|r\| < 1$	$\frac{a}{1-r}$
$\frac{1}{n}$	diverges
$(-1)^{n-1}\frac{1}{n}$	$\ln 2$
$\frac{1}{n^2}$	2

Common Expansions

e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$
$\sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
$\ln(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$
$\frac{1}{1+x}$	$\sum_{n=0}^{\infty} (-1)^n x^n$
$\frac{1}{1+x^2}$	$\sum_{n=0}^{\infty} (-1)^n x^{2n}$
$(1+x)^n, \|x\| < 1$	$\sum_{k=0}^{n} \binom{n}{k} x^k$
$(a+b)^n, n > 1$	$\sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

5. Vectors

Projection of $\mathbf{b}$ onto $\mathbf{a}$ is given by:

$$\text{proj}_{\mathbf{a}}\, \mathbf{b} = \text{comp}_{\mathbf{a}}\, \mathbf{b} \times \hat{\mathbf{a}} = (\mathbf{b} \cdot \hat{\mathbf{a}})\hat{\mathbf{a}}$$

Perpendicular distance from position vector $\mathbf{b}$ to $\mathbf{a}$ is given by:

$$\|\mathbf{b} \times \hat{\mathbf{a}}\|$$

Projection of $\mathbf{b}$ onto plane $\Pi : \mathbf{r} \cdot \mathbf{n} = D$:

$$\begin{aligned} \text{proj}_{\Pi}\, \mathbf{b} &= \mathbf{b} - \text{proj}_{\hat{\mathbf{n}}}\, \mathbf{b} \\ &= \mathbf{b} - (\mathbf{b} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} \\ \|\text{proj}_{\Pi}\, \mathbf{b}\| &= \|\mathbf{b} \times \hat{\mathbf{n}}\| \end{aligned}$$

Perpendicular distance from position vector $\mathbf{b}$ to plane $\mathbf{r} \cdot \mathbf{n} = D$:

$$\frac{|D - \mathbf{b} \cdot \mathbf{n}|}{\|\mathbf{n}\|}, \quad \frac{|D - D_1|}{\|\mathbf{n}\|} \text{ (to plane)}$$

Dot and Cross product are given by:

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta \\ \|\mathbf{a} \times \mathbf{b}\| &= \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta \\ \mathbf{a} \times \mathbf{b} &= \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \end{aligned}$$

Vector-valued Functions

Derivative of vector-valued function $\mathbf{r}(t)$ is given by:

$$\mathbf{r}'(t) = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t}$$

Derivative of $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is given by:

$$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

Arc length of a path measured along t:

$$l = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}dt$$

6. Multivariate Calculus

Derivative of parametric function $z = f(x, y)$ where $x = g(t)$ and $y = h(t)$ is given by:

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Derivative of parametric function $z = f(x, y)$ where $x = g(s, t)$, $y = h(s, t)$ is given by:

$$\frac{\partial z}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}, \quad \frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

Derivative of z in implicit function $F(x, y, z) = 0$ is given by:

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Directional derivative of f at $P = (x_0, y_0)$ in direction of unit vector $\hat{\mathbf{u}}$ making angle θ with ∇f is given by:

$$\begin{aligned} D_{\hat{\mathbf{u}}} f(P) &= \nabla f(x_0, y_0) \cdot \hat{\mathbf{u}} \\ &= \|\nabla f(x_0, y_0)\| \cos\theta \end{aligned}$$

where gradient vector ∇f is given by:

$$\nabla f = \langle f_x, f_y \rangle$$

and rate of change is optimized at:

i. $\|\nabla f(P)\|$ in direction $\nabla f(P)$ (Max.)
ii. $-\|\nabla f(P)\|$ in direction $-\nabla f(P)$ (Min.)

Critical points at (a, b) of function f are non-endpoints where $f_x(a, b) = f_y(a, b) = 0$ or a partial derivative does not exist.

Second Derivative Test:

$$D = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

i. $D > 0$ and $f_{xx}(a, b) < 0$ (Local max.)
ii. $D > 0$ and $f_{xx}(a, b) > 0$ (Local min.)
iii. $D < 0$ (Saddle point)
iv. $D = 0$ (Inconclusive)

Increments and Differentials

Increment of $z = f(x, y)$, where Δx and Δy are increments in x and y is given by:

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

Differentials dx and dy are defined as:

$$dx = \Delta x, \quad dy = \Delta y$$

Total differential dz is the linear approximation of the increment Δz and is given by:

$$\Delta z \approx dz = f_x(x, y)dx + f_y(x, y)dy$$

Level Curves/Surfaces vs. ∇f

$\nabla f(x_0, y_0)$ is normal to the level curve of $f(x, y) = k$ at (x_0, y_0).

$\nabla F(x_0, y_0, z_0)$ is normal to the level surface of $F(x, y, z) = k$ at (x_0, y_0, z_0).
If $\mathbf{r}(t_0) = \langle x_0, y_0, z_0 \rangle$, then $\nabla F(x_0, y_0, z_0) \cdot \mathbf{r}(t_0) = 0$

Tangent Planes

Tangent plane to surface $z = f(x, y)$ at (x_0, y_0) has normal vector $\langle f_x(x_0, y_0), f_y(x_0, y_0), -1 \rangle$ with equation:

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - f(x_0, y_0)) = 0$$

Tangent plane to level surface $F(x, y, z) = k$ at (x_0, y_0, z_0) has normal vector $\nabla F(x_0, y_0, z_0)$ with equation:

$$\nabla F(x_0, y_0, z_0) \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

7. Double Integrals

Double Integral $\iint_R f(x, y)dA$ over rectangular region $R = [a, b] \times [c, d]$:

$$\int_a^b \int_c^d f(x, y)dydx = \int_c^d \int_a^b f(x, y)dxdy$$

with special case when $f(x, y) = g(x)h(y)$:

$$\int_a^b g(x)dx \cdot \int_c^d h(y)dy$$

Area of general plane region D: $\iint_D dA$

Surface area: $\iint_D \sqrt{f_x^2 + f_y^2 + 1}dA$

Polar coordinates: $\iint_D f(r\cos\theta, r\sin\theta) r d\theta dr$

8. ODEs

Separable ODEs, reducing if necessary by $v = \dfrac{y}{x}$ or $u = ax + by$:

$$\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{1}{g(y)}dy = \int f(x)dx + C$$

Linear ODEs using $I(x) = e^{\int P(x)dx}$:

$$\frac{dy}{dx} + P(x)y = Q(x) \Rightarrow yI(x) = \int Q(x)I(x)dx$$

Bernoulli equation using $u = y^{1-n}$:

$$\begin{aligned} &\frac{dy}{dx} + P(x)y = Q(x)y^n \\ \implies &\frac{du}{dx} + (1 - n)P(x)u = (1 - n)Q(x) \end{aligned}$$

Appendix

Trignometric Identities

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$
$$\sin P + \sin Q = 2 \sin \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$
$$\sin P - \sin Q = 2 \cos \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$
$$\cos P + \cos Q = 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$
$$\cos P - \cos Q = -2 \sin \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$
$$\sin A \cos B = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$$
$$\cos A \sin B = \frac{1}{2}\sin(A+B) - \frac{1}{2}\sin(A-B)$$
$$\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$
$$\sin A \sin B = -\frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

Partial Fractions

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$
$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$$
$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$$

Cylinders and Quadric Surfaces

Cylinders are planes such that all other parallel planes intersect the surface in the same curve. Any equation in x, y, z with a missing variable is a cylinder such as:

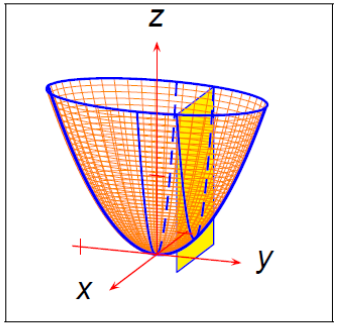
$$y^2 + z^2 = 1$$
$$z = x^2$$

Elliptic parabolids are open quadraic surfaces symmetric about the z-axis. If $c > 0$, it opens up in the positive z-axis. If $c < 0$, it opens down to the negative z-axis. General equation is given by:

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = \frac{z}{c}$$

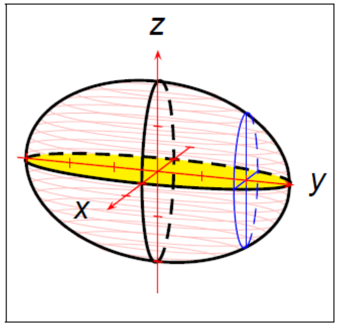
Ellipsoids are closed quadric surfaces. If $a = b = c$, it is a sphere. General equation is given by:

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} + \frac{(z-z_0)^2}{c^2} = 1$$



Elliptic Paraboloid

$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$



Ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Forming ODEs

General rate of change is given by:

Rate of change = rate of increase − rate of decrease

Example: At time $t = 0$, a tank contains 20kg of salt dissolved in 100 litres of water. Assume that water containing $\frac{1}{4}$kg of salt per litre is entering the tank at the rate of 3 litre per min, and the well-stirred solution is leaving the tank at the rate of 4 litre per min. Find the amount of salt at any time t.

Let S denote the amount of salt in kg at time t

$$\frac{dS}{dt} = \text{ salt input } - \text{ salt output}$$
$$= (3 \times \frac{1}{4}) - (\frac{3 \times S}{100 - (4+3)t})$$

which resolves to an ODE

Example: Formulae for half-life of a radioactive y substance with half-life T and initial amount y_0 with respect to time t is given by:

$$y = y_0(\frac{1}{2})^{\frac{t}{T}}, \quad \text{or } y = y_0 e^{-\frac{\ln 2t}{T}}$$