CS3230 Tutorial 9

AY 25/26 Sem 1 — github/omgeta

Q1). (a.) True; reduce to solving for optimum and checking if $\leq k$

(b.) True; reduce to solving decision problem $O(n)$ times

(c.) True; follows from contraposition in (a) of polynomial-reduction

(d.) True; follows from contraposition in (b) of polynomial-reduction

Q2). (c); this reduction ignores the preservation of set S

Q3). (a); trivial
(b); if a subset sums to $\frac{S}{2}$, just choosing those items for the knapsack satisfies $W \leq \frac{S}{2} \leq V$
(c); since every item has same weight as value, and we solve for $W \leq \frac{S}{2} \leq V$, we force $W = V = \frac{S}{2}$ so the knapsack contains exactly half the sum $\rightarrow$ Partition YES

Q4). Algorithm: transform unweighted graph G into complete weighted G' with

$$w(u,v) = \begin{cases} 1, & \text{if } (u,v) \in G \\ 2, & \text{otherwise} \end{cases}$$

Transformation is $O(n^2)$ by building edges between each of the n edges, and comparing with original graph in $O(1)$ each.

Suppose HC-Yes, then the same tour in G' costs $|V|$ due to each edge only costing 1 $\rightarrow$ TSP-YES for $|V|$

Suppose TSP-Yes, then the tour must have all edges cost 1 to keep total cost $\leq |V| \rightarrow$ HC-YES