

Let us consider the design problem of a constant speed DC motor with nominal values of 2700 rev/min, 1.5 kW, and 110 V.

### Speed

$$n = N(\text{rev/min})/60 = 2700/6 = 45 \text{ rev/s}$$

### output power

$$kw \text{ output}/n \times 10^3 = 1.5/45 \times 10^3 = 33.3$$

From the corresponding curves we can obtain the value of  $B_g$ , the specific magnetic loading, and  $a_c$ , the specific electric loading:

$$B_g = .42 \text{ T}$$

$$a_c = 9 \text{ kA/m}$$

We assume that the ratio of the pole arc to the pole pitch is 0.7:

$$\psi = \frac{\text{Bow pole}}{\text{Pole step}}$$

The average air gap density will be .294:

$$B_{av} = B_g \times \psi = .42 \times .7 = .294 \text{ T}$$

Then the output coefficient is calculated:

$$C_0 = \pi \times B_{av} a_c \times 10^{-3} = \pi \times 0.294 \times 9000 \times 10^{-3} = 26.11$$

$P_a$  = Then the product of the diameter and length will be calculated.  $c_0 D^2 L n$

$$D^2 L = P_a / C_0 n = 1.5 / 26.11 \times 45 = 1.276 \times 10^{-3} \text{ m}^3$$

Assuming that the ratio of the length to the diameter of the core is 1/3, we have:

$$D = 100 \text{ mm} \quad , \quad L = 128 \text{ mm}$$

Now it is necessary to test the ratio of core length to pole pitch and armature tangential speed. The rotor speed is 45 rev/s. If we choose the number of poles more than 2, the armature frequency and also its core losses will increase. By choosing a two-pole machine, the pole pitch is equal to:

$$\tau = \pi D/p = \pi \times 100/2 = 157 \text{ mm}$$

For DC motors between 6. to 9.

$$L/\tau = 128/157 = 0.82$$

The tangential velocity of the armature is equal to:  $V_a = \pi D n = \pi \times 0.1 \times 45 = 14.12 \text{ m/s}$

**which is within the specified range of 8 to 25 m/s.**

**Armature design:**

**The average flux in each pole is equal to:**

$$\phi = B_{av} \tau L = 0.294 \times 0.157 \times 0.128 = 5.9 \text{ mWb}$$

For motors, the no-load voltage is equal to the terminal voltage. Here we need to estimate the no-load speed. Let's assume the no-load speed is 2800 rev/min or 46.67 rev/s. Total number of armature conductors:

$$Z = E_a / \phi n_0 = 110 \times 2 / 5.9 \times 10^{-3} \times 46.67 \times 2 = 399$$

The number of poles is 2. The number of parallel paths, whether it is a simple ring or wave wiring, will be 2. The voltage induced in each conductor is:

$$e_c = \phi n_0 = 5.9 \times 10^{-3} \times 46.67 = 0.275 \text{ V}$$

Since this value is very small, a simple wave wiring, which is preferable, can be used.

The number of grooves between the edges of the pole must be at least 3. Therefore, this condition can be met with a number of grooves equal to or greater than 20. On the other hand, an odd number of grooves causes the grooves of each pole to add up to a whole number, which reduces the flux pulsations in the air gap. Therefore, this condition is met with an odd number of grooves equal to or greater than 21.

In selecting the sheets, existing sheets should be used as much as possible. In this case, we assume that sheets with a diameter of 100 mm and with 16, 27 or 36 grooves are available. Although we only need 21 grooves, it is more economical to use existing sheets with a larger number of grooves. Obviously, this will increase the cost of insulating the groove, but it eliminates the cost of tooling for new sheets.

The 27-groove sheet has a core depth of 16.6 mm, a tooth width of 3.5 mm, and a groove depth of 20.9 mm. We can use these sheets if the flux density in the teeth is less than the saturation value, the number of conductors in each groove can be considered an even number without causing a significant change in the electrical loading, especially an even number, and if the groove area is large enough to accommodate the number of conductors in each groove along with the conductor insulation and the groove.

The tooth flux, without taking into account the groove leakage, is equal to:

$$\phi_t = p\phi / \psi s$$

Where s is the number of armature slots (in this case 27). The flux density in the teeth is:

$$B_t = \phi_t / w_t L_i$$

The length of the armature iron can be considered equal to 0.95 of the armature length. Since the grooves are trapezoidal and the teeth have parallel sides, the tooth width is fixed at 3.5mm. Therefore:

$$B_t = 2 \times 5.9 \times 10^{-3} / 0.7 \times 27 \times 3.5 \times 10^{-3} \times 0.95 \times 0.128 = 1/47 \text{ T}$$

It should be noted that if we wanted to design another motor with the same rated power but with a full load speed of, say, 1300 rev/min, it would be possible to design it on a 4-pole basis without increasing the armature frequency beyond 50 Hz and thus reducing the core flux, allowing for a higher specific magnetic loading. It is obvious that the specific loading and output coefficient of the 1300 rev/min motor will be greater than the corresponding values for the 2700 rev/min motor. The product of the product for the lower speed machine will also be greater than that for the higher speed machine. This is because the product is inversely proportional to the product of the speed and the output coefficient. Moreover, the increase in the output coefficient is always less than the decrease in the speed. The diameter and length of the 1300 rev/min motor are about 100 and 150 mm respectively. The area of the slot of the shown sheets is: The number of conductors in each slot is:

$$z/s = 399/27 = 14.78$$

To avoid using virtual windings in the armature, the number of turns per slot should be a multiple of 2 or 6, respectively. If we consider 12 conductors per slot, we can design the wiring with either 2 or 6 winding arms per slot. However, this will cause a significant reduction in the specific electrical loading. Therefore, in this case, we consider 2 winding arms per slot and we can use either 14 or 16 conductors per slot. There are 7 or 8 turns per winding. The motor terminal current is equal to:

$$I = p_0 / V \eta$$

Assuming an efficiency of 85 percent (p.u. 85), the motor terminal current will be 16 A. According to Figure 20-4, the shunt field current will be approximately 1.34 percent of the rated motor current, or 0.2 A:

$$I_a = 16 - 0.2 = 15.8 \text{ A}, \quad I_z = I_a / a = 15.8/2 = 7.9 \text{ A}$$

We assume that the current density of the conductor is equal to . The cross-sectional area of the conductor is equal to:

$$a_z = I_z / \delta_a = 7.9/3.5 = 2.26 \text{ mm}^2$$

The diameter of the conductor is equal to:

$$d_a = \sqrt{P \times a_z / \pi} = \sqrt{4 \times 2.26 / \pi} = 1.69 \text{ mm}$$

The nearest diameter in the standard copper wire table is 1.7 mm and the insulated conductor diameter is 1.83 mm. The total cross-sectional area is 16 conductors. This will give us the slot occupancy factor. So 16 conductors and certainly 14 conductors can be easily accommodated inside the slot.

If we use these sheets, the slot utilization will be very bad and less than necessary. What we really need is sheets with smaller slots and deeper cores. With such sheets, the copper to slot area ratio will increase. Slot utilization will be better and core saturation will be avoided.

Now let us examine the sheet shown in Figure 4-25. Its outer diameter is 95 mm. Slot area . The core depth is 20 mm, the tooth width is 4.5 mm, and the groove depth is 15 mm. The groove occupancy factor for 16 and 14 conductors per groove will be 0.48 and 0.42, respectively. The smaller groove depth (15 mm compared to 20.9 mm for the 27-groove sheet) results in fewer ampere-turns required for the flux to pass through the teeth. Therefore, the 25-groove sheet is more suitable for this design. We use the sheet of Figure 4-25, and with 14 conductors per groove the specific electrical loading will be 9264 conductor amperes per meter. The total number of armature conductors is 350, and the average flux per pole at a no-load voltage equal to the terminal voltage of 110 V is required. 6.73 mwb. We choose the armature length so that the flux density generated in the core is 1.5 T. This gives an armature length of 118 mm.

Now the maximum flux density in the air gap is about .55 T. But the flux density in the teeth and core is 1.5 T and 1.53 T respectively which is quite acceptable.

The ampere turns per meter. for both the teeth and the core of the armature is obtained from the magnetization curve of the sheet material given in Appendix 1. Therefore:

$$At_t = 700 \text{ A/m} \quad , \quad at_c = 800 \text{ A/m}$$

The groove depth, including the groove opening, is 15 mm. The flux path length in the core is calculated from equation (A-152) in Appendix 2 and is equal to 35.4 mm. Therefore, the ampere-turns of the teeth and core for each pole are:

$$AT_t = 700 \times 0.015 = 11 \quad , \quad AT_c = 800 \times 35.4 = 28$$

There are 14 conductors in each slot. The number of coil arms in each slot is 2 and there are 7 turns in each coil. Therefore, the number of armature coils and commutator blades will be equal to the number of slots, i.e. 25. Now we will examine the commutator pitch.

Let us assume that the commutator diameter is 60% of the armature diameter. Therefore:

$$\beta_c = \pi D_c / C = \pi \times 0.6 \times 95 / 25 = 7.16 \text{ mm}$$

Which is quite acceptable:

The tangential speed of the commutator in the no-load state is about 8 m/s, which is within the limits given in Section 8.3.4. The voltage between adjacent blades of the commutator is also calculated by Equation (7-4) and is about 12.2 V. Again, these values are satisfactory and acceptable.

The average length of the armature winding is obtained from Equation (11-4) as follows:

$$L_{ma} = 2 \times L + 2.3 \times \tau + 5d_s =$$

$$2 \times 0.118 + 2.3 \times 0.149 + 5 \times 0.015 = 0.654 \text{ m}$$

And the armature resistance is obtained from equation (12-4):

$$R_a = z / (2a^2) (\rho L_{ma}) / (a_z 10^{-6}) =$$

$$350/2 \times 2^2 \times 1.8 \times 10^{-8} \times 0.654 / 2.27 \times 10^{-6} = 0.227 \Omega$$

The armature resistance voltage drop is 3.6 V. This value is about 3.3 percent of the terminal voltage. For  $n \cdot k_w = 5.67$ , the armature circuit voltage drop is about 7.2 percent or 7.9 V. This indicates that the armature design is satisfactory and acceptable.

### **Air gap and pole ampere-turns:**

In order to prevent the leading (agreeing) pole edge of the motor from being completely demagnetized, the ampere-turns per pole for the air gap and armature teeth should not be less than or equal to the armature reaction mmf at the pole edge. Therefore, we assume that the ampere-turns required to pass flux through the air gap and armature teeth are approximately 0.75 times the armature ampere-turns per pole.

The armature ampere-turns per pole are equal to:

$$AT_a = I_z / 2p = 7.9 \times 350 / 2 \times 2 = 691$$

Also:

$$AT_g = 0.75 AT_a - AT_t = 519 - 11 = 508$$

But:

$$AT_g = B_g / \mu_0 \times I_g k_g$$

So that the air gap correction factor is and in this case is equal to 1.15.

The air gap length obtained in this method is about 1 mm. The ampere turns required in each pole to send flux through the pole and yoke at full load can be considered equal to 2000 times the pole pitch. Where the pole pitch is in meters. Therefore:

$$AT_p + AT_y = 2000 \times 0.149 = 298$$

If it is chosen to be 1mm, the ampere-turns per pole at full load will be about 845, and this temporary and conditional value is about 22 percent greater than the armature reaction ampere-turns.

### **Field and Pole Winding Design:**

To determine the pole dimensions, we assume that the flux density in the pole body for the sag pole is 1.8T. The length of the pure iron of the pole is 0.95 times the length of the armature. Therefore:

$$B_p = 1.8T, \quad l_{pi} = 0.95 \times 0.118 = 0.112m$$

We assume the pole leakage coefficient to be 1.1. The flux in the pole body is equal to:

$$\phi_p = 6.73 \times 1.1 = 7.4 \text{ mwb}$$

The cross-sectional area is equal to:

$$A_p = \phi_p / B_p = 7.4 \times 10^{-3} / 1.8 = 4.11 \times 10^{-3} m^2$$

The width of the pole body is equal to:

$$b_p = A_p / l_{pi} = 4.11 \times 10^{-3} / 0.112 = 0.036 \text{ m}$$

The height of the pole can be assumed to be between 1 and 1.5 times the width of the pole. Let the height of the pole including the pole cap be 36mm. The amperes per meter of pole height are obtained from the magnetization characteristics given in Appendix 1 times 8000. The ampere-turns required to pass the flux through the pole body are:

$$AT_p = 8000 \times 0.036 = 288$$

We assume that the flux density in the yoke is 1.5 T. The depth of the yoke is about 22 mm. The length of the .attached flux path is 2.15 m. The ampere-turns per meter of flux path in the yoke are 700

The ampere-turns required to pass the flux through the yoke are:

$$AT_y = at_y \times I_y = 700 \times 0.15 = 105$$

Therefore, the ampere-turns required in each pole to pass the flux through the pole body and yoke are 393. Thus, .the total ampere-turns in each pole at full load will be 940

If the field winding can be accommodated, the pole design will be acceptable and satisfactory. The height of the field winding is slightly less than the height of the pole. The average height of the field winding is assumed to be 28mm. We also assume that the depth of the winding is 40mm and the field winding occupancy factor is .55. .Calculate the current density that we use in the field conductors

The average length of the field winding is calculated using equation:(21-4)

$$L_{fc} = 2(l_p + 2b_{fc} + b_p) = 2(0.118 + 2 \times 0.04 + 0.036) = 0.468m$$

The cross-sectional area of the conductor is calculated using equation (34-4):

$$a_{fc} = (PAT_{fl} \rho l_{fc}) / (VV_f 10^{-6}) =$$

$$2 \times 940 \times 1.8 \times 10^{-8} \times 0.468 / 110 \times 10^{-6} = 1.44 \times 10^{-7} m^2 = 0.144 mm^2$$

The nearest standard wire has a diameter of .45 mm. The diameter of the re-insulated wire is .516.

Now a more accurate value for the field winding occupancy factor can be obtained as follows:

$$f_{cf} = 0.75 \times (0.45 / 0.516)^2 = 0.57$$

Which is very close to the assumed value of 0.55. The number of turns on each field coil is equal to:

$$N_f = hf b_{fc} f_{cf} / a_{fc} = 28 \times 40 \times 0.57 / 0.159 = 4015$$

The field winding resistance is obtained from equation:(33-4)

$$R_f = P N_f \rho l_{fc} / a_{fc} 10^{-6} =$$

$$2 \times 4015 \times 1.8 \times 10^{-8} \times 0.468 / 0.159 \times 10^{-6} = 426 \Omega$$

## Design of commutation poles

The width of the auxiliary pole depends primarily on the width of the commutation area. Therefore, before designing the auxiliary poles, we design the commutator and brush accessories.

$$17.9 \leq w_{ip} \leq 22.3 \text{ mm}$$

For wave-wound motors, the commutation zone width is given by the equation: In the case under consideration, there are 2 winding arms per slot and 2 poles. The commutation zone width is:

$$W_c = (1/(0.6))(w_b - w_m)$$

Due to the high speed of this machine, we use graphite brushes mixed with resin. The commutator pitch is about 7mm. A brush with a width of 16mm along the commutator surface covers only 2 commutator blades. The thickness of the insulating separator can also be considered as .8mm. Therefore, the width obtained for the commutation area is 24mm, which is acceptable. The axial length of the brush is determined from its current density and the armature current. The current density of graphite mixed with resin is as given in Table 4-1.

Therefore, the axial length of the brush is 10mm, which will be acceptable. The radial length according to the standard brush size is 32mm.

For the auxiliary poles, the same air gap of 1 mm is used as for the main poles. To allow the flux to swell at the edges of the auxiliary pole, the width of the auxiliary pole is chosen to be 22 mm. The axial length of the auxiliary pole is also the same as the main pole, or 118 mm. The ampere-turns of the armature in each pole is 691. We consider the ampere-turns of the auxiliary pole to be 20% greater than the mmf of the armature in each pole, that is:

$$AT_{ip} = 1.2 \times 691 = 829$$

And the number of turns of the auxiliary pole is equal to:

$$N_{ip} = AT_{ip} / I_a = (829) / (15.8) = 52$$

The nearest standard wire has a copper diameter of 3mm and a cross-sectional area. The diameter of the double insulated conductor is 3.14mm. The copper area in 52 turns of the coil is . We assume the occupancy factor for the auxiliary pole coil to be .5. Let us assume the average depth of the auxiliary pole coil to be about 22mm and its average height to be 30mm.

The average lap length is equal to:

$$l_{ic} = 2(l_p + 2b_{fc} + b_p) = 2(0.118 + 2 \times 0.03 + 0.022) = 0.4 \text{ m}$$

The total resistance of the commutation field winding is obtained using equation (4-44) as follows:

$$R_{ip} = (\rho l_{ic} N_{ip} / a_{ip} 10^{-6}) \times P =$$

$$1.8 \times 10^{-8} \times 0.4 \times 52 / 7.069 \times 10^{-6} \times 2 = 0.106 \Omega$$

The voltage drop in the auxiliary pole winding is about 1.7V.

## Losses and Efficiency:

### Copper losses

The resistance of the armature winding, the shunt field winding and the commutation winding are respectively. These values are obtained at a temperature of 25 ° C. Their corresponding values at a temperature of 75 ° C are:

$$R_a=0.27 \, \Omega \quad , \quad R_f=507 \, \Omega \quad , \quad R_{ip}=0.126 \, \Omega$$

The field current at a temperature of 75°C is:

$$I_f=V_f/R_f =110/507=0.22 \, A$$

And the armature current at temperature is 15.78A. The armature copper losses are:

$$W_a=I_a^2 \times R_a = (15.78)^2 \times 0.27=67.2 \, W$$

The commutation field winding copper losses are equal to:

$$W_i=I_a^2 \times R_{ip} = (15.78)^2 \times 0.126=31.4 \, W$$

The shunt field winding copper losses are equal to:

$$W_f = I_f^2 \times R_f = (0.22)^2 \times 507 = 24.5 \, W$$

The brush junction voltage drop for resin-impregnated graphite brushes is assumed to be 2 V. The brush junction losses are:

$$W_b=2 \times 15.78=31.7 \, W$$

The total copper losses are w154.8.

#### **Core losses:**

The weights of the armature teeth and the core are obtained using equations (4-49) and (4-50) as follows:

$$G_t= S W_t l_{ia} d_s \gamma =25 \times 0.0045 \times 0.95 \times 0.118 \times 0.015 \times 7750=1.47 \, Kg$$

$$G_c=\pi/4[(D - 2d_s)^2 - D_i^2] l_{ia} \gamma =$$

$$\pi/4[(0.095-2 \times 0.015)^2 - (0.025)^2] 0.95 \times 0.118 \times 7750 = 2.46 \, Kg$$

The specific iron loss values at 50Hz and at 1.53T and 1.5T for the teeth and the armature core, respectively, are obtained from the loss curve of the lamination core. The iron loss values in Kg/W at 50Hz for the teeth and the armature core are 2.4 and 1.4, respectively. At 45Hz, these values are 3.8 kg/w, 3.7 kg/w and 3.7 kg/w.

The iron losses of the armature core and teeth are:

$$1.47 \times 3.8 + 2.46 \times 3.7=14.7 \, W$$

The total iron loss is estimated by multiplying the core and armature teeth losses by 2.5. Therefore, the total core loss is 37 W.

**Brush Friction Losses:** Friction losses for graphite brushes are determined by the total brush contact area and the commutator tangential velocity as discussed earlier. The commutator velocity is 8m/s and the total brush contact area is . Therefore:



$$W_{bf} = 2440 A_B V_C = 2440 \times 320 \times 10^{-6} \times 8 = 6.3 \text{ W}$$

#### **Bearing and Windage Friction Losses:**

When test data on similar machines are not available, we estimate these losses by dividing them by 50% of the output power. The tangential speed of the armature is about 13.4m/s. The bearing and windage friction losses should be approximately 4W.

#### **Efficiency:**

The total losses in the motor are:

$$\text{كل تلفات} = 154.8 + 37 + 6.3 + 4 = 202.1 \text{ W}$$

And the efficiency is equal to:

$$\eta = 1500 / (1500 + 202.1) \times 100 = 88.1\%$$