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Medical Image Processing Course

2022-2023 Spring Semester

HW02

Morphological Image Processing and Denosing

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Contents

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1 Theoretical Problems

1

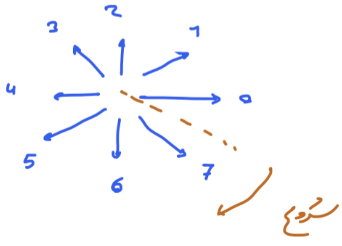
الف) زمانی که rotation با ضریب 45° رخ دهد، اعداد دنباله chain code ما همگی یک جمع و باقی‌مانده بر 8 برایشان رخ می‌دهد. در نتیجه در هیستوگرام دنباله صرفاً یک شیفت رخ می‌دهد.

- زمانی که scaling را اعمال می‌کنیم، تعداد هر کدام از code ها به یک نسبت ضرب می‌شود. در نتیجه هیستوگرام اگر نرمالیزه شده باشد هیچ تغییری رخ نمی‌دهد.

- در صورت تغییر starting point نیز صرفاً ترتیب دنباله عوض می‌شود اما میزان تکرار هر کد تغییری نمی‌کند، در نتیجه هیستوگرام هیچ تغییری نمی‌کند.

شخصاً chain code توصیف‌گر بهتری برای مرز تصویر است چرا که هیستوگرام صرفاً یک آماره از تکرار هر کدام از کدها بوده و ترتیب قرارگرفتن کدها را برای تشکیل شکل را به ما نمی‌دهد.

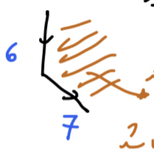
ب) اگر از بالاترین نقطه و در صورت وجود چند نقطه بالا در شکل، از راست‌ترین آنها شروع کرده و به صورت ساعتگرد بچرخیم، در این صورت لزوماً باید اعداد دنباله نزولی باشد و حداکثر پرش اعداد 3 واحد کم گردد.



اولاً اگر دنباله نزولی باشد و پیوسته (حداکثر 3 واحد کم شود)، تمام زوایای بین اضلاع شکل، چون ساعتگرد می‌چرخیم، کمتر از 180° بوده و در نتیجه شکل محدب می‌باشد.

چرخش پادساعتگرد

از طرفی اگر پیوسته عدد بیشتر از 3 باشد، چرخش پادساعتگرد بوده، و از طرفی اگر جایی دنباله صعود کند، چون شکل را ساعتگرد می‌چرخیم، پس ناحیه‌ی هاشور اگر بخواهد داخل شکل باشد، این جا به جایی پادساعتگرد بوده و در تناقض است. ■



اگر شکل بخواهد محدب باشد باید این خط در شکل باشد

2

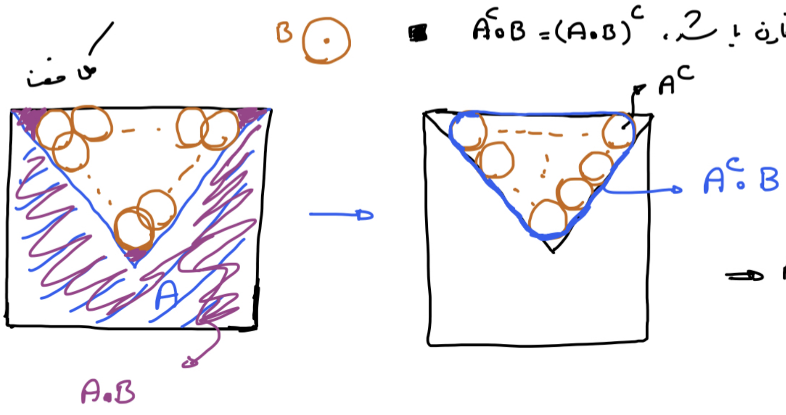
برای خواص erosion و dilation داریم:

$$\begin{cases} (A\oplus B)^c = A^c \ominus \breve{B} & \text{I} \\ (A\ominus B)^c = A^c \oplus \breve{B} & \text{II} \end{cases}$$

$$A^c \circ B = (A^c \ominus B)\oplus B$$

$$(A\bullet B)^c = (\boxed{(A\oplus B)}\ominus \boxed{B})^c \xrightarrow{\text{II}} \boxed{(A\oplus B)^c}\oplus \breve{B} \xrightarrow{\text{I}} (A^c\ominus \breve{B})\oplus \breve{B}$$

که در نتیجه زمانی که کرنل B متقارن باشد، $A^c\circ B = (A\bullet B)^c$ ■



$$\Rightarrow A^c\circ B = (A\bullet B)^c$$

2 Problem 1

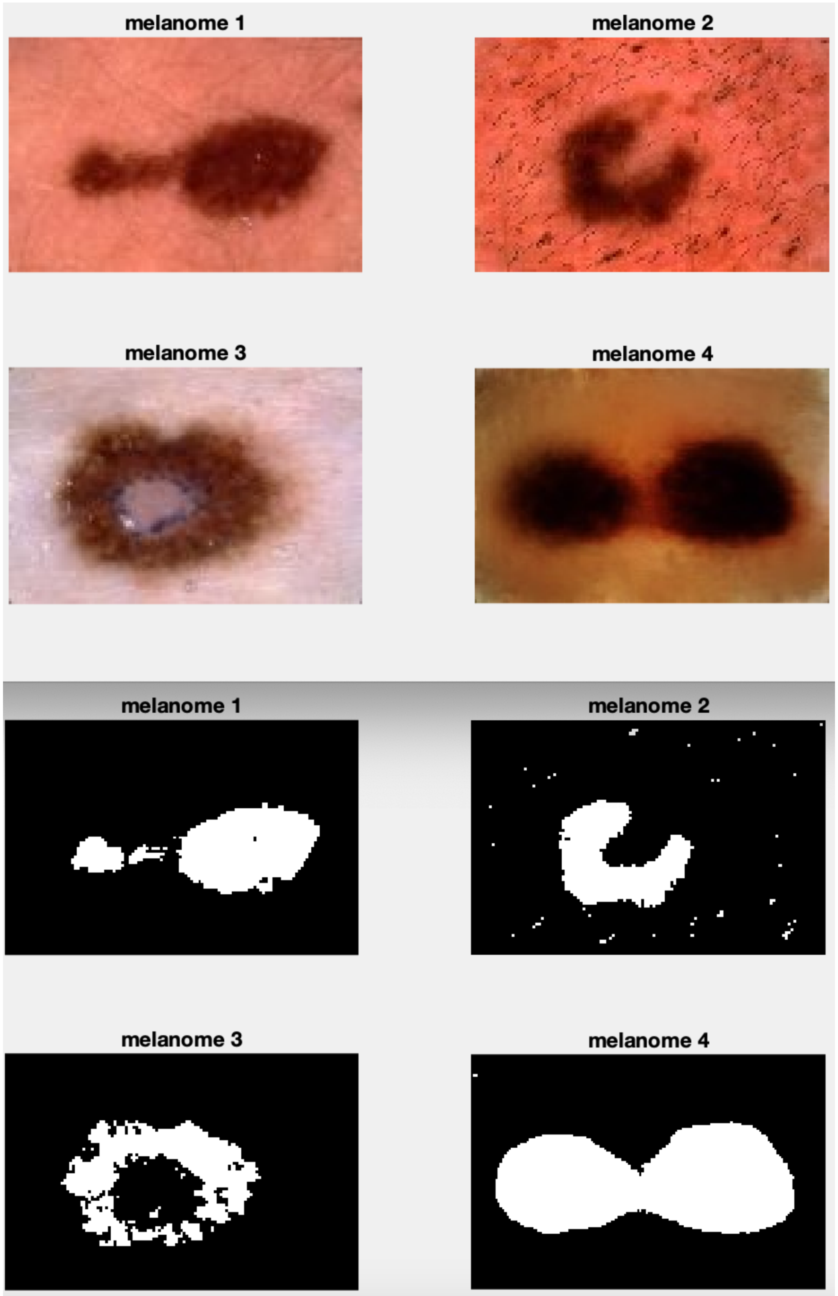


Figure 1: Binarized Images

2.1 A

For gaining one connected component from the binarized image, I used closing algorithm with disk structuring element size 3.

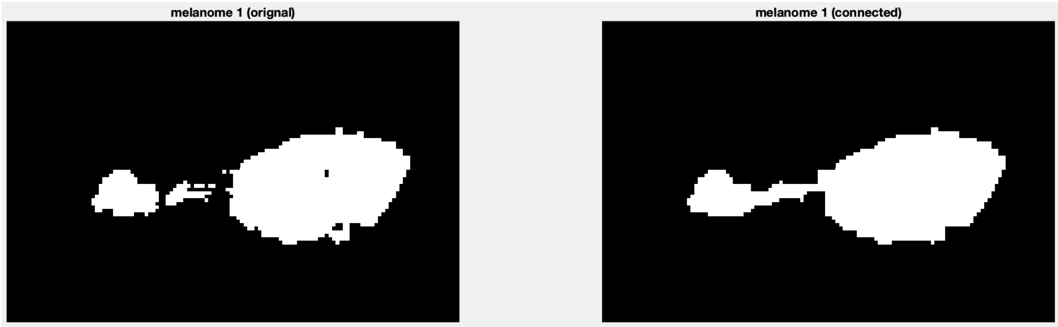


Figure 2: Closing Algorithm on melanome 1

2.2 B

First for removing noise of small lines, we use opening with disk structuring element size 3. Then for getting its boundaries, we erode the denoised image and remove it from the denoised image.

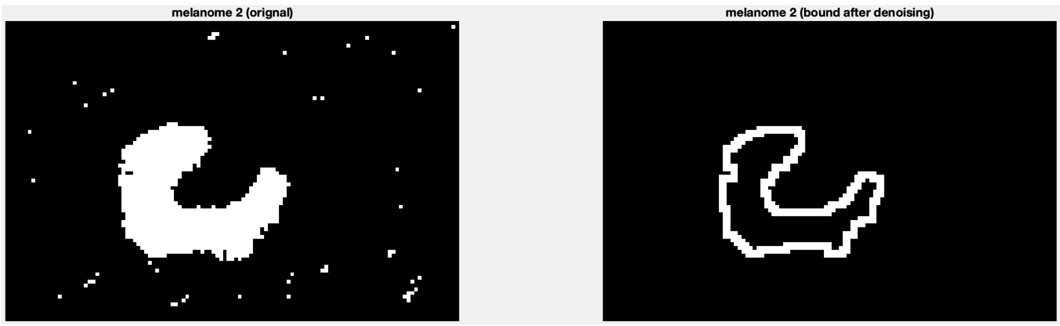


Figure 3: Bounds of melanome 2 after denoising

2.3 C

For hole filling, first we should make a connected region from the melanoma. So I used closing using disk structuring element size 3. Then I used imfill function of matlab.

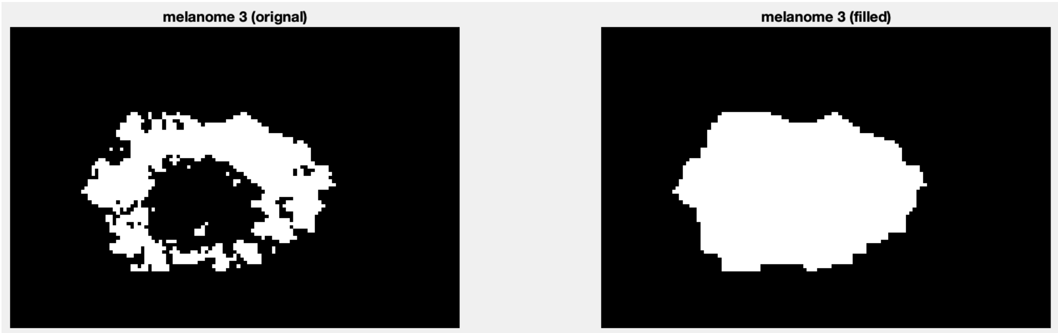
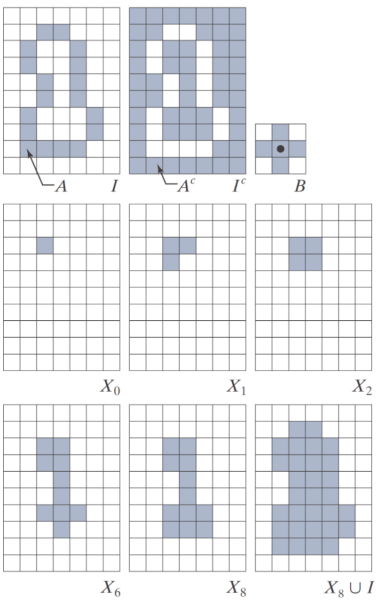


Figure 4: Filled melanome 3

If we did not want to use imfill function of matlab, we should use an iterative algorithm. First we start with a pixel inside the hole. Then update the region using :

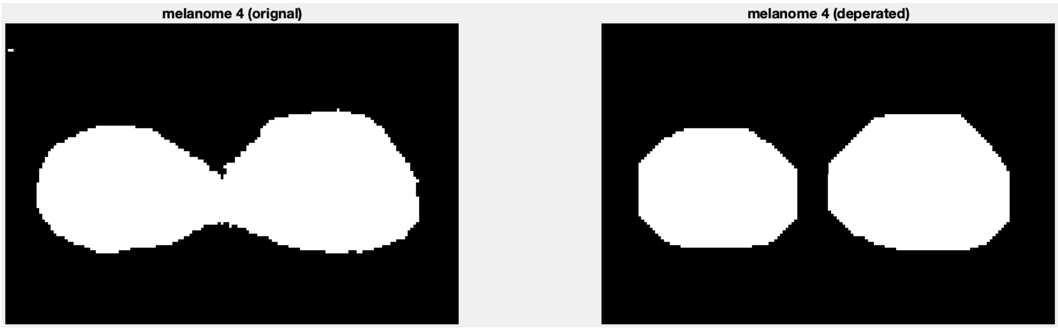
$$X_k = (X_{k-1} \oplus B) \cap I^c$$

And repeat until convergence.

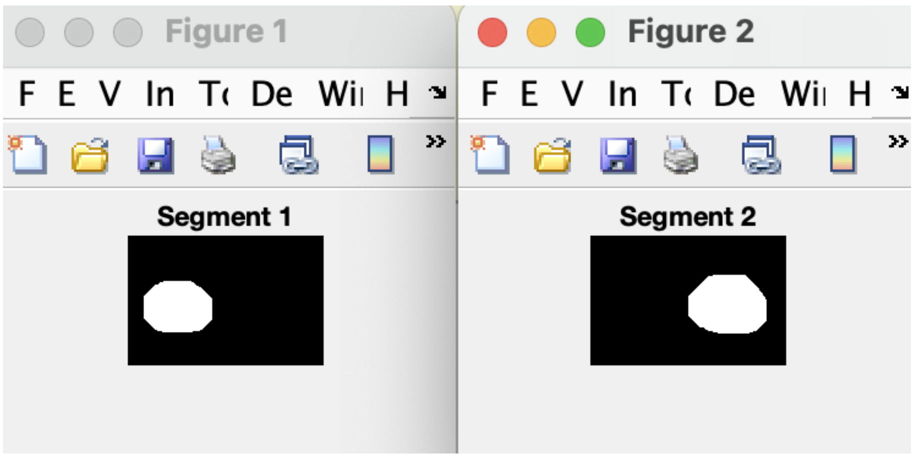


2.4 D

For making two parts apart, we should us opening with large SE size. If you use very large SE then it removes the main parts of image, also if we use not enough large, then it can not make two separate parts. I used disk SE with size 17 and the result is good.

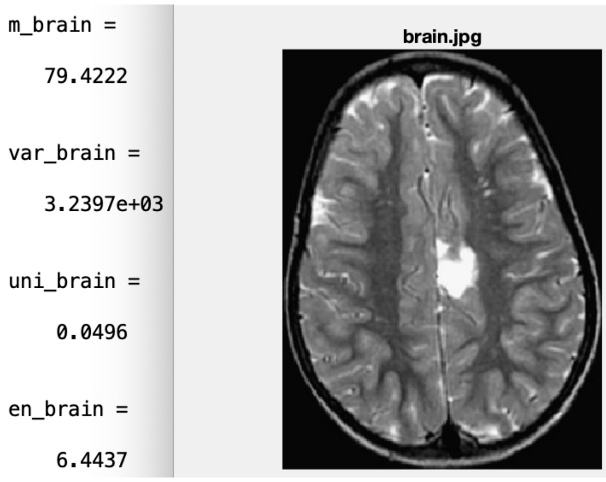


2.5 E



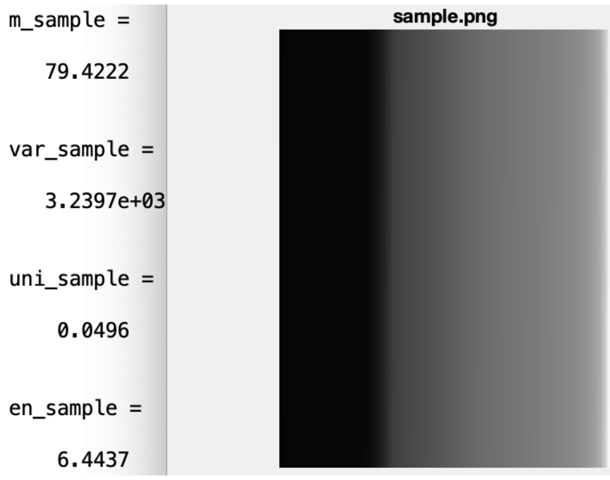
3 Problem 2

3.1 A



3.2 B

Yes, the histograms should be equal, so just by shuffling we will get same histogram and same parameters.



3.3 C

When we do not quantize the level, then the GLCM matrix have 256 levels so the matrix is 256*256

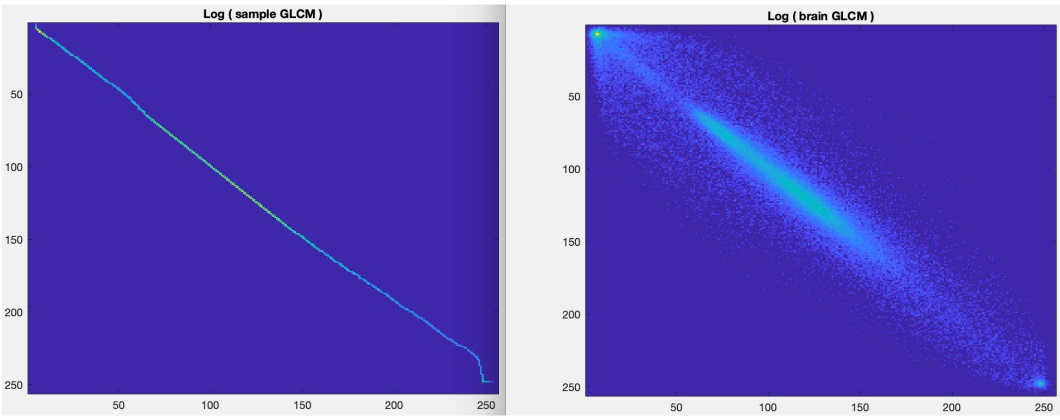


Figure 5: Full GLCM

3.4 D

As we can see, the entropy and the contrast that we calculate from the GLCM matrix for the sample image is less than the one for the brain. We expected this result based on the gradient form of the sample image pixels.

```
- Brain -
Contrast: 114.350583
Uniformity: 0.037457
Homogeneity: 0.435542
Entropy: 10.367313

 - Sample -
Contrast: 2.984043
Uniformity: 0.046276
Homogeneity: 0.752596
Entropy: 7.154491
```

3.5 E

For compressing the GLCM matrix by 4, we should quantize the gray level by factor of 2, so the final GLCM matrix become 128*128 instead of 256*256. This can help us for memory resource we use and somehow ignoring some effects of noise.

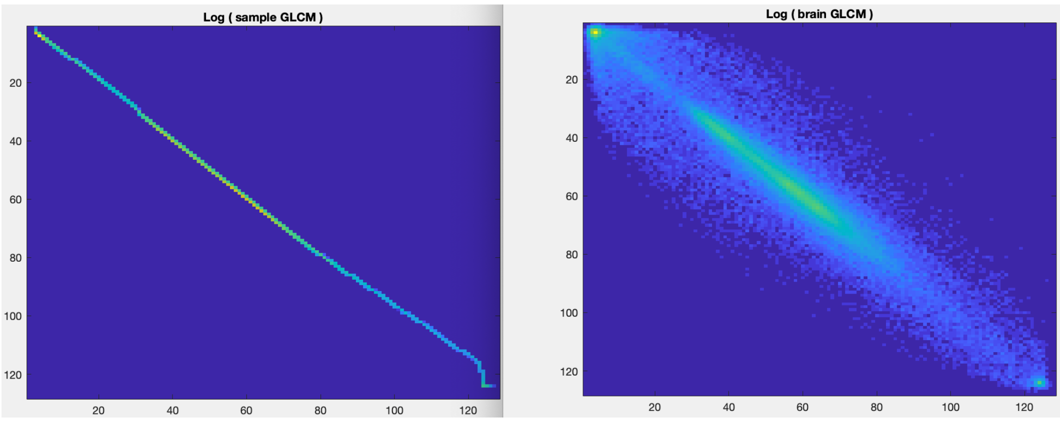


Figure 6: Compressed GLCM

As you can see the above images, we have squares in the GLCM matrix image and because of this down-sampling, both contrast and entropy reduces. On the other hand, uniformity and homogeneity should increase. It is important that the ratio of the brain image and the sample image parameters roughly remains the same in the full and compressed GLCM matrices.

```
- Brain -
Contrast: 28.687675
Uniformity: 0.043405
Homogeneity: 0.537826
Entropy: 8.809067

 - Sample -
Contrast: 0.837020
Uniformity: 0.057578
Homogeneity: 0.849572
Entropy: 6.177529
```

3.6 F

- GLDM : The GLDM is based on the occurrence of two pixels which have a given absolute difference in gray level and which are separated by a specific displacement.

  For example, for given:

  $$\delta = (\Delta x, \Delta y)$$

  we should calculate :

  $$D(i|\delta) = Prob[|S(x, y) - S(x + \Delta x, y + \Delta y)| = i]$$

- GLRLM : The GLRLM is based on computing the number of gray-level runs of various lengths. A gray-level run is a set of consecutive and collinear pixel points having the same gray-level value. The length of the run is the number of pixel points in the run.

  So we should calculate :

  $$R(\theta) = [g(i, j|\theta)]$$

  that means number of times the picture have runs with length j from gray level i in the direction of the angle θ.

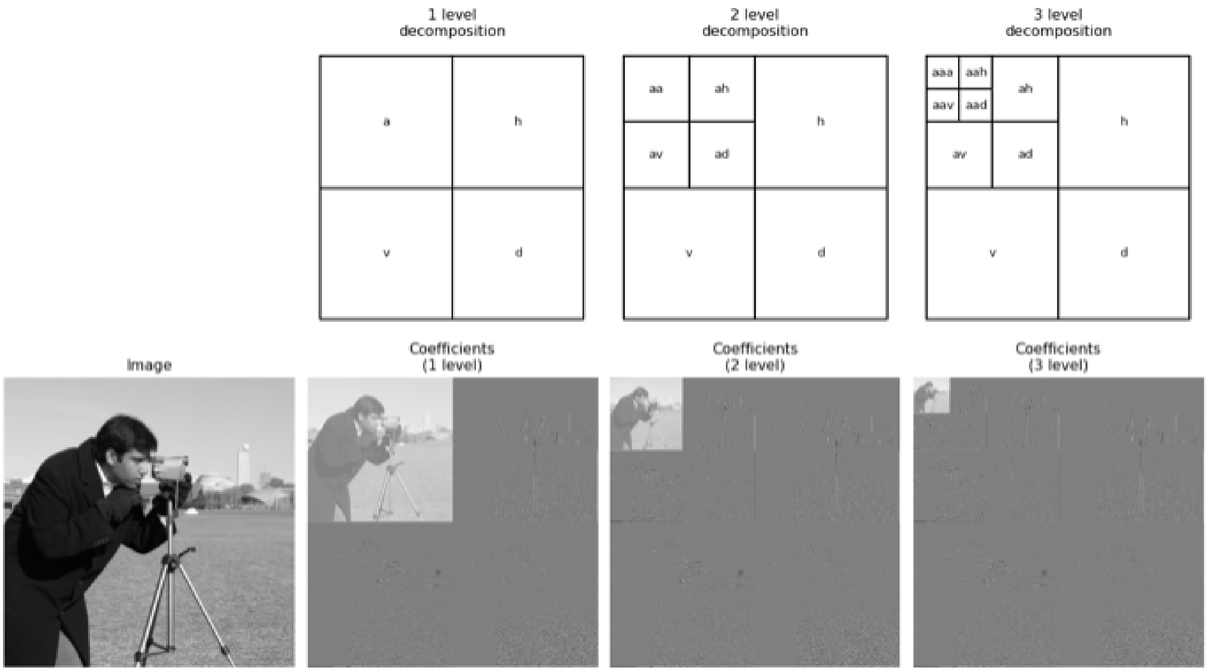
4 Problem 3

4.1 A

The wavelet analysis method is a time-frequency analysis method which selects the appropriate frequency band adaptively based on the characteristics of the signal. Then the frequency band matches the spectrum which improves the time-frequency resolution. The wavelet analysis method has an obvious effect on the removal of noise in the signal.

Wavelets are functions that are concentrated in time and frequency around a certain point. This transformation technique is used to overcome the drawbacks of fourier method. Fourier transformation, although it deals with frequencies, does not provide temporal details. According to Heisenberg's Uncertainty Principle, we can either have high frequency resolution and poor temporal resolution or vice versa.

This wavelet transform finds its most appropriate use in non-stationary signals. This transformation achieves good frequency resolution for low-frequency components and high temporal resolution for high-frequency components.



The discrete wavelet transform has a huge number of applications in science, engineering, mathematics and computer science. Most notably, it is used for signal coding, to represent a discrete signal in a more redundant form, often as a preconditioning for data compression.

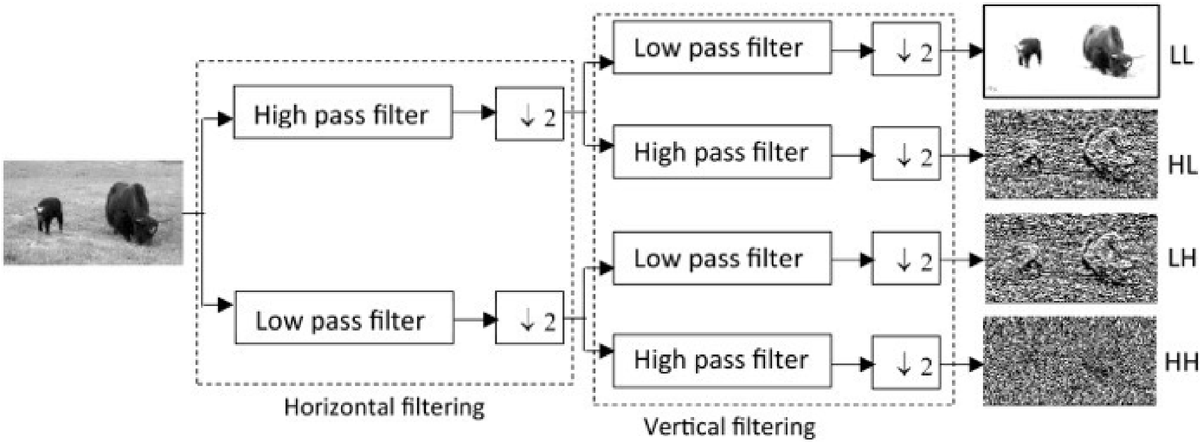
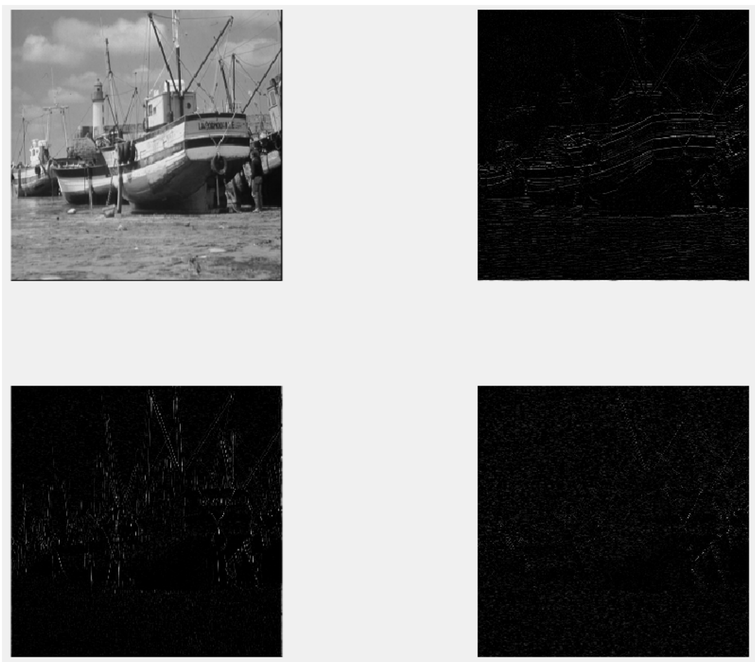


Figure 7: Decomposition of an image 2-D discrete wavelet transform (2-D DWT)

4.2 B



4.3 C

Figure 8: Reconstructed image

4.4 D

Figure 9: Reconstructed image using 95% of largest coefficients

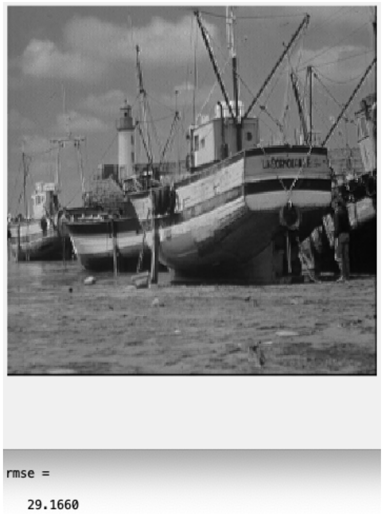


Figure 10: Reconstructed image using 40% of largest coefficients

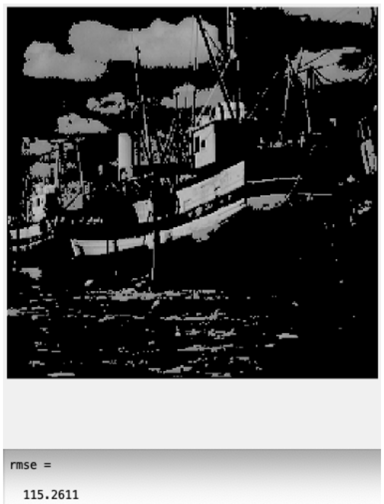


Figure 11: Reconstructed image using 5% of largest coefficients

4.5 E

For doing this part, we calculate and compare the rmse of the resulting images for DFT and DWT in different compression ratio.

CR	1.25	1.5	2	3	5	10	20	50	100
DWT	29.17	29.17	29.17	29.17	41.40	93.97	115.26	127.98	132.6
DFT	0.6663	1.25	2.13	3.52	5.31	7.94	10.64	14.2	16.8

So in the term of rmse error, it seems that DFT works better in compression task. But we should consider two facts. First for compression, I calculated the Thr based on the CR rate and apply it on all of the four coefficient matrices the same, but for better output it is obvious that we should first make the HH coefficient indexes zero and then go through other matrices.

Also the ways both methods compress the image are different. In DFT method the image becomes blurred and we are removing the high frequency coefficients first. We are losing the frequency domain data and the edges of the image. The DWT works better in frequency domain but worse in spatial. So its hard to say which method is better than the other one.

5 Problem 4

5.1 A

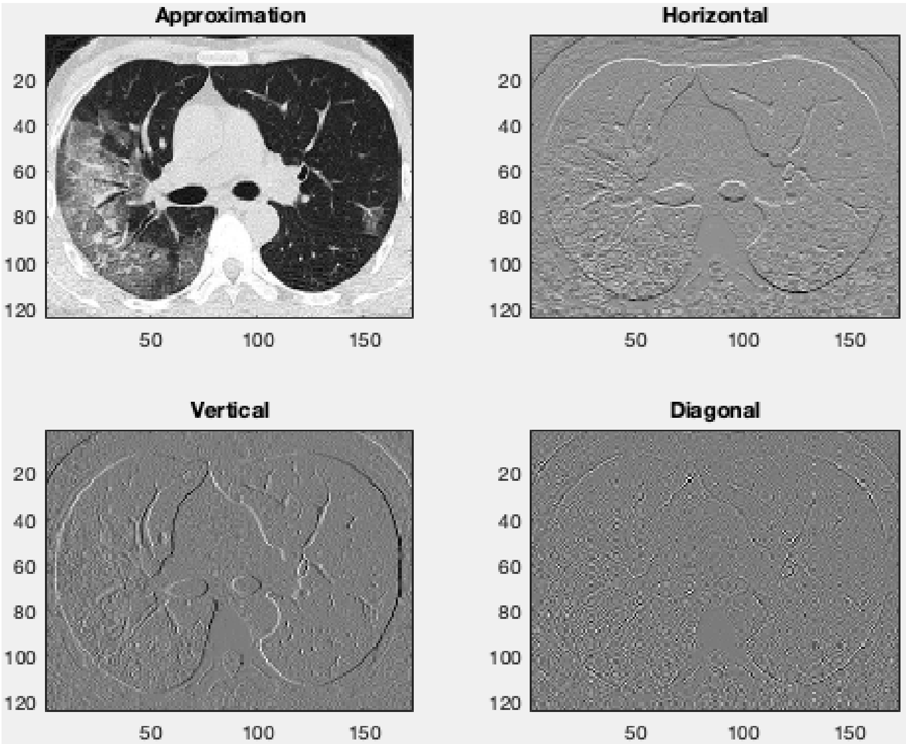


Figure 12: Haar dwt2

5.2 B

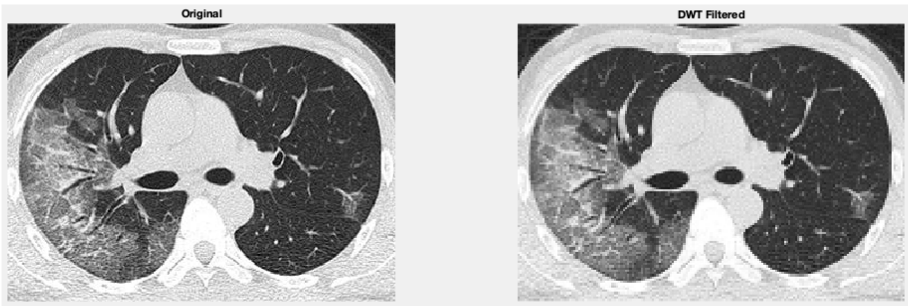


Figure 13: $\text{th}_{ch} = th_{cv} = 7, th_{cd} = 3$

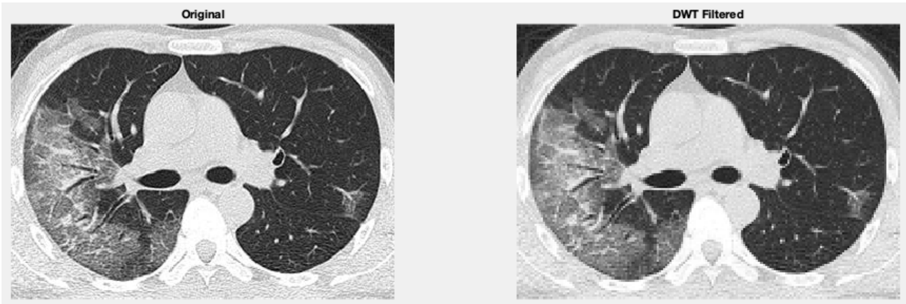


Figure 14: $\text{th}_{ch} = th_{cv} = 3, th_{cd} = 1$

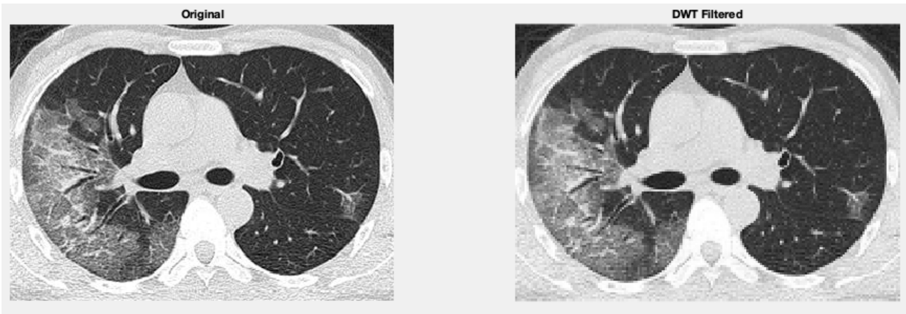


Figure 15: $\text{th}_{ch} = th_{cv} = 1, th_{cd} = 1$

5.3 C

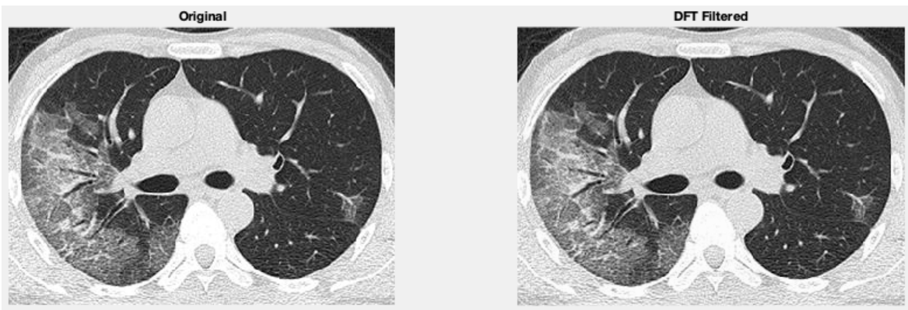


Figure 16: th = 926

Fourier-based transforms (e.g. DFT) are efficient in exploiting the low frequency nature of an image. However, a major disadvantage of these transforms is that the basis functions are very long. If a transform coefficient is quantized, the effect is visible throughout the image. This does not create much problem for the low frequency coefficients that are coded with higher precision. However, the high frequency coefficients

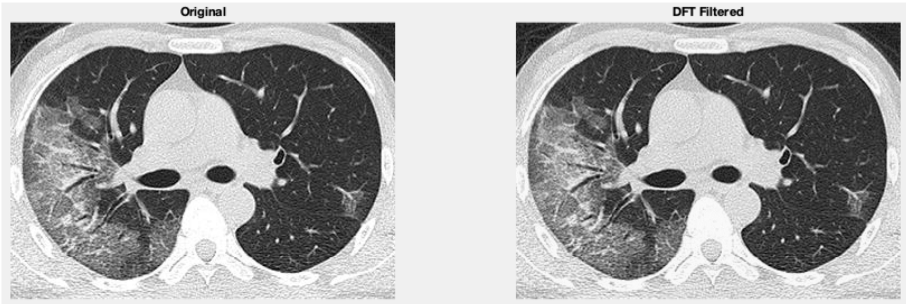


Figure 17: th = 1450

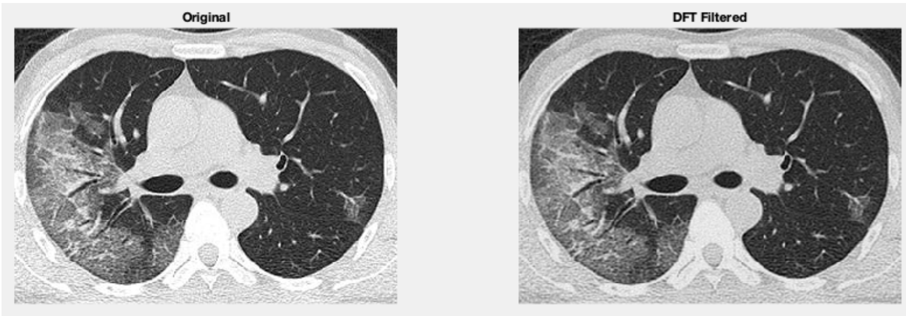


Figure 18: th = 6096

are quantized, and hence the reconstructed quality of the image at the edges will have poor quality.

Another method of decomposing signals that has gained a great deal of popularity in recent years is the use of WAVELETs. Decomposing a signal in terms of its frequency content using sinusoids results in a very fine resolution in the frequency domain, down to the individual frequencies. However, a sinusoid theoretically lasts forever; therefore, individual frequency components give no temporal resolution. In other words, the time resolution of the Fourier series representation is not very good. In a WAVELET representation, we represent our signal in terms of functions that are localized both in time and frequency.

Any small change in the WAVELET representation produces a correspondingly small change in the original signal, so as we have seen in the results, when we apply small threshold on HL, LH, and HH then we can see distortion on the image, but using DFT, when we remove lots of high frequency components of DFT, the image becomes blurred faster but the total distortion increase slower than its rate using dwt2 compression.