Problem 1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

a. $\int_{-\infty}^{0} \frac{dx}{3-4x}$ *b.* $\int_{1}^{\infty} \frac{dx}{(2x+1)^3}$

c. $\int_{1}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$ *d.* $\int_{0}^{2} z^2 \ln(z)\, dz$

e. $\int_{0}^{5} \frac{w}{w-2} dw$ *f.* $\int_{0}^{1} \frac{\ln(x)}{\sqrt{x}} dx$

Problem 2. Use the Comparison Theorem to determine whether each integral is convergent or divergent.

a. $\int_{0}^{\infty} \frac{x}{x^3+1} dx$ *b.* $\int_{1}^{\infty} \frac{2+e^{-x}}{x} dx$

c. $\int_{1}^{\infty} \frac{x+1}{\sqrt{x^4-x}} dx$ *d.* $\int_{0}^{\infty} \frac{\arctan x}{2+e^x} dx$

e. $\int_{0}^{1} \frac{\sec^2 x}{x\sqrt{x}} dx$ *f.* $\int_{0}^{\pi} \frac{\sin^2 x}{\sqrt{x}} dx$

Problem 3. Find the value of the constant C for which the integral

$$\int_{0}^{\infty} \left(\frac{x}{x^2+1} - \frac{C}{3x+1} \right) dx$$

converges. Evaluate the integral for this value of C.

Problem 4. Find the value of p for which the integral converges and evaluate the integral for those values of p.

a. $\int_{0}^{1} \frac{1}{x^p} dx$ *b.* $\int_{e}^{\infty} \frac{1}{x(\ln x)^p} dx$ *c.* $\int_{0}^{1} x^p \ln x dx$

Problem 5. Find the exact length of the following curves.

1. $y = 1 + 6x^{\frac{3}{2}}$ $0 \le x \le 1$
2. $y^2 = 4(x+4)^3$ $0 \le x \le 2,$ $y > 0$
3. $y = \ln(\cos x)$ $0 \le x \le \frac{\pi}{3}$
4. $y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x})$

Problem 6. Find the length of the arc of the curve from point P to point Q.

1. $y = \frac{x^2}{2}$ $P(-1, \frac{1}{2}),$ $Q\left(1, \frac{1}{2}\right)$
2. $x^2 = (y-4)^3$ $P(1,5),$ $Q(8,8)$