Assignment 4: Due Wednesday November 28th 2018 at 11:59 PM

Note: Assignments must be uploaded on **myCourses** in a single **PDF format** file.

Q1 You are visiting a small town with buses whose license plates show their numbers consecutively from 1 up to how many there are. In your mind the number of buses could be anything from 1 to 5, with all possibilities equally likely. Whilst touring the town you first happen to see Bus 3. (**Hint**: $f(y|\theta) = \frac{1}{\theta}$ for $y = 1, ..., \theta$ and $\pi(\theta) = \frac{1}{5}$ for $\theta = 1, ..., 5$)

(a) Assuming that at any point in time you are equally likely to see any of the buses in the town, how likely is it that the next bus number you see will be at least 4?

(b) What is the expected value of the bus number that you will next see?

Q2 (Normal-Normal-Gamma Model): Let $Y_1, ..., Y_n$ be a random sample from $N(\mu, 1/\lambda)$ and $\pi(\mu, \lambda) \propto 1/\lambda$ where $\mu \in \mathbb{R}$ and $\lambda > 0$. It can be shown that $\left(\frac{\mu-\bar{y}}{s/\sqrt{n}}|y\right) \sim t_{(n-1)}$. Suppose we observe the data $y = (2.1, 3.2, 5.2, 1.7)$.

(a) Generate $J = 1000$ samples from the posterior distribution of μ. Use this sample to perform M.C inference on μ. (Find the M.C estimate of μ, 95% C.I for $\hat{\mu}$ and 95% credible interval for μ).

(b) Compare the M.C estimates and the exact posterior estimates and intervals.

(c) Illustrate your inferences with a suitable graph.

Q3 Find the Jeffreys' prior for λ when $Y_1, ..., Y_n$ are a random sample from $N(\mu, 1/\lambda)$ and μ is known.

Q4 A biologist has hypothesized that high concentrations of actinomycin D inhibit RNA synthesis in cells and thereby inhibit the production of proteins. An experiment conducted to test this theory compared the RNA synthesis in cells treated with two concentrations of actinomycin D: 0.6 and 0.7 micrograms per liter. Cells treated with the lower concentration (0.6) of actinomycin D yielded that 55 out of 70 developed normally whereas only 23 out of 70 appeared to develop normally for the higher concentration (0.7).

(a) Do these data indicate that the rate of normal RNA synthesis is lower for cells exposed to the higher concentrations of actinomycin D?

(b) Find the p-value for the test.

(c) If you chose to use $\alpha = 0.05$ what is your conclusion?

Q5 The tremendous growth of the Florida lobster (called spiny lobster) industry over the past 20 years has made it the state's second most valuable fishery industry. A declaration by the Bahamian government that prohibits U.S. lobsterers from fishing on the Bahamian portion of the continental shelf was expected to reduce dramatically the landings in pounds per lobster trap. According to the records, the prior mean landings per trap was 30.31 pounds. A random sampling of 20 lobster traps since the Bahamian fishing restriction went into effect gave the following results (in pounds):

17.4, 18.9, 39.6, 34.4, 19.6, 33.7, 37.2, 43.4, 41.7, 27.5, 24.1, 39.6, 12.2, 25.5, 22.1, 29.3, 21.1, 23.8, 43.2, 24.4

(a) Do these landings provide sufficient evidence to support the contention that the mean landings per trap has decreased since imposition of the Bahamian restrictions? Test using $\alpha = 0.05$.

Q6 Suppose that $Y_1, ..., Y_n$ constitute a random sample from a normal distribution with known mean μ and unknown variance σ^2.

(a) Find the most powerful α-level test of $H_0 : \sigma^2 = \sigma_0^2$ vs $H_a : \sigma^2 = \sigma_1^2$, where $\sigma_1^2 > \sigma_0^2$.

(b) Is the test uniformly most powerful for $H_a : \sigma^2 > \sigma_0^2$?

Q7 Suppose Y is a random sample of size 1 from a population with density function

$$f(y|\theta) = \begin{cases} \theta y^{\theta-1} & 0 < y < 1; \theta > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Sketch the power function of the test with rejection region: $Y > 0.5$.

(b) Based on the single observation Y, find a uniformly most powerful test of size α for testing $H_0 : \theta = 1$ vs $H_a : \theta > 1$

Q8 A survey of voter sentiment was conducted in four midcity political wards to compare the fraction of voters favoring candidate A. Random samples of 200 voters were polled in each of the four wards, with the results as shown in the accompanying table. The numbers of voters favoring A in the four samples can be regarded as four independent binomial random variables.

	Ward				
Opinion	1	2	3	4	Total
Favor A	76	53	59	48	236
Do not favor A	124	147	141	152	564
Total	200	200	200	200	800

(a) Construct a likelihood ratio test of the hypothesis that the fractions of voters favoring candidate A are the same in all four wards. Use $\alpha = 0.05$.