Problem 1. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis or line.

1. $y = \sqrt[3]{x}$, $y = 0$, $x = 1$　　　　　　about the y axis.

2. $y = \sqrt{x}$, $x = 0$, $y = 2$　　　　　　about the x axis.

3. $y = x^2$, $y = 2 - x^2$　　　　　　about the $x = 1$ axis.

4. $x = y^2 + 1$, $x = 2$　　　　　　about the $y = -2$ axis.

Problem 2. Find the avarage value of the following functions on the given intervals.

1. $f(x) = 4x - x^2$　　　　　　　　　　$[0, 4]$

2. $g(x) = \frac{x}{\sqrt{3 + x^2}}$　　　　　　　　　　$[1, 3]$

3. $h(x) = \cos^4(x) \sin(x)$　　　　　　$[0, \pi]$

4. $f(x) = (3 - 2x)^{-1}$　　　　　　　$[-1, 1]$

Problem 3. Evaluate the following integrals by using the Integral by Part technique.

a. $\int \cos\sqrt{x}\, dx$　　　　　　　　　　b. $\int_0^\pi e^{\cos t} \sin 2t\, dt$

c. $\int x \ln(1 + x)\, dx$　　　　　　　　d. $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$

e. $\int \sin(\ln x)\, dx$　　　　　　　　f. $\int_1^2 x^4 (\ln x)^2\, dx$

Problem 4. Evaluate the following trigonometric integrals.

a. $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta\, d\theta$　　　　　　　　b. $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\, dx$

c. $\int_0^\pi \cos^4(2t)\, dt$　　　　　　　　d. $\int \tan^2\theta \sec^4\theta\, d\theta$

e. $\int x \sec x \tan x\, dx$　　　　　　　f. $\int \cos(\pi x) \cos(4\pi x)\, dx$

g. $\int_0^{\frac{\pi}{4}} \sqrt{1 - \cos(4\theta)}\, d\theta$　　　　　　h. $\int \frac{dx}{\cos x - 1}$