MATH 141 – practice problems 12 – Solution.

problem 1.

(a) $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$, Suppose $b_n = \frac{n}{2n^3}$

$a_n = \frac{n}{2n^3+1} < \frac{n}{2n^3} = b_n$

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{2n^2} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^2}$ is Convergent Since $p=2>1$

So, $\sum_{n=1}^{\infty} a_n$ is Convergent as well

b) $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ Suppose $b_n = \frac{9^n}{10^n}$

$a_n = \frac{9^n}{3+10^n} < \frac{9^n}{10^n} = b_n$

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$ is convergent because of geometric series $\frac{9}{10} < 1$

So, $\sum_{n=1}^{\infty} a_n$ is Convergent as well

problem 2.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$ $a_n = \frac{1}{2n+1}$ 1) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2n+1} = 0$

2) $a_{n+1} \le a_n$

$f(n) = \frac{1}{2n+1} \Rightarrow f'(n) = -\frac{2}{(2n+1)^2} < 0$

So, $\sum_{n=1} a_n$ is Conditionally Convergent

b) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$ $\quad a_n = \frac{n^n}{n!}$ $\quad$ 1) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^n}{n!} = \infty$

use the squeeze theorem

So, $\sum_{n=1}^{\infty} a_n$ is divergent

Problem 3

(a) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n^2}$

using the ratio test $\lim_{n\to\infty} \left| \frac{2^{n+1}/(n+1)^2}{2^n/n^2} \right| = \lim_{n\to\infty} \frac{n^2}{(n+1)^2} 2$

$= 2 > 1$

So, It is divergent.

c) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$ $\quad$ using the ratio test

$\lim \left| \frac{(n+1)!/100^{n+1}}{n!/100^n} \right| = 100 \lim_{n\to\infty} \left| \frac{(n+1)!}{n!} \right|$

$= \infty$

So, It is divergent.