Solution Practice problem 3

problem 1

a) $$\int (x+1)\sqrt{2x+x^2}\,dx = \frac{1}{2}\int (2x+2)\sqrt{2x+x^2}\,dx$$

$u = 2x+x^2$

$du = 2+2x$

$$= \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{3}(2x+x^2)^{3/2} + C$$

b) $$\int \frac{e^u}{(1+e^u)^2}\,du = \int \frac{dz}{z^2} = -\frac{1}{z} + C = -\frac{1}{1+e^u} + C$$

$z = 1+e^u$

$dz = e^u\,du$

c) $$\int \frac{\sin 2x}{1+\cos^2 x}\,dx = \int \frac{2\sin x \cos x}{1+\cos^2 x}\,dx = -\int \frac{2u\,du}{1+u^2}$$

$\cos x = u$

$-\sin x\,dx = du$

$$= -\int \frac{dv}{v} = -\ln|v| + C = -\ln|1+u^2| +$$

$$= -\ln|1+\cos^2 x| + C$$

$v = 1+u^2$

$dv = 2u\,du$

d) $$\int \frac{1+x}{1+x^2}\,dx = \int \frac{dx}{1+x^2} + \int \frac{x\,dx}{1+x^2} = \tan^{-1}x + \frac{1}{2}\ln|1+x^2| + $$

$1+x^2 = u \Rightarrow 2x\,dx = du$

problem 2

a) $\int x^3(2+x^4)^5 dx = \frac{1}{4}\int 4x^3(2+x^4)^5 dx = \frac{1}{4}\int (u)^5 du$

$u = 2+x^4$
$du = 4x^3 dx$

$= \frac{1}{4}\frac{u^6}{6} + c = \frac{1}{24}(2+x^4)^6 + c$

b) $\int (3t+2)^{2.4} dt = \frac{1}{3}\int (3t+2)^{2.4}\, 3dt$

$3t+2 = u$
$3dt = du$

$= \frac{1}{3}\int u^{2.4} du = \frac{1}{3}\frac{u^{3.4}}{3.4} + c$

$= \frac{1}{10.2}(3t+2)^{3.4} + c$

c) $\int \frac{\sec^2(\frac{1}{x})}{x^2} dx = -\int \frac{\sec^2(\frac{1}{x})}{-x^2} dx = -\int \sec^2(u)\, du$

$1/x = u$
$-\frac{dx}{x^2} = du$

$= -\tan(u) + C = -\tan(\frac{1}{x}) + C$

d) $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}} dx = \frac{1}{3}\int \frac{3a+3bx^2}{\sqrt{3ax+bx^3}} dx = \frac{1}{3}\int \frac{du}{\sqrt{u}}$

$u = 3ax+bx^3$
$du = (3a+3bx^2)dx$

$= \frac{1}{3}\int u^{-\frac{1}{2}} du = \frac{1}{3}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{3}u^{\frac{1}{2}} + c$

$= \frac{2}{3}(3ax+bx^3)^{\frac{1}{2}} + C$

$f1$ $\displaystyle\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = \int u\,du = \frac{u^2}{2} + c = \frac{(\sin^{-1}x)^2}{2} + c$

$u = \sin^{-1}x$

$du = \dfrac{dx}{\sqrt{1-x^2}}$

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problem 4:

$$\int_0^{\pi} x f(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$$

LHS

$$\int_0^{\pi} x f(\sin x)\,dx = -\int_{\pi}^{0} (\pi-u) f(\sin(\pi-u))\,du$$

$u = \pi - x \rightarrow x = \pi - u$

$du = -dx$

$$= -\pi\int_{\pi}^{0} f(\underbrace{\sin(\pi-u)}_{\sin u})\,du + \int_{\pi}^{0} u f(\underbrace{\sin(\pi-u)}_{\sin u})$$

$$= \pi\int_0^{\pi} f(\sin(u))\,du - \int_0^{\pi} u f(\sin u)\,du$$

Since we have definite integral we can replace u with any other variable including x

$$\Rightarrow \int_0^{\pi} x f(\sin x)\,dx = \pi\int_0^{\pi} f(\sin x)\,dx - \int_0^{\pi} x f(\sin x)\,dx$$

Rearrange the equation $2\int_0^{\pi} x f(\sin x)\,dx = \pi\int_0^{\pi} f(\sin x)\,dx$

$\Rightarrow$

So, $\displaystyle\int_0^{\pi} x f(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$