 Assignment 3: Due Friday November 9th 2018 at 11:59 PM

Note: Assignments must be uploaded on **myCourses** in a single **PDF format** file.

Q1 Let $Y_1, ..., Y_n$ denote a random sample from the uniform distribution on the interval $(\theta, \theta + 1)$. Let $\hat{\theta}_1 = \bar{Y} - \frac{1}{2}$ and $\hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}$.

(a) Show that $\hat{\theta}_1$ and $\hat{\theta}_2$ are both unbiased etimators of θ

(b) Find the relative efficiency of $\hat{\theta}_1$ and $\hat{\theta}_2$.

(c) Which estimator is more efficient?

Q2 Suppose that $Y_1, Y_2, ..., Y_n$ is a random sample of size n from a Poisson-distributed population with mean λ. Assume that $n = 2k$ for some integer k. Consider

$$\hat{\lambda} = \frac{1}{2k} \sum_{i=1}^{k} (Y_{2i} - Y_{2i-1})^2$$

(a) Show that $\hat{\lambda}$ is an unbiased estimator for λ.

(b) Show that $\hat{\lambda}$ is a consistent estimator for λ.

Q3 The number of breakdowns Y per day for a certain machine is a Poisson random variable with parameter λ. The daily cost of repairing these breakdowns is given by $C = 3Y^2$. Let $Y_1, ..., Y_n$ denote the observed number of breakdowns for n independently selected days. Find the MVUE of $E(C)$.

Q4 Let $Y_1, ..., Y_n$ denote a random sample from the probability density function

$$f(y|\theta) = \begin{cases} (\theta + 1)y^\theta & 0 < y < 1; \theta > -1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find an estimator for θ by the method of moments.

(b) Show that the estimator is consistent.

(c) Find the minimal sufficient statistic for θ

(d) Comment on the efficiency of the estimator for θ in (a)

Q5 Suppose that $Y_1, ..., Y_n$ denote a random sample from the Poisson distribution with parameter λ.

(a) Find the MLE of λ

(b) Find the MLE of $e^{-\lambda}$

(c) Find the MVUE of $e^{-\lambda}$.

Q6 A random sample of 100 men produced a total of 25 who favored a controversial local issue. An independent random sample of 100 women produced a total of 30 who favored the issue. Assume that p_M is the true underlying proportion of men who favor the issue and that p_W is the true underlying proportion of women who favor of the issue. If it actually is true that $p_W = p_M = p$. Find the MLE of the common proportion p.

Q7 Suppose that $X_1, ..., X_m$, representing yields per acre for corn variety A, constitute a random sample from a normal distribution with mean μ_1 and variance σ^2. Also, $Y_1, ..., Y_n$, representing yields for corn variety B, constitute a random sample from a normal distribution with mean μ_2 and variance σ^2. Assume that the X's and Y's are independent. Find the MLE for μ_1 and μ_2 and the common variance σ^2.

Q8 Conduct a simulation study to show the sensitivity of the 95% C.I for σ^2 to normality of the random sample. (**Hint**: Similar to the simulation study for the large and small sample C.I for μ in class.)

Q9 Use the *optim* function in R to obtain the MLE of θ given a random sample of $n = 100$ from $U(0, \theta)$. (Take $\theta = 3$, provide the numerical values as well as the curve of the likelihood function with a dashed line to represent the maximizer.)

Q10 In a research program on human health risk from recreational contact with water contaminated with pathogenic microbiological material, the National Institute of Water and Atmosphere (NIWA) instituted a study to determine the quality of NZ stream water at a variety of catchment types. This study is documented in McBride et al. (2002) where $n = 116$ one-liter water samples from sites identified as having a heavy environmental impact from birds (seagulls) and waterfowl. Out of these samples, $x = 17$ samples contained Giardia cysts. Let θ denote the true probability that a one-liter water sample from this type of site contains Giardia cysts.

(a) What is the conditional distribution of X, the number of samples containing Giardia cysts, given θ?

(b) Before the experiment, the NIWA scientists elicited that the expected value of θ is 0.2 with a standard deviation of 0.16. Determine the parameters α and β of a Beta prior distribution for θ with this prior mean and standard deviation. (Round α and β to the nearest integer).

(c) Find the posterior distribution of θ and summarize it by its posterior mean and standard deviation.

(d) Verify that the posterior mean is a weighted average of prior mean and data mean. Specify the respective weights.

(e) Plot the prior, posterior and normalized likelihood in one display.

(f) Find the posterior probability that $\theta < 0.1$ (use the R function).

(g) Find a central 95% posterior credible interval for θ.