Problem 1

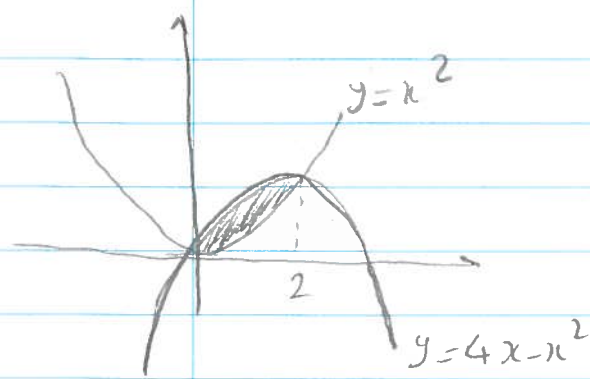a) $y = x^2$ $\qquad$ $y = 4x - x^2$ $\qquad \Rightarrow x^2 = 4x - x^2 \Rightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$
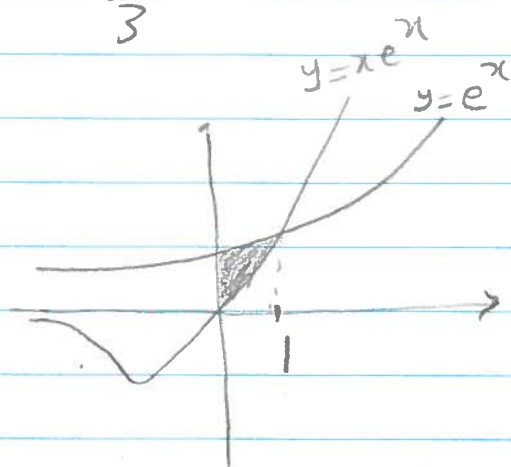

$$\int_0^2 (4x - x^2 - x^2)\, dx = 2x^2 - \frac{2}{3}x^3 \Big|_0^2$$

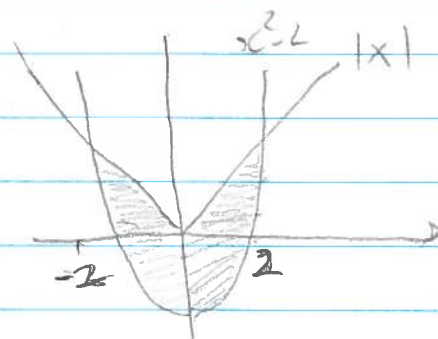$$= 8 - \frac{16}{3} = \frac{8}{3}$$

b) $y = e^x$, $y = xe^x$, $x = 0$

$$e^x = xe^x \Rightarrow \boxed{x = 1}$$

$$\int_0^1 (e^x - xe^x)\, dx = (1-x)e^x + e^x \Big|_0^1 = e - 2$$

integral by part
(not in quiz 2)

d) $y = |x|$, $y = x^2 - 2$

$$x^2 - x - 2 = 0 \Rightarrow \left(x - \tfrac{1}{2}\right)^2 - \frac{9}{4} = 0$$

$$x - \frac{1}{2} = \pm\frac{3}{2} \Rightarrow \boxed{x = 2}$$

$$2\int_0^2 (|x| - x^2 + 2)\, dx = 2\int_0^2 (x - x^2 + 2)\, dx = 2\left(\frac{x^2}{2} - \frac{x^3}{3} + 2x\right)\Big|$$
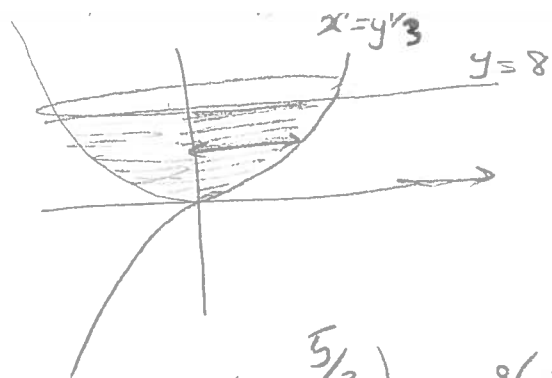
$$= 2\left(2 - 8/3 + 4\right) = 2\left(\frac{10}{3}\right) = \frac{20}{3}$$

problem 2

a) $y=x^3$, $y=8$, $x=0$

$$V = \int_0^8 \pi\left(\sqrt[3]{y}\right)^2 dy$$

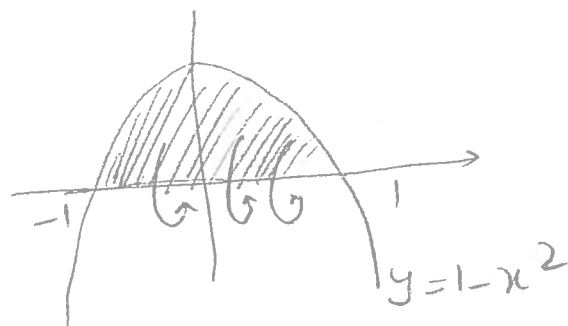$$= \pi \int_0^8 y^{2/3}\, dy = \pi\, \frac{y^{5/3}}{5/3}\bigg|_0^8 = \frac{3\pi}{5}\left(8^{5/3}\right) = \frac{96\pi}{5}$$

b) $y = 1-x^2$

$$V = \int_{-1}^1 \pi\left(1-x^2\right)^2 dx$$

$$= \pi\int_{-1}^1 \left(1+x^4-2x^2\right)dx = 2\pi\int_0^1 \left(1+x^4-2x^2\right)dx$$

$$= 2\pi\left(1+\tfrac{1}{5}-\tfrac{2}{3}\right) = 2\pi\left(\tfrac{1}{3}+\tfrac{1}{5}\right) = 2\pi\left(\frac{8}{15}\right)$$
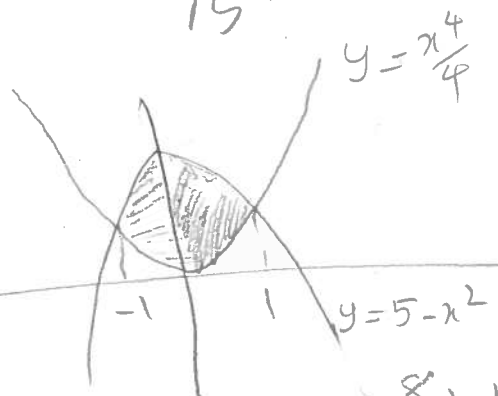
$$= \frac{16\pi}{15}$$

c) $y = \dfrac{x^4}{4}$, $y = 5-x^2$

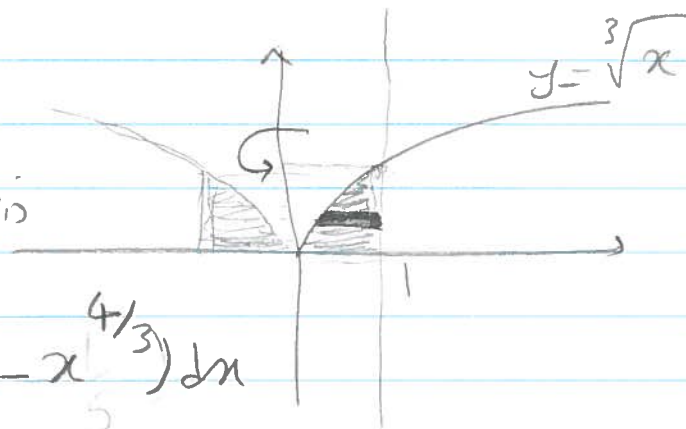$$\frac{x^4}{4} = 5-x^2 \Rightarrow \boxed{\begin{array}{l} x=1 \\ x=-1 \end{array}}$$

$$V = \int_{-1}^1 \pi\left(\left(5-x^2\right)^2 - \left(\tfrac{x^4}{4}\right)^2\right)dx = 2\pi\int_0^1\left(25+x^4-10x^2-\frac{x^8}{16}\right)dx$$

$$= 2\pi\left(25+\tfrac{1}{5}-\tfrac{10}{3}-\tfrac{1}{144}\right)$$

problem 3

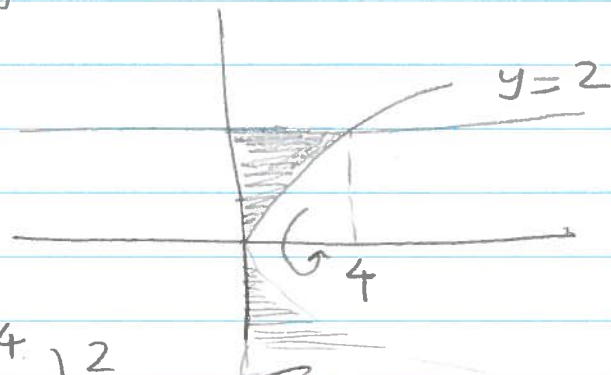a) $y = \sqrt[3]{x}$ $y = 0$, $x = 1$, y-axis

$y = \sqrt[3]{x}$

$V = \int_0^1 2\pi x (1 - x^{1/3}) \, dx = 2\pi \int_0^1 (x - x^{4/3}) \, dx$

$= 2\pi \left(\frac{1}{2} - \frac{1}{7/3} \right) = 2\pi \left(\frac{1}{2} - \frac{3}{7} \right) = 2\pi \left(\frac{1}{14} \right)$

$= \frac{\pi}{7}$

b) $y = \sqrt{x}$ $x = 0$, $y = 2$, x-axis
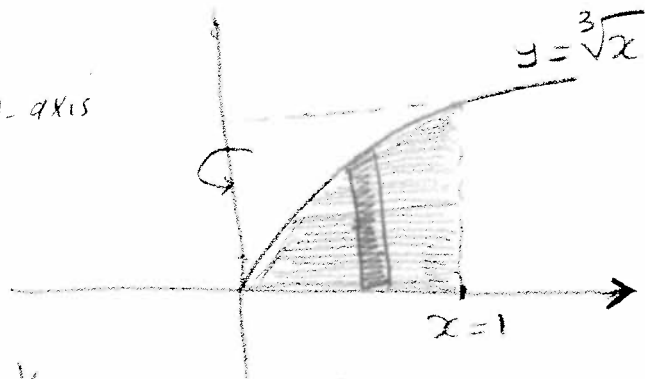
$y = 2$

$\Rightarrow V = \int_0^2 2\pi y \, f(y) \, dy$

$= \int_0^2 2\pi y (y^2) \, dy = 2\pi \left(\frac{y^4}{4} \right) \Big|_0^2 = 8\pi$

Problem 3.

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis

1 $y = \sqrt[3]{x}$, $y = 0$, $x = 1$, y-axis

shells method

$$V = \int_0^1 2\pi x \, f(x) \, dx = \int_0^1 2\pi x \, x^{1/3} \, dx = 2\pi \frac{x^{7/3}}{7/3} \Big|_0^1$$

$$= 6 \frac{\pi}{7}$$

Washer's method

$$V = \int_0^1 \pi (1 - (y^3)^2) \, dy = \pi \left(1 - \frac{1}{7}\right) = \frac{6\pi}{7}$$
$$\quad\quad\quad\quad y^6$$