Problem 1. Use the properties of integral to verify the following inequalities.

a. $\int_0^4 \left(x^2 - 4x + 4\right) dx \geq 0$

b. $2 \leq \int_{-1}^{1} \sqrt{1+x^2}\, dx \leq 2\sqrt{2}$

c. $\int_1^3 \sqrt{x^4+1}\, dx \geq \frac{26}{3}$

d. $\int_0^{\frac{\pi}{2}} x \sin(x)\, dx \leq \frac{\pi^2}{8}$

Problem 2. Use Part 1 of the fundamental Theorem of Calculus (FTC) to find the derivative of $g(x)$.

a. $g(x) = \int_1^x \frac{1}{t^3+1} dt$

b. $g(x) = \int_x^{\pi} \sqrt{1+\sec(t)}\, dt$

c. $g(x) = \int_1^{e^x} \ln(t)\, dt$

d. $g(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^2+1} dz$

e. $g(x) = \int_{\sin(x)}^{1} \sqrt{1+t^2}\, dt$

f. $g(x) = \int_0^{x^4} \cos^2(\theta)\, d\theta$

Problem 3. Find a continuous function f and a number a such that

$$2 + \int_a^x \frac{f(t)}{t^7} dt = 6x^{-5}$$

Problem 4. Evaluate the following definite integrals using Part 2 FTC.

a. $\int_{-1}^{2} \left(1 + \frac{1}{2}u^{\frac{1}{3}} - \frac{2}{5}u^9\right) du$

b. $\int_0^2 (y-1)(2y+1)\, dy$

c. $\int_0^{\frac{\pi}{4}} \sec(\theta)\tan(\theta)\, d\theta$

d. $\int_1^2 (2y+1)^2 dy$

e. $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}} dx$

f. $\int_{-2}^{2} f(x)\, dx$ Where $f(x) = \begin{cases} 2 & -2 \leq x \leq 0 \\ 4 - x^2 & 0 \leq x \leq 2 \end{cases}$