practice problem 8 – Solution

Problem 1)

1) $9x = y^2 + 18 \qquad 2 \le x \le 6$ about x-axis

$$A = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^6 2\pi y \sqrt{1 + \left(\frac{9}{2y}\right)^2}\, \frac{2y}{9}\, dy$$

($f(x)$ = y, dx = $\frac{2y}{9}\,dy$)

$9\,dx = 2y\,dy \rightarrow \frac{dy}{dx} = \frac{9}{2y}$

$\rightarrow dx = \frac{2y\,dy}{9}$

$2 \le x \le 6 \xrightarrow{9x = y^2+18} 0 \le y \le 6$

$$= \int_0^6 2\pi y \sqrt{\frac{4y^2+81}{4y^2}}\, \frac{2y}{9}\, dy$$

$$= \int_0^6 \frac{2\pi y}{9} \sqrt{4y^2+81}\, dy$$

u substitution

$4y^2 + 81 = u$

$8y\,dy = du$

$$= \int_{81}^{225} \frac{2\pi}{72} \sqrt{u}\, du$$

$$= \frac{2\pi}{72} \frac{u^{3/2}}{3/2} \Big|_{81}^{225} = \frac{4\pi}{216}\left(225^{3/2} - 81^{3/2}\right)$$

$$= \frac{\pi}{54}\left(15^3 - 9^3\right)$$

4) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$ $\quad 1 \le x \le 2$ about y-axis

$$A = \int_a^b 2\pi x \sqrt{1+(f'(x))^2}\,dx = \int_1^2 2\pi x \sqrt{1+\left(\frac{x}{2}-\frac{1}{2x}\right)^2}\,dx$$

$\frac{dy}{dx} = \frac{x}{2} - \frac{1}{2x}$

$$= \int_1^2 2\pi x \sqrt{1+\left(\frac{x^2-1}{2x}\right)^2}\,dx$$

$$= \int_1^2 2\pi x \sqrt{\frac{4x^2+(x^2-1)^2}{4x^2}}\,dx$$

$$= \pi \int_1^2 \sqrt{x^4+2x^2+1}\,dx$$

$$= \pi \int_1^2 \sqrt{(x^2+1)^2}\,dx$$

$$= \pi \int_1^2 (x^2+1)\,dx = \pi\left(\frac{x^3}{3}+x\right)_1^2$$

$$= \pi\left(\left(\frac{8}{3}+2\right)-\left(\frac{1}{3}+1\right)\right)$$

$$= \frac{10\pi}{3}$$