 Assignment 2: Due Friday October 19th 2018 at 11:59 PM

Note: Assignments must be uploaded on **myCourses** in a single file **PDF format**.

Q1 Let Y_1 and Y_2 denote the proportions of two different types of components in a sample from a mixture of chemicals used as an insecticide. Suppose that Y_1 and Y_2 have the joint density function given by

$$f(y_1, y_2) = \begin{cases} 2 & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1, 0 \leq y_1 + y_2 \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find $P(Y_1 \leq 3/4, Y_2 \leq 3/4)$

(b) Find $P(Y_1 < Y_2)$

(c) Find the marginal density functions of Y_1 and Y_2

(d) Find the conditional density function of Y_1 given Y_2

Q2 A quality control plan for an assembly line involves sampling $n = 10$ finished items per day and counting Y, the number of defectives. If p denotes the probability of observing a defective, then Y has a binomial distribution, assuming that a large number of items are produced by the line. But p varies from day to day and is assumed to have a uniform distribution on the interval from 0 to 1/4.

(a) Find $E(Y)$

(b) Find $V(Y)$

Q3 The National Fire Incident Reporting Service stated that, among residential fires, 73% are in family homes, 20% are in apartments, and 7% are in other types of dwellings. If four residential fires are independently reported on a single day, what is the probability that two are in family homes, one is in an apartment, and one is in another type of dwelling?

Q4 Let $Z_1, ..., Z_n$ be independent and identically distributed random variables from the standard Normal distribution.

(a) Use the method of distribution function to show $W_i = Z_i^2$ has the Chi-square distribution with 1 degree of freedom.

(b) Use the method of Moment-generating function to show $V = \sum_{i=1}^{n} W_i$ has Chi-square distribution with n degrees of freedom.

Q5 Let Y have the Exponential distribution with mean β.

(a) Show that $U = [Y]$ (the greatest integer value smaller or equal to Y) has the Geometric distribution. (Find the probability function of U)

(b) Use (a) to generate data from the Geometric($p = 0.4$) and the Negative Binomial ($r = 10$, $p = 0.4$) distributions by the Uniform distribution in $(0, 1)$ (Explain the steps, Write R code, Create the histograms)

(c) Repeat part (b) for both the Geometric and the Negative Binomial distributions $N = 10, 50, 100, 500$ times. Create the histogram of $\bar{X}_n$

(d) Comment on part (c)

Q6 Use the Uniform distribution to generate data from

(a) Two independent random variables $X \sim Gamma(5, 3)$ and $Y \sim Gamma(8, 3)$ (Explain the transormations and the steps, Write R code, Create the histogram).

(b) Use the generated data in (a) to generate data from $Beta(5, 8)$ (Explain the transformation and the steps, Write R code, Create the histogram)

Q7 The probability distribution shown here describes a population of measurements that can assume values of 0, 1, and 4, each of which occurs with the same probability as:

x	0	1	4
$p(x)$	1/3	1/3	1/3

(a) Find $E(X)$ and $V(X)$

(b) List all the different samples of $n = 2$ measurements that can be selected from this population.

(c) Construct the sampling distribution of $\bar{X}_n$ and S_n^2.

(d) Find $E(\bar{X}_n)$, $V(\bar{X}_n)$, $E(S_n^2)$, and create the histogram of $\bar{X}_n$.

(e) Comment on part (d)?

Q8 Use Monte Carlo integration to solve for these integrals and see that its close to the actual value.

(a) Let $h(x) = x^3$. Then $\mathcal{I} = \int_0^1 x^3 dx = 1/4$.

(b) $\mathcal{I} = \Phi(1.25) = \int_{-\infty}^{1.25} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$. Verify your answer with `pnorm`

(c) $\mathcal{I} = \int_{0.25}^{0.75} \frac{4}{1+x^2} dx$. Verify your answer with `integrate`

Q9 A chemist who has prepared a product designed to kill 60% of a particular type of insect wants to evaluate the kill rate of her preparation. What sample size should she use if she wishes to be 95% confident that her experimental results fall within 0.02 of the true fraction of insects killed?

Q10 A chemical process has produced, on the average, 800 tons of chemical per day. The daily yields for the past week are 785, 805, 790, 793, and 802 tons. Estimate the mean daily yield, with confidence coefficient 0.90, from the data. What assumptions did you make?

Q11 Two new drugs were given to patients with hypertension. The first drug lowered the blood pressure of 16 patients an average of 11 points, with a standard deviation of 6 points. The second drug lowered the blood pressure of 20 other patients an average of 12 points, with a standard deviation of 8 points. Determine a 95% confidence interval for the difference in the mean reductions in blood pressure, assuming that the measurements are normally distributed with equal variances.