Solution practice problem 5

a) $\int \cos\sqrt{x}\, dx = 2\int z \cos z\, dz = 2\left[z\sin z - \int \sin z\, dz\right] + C$

$\sqrt{x} = z \Rightarrow x = z^2 \Rightarrow dx = 2z\,dz$

$= 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C$

$z = u \Rightarrow dz = du$

$\cos z\, dz = dv \Rightarrow \sin z = v$

b) $\int_0^{\pi} e^{\cos t} \sin 2t\, dt = \int_0^{\pi} e^{\cos t}\, 2\sin t \cos t\, dt = -\int_1^{-1} 2e^z z\, dz = \int_{-1}^{1} 2z e^z dz$

$\cos t = z \Rightarrow -\sin t\, dt = dz$

$= +2\left[z e^z \Big|_{-1}^{1} - \int_{-1}^{1} e^z dz\right]$

$z = u \Rightarrow dz = du$

$e^z dz = dv \Rightarrow e^z = v$

$= 2\left[e + e^{-1} - (e - e^{-1})\right]$

$= 4e^{-1}$

c) $\int x \ln(1+x)\, dx = \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\int \frac{x^2}{x+1} dx + C$

$\ln(x+1) = u \Rightarrow \frac{dx}{x+1} = du$

$x\,dx = dv \Rightarrow \frac{x^2}{2} = v$

$= \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\int \frac{x^2 - 1 + 1}{x+1} dx + C$

$= \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\left[\int (x-1)\,dx + \int \frac{dx}{x+1}\right] + C$

$= \frac{x^2}{2}\ln(x+1) - \frac{(x-1)^2}{4} - \frac{1}{2}\ln(x+1) + C$

d) $\int_1^{\sqrt{3}} \tan^{-1}\left(\frac{1}{x}\right) dx = x \tan^{-1}\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} \frac{-x^{-1}}{1+\left(\frac{1}{x}\right)^2} dx$

$\tan^{-1}\left(\frac{1}{x}\right) = u \Rightarrow \frac{-x^{-2}}{1+\left(\frac{1}{x}\right)^2} dx = du$

$dx = dv \Rightarrow x = v$

$= \sqrt{3} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}(1) + \int_1^{\sqrt{3}} \frac{x}{1+x^2}$

$= \sqrt{3}\ \frac{\pi}{6} - \frac{\pi}{4} + \frac{1}{2}\ln(x^2+1)\Big|_1^{\sqrt{3}}$

$= \frac{\pi\sqrt{3}}{6} - \pi/4 + \frac{1}{2}\ln(4) - \frac{1}{2}\ln(2)$

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e) $\int \sin(\ln x)\, dx = x\sin(\ln x) - \int \cos(\ln x)\, dx$

$\sin(\ln x) = u \Rightarrow \frac{1}{x}\cos(\ln x)\, dx = du$

$dx = dv \Rightarrow x = v$

$= x\sin(\ln x) - \left[x\cos(\ln x) + \int \sin(\ln x)\right] + C$

$\cos(\ln x) = u \Rightarrow -\frac{1}{x}\sin(\ln x) = du$

$dx = dv \Rightarrow x = v$

$\Rightarrow \int \sin(\ln x)\, dx = \frac{1}{2} x\sin(\ln x) - \frac{x}{2}\cos(\ln x) + C$

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f) $\int_1^2 x^4 (\ln x)^2 dx = \int_0^{\ln 2} e^{4u} u^2 e^u du = \int_0^{\ln 2} u^2 e^{5u} du$

$\ln x = u \Rightarrow x = e^u \Rightarrow dx = e^u du$

$= - - - - -$