Assignment 1: Due Friday Sept 28th 2018 at 11:59 PM

Note: Assignments must be uploaded on **myCourses** in a single file **PDF format**.

Q1 Red snapper is a rare and expensive reef fish served at upscale restaurants. Federal law prohibits restaurants from serving a cheaper, look-alike variety of fish (e.g., vermillion snapper or lane snapper) to customers who order red snapper. Researchers at the University of North Carolina used DNA analysis to examine fish specimens labeled "red snapper" that were purchased form vendors across the country (Nature, July 15, 2004). The DNA tests revealed that 77% of the specimens were not red snapper, but the cheaper, look-alike variety of fish.

(a) Assuming that the results of the DNA analysis are valid, what is the probability that you are actually served red snapper the next time you order it at a restaurant?

(b) If there are five customers at a restaurant, all who have ordered red snapper, what is the probability that at least one customer is actually served red snapper?

Q2 Nondestructive evaluation (NDE) describes methods that quantitatively characterize materials, tissues, and structures by noninvasive means, such as X-ray computed tomography, ultrasonics, and acoustic emission. Recently, NDE was used to detect defects in steel castings (JOM, May 2005). Assume that the probability that NDE detects a "hit" (i.e., predicts a defect in a steel casting) when, in fact, a defect exists is 0.97 (This is often called the probability of detection.). Assume also that the probability that NDE detects a "hit" when, in fact, no defect exists is 0.005 (This is called the probability of a false call.). Past experience has shown that a defect occurs once in every 100 steel castings. If NDE detects a "hit" for a particular steel casting, what is the probability that an actual defect exists?

Q3 HIV testing and false positives. In North America, the probability of a person having HIV is 0.008. A test for HIV yields either a positive or negative result. Given that a person has HIV, the probability of a positive test result is 0.99 (This probability is called the sensitivity of the test.). Given that a person does not have HIV, the probability of a negative test result is also 0.99 (This probability is called the specificity of the test.). The authors of the article are interested in the probability that a person actually has HIV given that the test is positive.

(a) Find the probability of interest for a North American by using Bayes' rule.

(b) In East Asia, the probability of a person having HIV is only 0.001. Find the probability of interest for an East Asian by using Bayes's rule (Assume that both the sensitivity and specificity of the test are 0.99).

(c) Typically, if one tests positive for HIV, a follow-up test is administered. What is the probability that a North American has HIV given that both tests are positive? (Assume that the tests are independent.)

(d) Repeat part c for an East Asian.

Q4 Using the axioms of probability, show that for any sequence of events $A_1, A_2, ...$

$$P(\bigcup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i)$$

Q5 Tay-Sachs disease is a genetic disorder that is usually fatal in young children. If both parents are carriers of the disease, the probability that their offspring will develop the disease is approximately 0.25. Suppose that a husband and wife are both carriers and that they have three children. If the outcomes of the three pregnancies are mutually independent, what are the probabilities of the following events?

(a) All three children develop Tay-Sachs.

(b) Only one child develops Tay-Sachs.

(c) The third child develops Tay-Sachs, given that the first two did not.

Q6 The employees of a firm that manufactures insulation are being tested for indications of asbestos in their lungs. The firm is requested to send three employees who have positive indications of asbestos on to a medical center for further testing. If 40% of the employees have positive indications of asbestos in their lungs, find the probability that ten employees must be tested in order to find three positives.

Q7 Seed are often treated with fungicides to protect them in poor draining, wet environments. A small-scale trial, involving five treated and five untreated seeds, was conducted prior to a large-scale experiment to explore how much fungicide to apply. The seeds were planted in wet soil, and the number of emerging plants were counted. If the solution was not effective and four plants actually sprouted, what is the probability that

(a) All four plants emerged from treated seeds?

(b) Three or fewer emerged from treated seeds?

(c) At least one emerged from untreated seeds?

Q8 A salesperson has found that the probability of a sale on a single contact is approximately 0.03. If the salesperson contacts 100 prospects, what is the approximate probability of making at least one sale?

Q9 A gas station operates two pumps, each of which can pump up to 10000 gallons of gas in a month. The total amount of gas pumped at the station in a month is a random variable Y, measured in 10000 gallons, with a probability density function given by

$$f(y) = \begin{cases} y & 0 \leq y < 1, \\ 2 - y & 1 \leq y < 2, \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find the cumulative distribution function of Y denoted by $F_Y(y)$.

(b) Find the probability that the station will pump between 8500 and 11500 gallons in a particular month.

(c) What is the expected monthly revenue if the station sells the gas at the price of 2.10$ per gallon?

Q10 Wires manufactured for use in a computer system are specified to have resistances between 0.12 and 0.14 ohms. The actual measured resistances of the wires produced by company A have a normal probability distribution with mean 0.13 ohm and standard deviation 0.005 ohm.

(a) What is the probability that a randomly selected wire from company A's production will meet the specifications?

(b) If four of these wires are used in each computer system and all are selected from company A, what is the probability that all four in a randomly selected system will meet the specifications?

Q11 A random variable Y has the density function $f(y) = e^y$ for $y < 0$.

(a) Find $E(e^{3Y/2})$

(b) Find the moment-generating function of Y.

(c) Find the variance of Y.