Problem 1. Evaluate the following trigonometric integrals.

a. $\int_0^{\frac{\pi}{2}} \sin^7\theta\cos^5\theta d\theta$

b. $\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}dx$

c. $\int_0^{\pi} \cos^4(2t)dt$

d. $\int \tan^2\theta\sec^4\theta d\theta$

e. $\int x\sec x\tan x dx$

f. $\int \cos(\pi x)\cos(4\pi x)dx$

g. $\int_0^{\frac{\pi}{4}} \sqrt{1-\cos(4\theta)}d\theta$

h. $\int \frac{dx}{\cos x - 1}$

Problem 2. Evaluate the following integrals using trigonometric substitution.

a. $\int \frac{dx}{x^2\sqrt{4-x^2}}$

b. $\int \frac{x^3}{\sqrt{x^2+4}}dx$

c. $\int \frac{\sqrt{x^2-4}}{x}dx$

d. $\int_0^1 x^3\sqrt{1-x^2}dx$

e. $\int \frac{\sqrt{1+x^2}}{x}dx$

f. $\int_0^1 \frac{dx}{(x^2+1)^2}$

Problem 3. Use trigonometric substitution to show

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)+c$$

Problem 4. Use partial fractions to solve the following integrals.

a. $\int \frac{y}{(y+4)(2y-1)}dy$

b. $\int_0^1 \frac{2}{2x^2+3x+1}dx$

c. $\int \frac{4x}{x^3+x^2+x+1}dx$

d. $\int \frac{3x^2+x+4}{x^4+3x^2+2}dx$

e. $\int \frac{x^4+3x^2+1}{x^5+5x^3+5x}dx$

f. $\int \frac{x^3+2x^2+3x-2}{(x^2+2x+2)}dx$

Problem 5. Make a substitution to express the integrand as a rational function and then evaluate it.

a. $\int \frac{dx}{2\sqrt{x+3}+x}$

b. $\int_0^1 \frac{dx}{1+\sqrt[3]{x}}$

c. $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$

d. $\int \frac{\sqrt{1+\sqrt{x}}}{x}dx$

e. $\int \frac{\sin(x)}{\cos^2(x)-3\cos(x)}dx$

f. $\int \frac{dx}{1+e^x}$

Problem 6. Use integration by parts together with partial fractions to evaluate the following integrals.

a. $\int \ln\left(x^2 - x + 2\right) dx$ *b.* $\int x\tan^{-1}(x)\, dx$

Problem 7. Evaluate the following integrals by completing the square.

a. $\int \frac{dx}{x^2-2x}$ *b.* $\int \frac{2x+1}{4x^2+12x-7} dx$