**Problem 1.** Find the critical numbers of each function .

*a.* $f(x) = x^3 + 6x^2 - 15x$ *b.* $f(x) = x^{4/5}(x-4)^2$

*d.* $f(x) = \frac{x-1}{x^2-x+1}$ *e.* $f(x) = 3x - \sin^{-1}(x)$

*f.* $f(x) = |3x-4|$ *g.* $f(x) = x^{-2}\ln(x)$

**Problem 2.** Find the local/absolute minimum and maximum of each function. Determine where they are increasing or decreasing. Determine where they are concave up or concave down.

*a.* $f(x) = 8x + \frac{10}{x}$ $x > 0$ *b.* $f(x) = x^3(x+2)^4$ $x \in [-14, 15]$

*c.* $f(x) = 1 - 2x^2$ $x \in [-5, 1]$ *d.* $f(x) = x\sqrt{x^2+16}$ $x \in [-4, 6]$

*e.* $f(x) = \frac{2x-6}{x+3}$ *f.* $f(x) = \frac{2x}{x^2-9}$

**Problem 3.** Use the Intermediate Value Theorem to show each statement.

- $f(x) = x^3 - x - 2$ has at least one root in $(0, 2)$.
- $f(x) = x^3 - 3x + 0.1$ has at least a root in the interval $(0, 1)$.

**Problem 4.** Use the Rolle 's Theorem to prove each statement.

- $f(x) = x^3 - 15x + c$ has at most one root in $[-2, 2]$. ($c$ is a constant)
- $f(x) = \sin(x) + 2x - 3$ has exactly one root.
- $f(x) = 3xe^x + 1$ has exactly two roots.

**Problem 5.** Find the number $c$ that satisfies the conclusion of the Mean Value Theorem on the given interval.

*a.* $f(x) = \sqrt{x}$ $[0, 4]$ *b.* $f(x) = e^{-x}$ $[0, 2]$

**Recall:**

1. **Intermediate Value Theorem** Suppose $f$ is continuous on the closed interval $[a, b]$ and $N$ be any number between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$, then there is a number $c$ in $(a, b)$ such that $f(c) = N$.

2. **Rolle 's Theorem** Let $f$ be a function satisfying the following conditions

   - $f$ is continuous on $[a, b]$
   - $f$ is differentiable on $(a, b)$
   - $f(a) = f(b)$

   Then there is a number $c$ in $(a, b)$ such that $f'(c) = 0$.

3. **Mean Value Theorem** Let $f$ be a function satisfying the following conditions

   - $f$ is continuous on $[a, b]$
   - $f$ is differentiable on $(a, b)$

   Then there is a number $c$ in $(a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$