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Digital Image processing

CHW2

Due: 1399/1/22

Machine Vision & Image Processing Lab.

THE 2-D DISCRETE FOURIER TRANSFORM AND ITS INVERSE:

A development similar to the material in Sections 4.3 and 4.4 of the Gonzalez book would yield the following 2-D discrete Fourier transform (DFT):

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j\,2\pi\,(\frac{ux}{M}+\frac{vy}{N})} \qquad 1$$

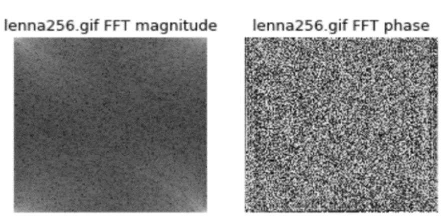
where $f(x, y)$ is a digital image of size $M * N$. As in the 1-D case, Eq.1 must be evaluated for values of the discrete variables u and v in the ranges u = 0, 1, 2,…,M -1and v = 0, 1, 2,…, N -1.

- **Part I:** Try to show the two images $f_1(x,y)$ (referred as Lena) and $f_2(x,y)$ (referred as Synthetic):

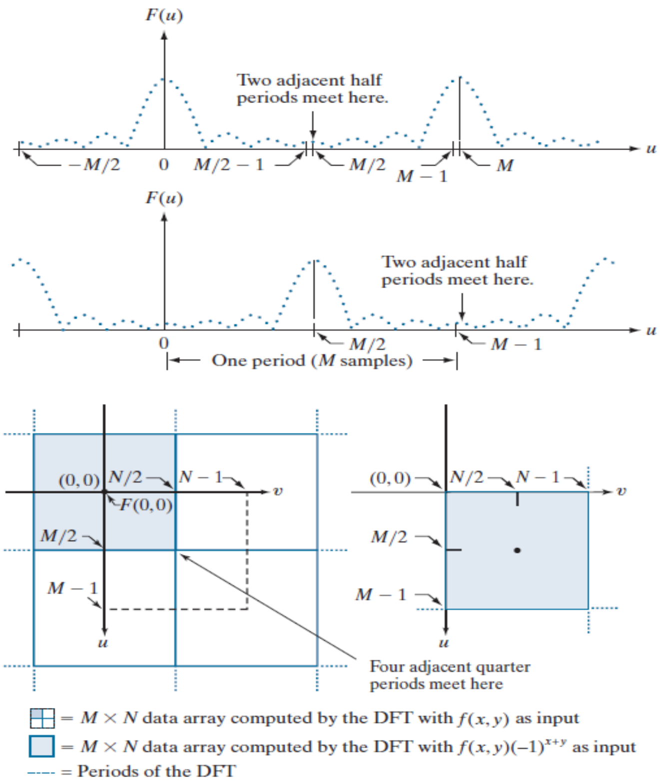
```python
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

```python
f1 = plt.imread("lenna256.gif")[:,:, 0] # we know this image is grayscale so we grab first channel's color
f2 = plt.imread('cir.tif')
plt.figure(1)
plt.subplot(121)
plt.axis('off')
plt.imshow(f1, cmap='gray')
plt.subplot(122)
plt.axis('off')
plt.imshow(f2, cmap='gray')
plt.show()
```

- **Part II:** After showing each image, compute the DFT using numpy's built-in FFT function and show phase and magnitude of each image. your output must be something like this (for Lena image):



Part III: The periodicities of the transform and its inverse are important issues in the implementation of DFT-based algorithms. As explained in Section 4.4 [see the footnote to Eq. (4-42)], the transform data in the interval from 0 to M - 1 consists of two half periods meeting at point $M/2$, but with the lower part of the period appearing at higher frequencies. For display and filtering purposes, it is more convenient to have in this interval a complete period of the transform in which the data are contiguous and ordered properly, as illustrated in the following Figure.

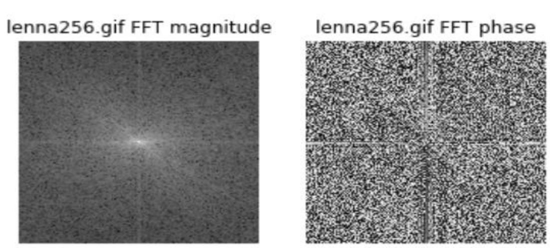


In other words, multiplying $f(x)$ by the exponential term $e^{\frac{j2\pi}{M}u_0 x}$ shifts the transform data so that the origin, $F(0)$, is moved to u_0. If we let $u_0 = M/2$, the exponential term will become $e^{j\pi x}$, which is equal to $(-1)^x$ where x is an integer between 0 and M-1. In this case, that is, multiplying $f(x)$ by $(-1)^x$ shifts the data so that $F(u)$ is centered in the interval $[0, M - 1]$, as desired.

In 2-D, the situation is more difficult to graph, but the principle is the same. Instead of two half periods, there are now four quarter periods meeting at the point $(M/2, N/2)$. As in the 1-D case, we want to shift the data so that $F(0,0)$ is shifted to $(M/2, N/2)$. Letting $(u_0, v_0) = (M/2, N/2)$ results in the expression:

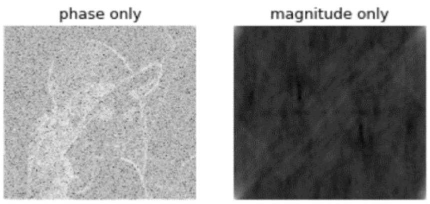
$$f(x, y)(-1)^{x+y} \overset{DFT}{\Longleftrightarrow} F(u - M/2, v - N/2)$$

- Using this equation shifts the data so that $F(0,0)$ is moved to the center of the *frequency rectangle* (i.e., the rectangle defined by the intervals $[0, M - 1]$ and $[0, N - 1]$ in the frequency domain). Your output (for Lena image) must be like this:



- What is the type of coordinate values in both the spatial and frequency domains (integers, float, …)? Why?

- **Part IV:** Reconstruct each of the two images using the phase and magnitude obtained in Part II. Your output must include 3 reconstructed images: using phase only, using magnitude only and using both phase and magnitude). Hereafter you see the reconstructed Lena image using only its DFT phase and only its DFT magnitude.



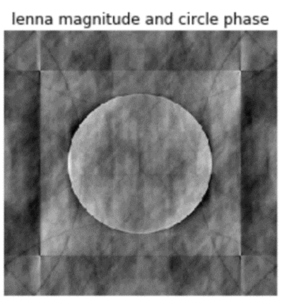
- What can be inferred from the output of the inverse Fourier transform of the phase? of the magnitude? and of both the phase and magnitude?

- **Part V:** Take the phase from one image and the magnitude from another image and generate the inverse DFT of the resulted spectrum.

$$G_1 = |F_1(u,v)|e^{-j\theta(F_2)}$$

$$G_2 = |F_1(u,v)|e^{-j\theta(F_1)}$$

The following figure shows the image reconstructed using the magnitude spectrum of Lena and phase spectrum of Synthetic image.



- What can be inferred from the output of the above inverse Fourier transforms?

TRANSLATION AND ROTATION:

The validity of the following Fourier transform pairs can be demonstrated by direct substitution into Eqs. (4-67) and (4-68) of Gonzalez:

$$f(x,y)e^{j\,2\pi\,(\frac{u_0 x}{M}+\frac{v_0 y}{N})} \Leftrightarrow F(u - u_0\,,\, \mathrm{v} - v_0) \qquad 3$$

and

$$F(u,v)e^{-j\,2\pi\,(\frac{u x_0}{M}+\frac{v y_0}{N})} \Leftrightarrow f(x - x_0\,,\, \mathrm{y} - y_0) \qquad 4$$

That is, multiplying $f(x, y)$ by the exponential shown shifts the origin of the DFT to (u_0, v_0) and, conversely, multiplying F(u,v) by the negative of that exponential shifts the origin of $f(x, y)$ to (x_0, y_0).

- **Part VI:** Reconstruct each of the two images using the phase and magnitude obtained in Part II after applying Eq. 4 (use $x_0 = \frac{M}{2}, y_0 = \frac{N}{2}$) and compare with the original images shown in part I.