I. Section a

Since the motion is in 30 degrees direction with a length of 20 pixels, the degradation function can be expressed as follows, as it has been discussed in previous assignments (HW7):

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))dt \tag{1}$$

The previous integration can be expressed in discrete form according to the definition. This new summation can be performed on a trajectory like $m(r,\theta)$. So, a new 2D filter is achieved and which can be implemented simply.

$$g(x,y) \cong \frac{1}{R}\sum_{r=-\frac{R}{2}}^{\frac{R}{2}} f(r' + r, \theta) = h(x,y) * \mathrm{f(x,y)} \tag{2}$$

The above summation gives the average of pixels on a specific trajectory, which can be defined as follows:

$$m(R,\theta) = 20\angle 30° \cong 18\hat{\imath} + 10\hat{\jmath}$$

So, the final kernel is a 10×18 matrix with ones on its diameter.

Another way to define a specific filter in the frequency spectra is to perform Fourier transform on the above integration. This will give the Fourier transform of the motion filter.

$$G(u,v) = \mathcal{F}\left[\int_0^T f(x - x_0(t), y - y_0(t))dt\right] \tag{3}$$

$$H(u,v) = \int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt = \frac{T}{\pi(ua+vb)}\sin\left(\pi(ua+vb)\right)e^{-j\pi(ua+vb)}. \tag{4}$$

where a and b are the total distance the image has been displaced.

In this assignment, the first method that is explained above is implemented using MATLAB. The motion-blurring simulation has been done with python too. But there were too many difficulties to go on with it.

The result of simulating the motion-blurring to the original image is as follows:

Figure 1. The left-sided image is the original image and the right-sided image is a simulation of the motion-blurred image.

II. Section b

In this section, the effect of the additive white Gaussian noise with zero mean and variance of 0.25 is simulated. The variance of 0.25 is too much for the image and destroys its information. Anyway, it is used to show the advantages of the Wiener filter.

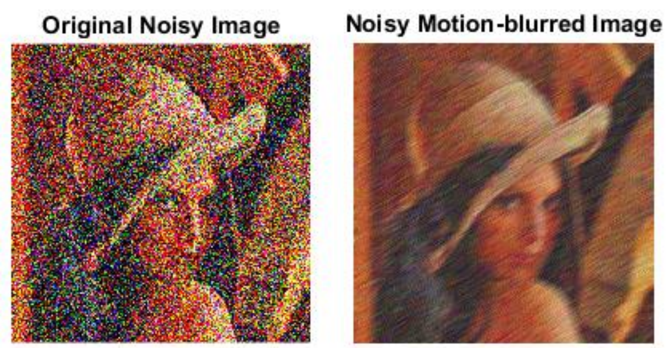

Figure 2. The left-sided image is the noisy image and the right-sided image is a simulation of the noisy motion-blurred image.

III. Section c

Since the motion-blurred image in section (a) had no noises at all, it is possible to perform an inverse filter on it. The result is compared with the original image in the following figure.

Figure 3. Original image compared with the result using the inverse filter as the restoration filter.

VI. Section d

To enhance the noisy motion-blurred image, the inverse filter and the Wiener filter can be implemented. The result shows that the noise will prevent the inverse filter to enhance the image. Instead, it will ruin the image information totally. But by performing the Wiener filter, a better result can be achieved.

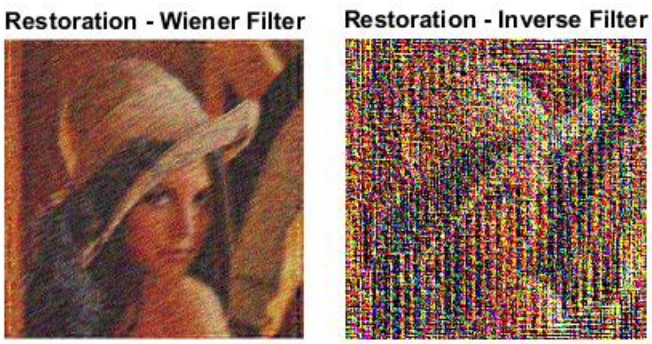

Figure 4. The Wiener filter compared to the inverse filtering to remove motion-blurring from the image.

The Wiener filter filters out the noise, while the inverse filter tries to achieve the original image through inverse filtering the noise. This will produce some an entirely different image.

V. Section e & g

To compute the signal to noise ration (SNR) of the image, it is possible to take the difference of the original image and the restored image as the noise. So, the SNR formula is as follows:

$$SNR = \frac{P_{Signal}}{P_{noise}} = \left(\frac{A_{Signal}}{A_{noise}}\right)^2 \tag{5}$$

where A is the root mean square (RMS) of the signal.

$$noise = Signal - Restored\ signal$$

The SNR in decibels is:

$$SNR_{db} = 10\log\left(SNR\right) \tag{6}$$

This criterion can show which filter can perform a better denoising and motion deblurring. The more the SNR is, the better the image restoration performed.

The following table compares Inverse filter, Wiener filter and least square error method using the SNR criterion.

TABLE I
A comparison between image restoration methods

Method	SNR_{dB}
Inverse Filter (noiseless image)	117
Inverse Filter (noisy image)	79
Wiener Filter	103
Least Square Error	84

The noiseless image has a better result, which was expected for the image has no noise at all. And for the noisy image, the Wiener filter has the best result between these methods.

V. Section f

The result of restoring the image using LSE method gives the following result. But accreting to the above table, it has a better result in comparison with the inverse filter. Yet, it is not desirable.

Figure 5. The result of applying LSE method to the noisy motion-blurred image.