Wavelet Transform

In many image transformation methods, the position and time information are lost due to moving to the frequency spectrum. Therefore, there should be a better way to having the position and frequency data at the same time.

Wavelet transform can extract the high/low frequency data with keeping the position of them. This approach can be used in decomposing an image into more detailed data to modify and manipulate it.

2D Wavelet Sub-band Coding

The Daubechies 8-tap (aka db4) wavelets are used to apply wavelet transformation to the test images. These signals can decompose and reconstruct an image and are as follows:

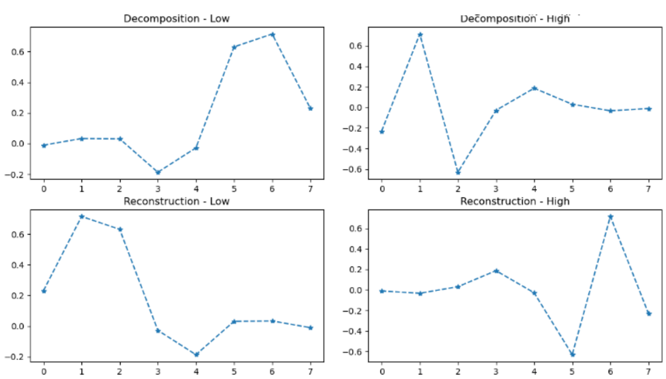

Figure 1. High/low frequency Daubechies wavelets used for decomposition and reconstruction in wavelet transform.

This wavelet transform is used to decompose and reconstruct test images. By computing the mean absolute error of the reconstructed image, the accuracy of the image reconstruction can be verified.

$$MAE = 1.162404 \times 10^{-7}$$

Three level of decomposition is applied to the original image, which gives some high/low frequency details in horizontal and vertical directions. The top-left block in these details is the approximation image which is the low frequency (in both horizontal and vertical directions) data of the original image.

The following figure shows the wavelet transform of the 'boat' image:

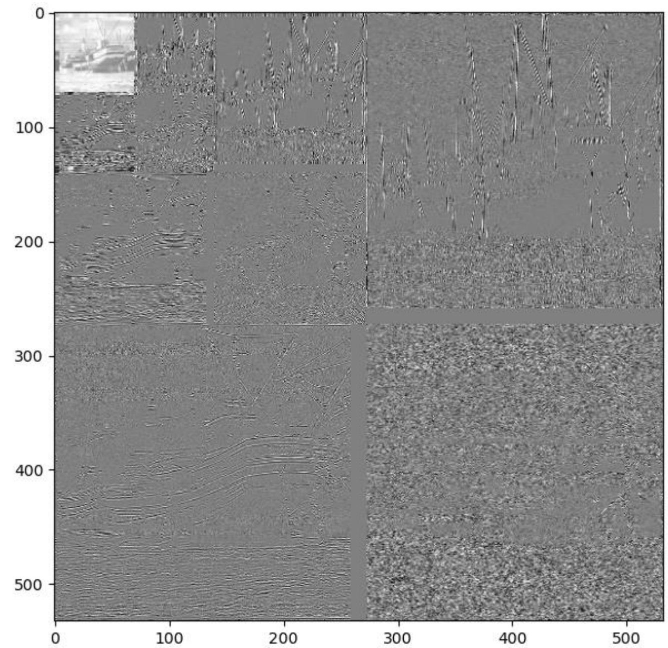

Figure 2. Three level wavelet decomposition using Daubechies wavelet. The coefficients are normalized to be more distinguishable.

The approximation blocks of the 3, 4, 5 and 6 level of wavelet decomposition are depicted in the following figure. It shows that by decomposing an image in higher levels, there will be less details in the approximation block due to the low-frequency filter.

Figure 3. Approximation block of 3, 4, 5 and 6 decomposition levels. The details are fading by increasing the level.

Wavelet Thresholding (Denoising)

Denoising can be performed by removing undesired frequencies. This denoising/smoothing can be done by estimating the variance of the noise and removing some coefficients that their absolute value is less than a specific threshold.

$$T = \sigma\sqrt{2\log(n)}$$

where n is the number of the detail coefficient.

The result of smoothing using the wavelet thresholding is as follows:

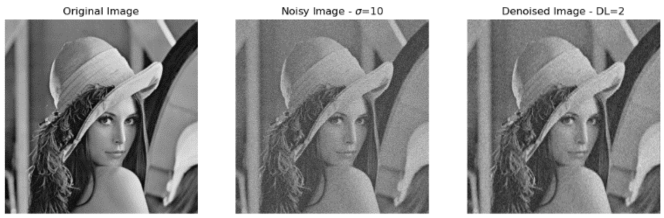

Figure 4. An example of smoothing and denoising the image by removing undesirable frequencies and details in wavelet decomposition. (Decomposition level=2, sigma=10)

A table of Peak signal to noise ratio (PSNR) is prepared for various noise deviation and decomposition levels to verify the accuracy of denoising:

TABLE I
PSNR of noisy and denoised images respect to the original image

	$\sigma = 1$	$\sigma = 10$	$\sigma = 100$
$DL = 1$	45.25	51.45	51.42
$DL = 2$	84.60	90.23	90.20
$DL = 3$	84.60	90.68	90.66

The noisy images with deviation of 1 did not improve and they have the same PSNR as the noisy image. So, the first column shows the PSNR of the noisy image. Since the second and third columns have bigger numbers respect to the first column, the image has been enhanced due to the this denoising method. The numbers are in decibels.

The codes can be applied to the rest of the test images. It is enough to change the name of the input image in the codes.