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Optimization Algorithms

Mini-batch gradient descent

entire training set for each iteration

a mini_batch for each itteration

Batch vs. mini-batch gradient descent

X, Y $\quad$ $X^{\{t\}}, Y^{\{t\}}$

Vectorization allows you to efficiently compute on m examples.

$$X = [\underbrace{x^{(1)}\ x^{(2)}\ x^{(3)} \cdots x^{(1000)}}_{X^{\{1\}}\ (n_x, 1000)} \mid \underbrace{x^{(1001)} \cdots x^{(2000)}}_{X^{\{2\}}\ (n_x, 1000)} \mid \cdots \mid \underbrace{\cdots x^{(m)}}_{X^{\{5,000\}}\ (n_x, 1000)}]$$

(n_x, m)

$$Y = [\underbrace{y^{(1)}\ y^{(2)}\ y^{(3)} \cdots y^{(1000)}}_{Y^{\{1\}}\ (1, 1000)} \mid \underbrace{y^{(1001)} \cdots y^{(2000)}}_{Y^{\{2\}}\ (1, 1000)} \mid \cdots \mid \underbrace{\cdots y^{(m)}}_{Y^{\{5,000\}}\ (1, 1000)}]$$

$(1, m)$

What if $m = 5{,}000{,}000$?

5,000 mini-batches of 1,000 each

Mini-batch t: $X^{\{t\}}, Y^{\{t\}}$

$x^{(i)}$ i-th training example

$z^{[l]}$ l-th layer

$X^{\{t\}}, Y^{\{t\}}$. t-th minibach

$$\begin{cases} X = (n_x, m) \\ Y = (1, m) \end{cases} \Rightarrow \begin{cases} X^{\{t\}} = (n_x, \frac{m}{k}) \\ Y^{\{t\}} = (1, \frac{m}{k}) \end{cases}$$

$k = \#$ mini batches

Algorithm

Mini-batch gradient descent

repeat {

for $t = 1, \ldots, 5000$ {

→ Forward prop on $X^{\{t\}}$.

$$Z^{[1]} = W^{[1]} X^{\{t\}} + b^{[1]}$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$\vdots$$
$$A^{[L]} = g^{[L]}(Z^{[L]})$$

Vectorized implementation (1000 examples)

→ Compute cost $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^{l} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2 \cdot 1000} \sum_{l} \|W^{[l]}\|_F^2$.

from $X^{\{t\}}, Y^{\{t\}}$

Backprop to compute gradients wrt $J^{\{t\}}$ (using $(X^{\{t\}}, Y^{\{t\}})$)

$W^{[l]} := W^{[l]} - \alpha dW^{[l]}$, $b^{[l]} := b^{[l]} - \alpha db^{[l]}$

}

}

One step of gradient descent using $X^{\{t\}}, Y^{\{t\}}$ (as if $m = 1000$)

Instead of X, Y use $X^{\{t\}}, Y^{\{t\}}$

"1 epoch" a single pass through train set.

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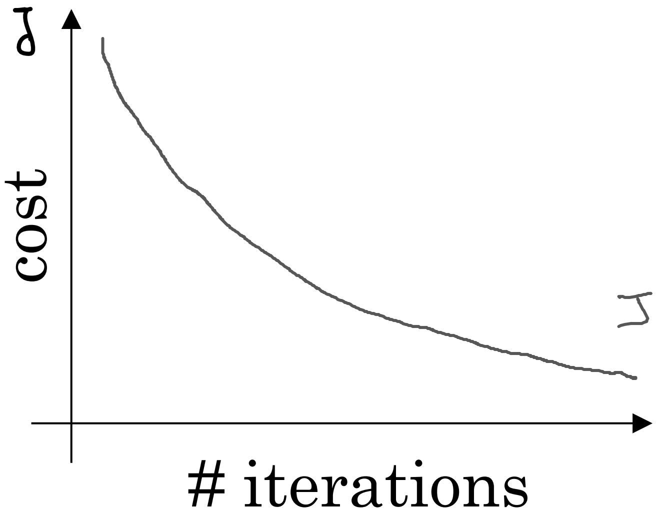
Optimization Algorithms

Understanding mini-batch gradient descent

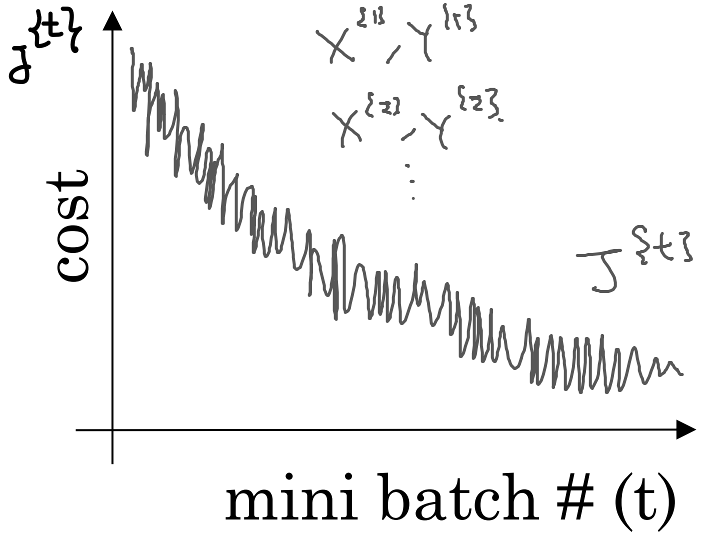
Training with mini batch gradient descent

کمی noisy خواهد شد.

Batch gradient descent



Mini-batch gradient descent



Plot $J^{\{t\}}$ computed using $X^{\{t\}}, Y^{\{t\}}$

If mini_batch size = m ≡ Batch gradient descent

If mini_batch size = 1 ≡ Stochastic gradient descent

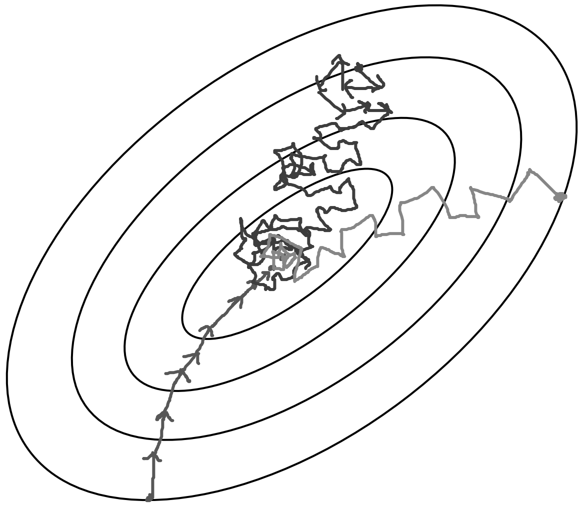
⇒ mini_batch size between 1 and m

Choosing your mini-batch size

→ If mini-batch size = m : Batch gradient descent. $(X^{\{1\}}, Y^{\{1\}}) = (X, Y)$.

→ If mini-batch size = 1 : Stochastic gradient descent. Every example is it own mini-batch.
$(X^{\{1\}}, Y^{\{1\}}) = (x^{(1)}, y^{(1)}) \dots (x^{(2)}, y^{(2)})$ mini-batch.

In practice: Somewhere in-between $\underline{1}$ and $\underline{m}$



Stochastic gradient descent
↓
Lose speedup from vectorization

In-between (mini-batch size not too big/small)
↓
Fastest learning.
- Vectorization. (~1000)
- Make progress without processing entire training set.

Batch gradient descent (mini-batch size = m)
↓
Too long per iteration

Choosing your mini-batch size

If small train set: Use batch gradient descent.
$(m \leq 2000)$

Typical mini-batch sizes:

$\longrightarrow \underbrace{\underset{2^6}{64}, \underset{2^7}{128}, \underset{2^8}{256}, \underset{2^9}{512}}$ $\quad \underset{2^{10}}{1024}$

Make sure mini-batch fit in CPU/GPU memory.
$X^{\{t\}}, Y^{\{t\}}$

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Optimization Algorithms

Exponentially weighted averages

هدف: یافتن روش‌های سریع‌تر از gradient desent برای optimization است.

Temperature in London

$\theta_1 = 40°F$ 4°C ←

$\theta_2 = 49°F$ 9°C

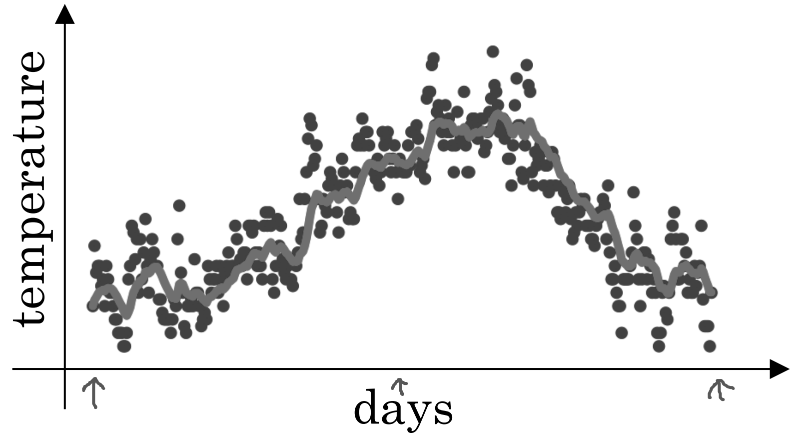
$\theta_3 = 45°F$ ⋮

⋮

$\theta_{180} = 60°F$ 15°C

$\theta_{181} = 56°F$ ⋮

⋮



$V_0 = 0$

$V_1 = 0.9\,V_0 + 0.1\,\theta_1$

$V_2 = 0.9\,V_1 + 0.1\,\theta_2$

$V_3 = 0.9\,V_2 + 0.1\,\theta_3$

⋮

$$\boxed{V_t = 0.9\,V_{t-1} + 0.1\,\theta_t}$$

$$V_t = \beta V_{t-1} + (1-\beta)\,\theta_t \;\Rightarrow\; \approx \frac{1}{\beta-1} \text{ Previous } \theta$$

exponentially weighted moving average
exponentially weight averages

Exponentially weighted (moving) averages

$$V_t = \beta V_{t-1} + (1-\beta)\,\theta_t \leftarrow$$

$\beta = 0.9$: ≈ 10 days' temperature.

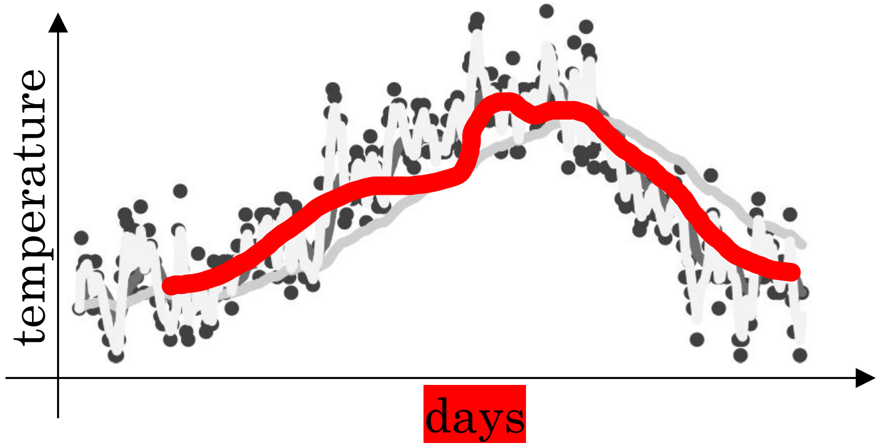
$\beta = 0.98$: ≈ 50 days

$\beta = 0.5$: ≈ 2 days

V_t as approximately average over

$\rightarrow \approx \frac{1}{1-\beta}$ days' temperature.

$$\frac{1}{1-0.98} = 50$$



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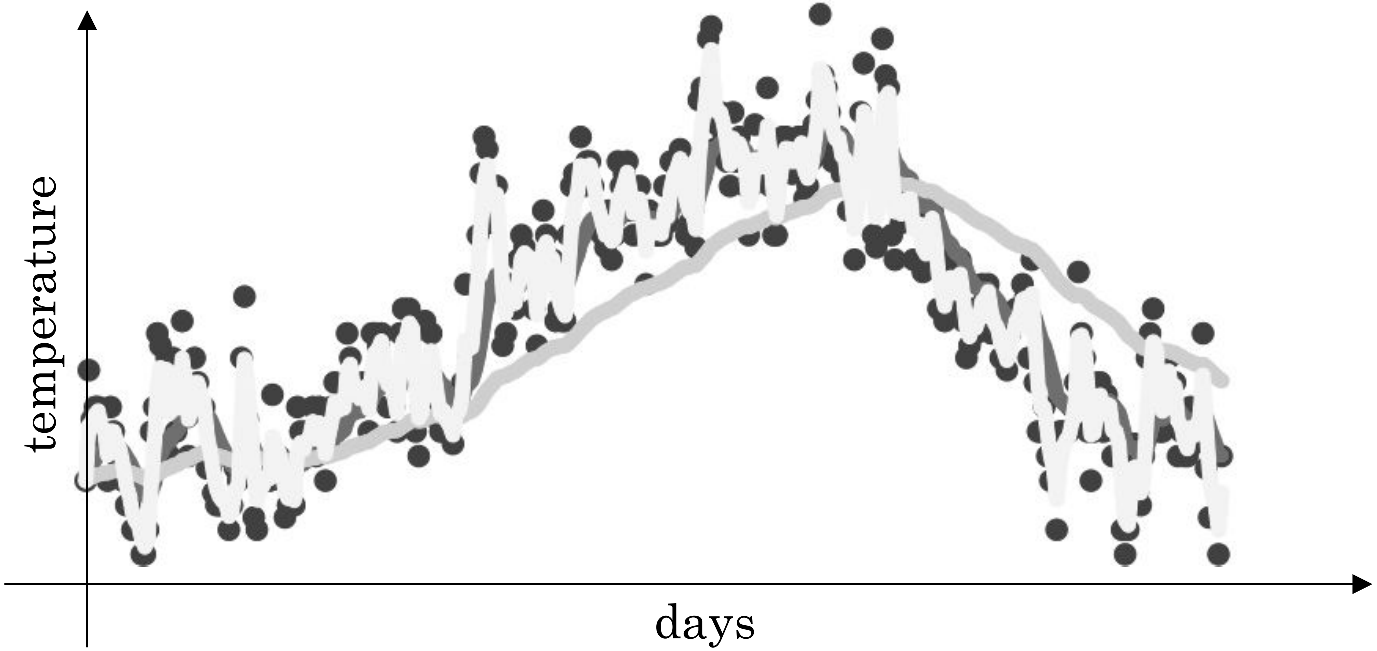
Optimization Algorithms

Understanding exponentially weighted averages

Exponentially weighted averages

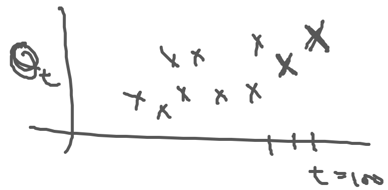
$$\boxed{v_t = \beta v_{t-1} + (1-\beta)\theta_t}$$

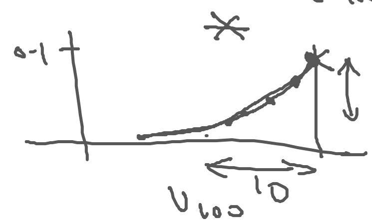
$\beta = 0.9$ $\quad$ 0.98 $\quad$ 0.5



Exponentially weighted averages

$$v_t = \beta v_{t-1} + (1-\beta)\theta_t$$





$\approx \frac{1}{1-\beta}$

$\varepsilon = 1-\beta$

$$v_{100} = 0.9v_{99} + 0.1\theta_{100}$$

$$\boxed{v_{99} = 0.9v_{98} + 0.1\theta_{99}}$$

$$\boxed{v_{98} = 0.9v_{97} + 0.1\theta_{98}}$$

...

$0.1\,\theta_{98} + 0.9\,v_{97}$

$$v_{100} = 0.1\,\theta_{100} + 0.9\,\cancel{v_{99}}\,(0.1\,\theta_{99} + 0.9\,\cancel{v_{98}})$$

$$\boxed{v_{100} = 0.1\,\theta_{100} + 0.1 \times 0.9 \cdot \theta_{99} + 0.1\,(0.9)^2\,\theta_{98} + 0.1\,(0.9)^3\,\theta_{97} + 0.1\,(0.9)^4\,\theta_{96} + \dots}$$

$0.9^{10} \approx 0.35 \approx \frac{1}{e}$

$\underbrace{(1-\varepsilon)}_{0.9}{}^{1/\varepsilon} = \frac{1}{e}$

$0.98^{?}$

$\varepsilon = 0.02 \rightarrow 0.98^{50} \approx \frac{1}{e}$

Implementing exponentially weighted averages

$$v_0 = 0$$
$$v_1 = \beta v_0 + (1-\beta)\,\theta_1$$
$$v_2 = \beta v_1 + (1-\beta)\,\theta_2$$
$$v_3 = \beta v_2 + (1-\beta)\,\theta_3$$
$$\dots$$

$$V_\theta := 0$$
$$V_\theta := \beta v + (1-\beta)\,\theta_1$$
$$V_\theta := \beta v + (1-\beta)\,\theta_2$$
$$\vdots$$

$V_\theta = 0$

Repeat {

  Get next θ_t

  $V_\theta := \beta V_\theta + (1-\beta)\,\theta_t$

}

Algorithm

راه مناسب برای محاسبه میانگین است ولی با خطای Bias correction

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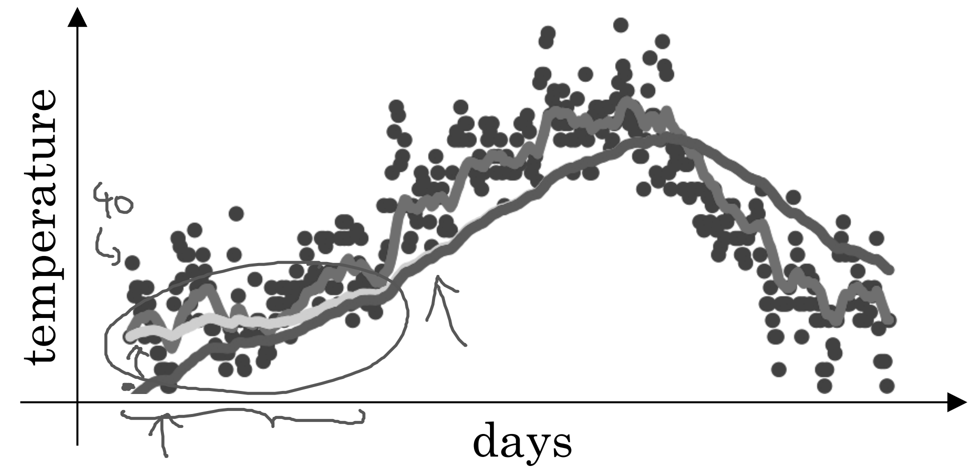
Optimization Algorithms

Bias correction in exponentially weighted average

$$\boxed{V_t = \frac{\beta V_{t-1} + (1-\beta)\theta_t}{1-\beta^t}}$$

$\Rightarrow$ If $t \gg 1$: $1-\beta^t \simeq 1$ $\Leftarrow$ Bias Correction کمک می‌کند تأثیر β در t های کوچک regularize شود

Bias correction



$\beta = 0.98$

$$\rightarrow v_t = \beta v_{t-1} + (1-\beta)\theta_t$$

$V_0 = 0$

$V_1 = \cancel{0.98 V_0 +} \ \underline{0.02\,\theta_1}$

$V_2 = 0.98\, V_1 + 0.02\,\theta_2$

$= 0.98 \times 0.02 \times \theta_1 + 0.02\,\theta_2$

$= 0.0196\,\underline{\theta_1} + 0.02\,\underline{\theta_2}$

$$\frac{V_t}{1-\beta^t}$$

$t=2:\quad 1-\beta^t = 1-(0.98)^2 = 0.0396$

$$\frac{V_2}{0.0396} = \frac{0.0196\,\theta_1 + 0.02\,\theta_2}{0.0396}$$

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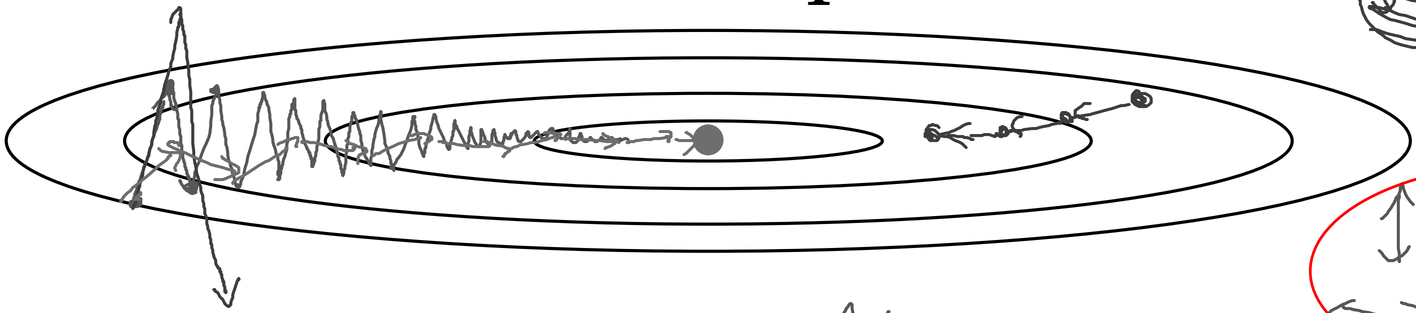
Optimization Algorithms

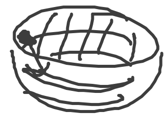
Gradient descent with momentum

Gradient descent with momentum

Idea: using exponentially weighted averages in gradient descents ⇒ always works better than gradient descent!

Gradient descent example





momentum leads to

↕ slower learning

↔ faster learning

Momentum:

On iteration t:

Compute dW, db on current mini-batch.

$$V_{dw} = \beta V_{dw} + (1-\beta)\, dW$$

$$V_{db} = \beta V_{db} + (1-\beta)\, db$$

friction, velocity, acceleration

"$V_\theta = \beta V_\theta + (1-\beta)\,\theta_t$"

$$W := W - \alpha V_{dw}, \quad b := b - \alpha V_{db}$$

momentum (algorithm)

Implementation details

$V_{dw} = 0$, $V_{db} = 0$

On iteration t:

Compute dW, db on the current mini-batch

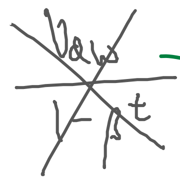
momentum

$$\rightarrow v_{dW} = \beta v_{dW} + \cancel{(1-\beta)} dW$$

$$\rightarrow v_{db} = \beta v_{db} + \cancel{(1-\beta)} db$$

$$W = W - \alpha v_{dW}, \quad b = b - \alpha v_{db}$$

$\underbrace{V_{dw} = \beta V_{dw} + dW}$ ←



نیازی به Bias Correction نیست!

Hyperparameters: α, β

. دو Hyperparam داریم

$\beta = 0.9$

average over last ≈ 10 gradients

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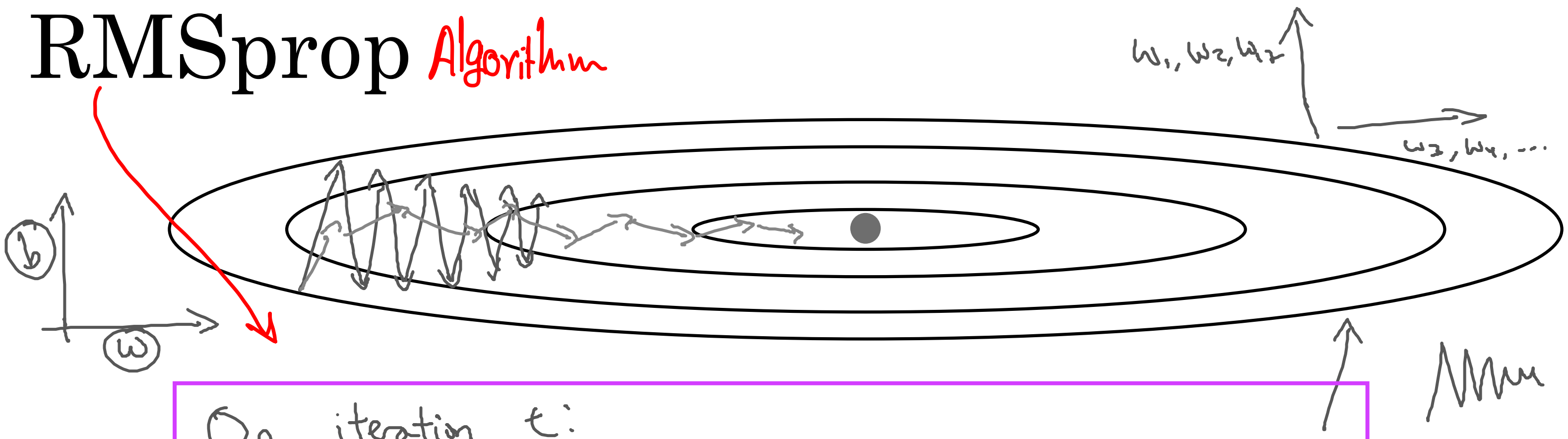
Optimization Algorithms

RMSprop

Root mean square Prop!

RMSprop Algorithm



leads to

slow

fast

On iteration t:

Compute dW, db on current mini-batch

element-wise

$$S_{dW} = \beta_2 S_{dW} + (1-\beta_2)\, dW^2 \leftarrow \text{small}$$

$$S_{db} = \beta_2 S_{db} + (1-\beta_2)\, db^2 \leftarrow \text{large}$$

$$W := W - \alpha \frac{dW}{\sqrt{S_{dW}+\varepsilon}} \qquad b := b - \alpha \frac{db}{\sqrt{S_{db}+\varepsilon}}$$

$$\varepsilon = 10^{-8}$$

⇒ Possible to use $\alpha \uparrow\uparrow$ without any diverging!

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Optimization Algorithms

Adam optimization algorithm

ترکیب RMSprop و momentum می‌شود Adam که از همه بهتر است.

yhat = np.array([.9, 0.2, 0.1, .4, .9])

Adam optimization algorithm

Algorithm

$V_{dw} = 0$, $S_{dw} = 0$. $V_{db} = 0$, $S_{db} = 0$

On iteration t:

Compute dW, db using current mini-batch

$V_{dw} = \beta_1 V_{dw} + (1-\beta_1) dW$, $V_{db} = \beta_1 V_{db} + (1-\beta_1) db$ ← "momentum" β_1

$S_{dw} = \beta_2 S_{dw} + (1-\beta_2) dW^2$, $S_{db} = \beta_2 S_{db} + (1-\beta_2) db^2$ ← "RMSprop" β_2

$V_{dw}^{corrected} = V_{dw} / (1-\beta_1^t)$, $V_{db}^{corrected} = V_{db} / (1-\beta_1^t)$

$S_{dw}^{corrected} = S_{dw} / (1-\beta_2^t)$, $S_{db}^{corrected} = S_{db} / (1-\beta_2^t)$

$$W := W - \alpha \frac{V_{dw}^{corrected}}{\sqrt{S_{dw}^{corrected}} + \varepsilon} \qquad b := b - \alpha \frac{V_{db}^{corrected}}{\sqrt{S_{db}^{corrected}} + \varepsilon}$$

4 تا دارد.

Hyperparameters choice:

learning rate

$\rightarrow \alpha$: needs to be tune

$\rightarrow \beta_1$: 0.9 $\longrightarrow$ ($\underline{dw}$)

$\rightarrow \beta_2$: 0.999 $\longrightarrow$ ($\underline{dw^2}$)

$\rightarrow \varepsilon$: 10^{-8}

Adam : Adaptive moment estimation

Adaptive moment estimation

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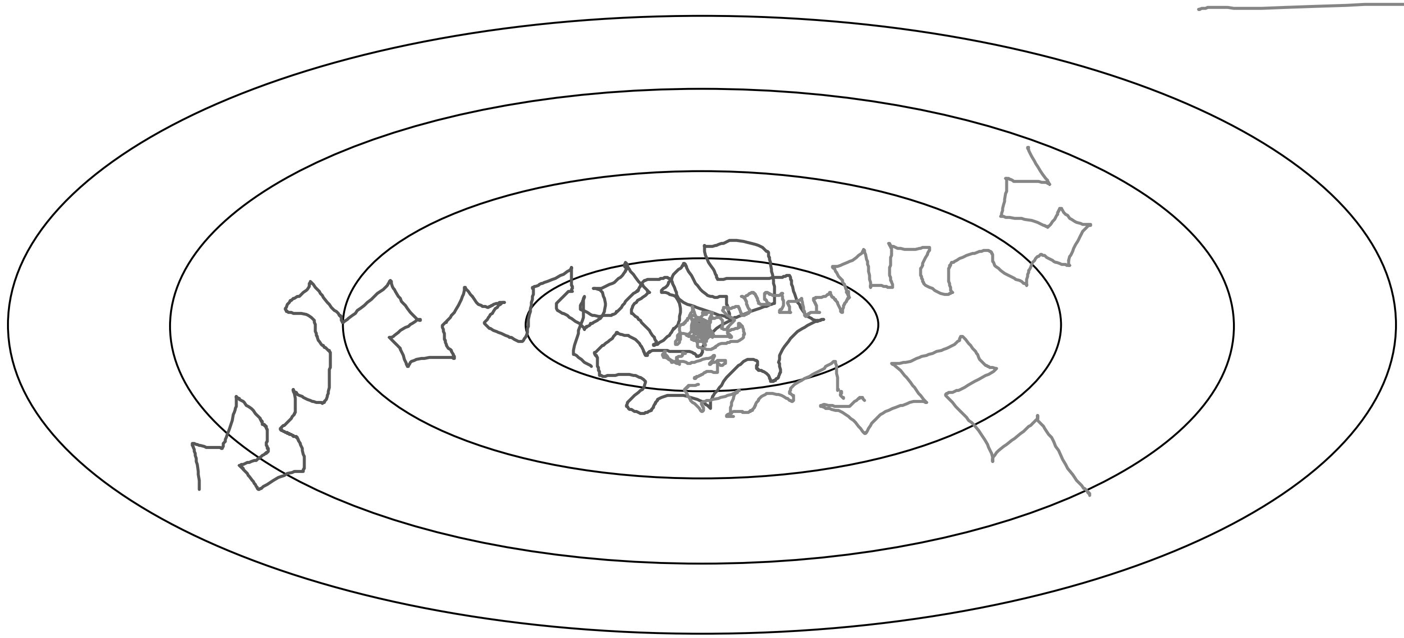
Optimization Algorithms

Learning rate decay

ایده: α را نسبت به زمان آرام آرام کم کنیم.

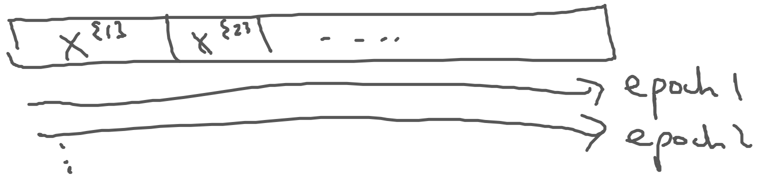
Learning rate decay

Slowly reduce α



Learning rate decay

1 epoch = 1 pass through data.



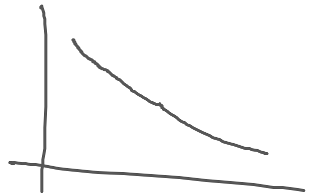
$$\alpha = \frac{1}{1 + \underline{\text{decay-rate}} * \text{epoch-num}} \alpha_0$$

Epoch	α
1	0.1
2	0.067
3	0.05
4	0.04
⋮	⋮

$\alpha \downarrow$

$\alpha_0 = 0.2$

decay-rate = 1



learning rate decay

$$\alpha = \frac{\overset{\text{initial } \alpha}{\alpha_0}}{1 + \underbrace{\text{decay-rate}}_{\text{Hyper parameter}} * \text{epoch\_num}}$$

شبیه

$$\alpha(t) = \frac{\alpha_0}{1 + \lambda t}$$

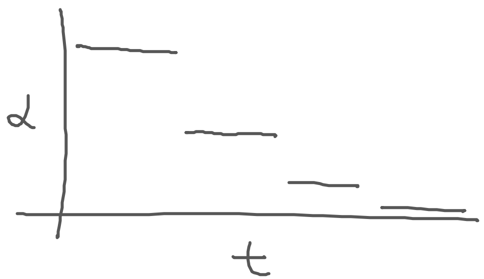
جدای از روش قبل، کارهای دیگر هم ممکن است ⇐ کافی است تابع $\alpha(t)$ نزولی تعریف کنیم!

Other learning rate decay methods

formula

$$\alpha = 0.95^{\text{epoch-num}} \cdot \alpha_0 \quad \text{— exponentially decay.}$$

$$\alpha = \frac{k}{\sqrt{\text{epoch-num}}} \cdot \alpha_0 \quad \text{or} \quad \frac{k}{\sqrt{t}} \cdot \alpha_0$$



discrete staircase

Manual decay.

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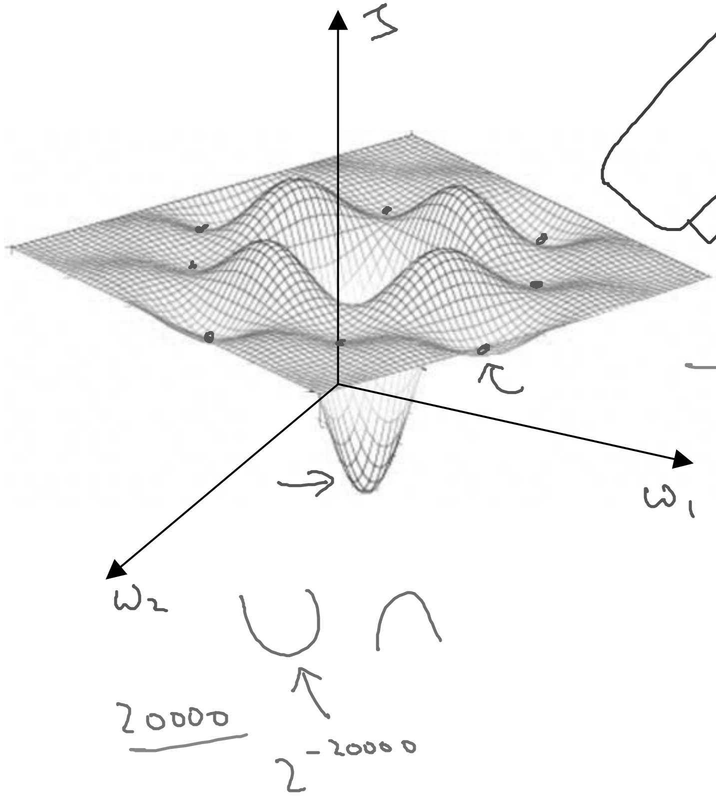
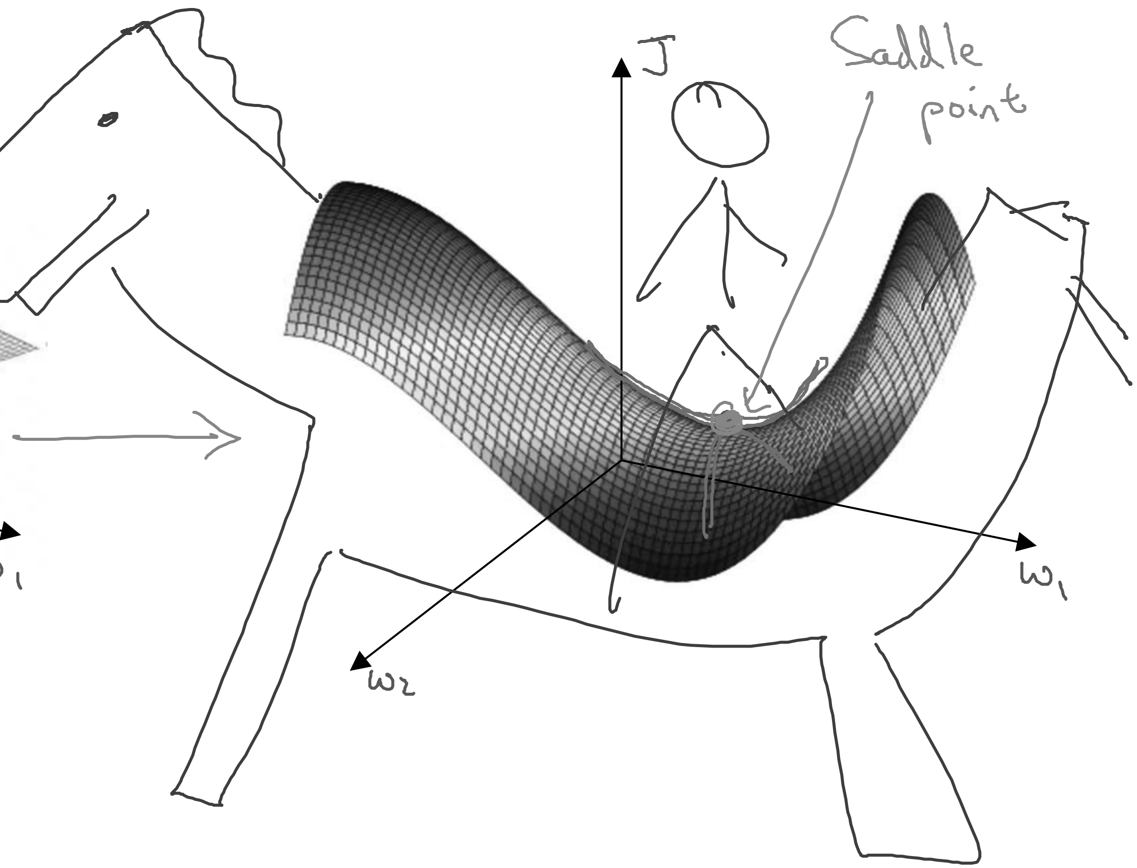
Optimization Algorithms

The problem of local optima

اکثر نقاطی که گرادیان صفر دارند ، saddle points هستند نه local min/max؛ چون احتمال اینکه در فضای n بعدی همه مشتقات جزئی هم علامت
شوند خیلی پایین است

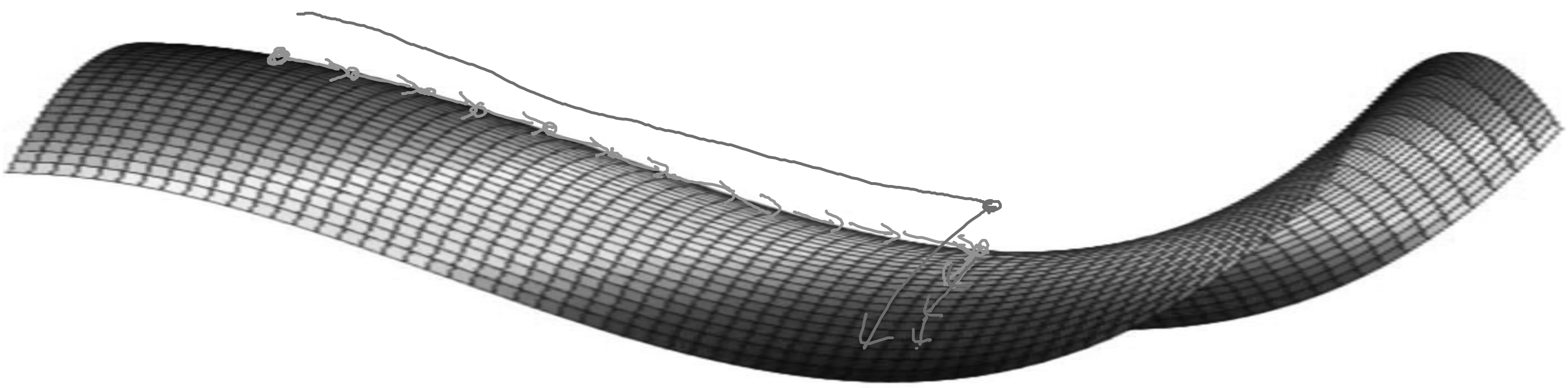
Local optima in neural networks





فلات

Problem of plateaus



- Unlikely to get stuck in a bad local optima
- Plateaus can make learning slow