Math 124 - Programming for Mathematical Applications

UC Berkeley, Spring 2021

Homework 12

Due Wednesday April 28

Problem 1

It is well known that $\sum_{i=1}^{n} i = \frac{1}{2} n (1 + n)$. Make a table with similar formulas for $\sum_{i=1}^{n} i^k$, with k ranging from 1 to 8.

Problem 2

Use the Factor function to prove that the product of four consecutive numbers plus one is always a squared number.

Problem 3

Show that the formula $n^2 + n + 41$ produces prime numbers for n from 0 to 39.

Problem 4

11 is the first prime number with all digits equal to 1. Find the next one (using a loop).

Problem 5

Define the function $f(x)$ as follows:

$$f(xy) = f(x) + f(y)$$
$$f(x^n) = nf(x)$$
$$f(n) = 0$$

where n is an integer. Show that

$$f\left(\prod_{k=1}^{20} k! \, (x_k)^k\right) = \sum_{k=1}^{20} k \, f(x_k)$$

Problem 6

a) Plot the function $f(x) = e^{-x}/(2 + \sin(x^2))$ and its tangent line $g(x)$ at $x = 1$ for $x \in [0, 3]$.
b) Calculate the integral of $f(x) - g(x)$ between $x = 0$ and $x = 1$ numerically with 100 digits.

Problem 7

Define the following piecewise function:

$$f(x) = \begin{cases} -x & \text{if } |x| < 1 \\ \sin(x) & \text{if } 1 \leq |x| < 2 \\ \cos(x) & \text{otherwise.} \end{cases}$$

a) Plot $f(x)$ between $x = -3$ and $x = 3$.

b) Calculate the integral of $1/(1 + f(x)^2)$ between $x = -3$ and $x = 3$ (symbolically).