Team Assignment 1

Instructor: Aninda Sinha (Indian Institute of Science & University of Calgary)

Deadline: Next Sunday, 11:59 PM IST (7+ days from today)

Problem Statement

This project explores the **classical and quantum dynamics of a free particle in an elliptic stadium**. You will use Python codes (with AI assistance, e.g. ChatGPT-5 or ChatGPT-4) to analyze the system. Your work should demonstrate both the capabilities of AI-assisted coding and your own physical insight via appropriate checks. You should freely use AI to find out about unfamiliar things in the instructions below—keep nagging the AI till you are happy with the explanations provided.

Elliptic Stadium Geometry

The stadium consists of a rectangle of half-length a and vertical extent $\pm b$, capped on both sides by half-ellipses of semi-axes (r_x, r_y). The boundary of an ellipse satisfies

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1.$$

Part A: Classical Motion (Newtonian Dynamics)

- Numerically integrate Newton's equations for a free particle in the elliptic stadium with **perfectly reflecting boundaries**.
- Plot trajectories for short times T (a few bounces) and for longer times T (where the trajectory begins to cover the stadium).
- Discuss how generic initial conditions can lead to ergodic-like coverage of the billiard.

Sanity checks:

1. Verify reflections: confirm the law of reflection using boundary normals at bounce points.
2. Test energy conservation.
3. Show sensitivity to initial conditions as a diagnostic of chaos.

Part B: Quantum Dynamics (TDSE)

- Solve the **time-dependent Schrödinger equation** for a Gaussian wave packet inside the stadium, with steep potential walls mimicking reflection.
- Plot $\langle x(t)\rangle$, $\langle y(t)\rangle$ and compare with the classical trajectory.

Sanity checks:

1. Normalization of the wavefunction must be preserved.
2. Probability density should vanish outside the stadium.

Part C: Preliminary Quantum Chaos Investigation

- Implement the **Loschmidt echo**:

$$M(t) = \left|\langle\psi_0|e^{+i(H+\delta H)t}\, e^{-iHt}|\psi_0\rangle\right|^2,$$

  with perturbation δH corresponding to a small change in the stadium geometry.

- Compare the decay of $M(t)$ for small and larger perturbations.
- Comment on the relation to quantum chaos.

Deliverables

- A report (main: 10–15 pages) including methods, results, plots, and explicit description of sanity checks.
- Discussion of ergodicity (classical) and breakdown of Ehrenfest correspondence (quantum).
- Preliminary results on Loschmidt echo.
- **Appendix:** This can be as long as 20 pages. all AI prompts used (with ChatGPT or other tools) and their outputs.

Submission

- **Deadline:** Next Sunday, 11:59 PM IST.
- Submit as a single PDF (code excerpts, plots, appendix).
- Teams of 3–4 students.

Grading

All memebers in the team will get the same grade. Grades are A(exceptional), B(very good), C(decent), D(could do better). Using teams, you should also upload a final well-commented python nb which Ujjwal will run and cross-verify.

Note: The **sanity checks** are as important as the results. You must show how you verified that the AI-assisted code is producing physically consistent outcomes.