This time: Fuzzy Logic and Fuzzy Inference

- Why use fuzzy logic?
- Tipping example
- Fuzzy set theory
- Fuzzy inference

What is fuzzy logic?

- A super set of Boolean logic
- Builds upon fuzzy set theory
- Graded truth. Truth values between True and False. Not everything is either/or, true/false, black/white, on/off etc.
- Grades of membership. Class of tall men, class of far cities, class of expensive things, etc.
- Lotfi Zadeh, UC/Berkely 1965. Introduced **FL to model uncertainty in natural language**. *Tall, far, nice, large, hot, …*
- Reasoning using linguistic terms. Natural to express expert knowledge.
  If the weather is cold then wear warm clothing

Why use fuzzy logic?

Pros:

- Conceptually easy to understand w/ "natural" maths
- Tolerant of imprecise data
- Universal approximation: can model arbitrary nonlinear functions
- Intuitive
- Based on linguistic terms
- Convenient way to express expert and common sense knowledge

Cons:

- Not a cure-all
- Crisp/precise models can be more efficient and even convenient
- Other approaches might be formally verified to work

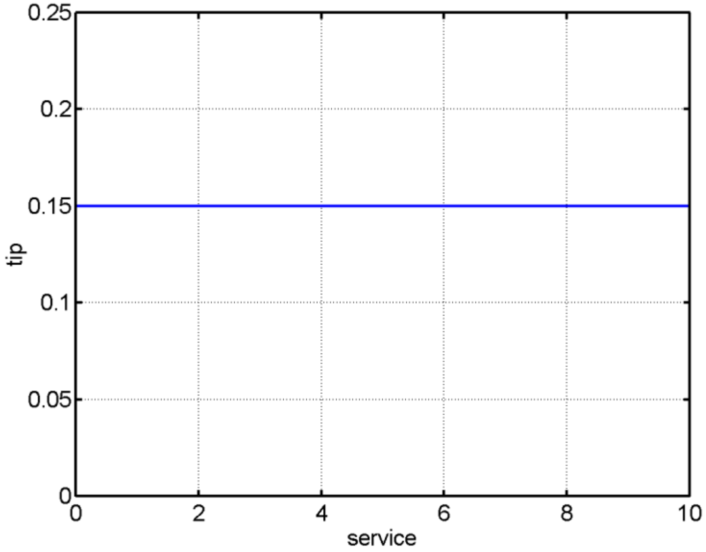
Tipping example

- **The Basic Tipping Problem:** Given a number between 0 and 10 that represents the quality of service at a restaurant what should the tip be?

  Cultural footnote: An average tip for a meal in the U.S. is 15%, which may vary depending on the quality of the service provided.

Tipping example: The non-fuzzy approach

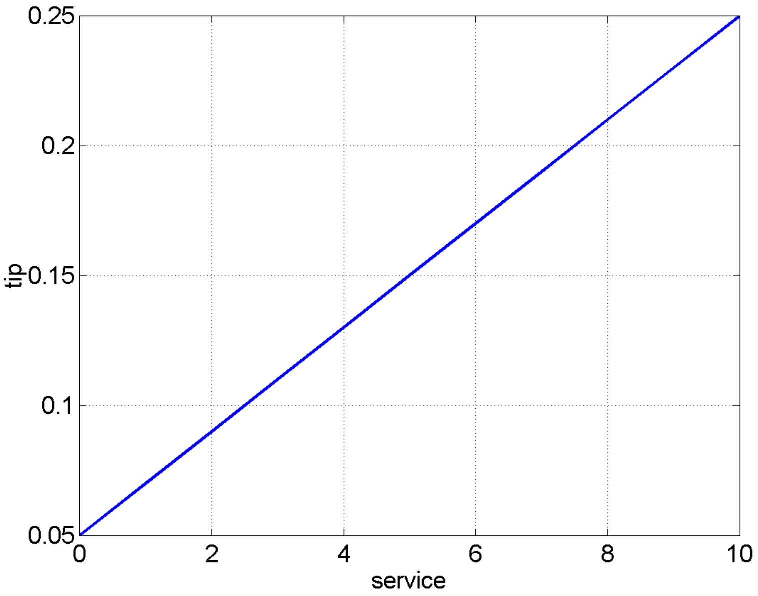
- Tip = 15% of total bill



- What about quality of service?

Tipping example: The non-fuzzy approach

- Tip = linearly proportional to service from 5% to 25%
  tip = 0.20/10*service+0.05



- What about quality of the food?

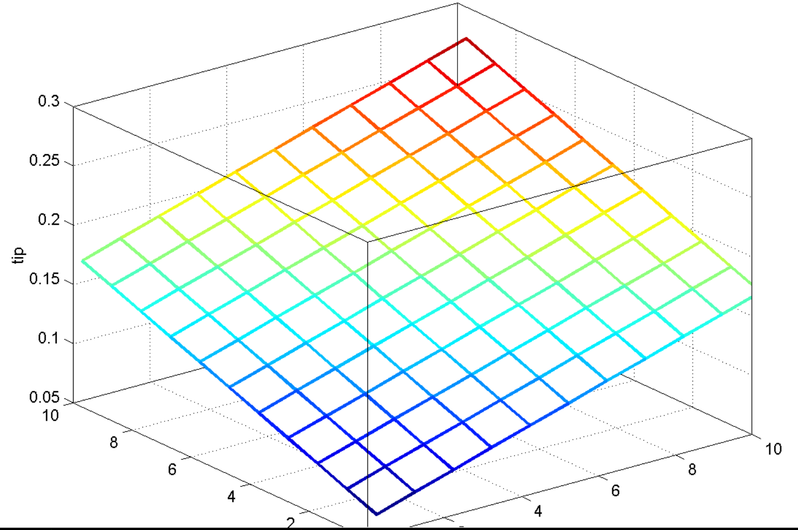
Tipping example: Extended

- **The Extended Tipping Problem:** Given a number between 0 and 10 that represents the quality of service and the quality of the food, at a restaurant, what should the tip be?

  How will this affect our tipping formula?

Tipping example: The non-fuzzy approach

- Tip = 0.20/20*(service+food)+0.05

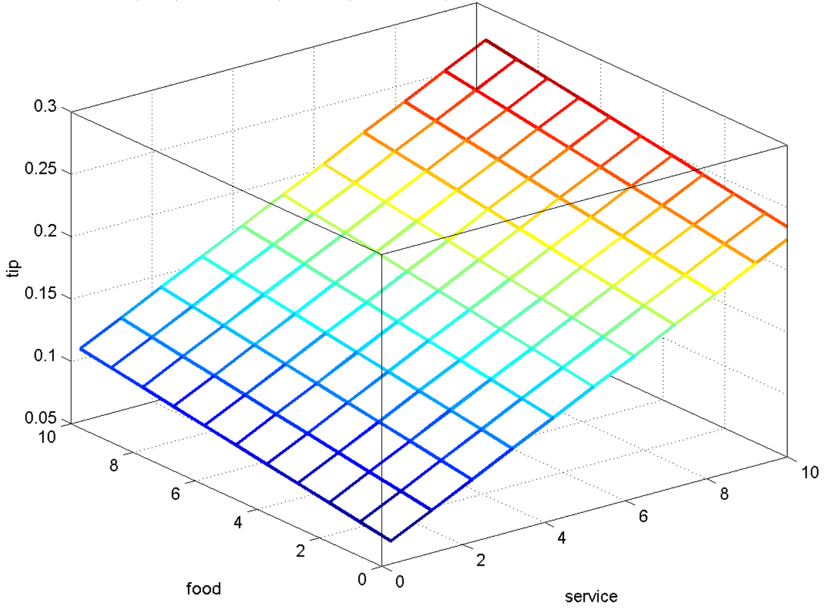


[redacted]

- We want service to be more important than food quality. E.g., 80% for service and 20% for food.

Tipping example: The non-fuzzy approach

- Tip = **servRatio*(.2/10*(service)+.05) + (1-servRatio)*(.2/10*(food)+0.05);** **servRatio = 80%**



- Seems too linear. Want 15% tip in general and deviation only for exceptionally good or bad service.

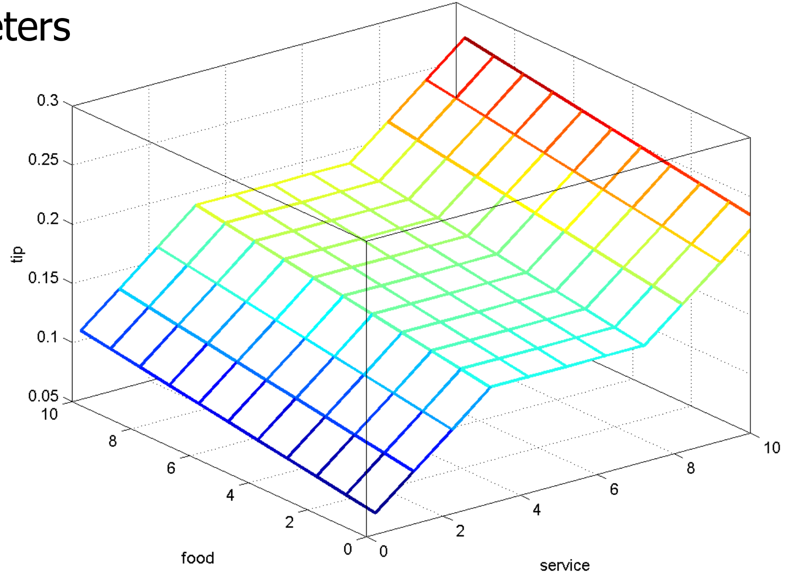
Tipping example: The non-fuzzy approach

```
if service < 3,
  tip(f+1,s+1) = servRatio*(.1/3*(s)+.05) + ...
                 (1-servRatio)*(.2/10*(f)+0.05);
elseif s < 7,
  tip(f+1,s+1) = servRatio*(.15) + ...
                 (1-servRatio)*(.2/10*(f)+0.05);
else,
  tip(f+1,s+1) = servRatio*(.1/3*(s-7)+.15) + ...
                 (1-servRatio)*(.2/10*(f)+0.05);
end;
```

Tipping example: The non-fuzzy approach

Nice plot <u>but</u>

- 'Complicated' function
- Not easy to modify
- Not intuitive
- Many hard-coded parameters
- Not easy to understand



Tipping problem: the fuzzy approach

What we want to express is:

1. *If service is poor then tip is cheap*
2. *If service is good the tip is average*
3. *If service is excellent then tip is generous*
4. *If food is rancid then tip is cheap*
5. *If food is delicious then tip is generous*

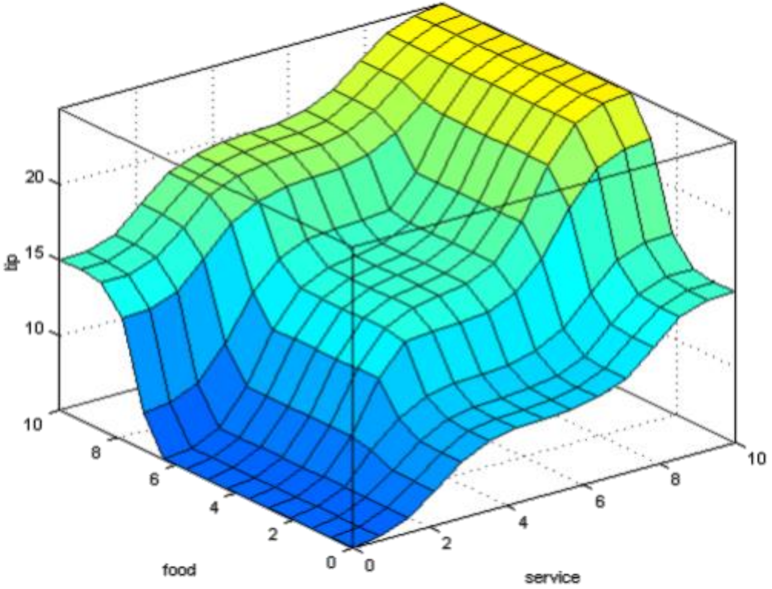
or

1. *If service is poor or the food is rancid then tip is cheap*
2. *If service is good then tip is average*
3. *If service is excellent or food is delicious then tip is generous*

We have just defined the rules for a fuzzy logic system.

Tipping problem: fuzzy solution

Decision function generated using the 3 rules.



Tipping problem: fuzzy solution

- Before we have a fuzzy solution we need to find out

a) how to define terms such as *poor, delicious, cheap, generous etc.*
b) how to combine terms using AND, OR and other connectives
c) how to combine all the rules into one final output

Fuzzy sets

- **Boolean/Crisp set A** is a mapping for the elements of S to the set {0, 1}, i.e., A: S → {0, 1}
- ***Characteristic function:***

$$\mu_A(x) = \begin{cases} 1 & \text{if x is an element of set A} \\ 0 & \text{if x is not an element of set A} \end{cases}$$

- **Fuzzy set F** is a mapping for the elements of S to the interval [0, 1], i.e., F: S → [0, 1]
- Characteristic function: $0 \leq \mu_F(x) \leq 1$
- 1 means full membership, 0 means no membership and anything in between, e.g., 0.5 is called **graded membership**

Example: Crisp set Tall

- Fuzzy sets and concepts are commonly used in natural language

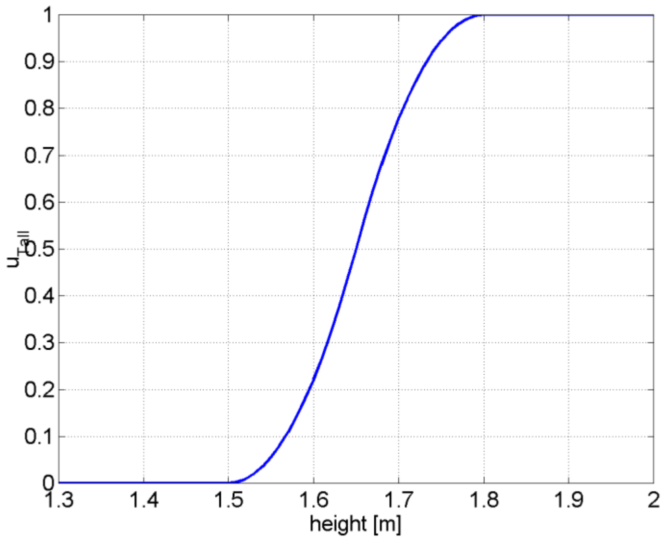
  *John is **tall***
  *Dan is **smart***
  *Alex is **happy***
  *The class is **hot***

- E.g., the crisp set ***Tall*** can be defined as $\{x \mid \text{height } x > 1.8 \text{ meters}\}$
  But what about a person with a height = 1.79 meters?
  What about 1.78 meters?
  …
  What about 1.52 meters?

Example: Fuzzy set Tall

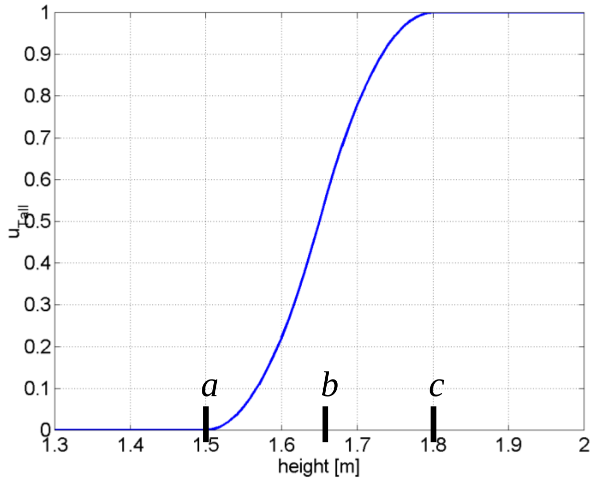
- In a fuzzy set a person with a height of 1.8 meters would be considered tall to a **high degree** A person with a height of 1.7 meters would be considered tall to a lesser degree etc.

- The function can change for basketball players, Danes, women, children etc.



Membership functions: S-function

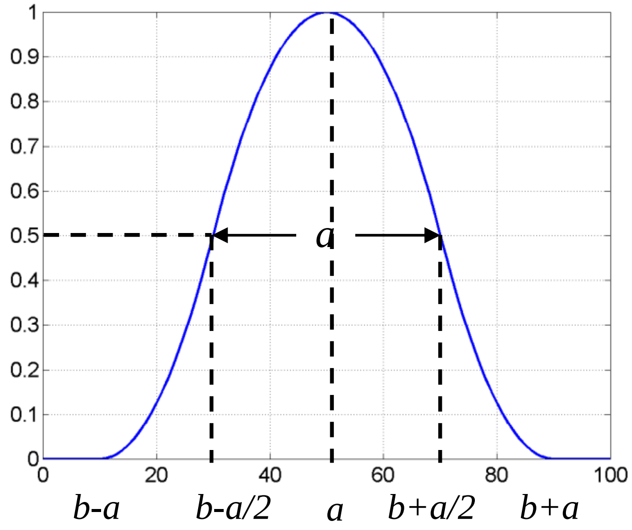
- The S-function can be used to define fuzzy sets
- S(x, a, b, c) =
  - 0 for $x \le a$
  - $2(x\text{-a/c-a})^2$ for $a \le x \le b$
  - $1 - 2(x\text{-}c/c\text{-}a)^2$ for $b \le x \le c$
  - 1 for $x \ge c$



Membership functions: Π–Function

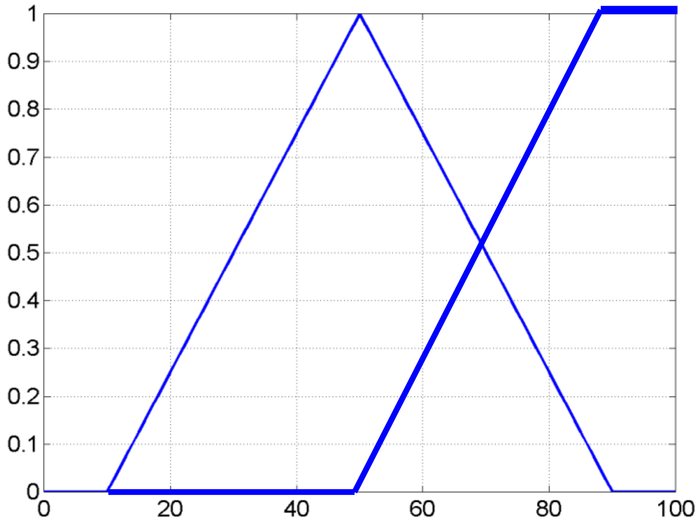
- $\Pi(x, a, b)$ =
  - $S(x, b\text{-}a, b\text{-}a/2, b)$ for $x \le b$
  - $1 - S(x, b, b+a/2, a+b)$ for $x \ge b$

E.g., ***close*** (to a)



Simple membership functions

- Piecewise linear: triangular etc.
- Easier to represent and calculate ⇒ saves computation



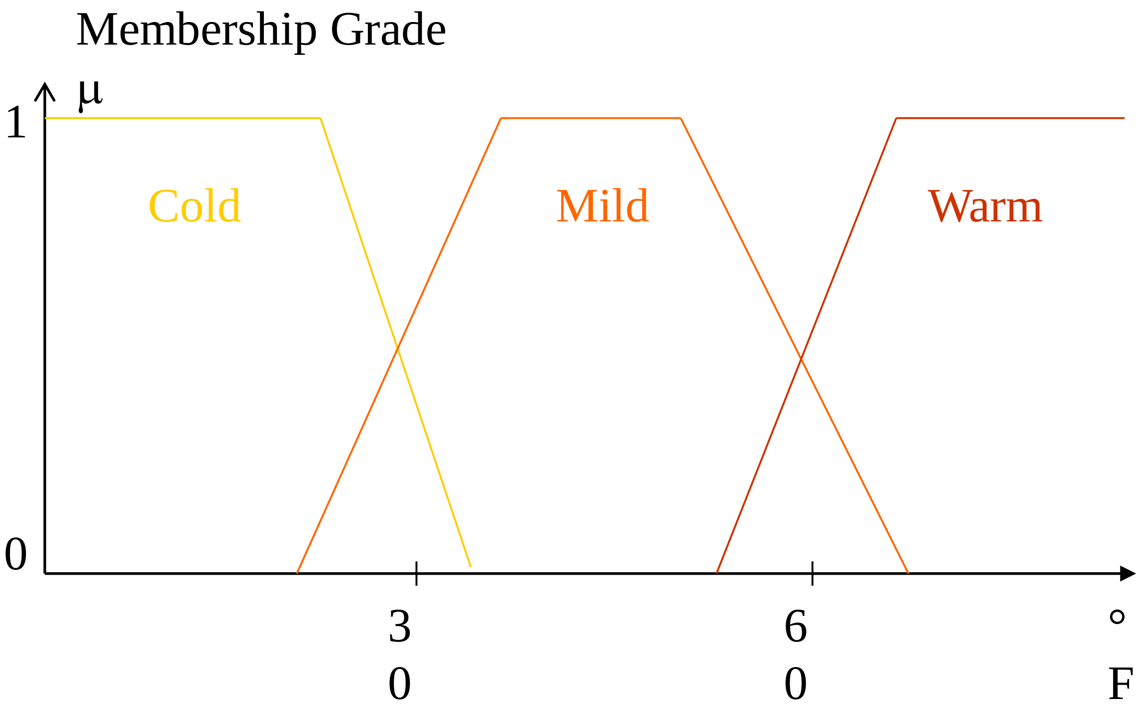
Fuzzy logic vs probability

- Fuzzy ≠ Probability => $\mu_A(x) \neq p_A(x)$
- Both map x to a value in [0,1].
- $P_A(x)$ measures our knowledge or ignorance of the truth of the event that x belongs to the set A.
  - Probability deals with uncertainty and likelihood.
- μA(x) measures the degree of belongingness of x to set A and there is no interest regarding the uncertainty behind the outcome of the event x. Event x has occurred and we are interested in only making observations regarding the degree to which x belongs to A.
  - Fuzzy logic deals with ambiguity and vagueness.

Fuzzy logic vs probability

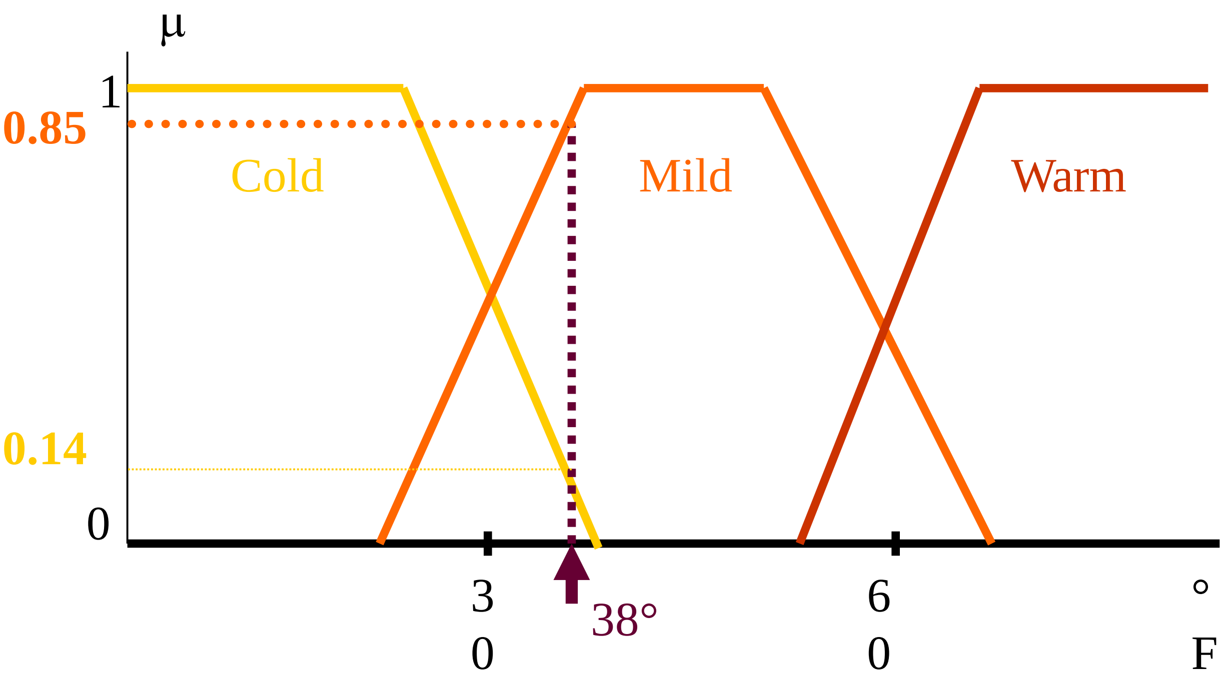
- A bottle of water
- 50% probability of being poisonous means 50% chance.
  - 50% water is clean.
  - 50% water is poisonous.
- 50% fuzzy membership of poisonous means that the water has poison.
  - Water is half poisonous.

Fuzzy Sets



Observation

An observed temperature of 38 is cold with a belief of 0.14,
Mild with a belief of 0.85 and warm with a belief of 0



Other representations of fuzzy sets

- A finite set of elements:

  $F = \mu_1/x_1 + \mu_2/x_2 + \ldots \mu_n/x_n$

  + means (Boolean) set union

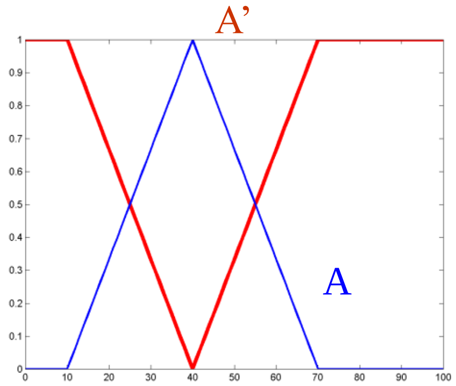
- For example:

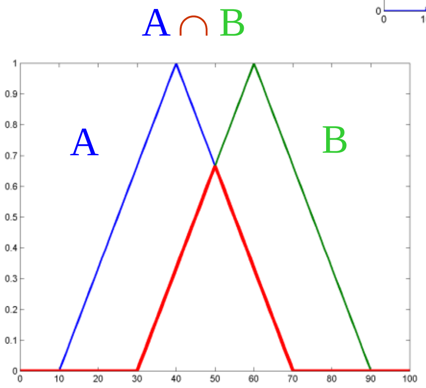
  TALL = {0/1.0, 0/1.2, 0/1.4, 0.2/1.6, 0.8/1.7, 1.0/1.8}

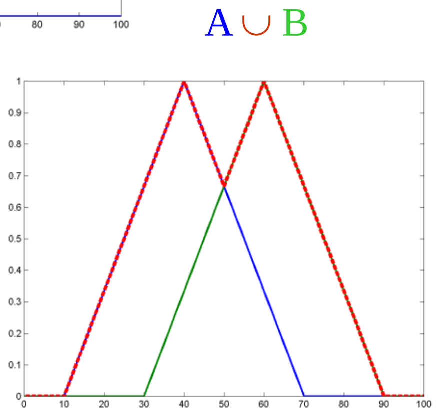
Fuzzy set operators

- Equality
  $A = B$
  $\mu_A(x) = \mu_B(x)$ for all $x \in X$
- Complement
  A'
  $\mu_{A'}(x) = 1 - \mu_A(x)$ for all $x \in X$
- Containment
  $A \subseteq B$
  $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$
- Union
  $A \cup B$
  $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$ for all $x \in X$
- Intersection
  $A \cap B$
  $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$ for all $x \in X$

Example fuzzy set operations



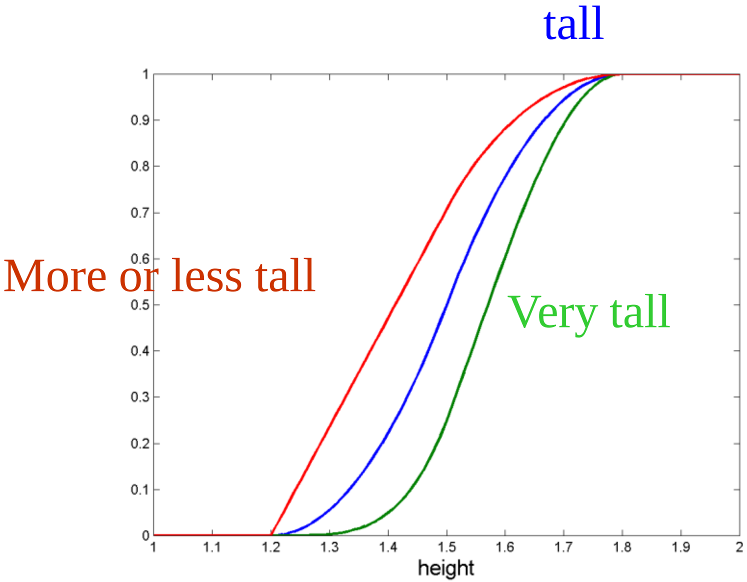




Linguistic Hedges

- Linguistic hedge = intensifier
- Modifying the meaning of a fuzzy set using hedges such as *very, more or less, slightly, etc.*

- *"Very F "= F^2*
- *"More or less F "= $F^{1/2}$*
- *etc.*



Fuzzy relations

- A fuzzy relation for N sets is defined as an extension of the crisp relation to include the membership grade.

  R = $\{\mu_R(x_1, x_2, \dots x_N)/(x_1, x_2, \dots x_N) \mid x_i \in X, i=1, \dots N\}$

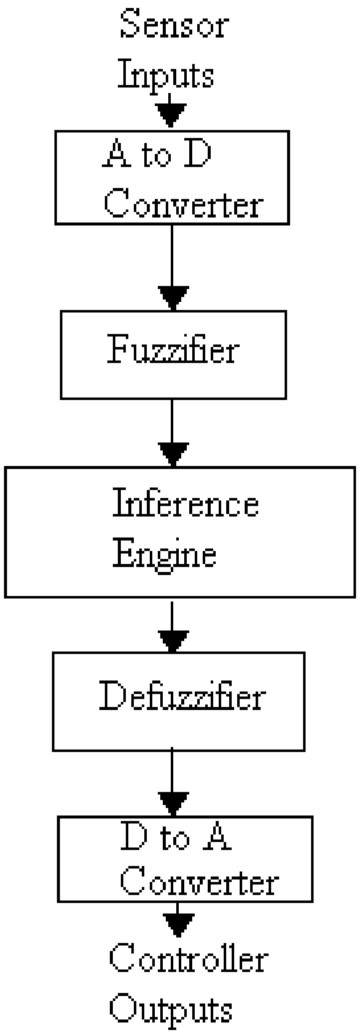
  which associates the membership grade, μ_R , of each tuple.

- E.g.

  Friend = {0.9/(Manos, Nacho), 0.1/(Manos, Dan),
  0.8/(Alex, Mike), 0.3/(Alex, John)}

Fuzzy inference

- Fuzzy logical operations
- Fuzzy rules
- Fuzzification
- Implication
- Aggregation
- Defuzzification



Fuzzy logical operations

- AND, OR, NOT, etc.

- **NOT** A = A′ = 1 - $\mu_A(x)$
- A **AND** B = $A \cap B$ = $\min(\mu_A(x), \mu_B(x))$
- A **OR** B = $A \cup B$ = $\max(\mu_A(x), \mu_B(x))$

From the following truth tables it is seen that fuzzy logic is a **superset** of Boolean logic.

min(A,B)

A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

max(A,B)

A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

1-A

A	not A
0	1
1	0

If-Then Rules

- Use fuzzy sets and fuzzy operators as the **subjects** and **verbs** of fuzzy logic to form rules.

  if x is A then y is B

  where A and B are linguistic terms defined by fuzzy sets on the sets X and Y respectively.

  This reads

  if <u>x == A</u> then <u>y = B</u>

Evaluation of fuzzy rules

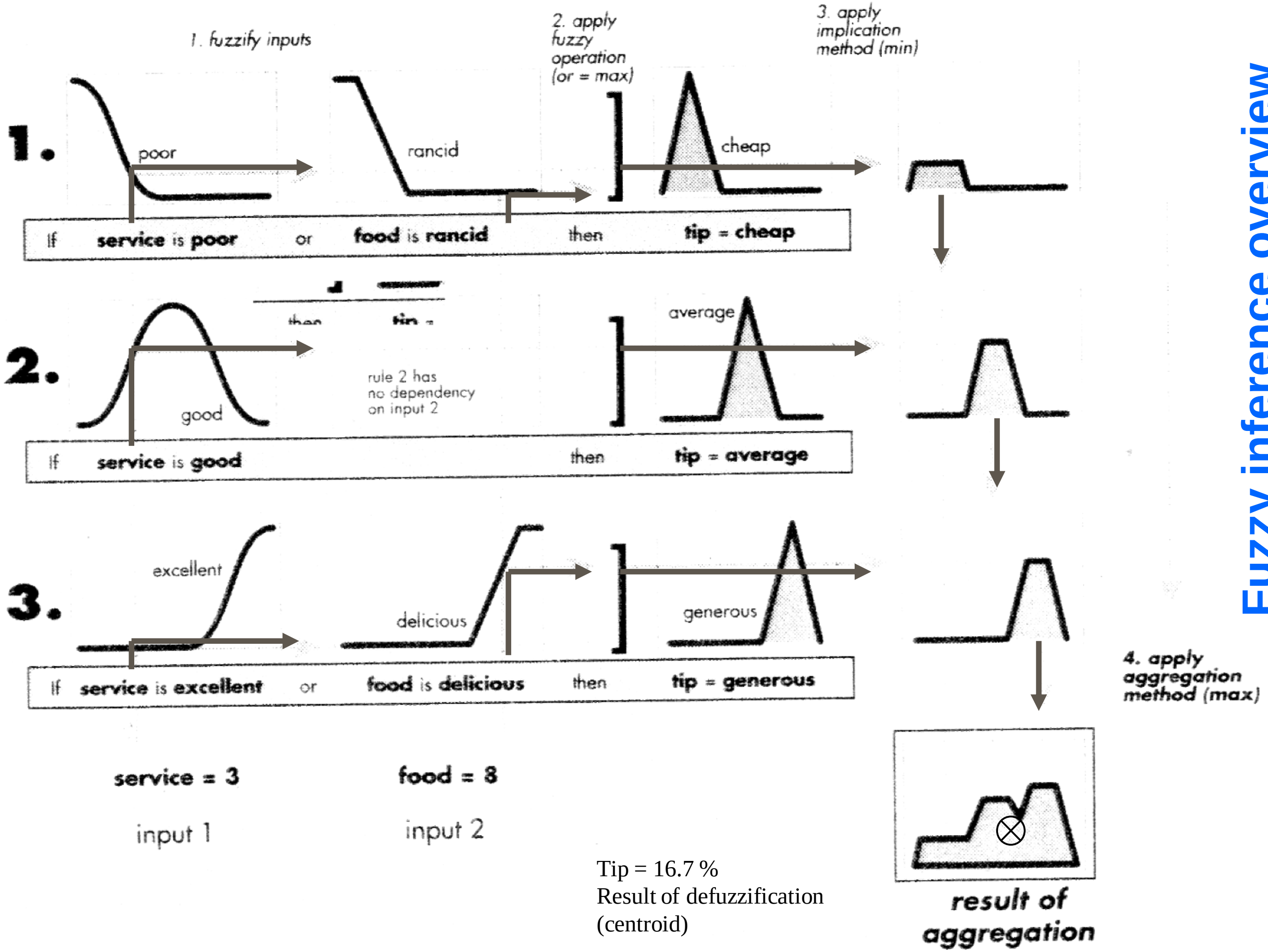
- In Boolean logic: $p \Rightarrow q$
  if p is true then q is true

- In fuzzy logic: $p \Rightarrow q$
  if p is true to some degree then q is true to some degree.

  0.5p => 0.5q (partial premise implies partially)

- How?

Fuzzy inference overview



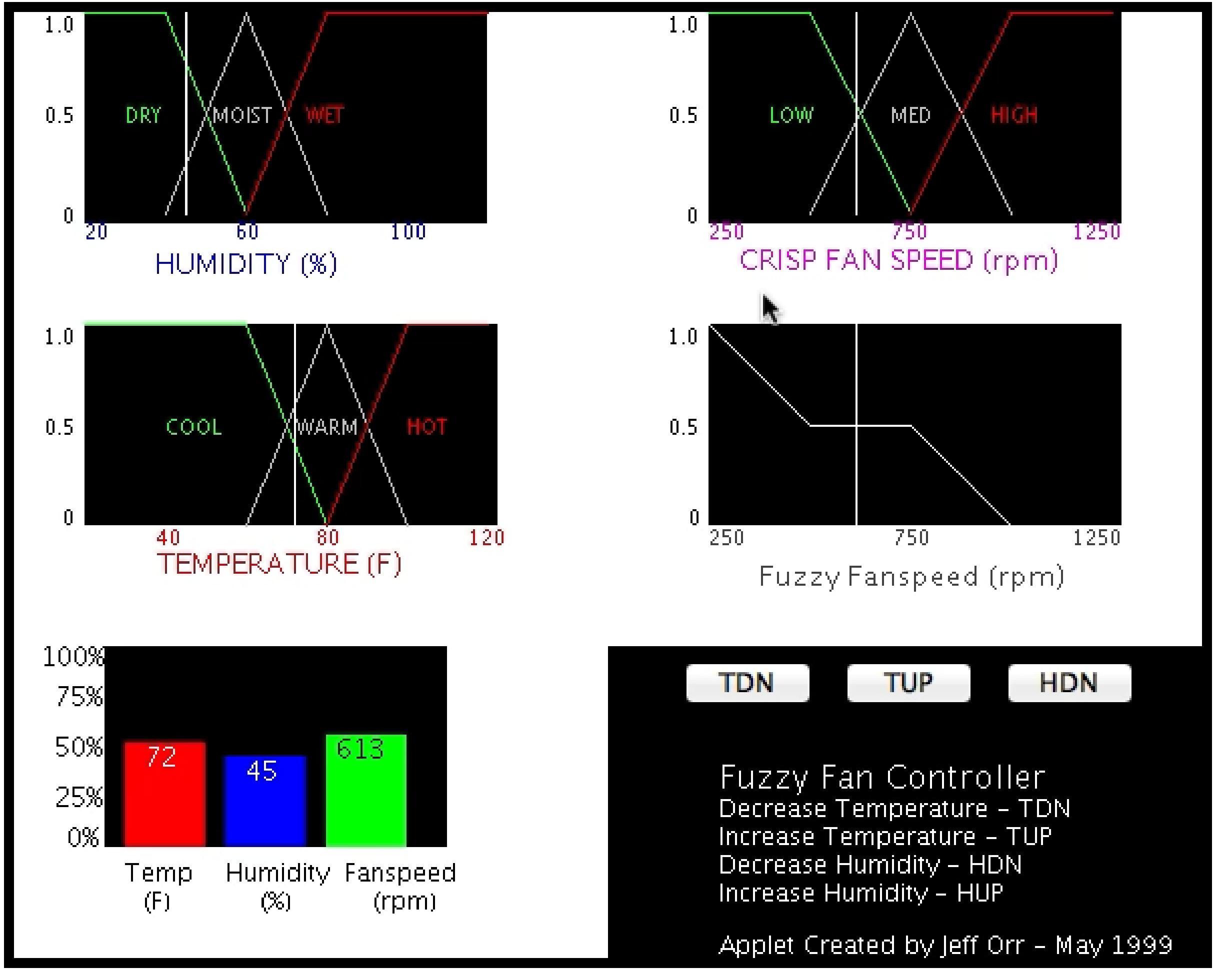
Tip = 16.7 %
Result of defuzzification
(centroid)

Demo

		Temperature		
		DRY	MOIST	WET
	COOL	MED	HIGH	HIGH
Temp.	WARM	LOW	MED	HIGH
	HOT	MED	HIGH	HIGH

The above linguistic matrix can be reduced to

```
IF H = WET  THEN FS = HIGH
IF T = COOL AND H = DRY   THEN FS = MED
IF T = COOL AND H = MOIST THEN FS = HIGH
IF T = WARM AND H = DRY   THEN FS = LOW
IF T = WARM AND H = MOIST THEN FS = MED
IF T = HOT  AND H = DRY   THEN FS = MED
IF T = HOT  AND H = MOIST THEN FS = HIGH
```

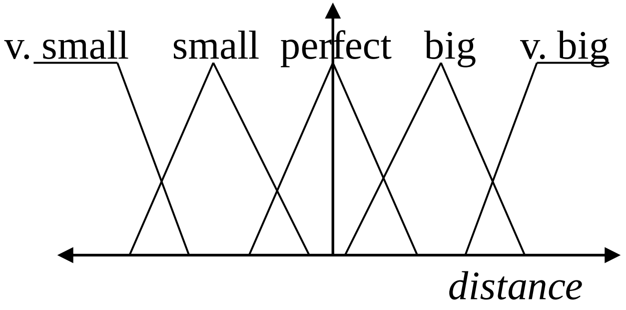


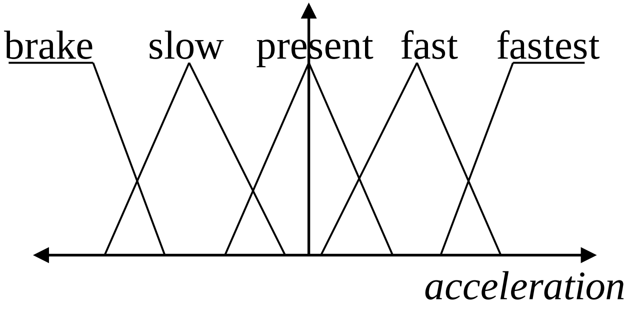
Fuzzy Rules

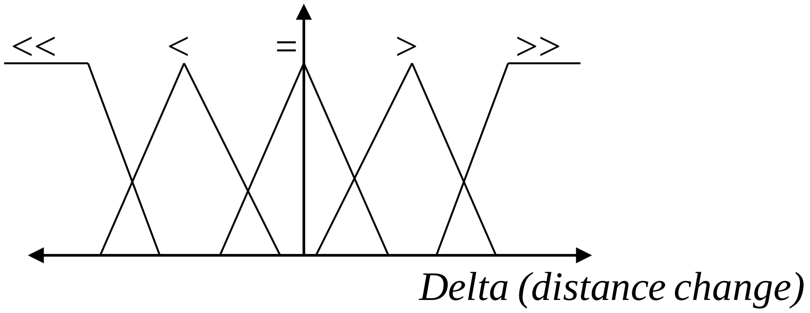
- Example: "If our distance to the car in front is small, and the distance is decreasing slowly, then decelerate quite hard"
  - Fuzzy variables in blue
  - Fuzzy sets in red

- *QUESTION:* Given the distance and the change in the distance, what acceleration should we select?

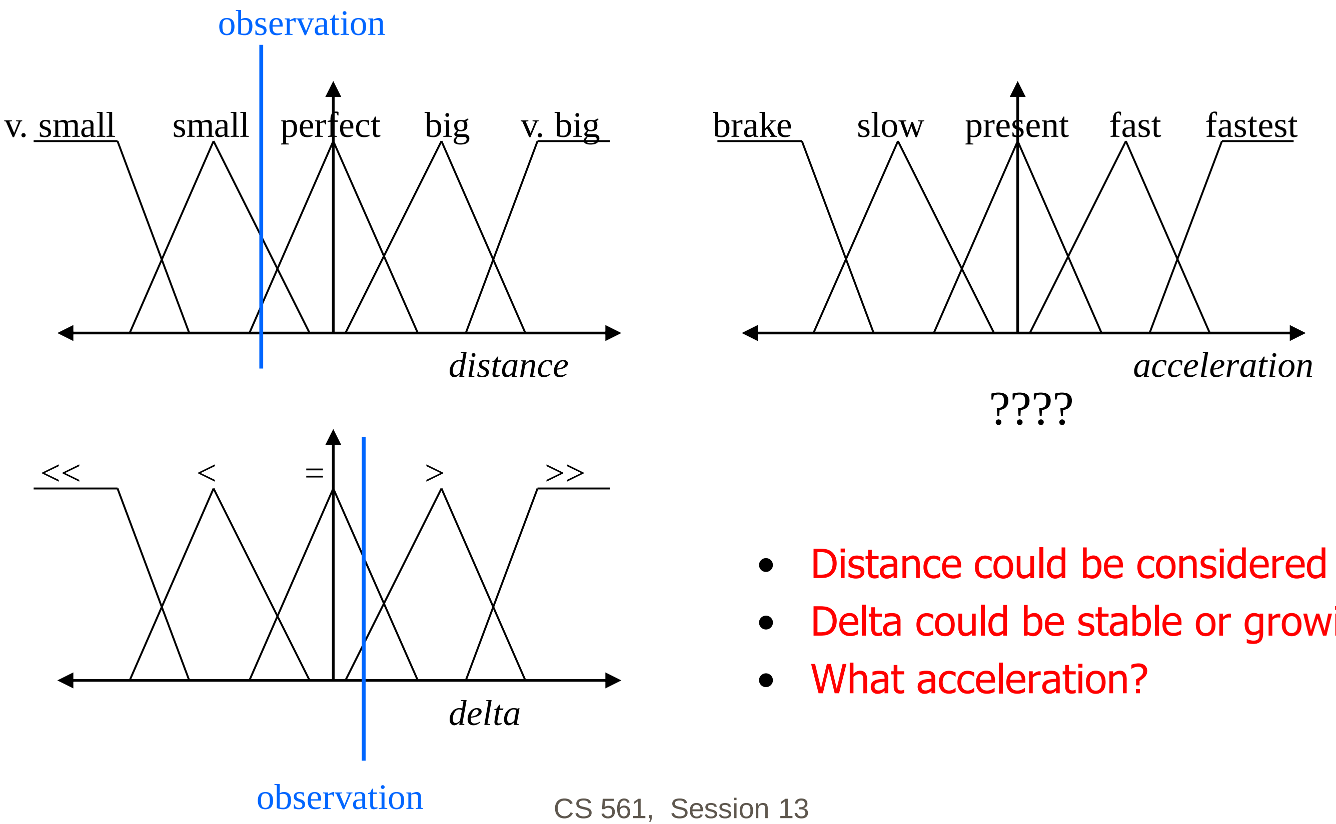
Fuzzification: Set Definitions





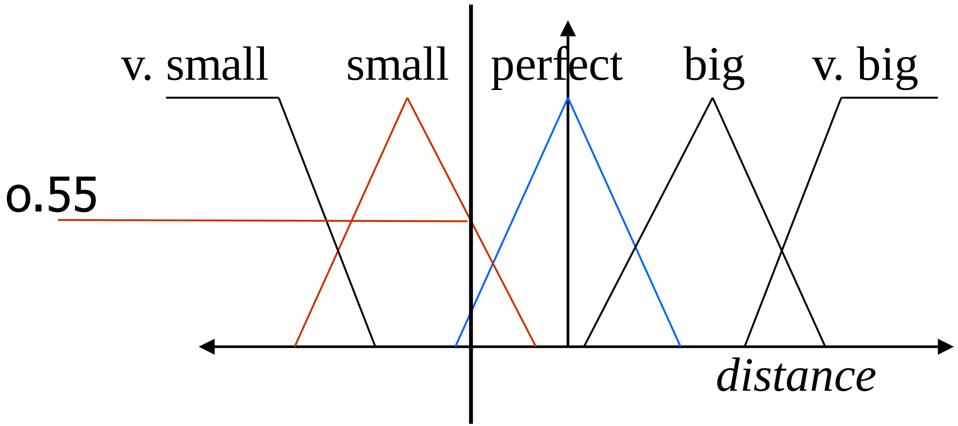


Fuzzification: Instance



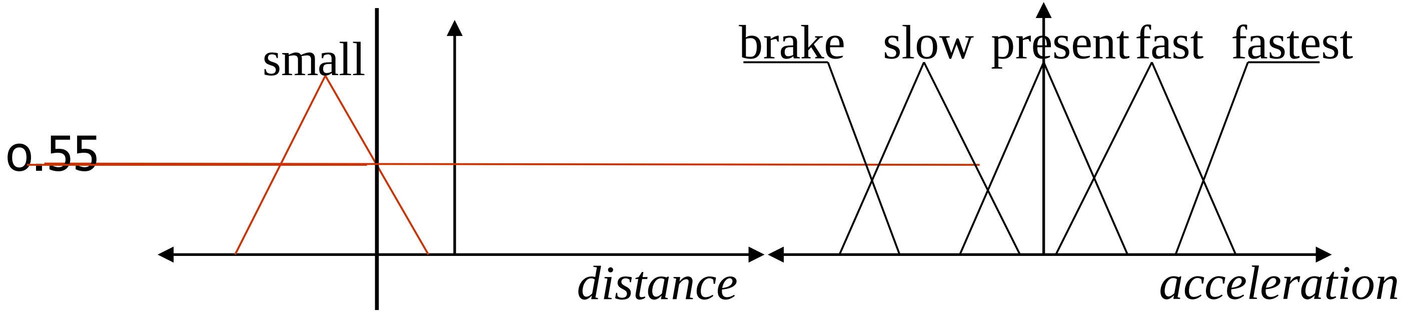
- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

Let's start with just one premise: fuzzification



IF distance is Small THEN Slow Down

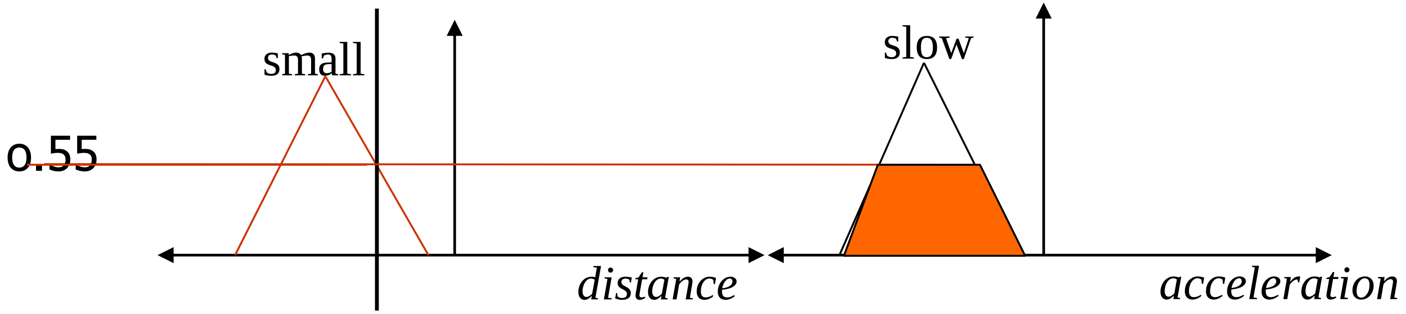
Rule Evaluation



If distance is small, then slow down.

Question: What is the weight on slow down?

Rule Evaluation



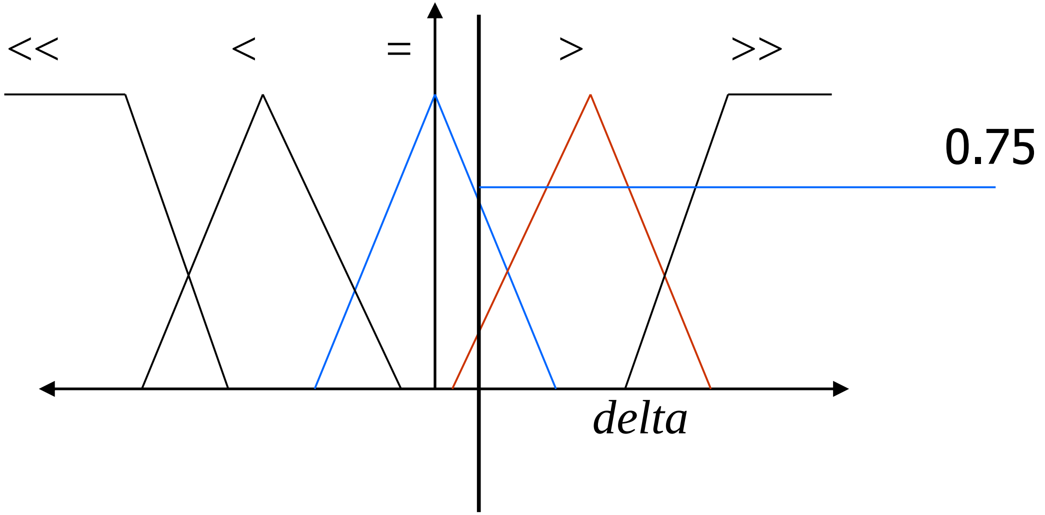
Clipping approach (others are possible):

Clip the fuzzy set for “slow” (the consequent) at the height given by our belief in the premises (0.55)

We will then consider the clipped AREA (orange) when making our final decision

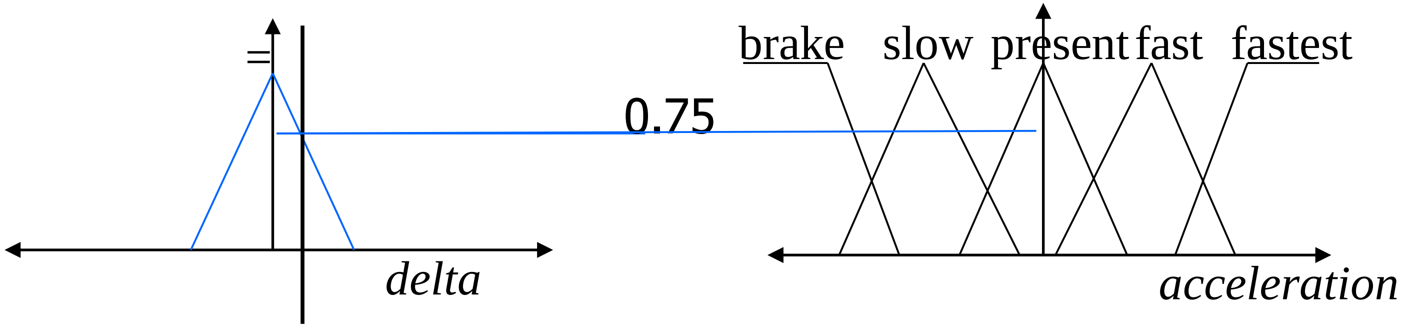
Rationale: if belief in premises is low, clipped area will be very small
But if belief is high it will be close to the whole unclipped area

Another rule to evaluate: fuzzification



IF change in distance is = THEN keep present acceleration

Rule Evaluation



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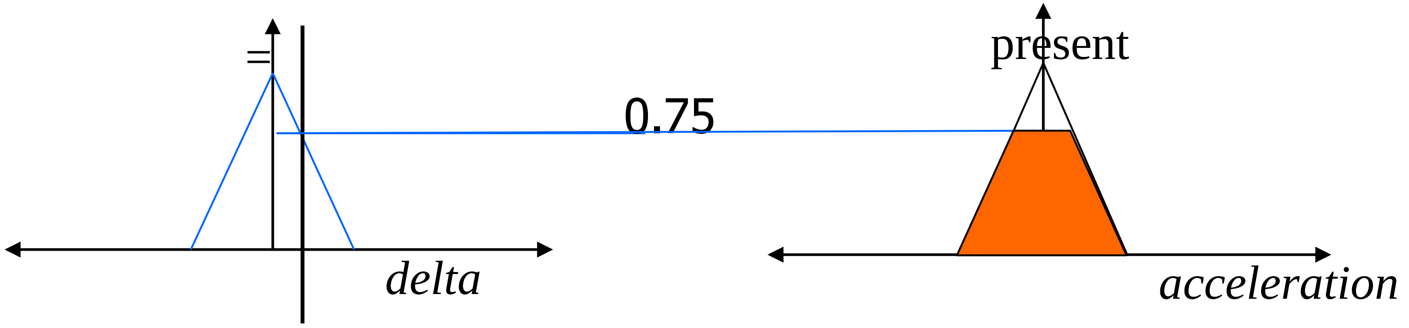
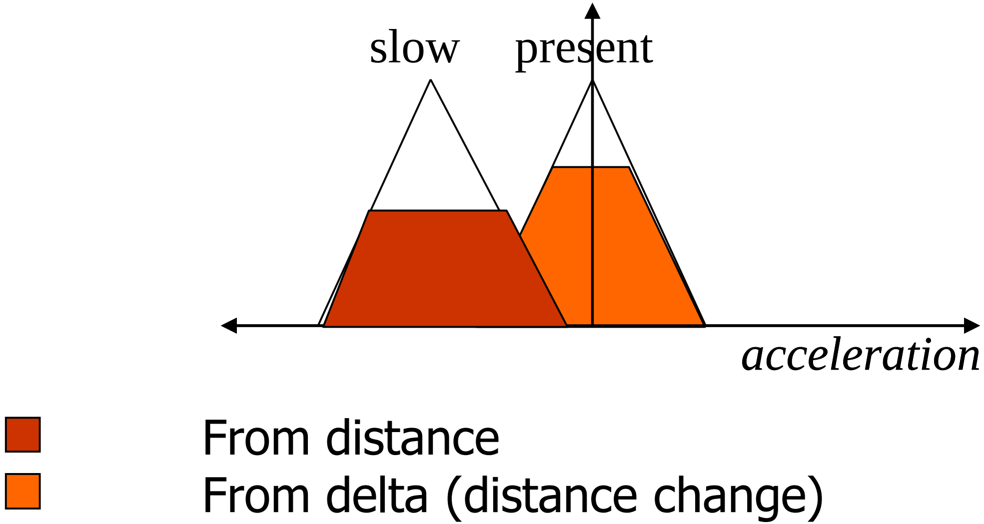


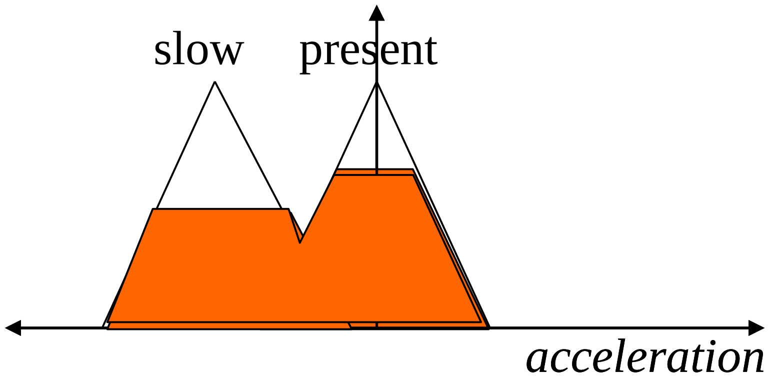
IF change in distance is = THEN keep present acceleration

Rule Aggregation

How do we make a final decision? From each rule we have obtained a clipped area. But in the end we want a single number output: our desired acceleration



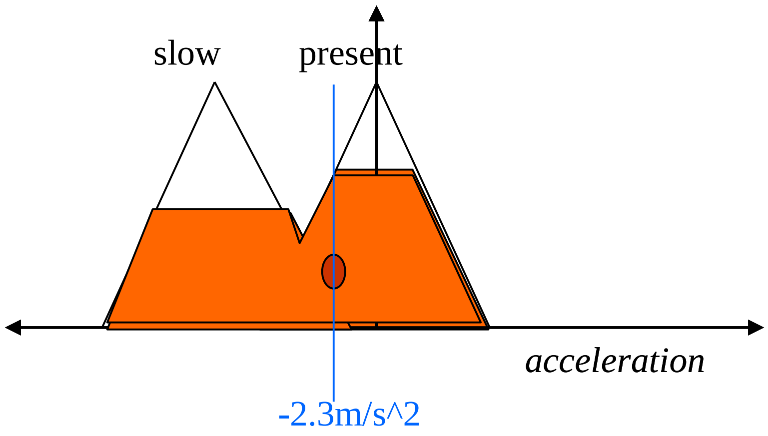
Rule Aggregation



In the rule aggregation step, we merge all clipped areas into one (taking the union).

Intuition: rules for which we had a strong belief that their premises were satisfied will tend to "pull" that merged area towards their own central value, since their clipped areas will be large

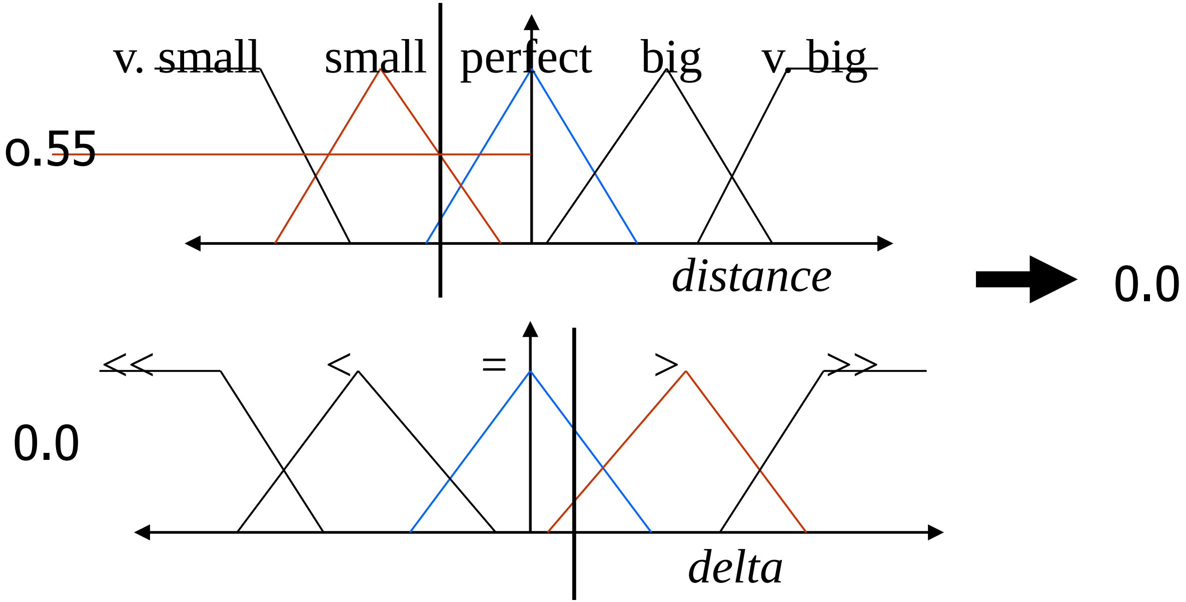
Defuzzification



In the last step, defuzzification, we return as our acceleration
Value the x coordinate of the center of mass of the merged area

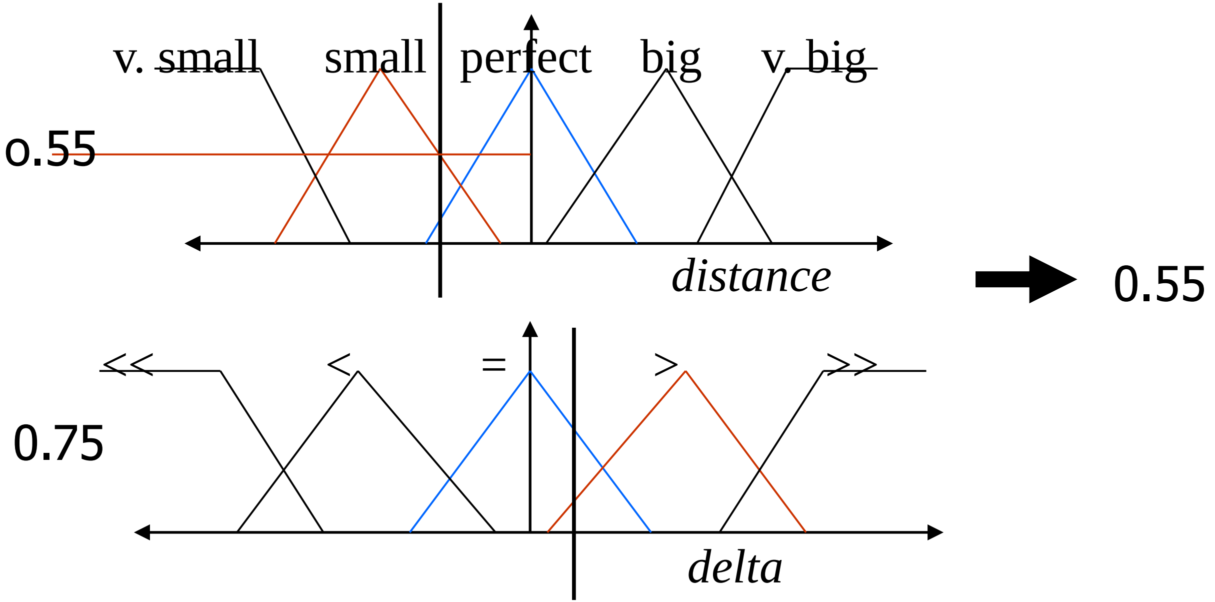
AND/OR Example

- IF Distance Small **AND** change in distance (delta) negative THEN high deceleration



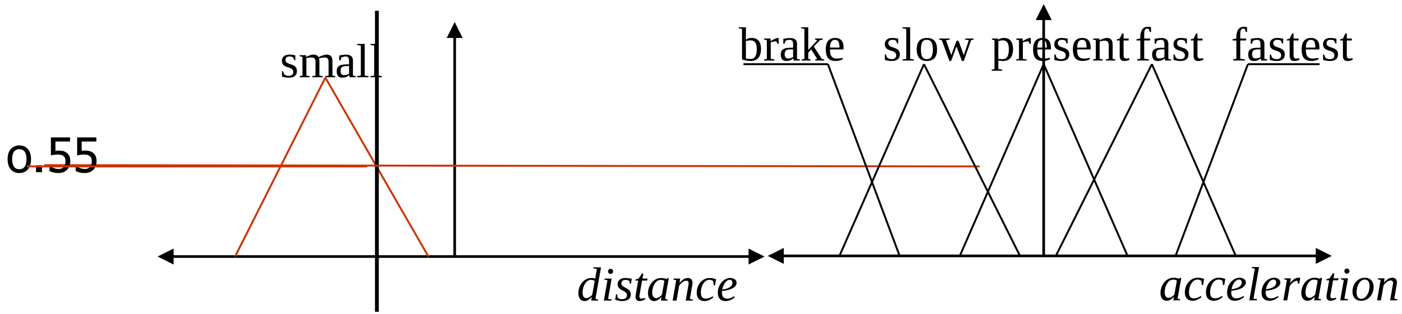
AND/OR Example

- IF Distance Small **AND** change in distance (delta) = THEN slow deceleration



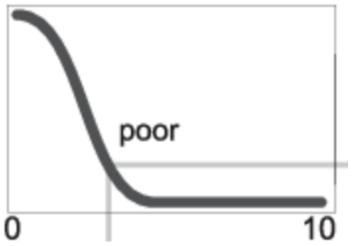
AND/OR Example

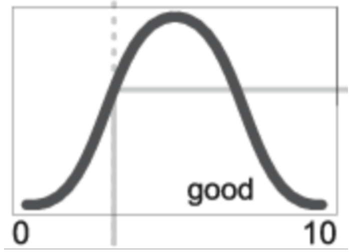
- IF Distance Small **AND** change in distance (delta) = THEN slow deceleration



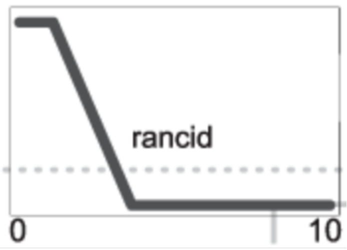
Tipping example

Define fuzzy concepts poor, good, and excellent for service:



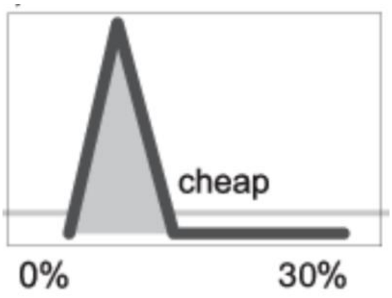


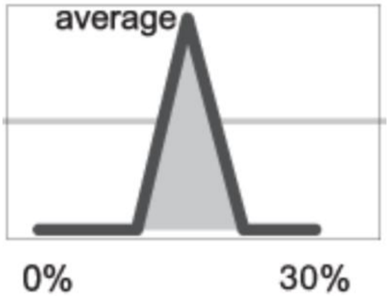
Define fuzzy concepts rancid and delicious for food:

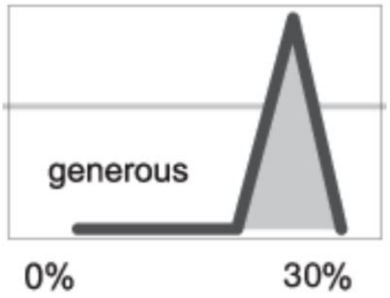


Tipping example

Define fuzzy concepts cheap, average, and generous for tip:

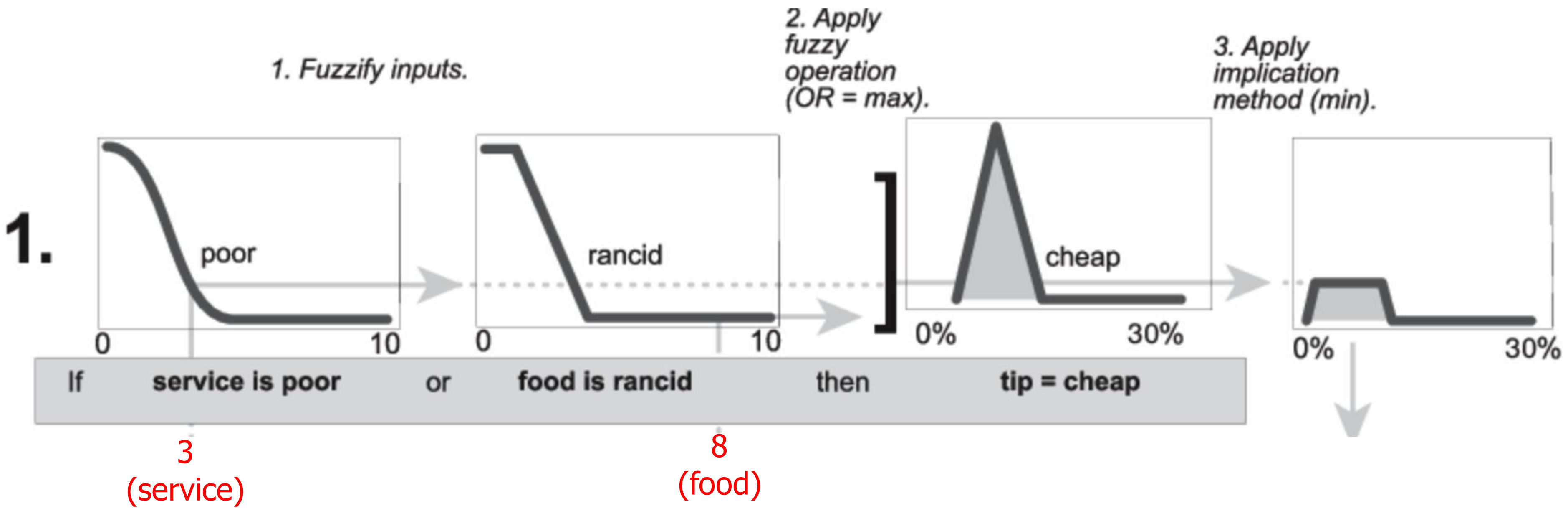




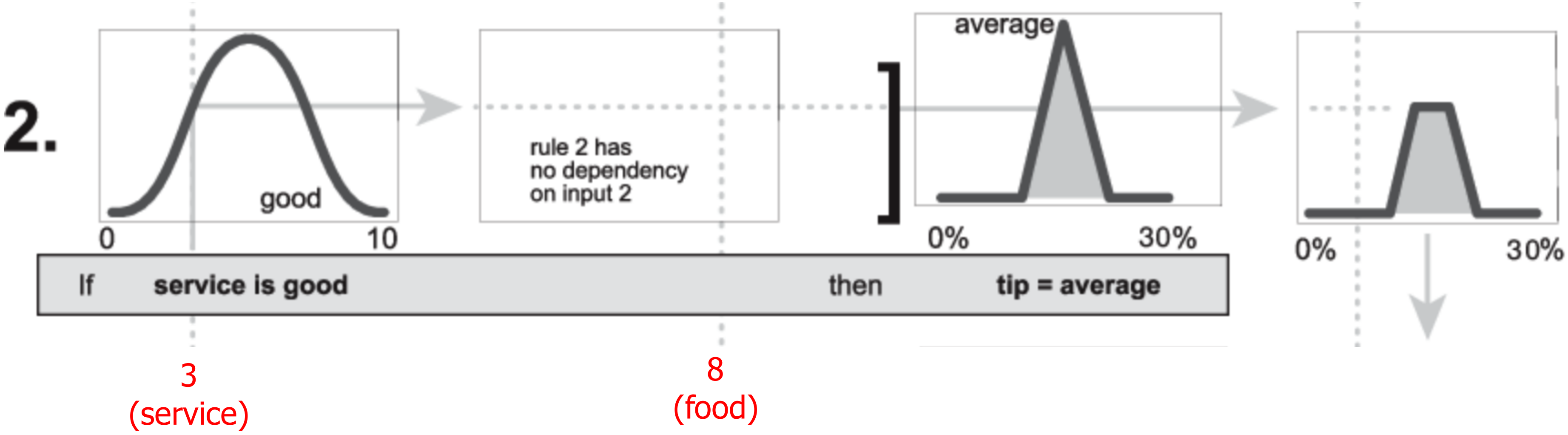


AND/OR Example

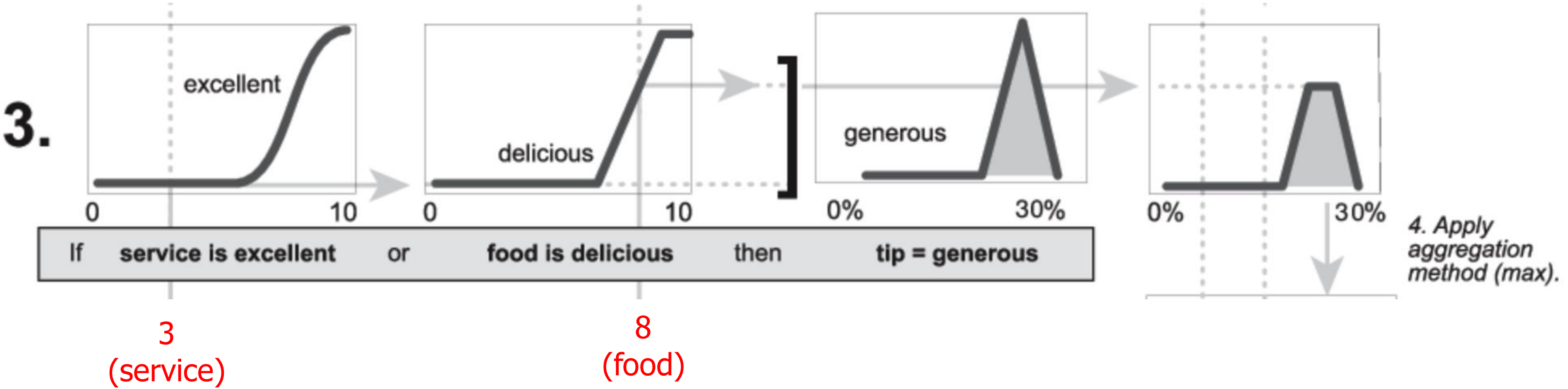
Assume that at today's dinner, service=3, food=8

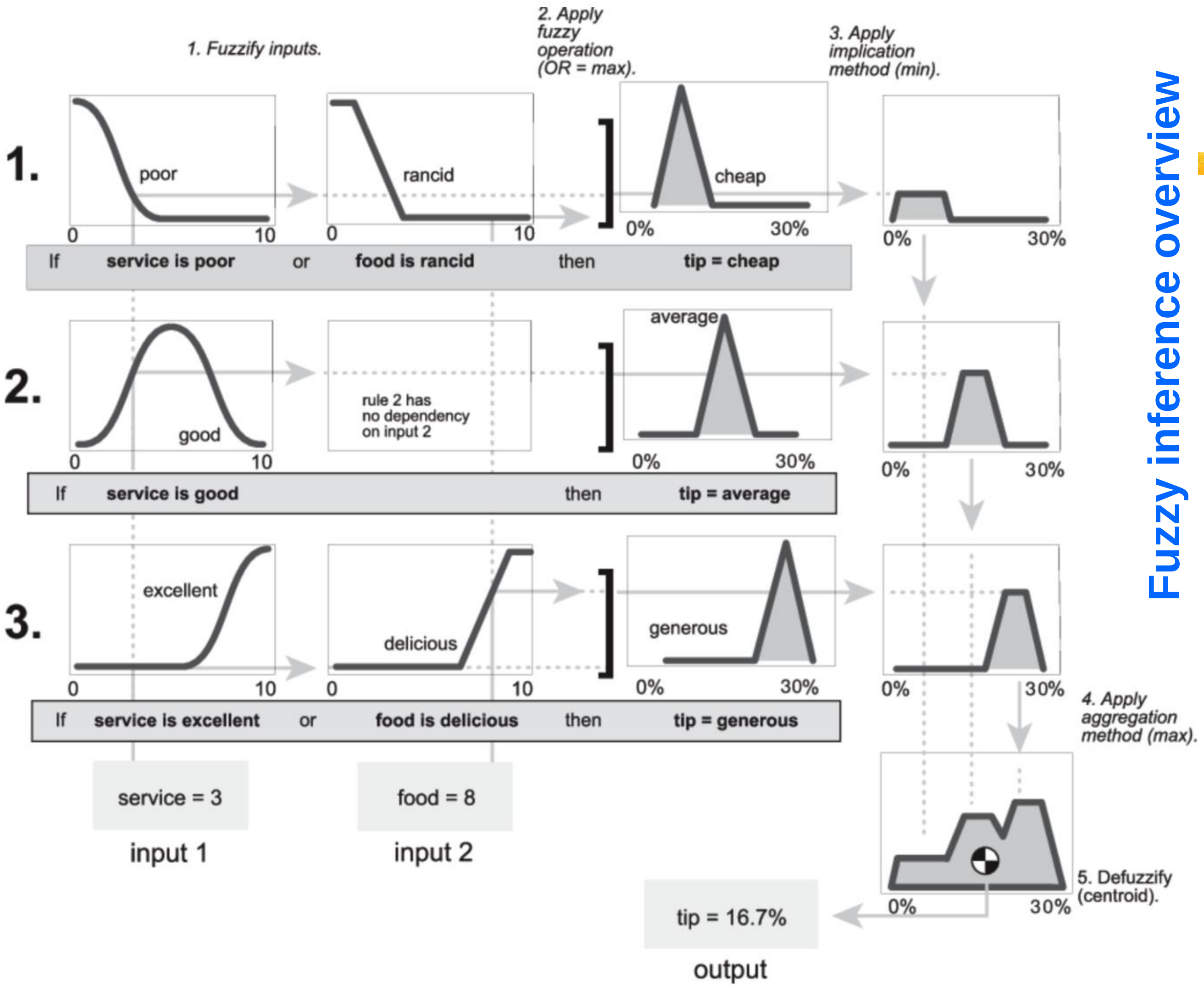


AND/OR Example



AND/OR Example

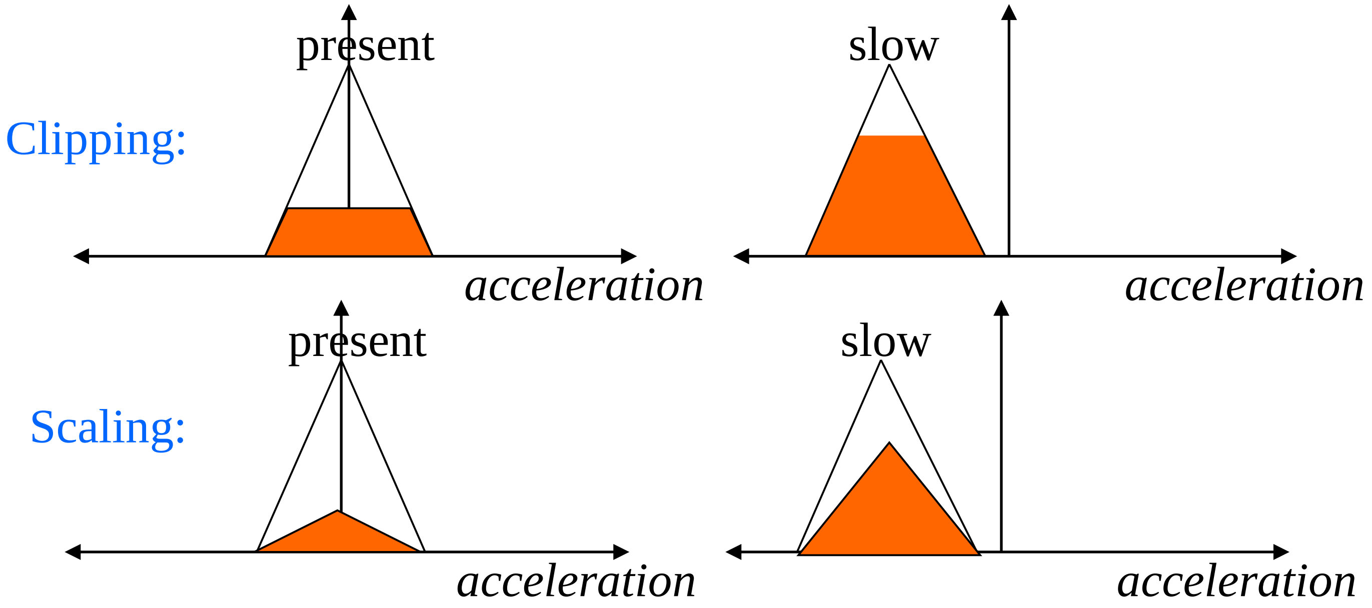




Fuzzy inference overview

Scaling vs. Clipping

Instead of clipping, another approach is to scale the fuzzy set by the belief in the premises



Limitations of fuzzy logic

- How to determine the membership functions? Usually requires fine-tuning of parameters
- Defuzzification can produce undesired results

Application areas

Product	Company	Fuzzy Logic
Anti-lock brakes	Nissan	Use fuzzy logic to controls brakes in hazardous cases depend on car speed, acceleration, wheel speed, and acceleration
Auto transmission	NOK/Nissan	Fuzzy logic is used to control the fuel injection and ignition based on throttle setting, cooling water temperature, RPM, etc.
Auto engine	Honda, Nissan	Use to select geat based on engine load, driving style, and road conditions.
Copy machine	Canon	Using for adjusting drum voltage based on picture density, humidity, and temperature.
Cruise control	Nissan, Isuzu, Mitsubishi	Use it to adjusts throttle setting to set car speed and acceleration
Dishwasher	Matsushita	Use for adjusting the cleaning cycle, rinse and wash strategies based depend upon the number of dishes and the amount of food served on the dishes.
Elevator control	Fujitec, Mitsubishi Electric, Toshiba	Use it to reduce waiting for time-based on passenger traffic
Golf diagnostic system	Maruman Golf	Selects golf club based on golfer's swing and physique.

https://www.guru99.com/what-is-fuzzy-logic.html

Application areas

Fitness management	Omron	Fuzzy rules implied by them to check the fitness of their employees.
Kiln control	Nippon Steel	Mixes cement
Microwave oven	Mitsubishi Chemical	Sets lunes power and cooking strategy
Palmtop computer	Hitachi, Sharp, Sanyo, Toshiba	Recognizes handwritten Kanji characters
Plasma etching	Mitsubishi Electric	Sets etch time and strategy

And many more!

Fuzzy tools and shells

- Matlab's Fuzzy Toolbox
- FuzzyClips
- Etc.