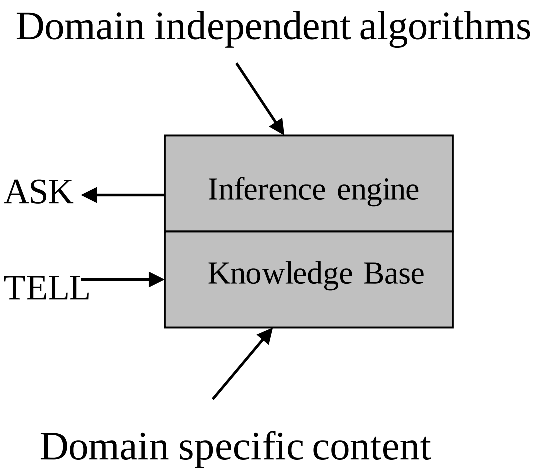
Knowledge and reasoning – second part

- Knowledge representation
- Logic and representation
- Propositional (Boolean) logic
- Normal forms
- Inference in propositional logic
- Wumpus world example

Knowledge-Based Agent



- Agent that uses **prior** or **acquired** knowledge to achieve its goals
  - Can make more efficient decisions
  - Can make informed decisions
- Knowledge Base (KB): contains a set of <u>representations</u> of facts about the Agent's environment
- Each representation is called a **sentence**
- Use some **knowledge representation language**, to TELL it what to know e.g., (temperature 72F)
- ASK agent to query what to do
- Agent can use inference to deduce new facts from TELLed facts

Generic knowledge-based agent

```
function KB-AGENT( percept) returns an action
   static: KB, a knowledge base
           t, a counter, initially 0, indicating time

   TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
   action ← ASK(KB, MAKE-ACTION-QUERY(t))
   TELL(KB, MAKE-ACTION-SENTENCE( action, t))
   t ← t + 1
   return action
```

1. TELL KB what was perceived
   Uses a KRL to insert new sentences, representations of facts, into KB

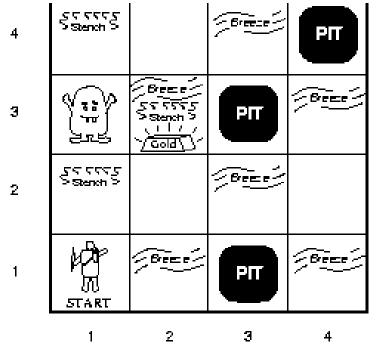
2. ASK KB what to do.
   Uses logical reasoning to examine actions and select best.

Wumpus world example

Percepts Breeze, Glitter, Smell

Actions Left turn, Right turn,
Forward, Grab, Release, Shoot

Goals Get gold back to start
without entering pit or wumpus square



Environment

Squares adjacent to wumpus are smelly
Squares adjacent to pit are breezy
Glitter if and only if gold is in the same square
Shooting kills the wumpus if you are facing it
Shooting uses up the only arrow
Grabbing picks up the gold if in the same square
Releasing drops the gold in the same square

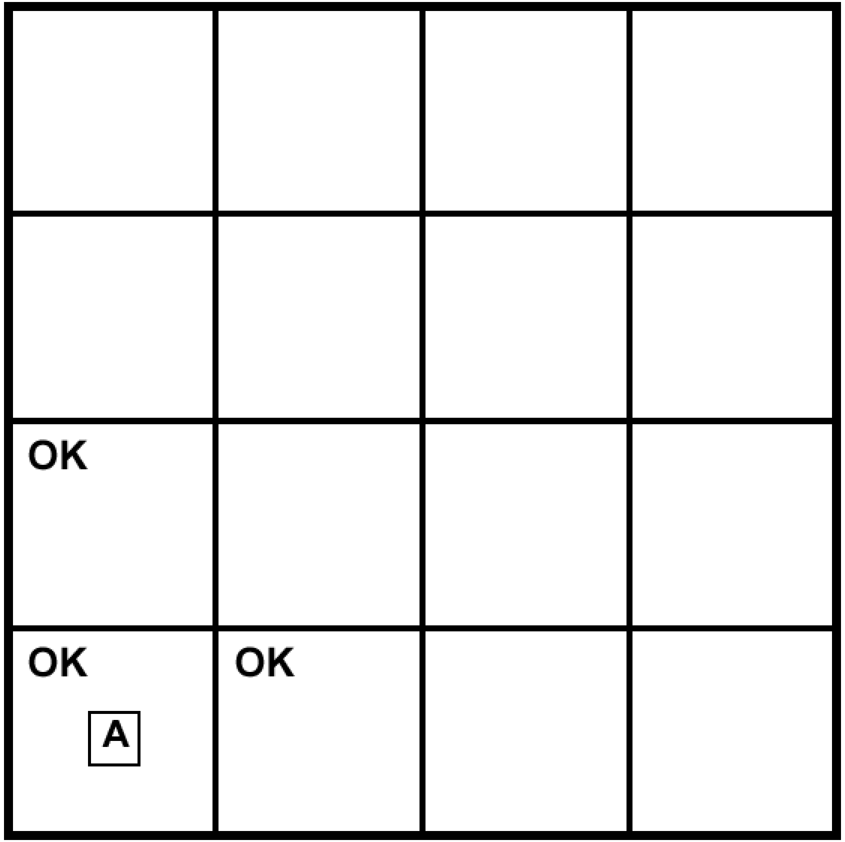
Wumpus world characterization

- Deterministic?
- Accessible?
- Static?
- Discrete?
- Episodic?

Wumpus world characterization

• Deterministic?	Yes – outcome exactly specified.
• Accessible?	No – only local perception.
• Static?	Yes – Wumpus and pits do not move.
• Discrete?	Yes
• Episodic?	(Yes) – because static.

Exploring a Wumpus world



A= Agent
B= Breeze
S= Smell
P= Pit
W= Wumpus
OK = Safe
V = Visited
G = Glitter

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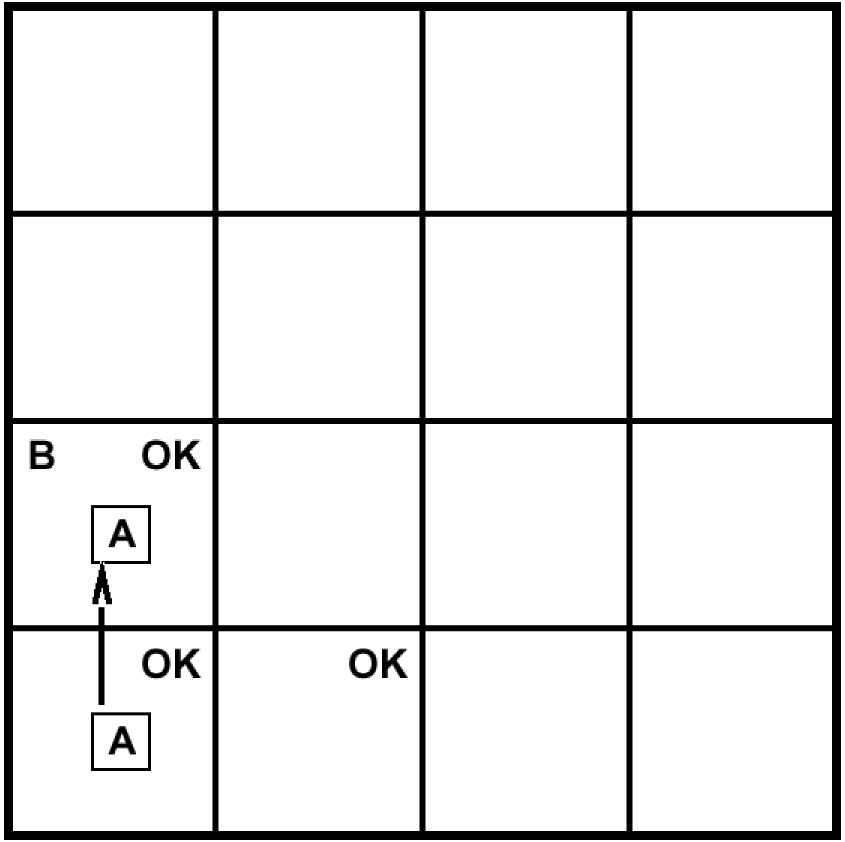

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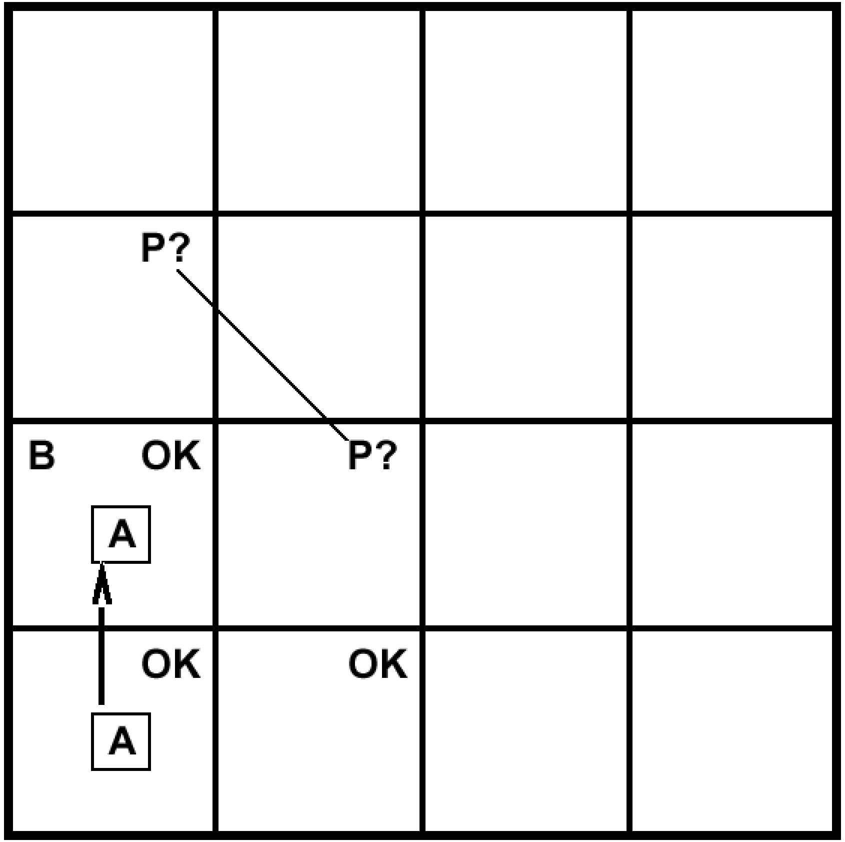

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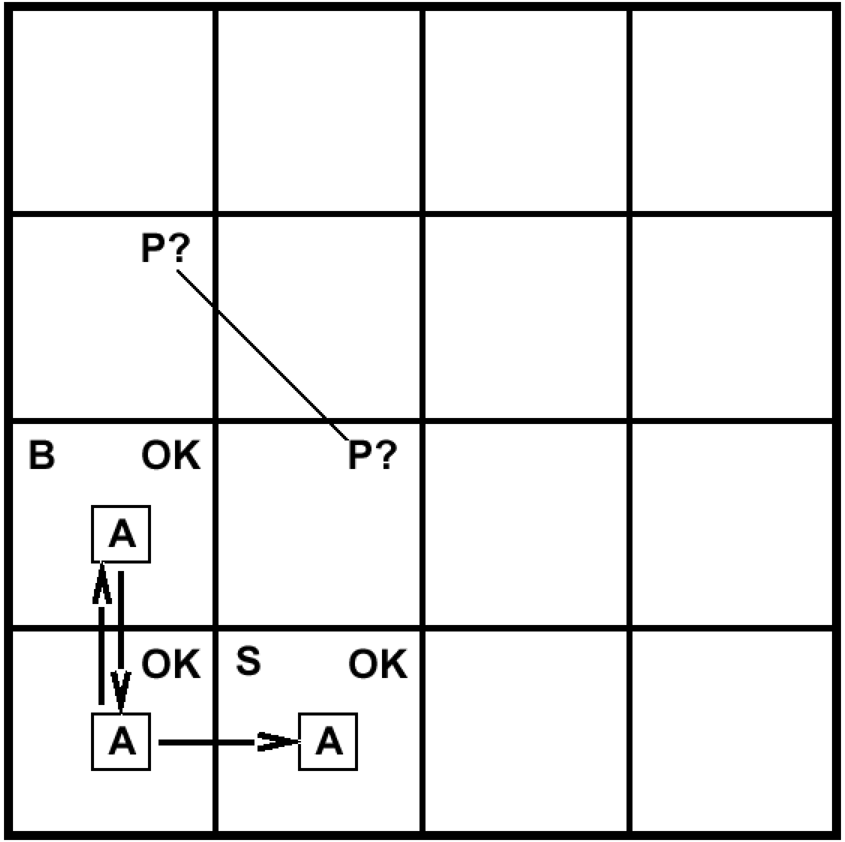

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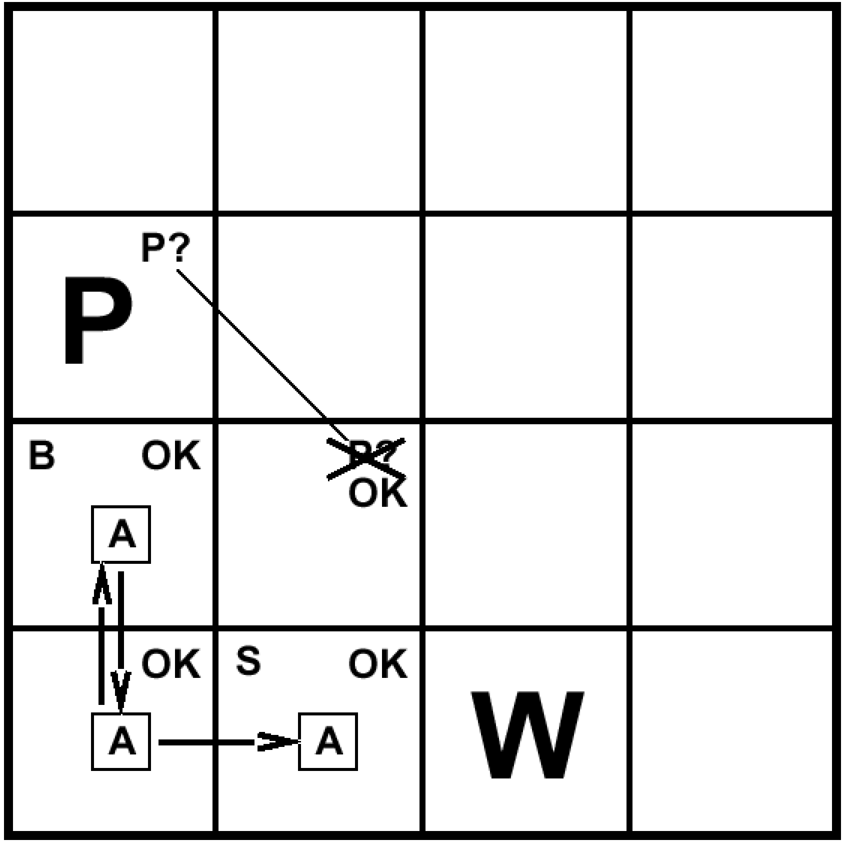

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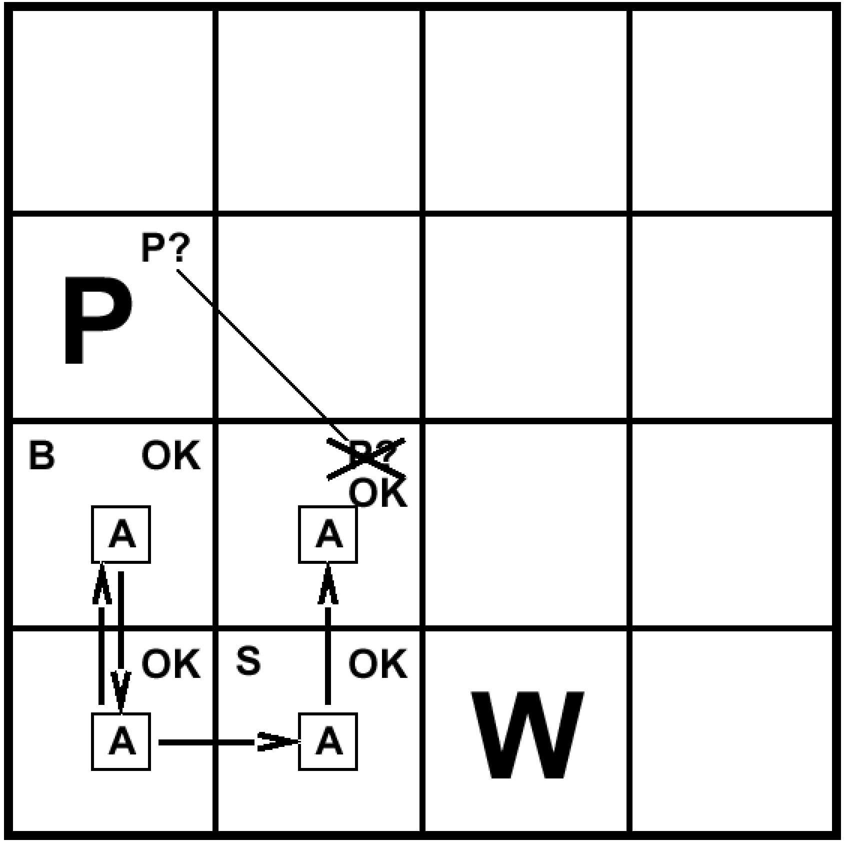

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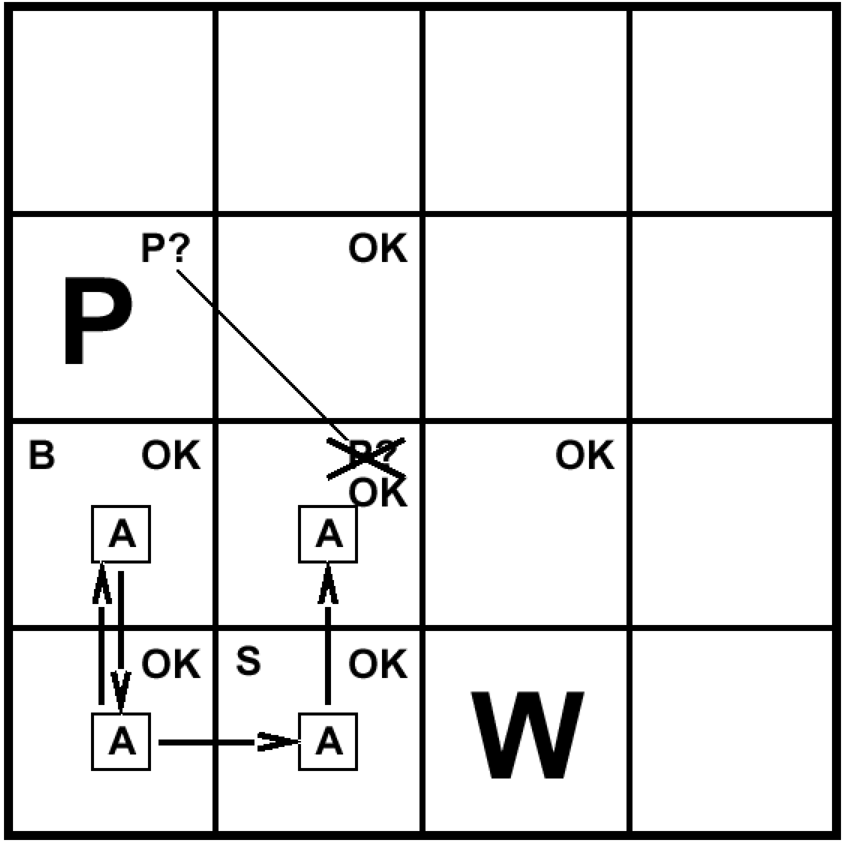

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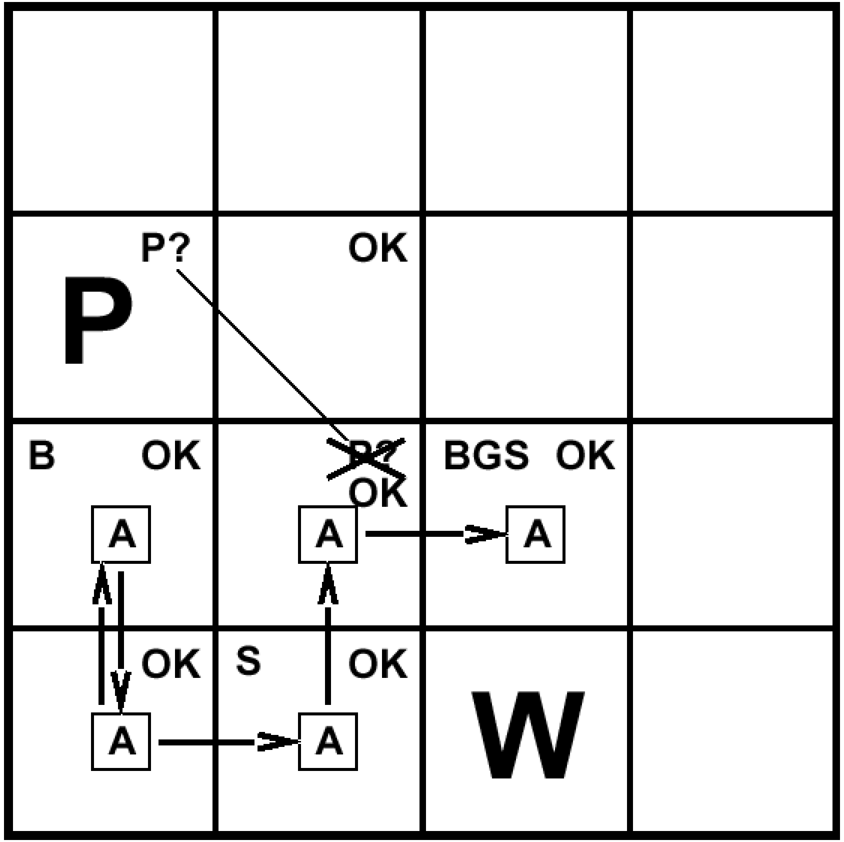

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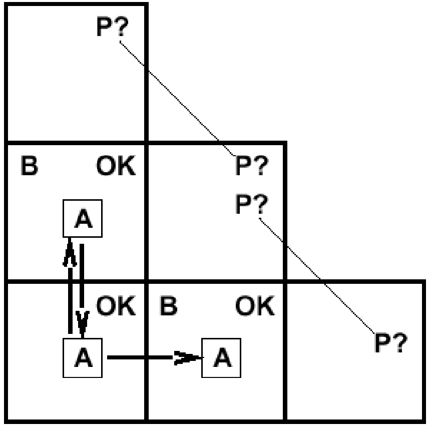
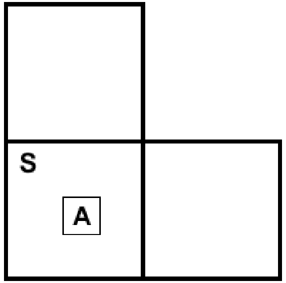
Other tight spots



Breeze in (1,2) and (2,1)
$\Rightarrow$ no safe actions

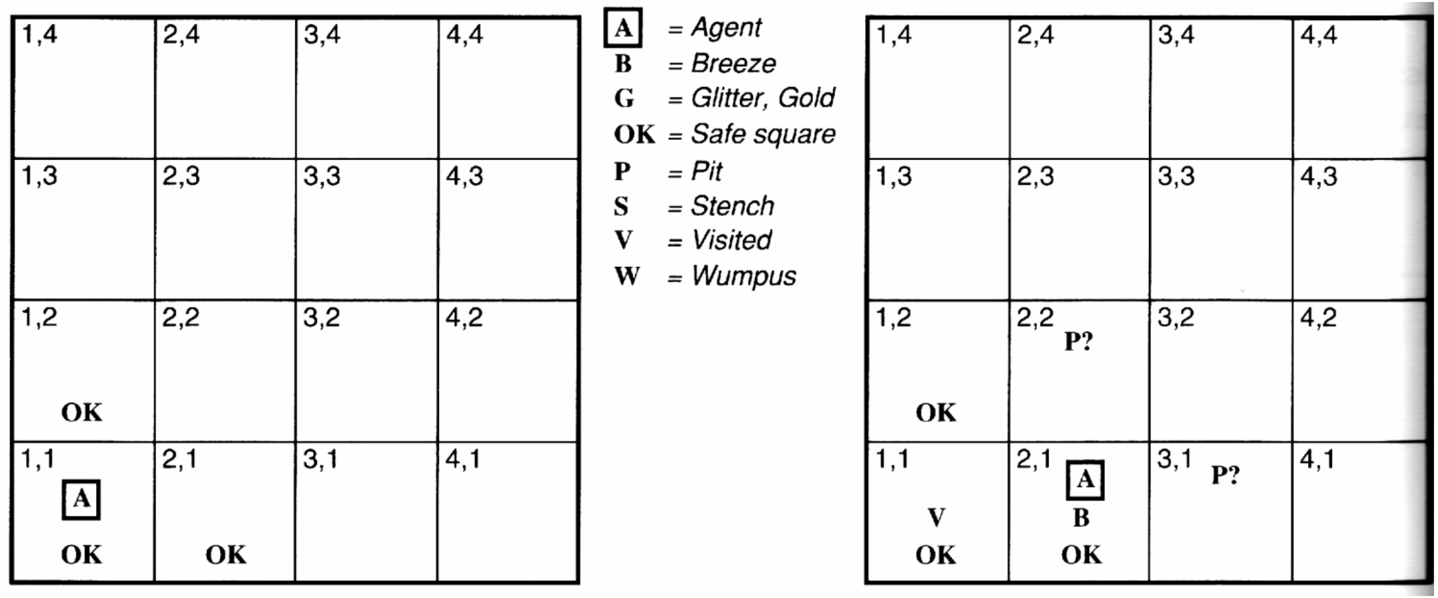
Assuming pits uniformly distributed,
(2,2) is most likely to have a pit



Smell in (1,1)
$\Rightarrow$ cannot move

Can use a strategy of <u>coercion</u>:
shoot straight ahead
wumpus was there $\Rightarrow$ dead $\Rightarrow$ safe
wumpus wasn't there $\Rightarrow$ safe

Another example solution



No perception → 1,2 and 2,1 OK

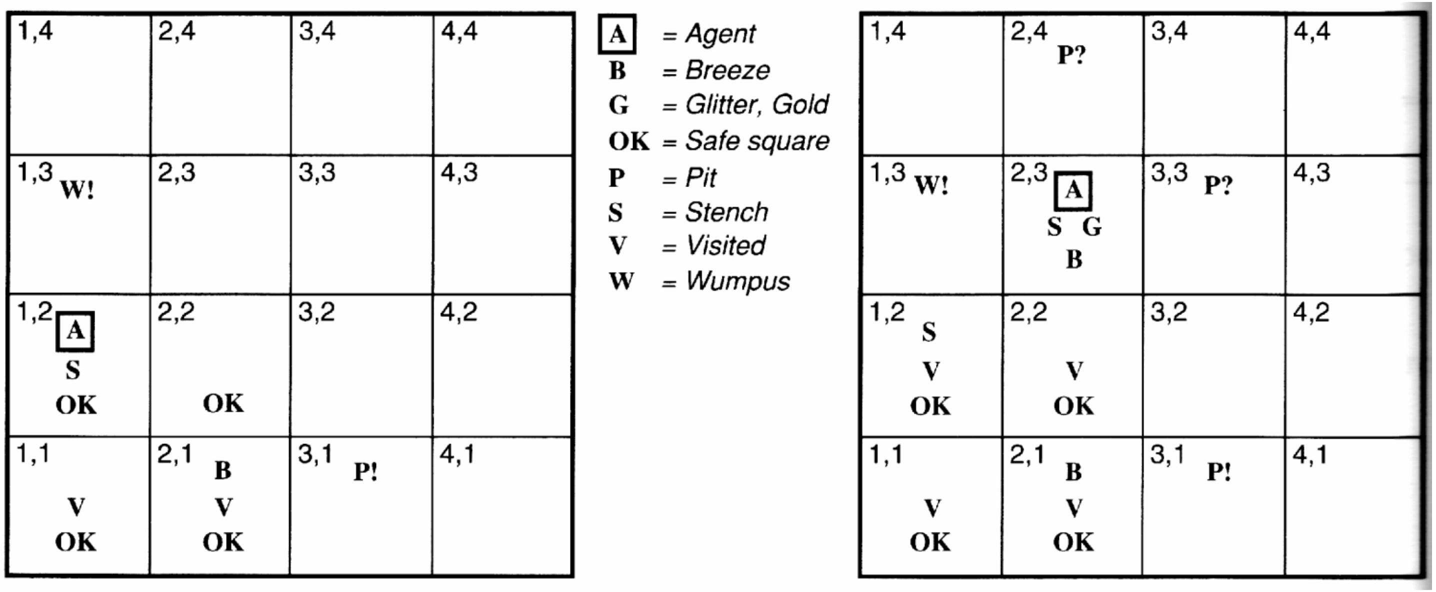
Move to 2,1

B in 2,1 → 2,2 or 3,1 P?

1,1 V → no P in 1,1

Move to 1,2 (only option)

Example solution



S and No S when in 2,1 → 1,3 or 1,2 has W

1,2 OK → 1,3 W

No B in 1,2 → 2,2 OK & 3,1 P

Logic in general

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Types of logic

Logics are characterized by what they commit to as "primitives"

Ontological commitment: what exists—facts? objects? time? beliefs?

Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $0 \ldots 1$
Fuzzy logic	degree of truth	degree of belief $0 \ldots 1$

The Semantic Wall

Physical Symbol System

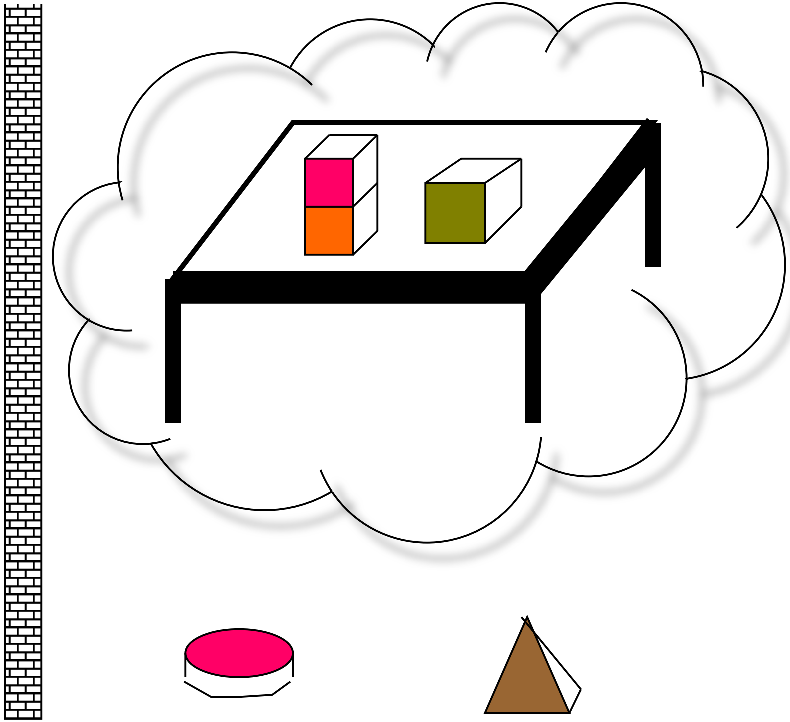
+BLOCKA+

+BLOCKB+

+BLOCKC+

P_1:(IS_ON +BLOCKA+ +BLOCKB+)
P_2:((IS_RED +BLOCKA+)

World



Truth depends on Interpretation

Representation 1

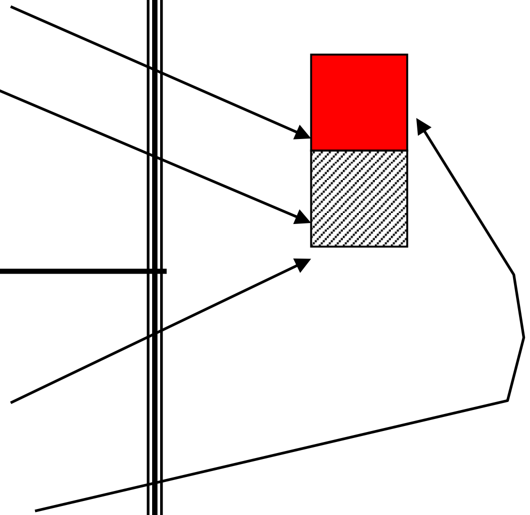
World

A

B

ON(A,B) T

ON(B,A) F



Representation 2

ON(A,B) F   A

ON(B,A) T   B

Entailment

$$KB \models \alpha$$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

Entailment is different than inference

Logic as a representation of the World

$$\begin{array}{lcc}
\text{Representation: Sentences} & \xrightarrow{\text{entails}} & \text{Sentence} \\
\text{Refers to (Semantics)} \quad \downarrow & & \downarrow \\
\text{World} \quad \text{Facts} & \xrightarrow{\text{follows}} & \text{Fact}
\end{array}$$

Models

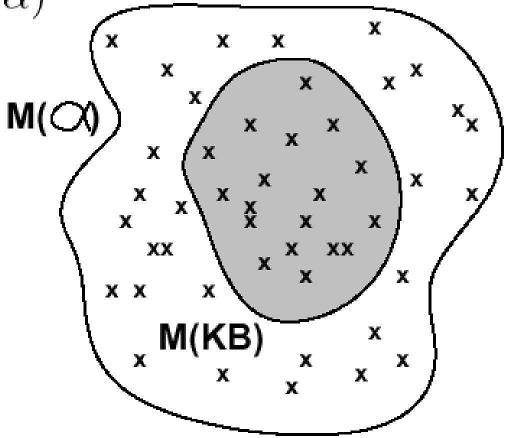
Logicians typically think in terms of <u>models</u>, which are formally structured worlds with respect to which truth can be evaluated

We say m is a <u>model</u> of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. $KB =$ Giants won and Reds won
$\alpha =$ Giants won



Inference

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Basic symbols

- Expressions only evaluate to either "true" or "false."

• P	"P is true"	
• $\neg$P	"P is false"	negation
• P V Q	"either P is true or Q is true or both"	disjunction
• P ^ Q	"both P and Q are true"	conjunction
• P => Q	"if P is true, then Q is true"	implication
• P ⇔ Q	"P and Q are either both true or both false"	equivalence

Propositional logic: syntax

Propositional logic is the simplest logic

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence

If S_1 and S_2 is a sentence, $S_1 \wedge S_2$ is a sentence

If S_1 and S_2 is a sentence, $S_1 \vee S_2$ is a sentence

If S_1 and S_2 is a sentence, $S_1 \Rightarrow S_2$ is a sentence

If S_1 and S_2 is a sentence, $S_1 \Leftrightarrow S_2$ is a sentence

Propositional logic: semantics

Each model specifies true/false for each proposition symbol

E.g. A B C
$True$ $True$ $False$

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true and	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true or	S_2	is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false or	S_2	is true
i.e.,	is false iff	S_1	is true and	S_2	is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true and	$S_2 \Rightarrow S_1$	is true

Truth tables

- Truth value: whether a statement is true or false.
- Truth table: complete list of truth values for a statement given all possible values of the individual atomic expressions.

Example:

P	Q	P V Q
T	T	T
T	F	T
F	T	T
F	F	F

Truth tables for basic connectives

P	Q	¬P	¬Q	P V Q	P ^ Q	P=>Q	P⇔Q
T	T	F	F	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	T	F	T	F
F	F	T	T	F	F	T	T

Propositional logic: basic manipulation rules

- $\neg(\neg A) = A$ — Double negation

- $\neg(A \wedge B) = (\neg A) \vee (\neg B)$ — Negated "and"
- $\neg(A \vee B) = (\neg A) \wedge (\neg B)$ — Negated "or"

- $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$ — Distributivity of ^ on V
- $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$ — Distributivity of V on ^
- $A \Rightarrow B = (\neg A) \vee B$ — by definition
- $\neg(A \Rightarrow B) = A \wedge (\neg B)$ — using negated or
- $A \Leftrightarrow B = (A \Rightarrow B) \wedge (B \Rightarrow A)$ — by definition
- $\neg(A \Leftrightarrow B) = (A \wedge (\neg B)) \vee (B \wedge (\neg A))$ — using negated and & or
- …

Propositional inference: enumeration method (truth table method)

Let $\alpha = A \vee B$ and $KB = (A \vee C) \wedge (B \vee \neg C)$

Is it the case that $KB \models \alpha$?

Check all possible models—α must be true wherever KB is true

A	B	C	$A \vee C$	$B \vee \neg C$	KB	α
$False$	$False$	$False$				
$False$	$False$	$True$				
$False$	$True$	$False$				
$False$	$True$	$True$				
$True$	$False$	$False$				
$True$	$False$	$True$				
$True$	$True$	$False$				
$True$	$True$	$True$				

Enumeration: Solution

A	B	C	$A \vee C$	$B \vee \neg C$	KB	α
$False$	$False$	$False$	$False$	$True$	$False$	$False$
$False$	$False$	$True$	$True$	$False$	$False$	$False$
$False$	$True$	$False$	$False$	$True$	$False$	$True$
$False$	$True$	$True$	$True$	$True$	$True$	$True$
$True$	$False$	$False$	$True$	$True$	$True$	$True$
$True$	$False$	$True$	$True$	$False$	$False$	$True$
$True$	$True$	$False$	$True$	$True$	$True$	$True$
$True$	$True$	$True$	$True$	$True$	$True$	$True$

Propositional inference: normal forms

Other approaches to inference use syntactic operations on sentences, often expressed in standardized forms

Conjunctive Normal Form (CNF—universal)

$$\textit{conjunction} \text{ of } \underbrace{\textit{disjunctions} \text{ of } \textit{literals}}_{\textit{clauses}}$$

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

"product of sums of simple variables or negated simple variables"

Disjunctive Normal Form (DNF—universal)

$$\textit{disjunction} \text{ of } \underbrace{\textit{conjunctions} \text{ of } \textit{literals}}_{\textit{terms}}$$

E.g., $(A \wedge B) \vee (A \wedge \neg C) \vee (A \wedge \neg D) \vee (\neg B \wedge \neg C) \vee (\neg B \wedge \neg D)$

"sum of products of simple variables or negated simple variables"

Horn Form (restricted)

conjunction of *Horn clauses* (clauses with ≤ 1 positive literal)

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Often written as set of implications:

$B \Rightarrow A$ and $(C \wedge D) \Rightarrow B$

Deriving expressions from functions

- Given a boolean function in truth table form, find a propositional logic expression for it that uses only V, ^ and ¬.
- **Idea:** We can easily do it by disjoining the "T" rows of the truth table.

Example: XOR function

P	Q	RESULT
T	T	F
T	F	T
F	T	T
F	F	F

RESULT = (

Deriving expressions from functions

- Given a boolean function in truth table form, find a propositional logic expression for it that uses only V, ^ and ¬.
- **Idea:** We can easily do it by disjoining the "T" rows of the truth table.

Example: XOR function

P	Q	RESULT	
T	T	F	
T	F	T	P ^ (¬Q)
F	T	T	(¬P) ^ Q
F	F	F	

RESULT = (P ^ (¬Q)) V ((¬P) ^ Q)

A more formal approach

- To construct a logical expression in disjunctive normal form from a truth table:

- Build a **“minterm”** for each row of the table, where:

    - For each variable whose value is T in that row, include the variable in the minterm
    - For each variable whose value is F in that row, include the negation of the variable in the minterm
    - Link variables in minterm by conjunctions

- The expression consists of the disjunction of all minterms.

Example: adder with carry

Takes 3 variables in: x, y and ci (carry-in); yields 2 results: sum (s) and carry-out (co). To get you used to other notations, here we assume T = 1, F = 0, V = OR, ^ = AND, ¬ = NOT.

x	y	ci	co	s	
0	0	0	0	0	
0	0	1	0	1	s : NOT x AND NOT y AND ci
0	1	0	0	1	s : NOT x AND y AND NOT ci
0	1	1	1	0	co: NOT x AND y AND ci
1	0	0	0	1	s : x AND NOT y AND NOT ci
1	0	1	1	0	co: x AND NOT y AND ci
1	1	0	1	0	co: x AND y AND NOT ci
1	1	1	1	1	co, s: x AND y AND ci

The logical expression for co is:

```
(NOT x AND y AND ci) OR (x AND NOT y AND ci) OR
(x AND y AND NOT ci) OR (x AND y AND ci)
```

The logical expression for s is:

```
(NOT x AND NOT y AND ci) OR (NOT x AND y AND NOT ci)
OR (x AND NOT y AND NOT ci) OR (x AND y AND ci)
```

Tautologies

- Logical expressions that are always true. Can be simplified out.

Examples:

T

T V A

A V (¬A)

¬(A ^ (¬A))

A ⇔ A

((P V Q) ⇔ P) V (¬P ^ Q)

(P ⇔ Q) => (P => Q)

Validity and satisfiability

A sentence is <u>valid</u> if it is true in <u>all</u> models

e.g., $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow$ ***B***

Validity is connected to inference via the <u>Deduction</u> <u>Theorem</u>

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is <u>satisfiable</u> if it is true in <u>some</u> model

e.g., $A \vee B$, C

A sentence is <u>unsatisfiable</u> if it is true in <u>no</u> models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Proof methods

Proof methods divide into (roughly) two kinds:

Model checking

truth table enumeration (sound and complete for propositional)

heuristic search in model space (sound but incomplete)

e.g., the GSAT algorithm (Ex. 6.15)

Application of inference rules

Legitimate (sound) generation of new sentences from old

Proof = a sequence of inference rule applications

Can use inference rules as operators in a standard search alg.

Inference Rules

- **Modus Ponens** or **Implication-Elimination**: (From an implication and the premise of the implication, you can infer the conclusion.)

$$\frac{\alpha \Rightarrow \beta, \qquad \alpha}{\beta}$$

Modus Tollens: $\dfrac{\alpha \Rightarrow \beta, \qquad \neg\beta}{\neg\alpha}$

- **And-Elimination**: (From a conjunction, you can infer any of the conjuncts.)

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

- **And-Introduction**: (From a list of sentences, you can infer their conjunction.)

$$\frac{\alpha_1, \ \alpha_2, \quad \ldots, \quad \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$

- **Or-Introduction**: (From a sentence, you can infer its disjunction with anything else at all.)

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$

Inference Rules

◇ **Double-Negation Elimination**: (From a doubly negated sentence, you can infer a positive sentence.)

$$\frac{\neg\neg\alpha}{\alpha}$$

◇ **Unit Resolution**: (From a disjunction, if one of the disjuncts is false, then you can infer the other one is true.)

$$\frac{\alpha \vee \beta, \qquad \neg\beta}{\alpha}$$

◇ **Resolution**: (This is the most difficult. Because β cannot be both true and false, one of the other disjuncts must be true in one of the premises. Or equivalently, implication is transitive.)

$$\frac{\alpha \vee \beta, \qquad \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or equivalently} \quad \frac{\neg\alpha \Rightarrow \beta, \qquad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

Inference example

Show that the hypotheses:

- It is not sunny this afternoon and it is colder than yesterday.
- We will go swimming only if it is sunny.
- If we do not go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

lead to the conclusion:

- We will be home by the sunset.

Inference example

Show that the hypotheses:

- It is not sunny this afternoon and it is colder than yesterday. $\neg s \wedge c$
- We will go swimming only if it is sunny. $w \rightarrow s$
- If we do not go swimming, then we will take a canoe trip. $\neg w \rightarrow t$
- If we take a canoe trip, then we will be home by sunset. $t \rightarrow h$

lead to the conclusion:

- We will be home by the sunset. h

Where:
s: "it is sunny this afternoon"
c: "it is colder than yesterday"
w: "we will go swimming"
t: "we will take a canoe trip.
h: "we will be home by the sunset."

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Step	Reason
1. $\neg s \wedge c$	hypothesis

Where:
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Step	Reason
1. $\neg s \wedge c$	hypothesis
2. $\neg s$	simplification

Where:
s: "it is sunny this afternoon"
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w: "we will go swimming"
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Show that the hypotheses:

- It is not sunny this afternoon and it is colder than yesterday. $\neg s \wedge c$
- We will go swimming only if it is sunny. $w \rightarrow s$
- If we do not go swimming, then we will take a canoe trip. $\neg w \rightarrow t$
- If we take a canoe trip, then we will be home by sunset. $t \rightarrow h$

lead to the conclusion:

- We will be home by the sunset. h

Step	Reason
1. $\neg s \wedge c$	hypothesis
2. $\neg s$	simplification
3. $w \rightarrow s$	hypothesis

Where:
s: "it is sunny this afternoon"
c: "it is colder than yesterday"
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Inference example

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Step	Reason
1. $\neg s \wedge c$	hypothesis
2. $\neg s$	simplification
3. $w \rightarrow s$	hypothesis
4. $\neg w$	modus tollens of 2 and 3

Where:
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3. $w \rightarrow s$	hypothesis
4. $\neg w$	modus tollens of 2 and 3
5. $\neg w \rightarrow t$	hypothesis

Where:
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Show that the hypotheses:

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lead to the conclusion:

- We will be home by the sunset. h

Step	Reason
1. $\neg s \wedge c$	hypothesis
2. $\neg s$	simplification
3. $w \to s$	hypothesis
4. $\neg w$	modus tollens of 2 and 3
5. $\neg w \to t$	hypothesis
6. t	modus ponens of 4 and 5

Where:
s: "it is sunny this afternoon"
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2. $\neg s$	simplification
3. $w \rightarrow s$	hypothesis
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5. $\neg w \rightarrow t$	hypothesis
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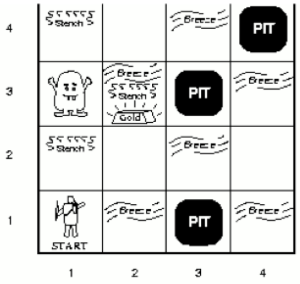
1. $\neg s \wedge c$	hypothesis
2. $\neg s$	simplification
3. $w \rightarrow s$	hypothesis
4. $\neg w$	modus tollens of 2 and 3
5. $\neg w \rightarrow t$	hypothesis
6. t	modus ponens of 4 and 5
7. $t \rightarrow h$	hypothesis
8. h	modus ponens of 6 and 7

Where:
s: "it is sunny this afternoon"
c: "it is colder than yesterday"
w: "we will go swimming"
t: "we will take a canoe trip.
h: "we will be home by the sunset."

Wumpus world: example

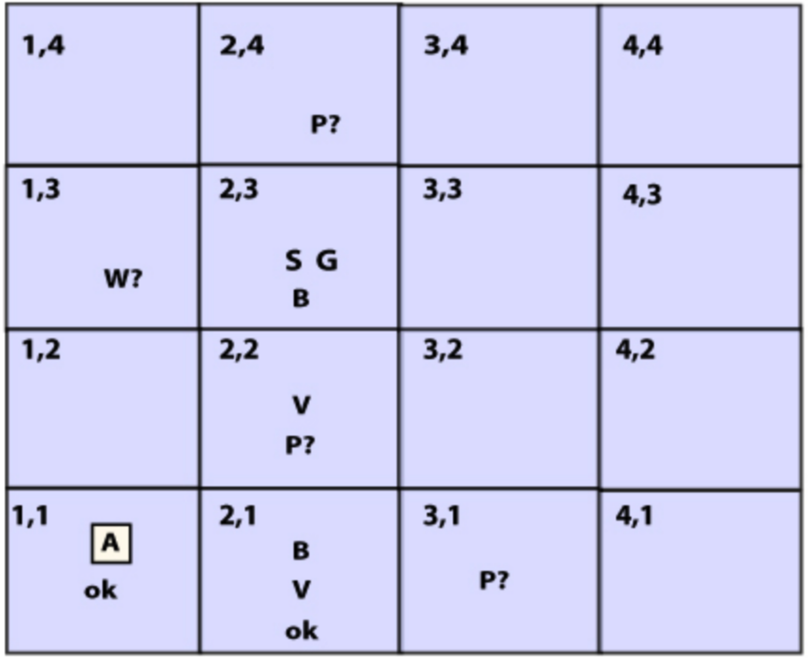


- **Facts:** Percepts inject (TELL) facts into the KB
  - [stench at 1,1 and 2,1] → S1,1 ; S2,1
- **Rules:** if square has no stench then neither the square or adjacent squares contain the wumpus
  - R1: $\neg S1,1 \Rightarrow \neg W1,1 \wedge \neg W1,2 \wedge \neg W2,1$
  - R2: $\neg S2,1 \Rightarrow \neg W1,1 \wedge \neg W2,1 \wedge \neg W2,2 \wedge \neg W3,1$
  - …
- **Inference:**
  - KB contains $\neg S1,1$ then using Modus Ponens we infer

    $\neg W1,1 \wedge \neg W1,2 \wedge \neg W2,1$
  - Using And-Elimination we get: $\neg W1,1 \quad \neg W1,2 \quad \neg W2,1$
  - …

Wumpus world proof example

The agent starts visiting from first square [1, 1], and we already know that this room is safe for the agent. To build a knowledge base for wumpus world, we will use some rules and atomic propositions. We need symbol [i, j] for each location in the wumpus world, where i is for the location of rows, and j for column location.



Wumpus world proof example

Atomic proposition variable for Wumpus world:

- Let $\mathbf{P}_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $\mathbf{B}_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $\mathbf{W}_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $\mathbf{S}_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $\mathbf{V}_{i,j}$ be true if that square[i, j] is visited.
- Let $\mathbf{G}_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $\mathbf{OK}_{i,j}$ be true if the room is safe.

Note: For a 4 * 4 square board, there will be 7*4*4= 122 propositional variables.

Wumpus world proof example

Some Propositional Rules for the wumpus world:

(R1) $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

(R2) $\neg S_{21} \rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$

(R3) $\neg S_{12} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{22} \wedge \neg W_{13}$

(R4) $S_{12} \rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$

Note: lack of variables gives us similar rules for each cell.

Wumpus world proof example

Representation of Knowledgebase for Wumpus world:

Following is the Simple KB for wumpus world when an agent moves from room [1, 1], to room [2,1]:

$\neg W_{11}$	$\neg S_{11}$	$\neg P_{11}$	$\neg B_{11}$	$\neg G_{11}$	V_{11}	OK_{11}
$\neg W_{12}$	----	$\neg P_{12}$	------	----	$\neg V_{12}$	OK_{12}
$\neg W_{21}$	$\neg S_{21}$	$\neg P_{21}$	B_{21}	$\neg G_{21}$	V_{21}	OK_{21}

Here in the first row, we have mentioned propositional variables for room[1,1], which is showing that room does not have wumpus($\neg W_{11}$), no stench (**$\neg S_{11}$**), no Pit(**$\neg P_{11}$**), no breeze(**$\neg B_{11}$**), no gold (**$\neg G_{11}$**), visited (**V_{11}**), and the room is Safe(**OK_{11}**).

In the second row, we have mentioned propositional variables for room [1,2], which is showing that there is no wumpus, stench and breeze are unknown as an agent has not visited room [1,2], no Pit, not visited yet, and the room is safe.

In the third row we have mentioned propositional variable for room[2,1], which is showing that there is no wumpus($\neg$ W21), no stench (**$\neg S_{21}$**), no Pit (**$\neg P_{21}$**), Perceives breeze(**B_{21}**), no glitter(**$\neg G_{21}$**), visited (**V_{21}**), and room is safe (**OK_{21}**).

https://www.javatpoint.com/ai-knowledge-base-for-wumpus-world

Wumpus world proof example

Prove that the Wumpus is in room (1,3) given this:

1,4	2,4 W? P?	3,4	4,4
1,3 W?	2,3 S G B	3,3 W?	4,3
1,2 S	2,2 W? V P?	3,2	4,2
1,1 A ok	2,1 B V ok	3,1 P?	4,1

https://www.javatpoint.com/ai-knowledge-base-for-wumpus-world

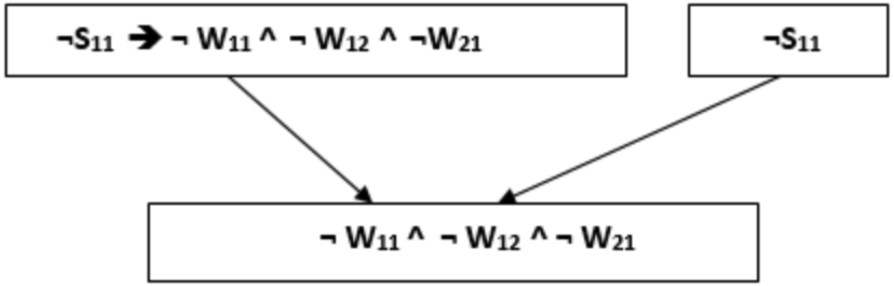
Wumpus world proof example

Prove that Wumpus is in the room (1, 3)

We can prove that wumpus is in the room (1, 3) using propositional rules which we have derived for the wumpus world and using inference rule.

- **Apply Modus Ponens with ¬S11 and R1:** (R1) $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

We will firstly apply MP rule with R1 which is $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, and $\mathbf{\neg S_{11}}$ which will give the output $\neg W_{11} \wedge W_{12} \wedge W_{12}$.



Wumpus world proof example

- **Apply And-Elimination Rule:**

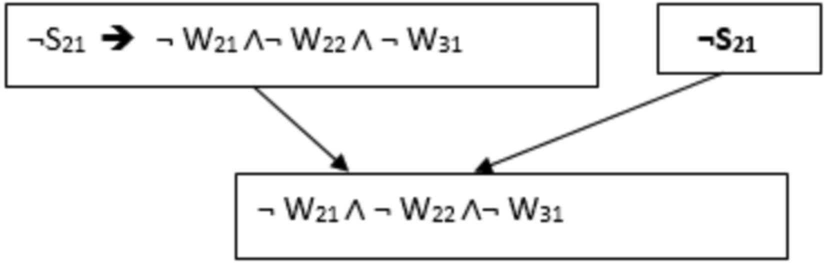
After applying And-elimination rule to $\neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, we will get three statements:

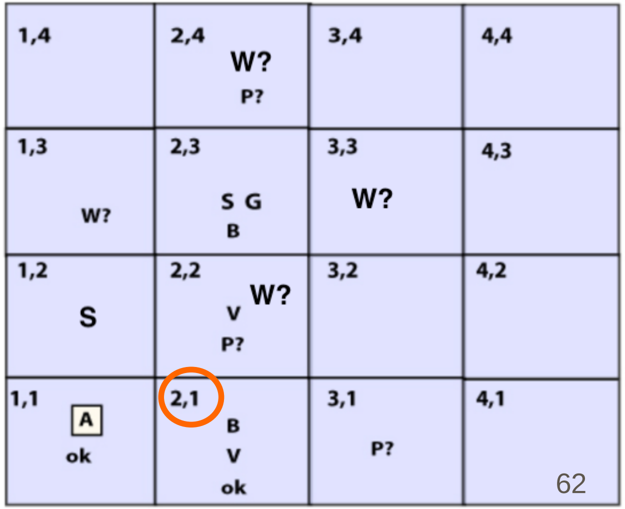
$\neg W_{11}$, $\neg W_{12}$, and $\neg W_{21}$.

Wumpus world proof example

- **Apply Modus Ponens to $\neg S_{21}$, and R2:**

Now we will apply Modus Ponens to $\neg S_{21}$ and R2 which is $\neg S_{21} \rightarrow \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$, which will give the Output as $\neg \mathbf{W_{21}} \wedge \neg \mathbf{W_{22}} \wedge \neg \mathbf{W_{31}}$





https://www.javatpoint.com/ai-knowledge-base-for-wumpus-world

Wumpus world proof example

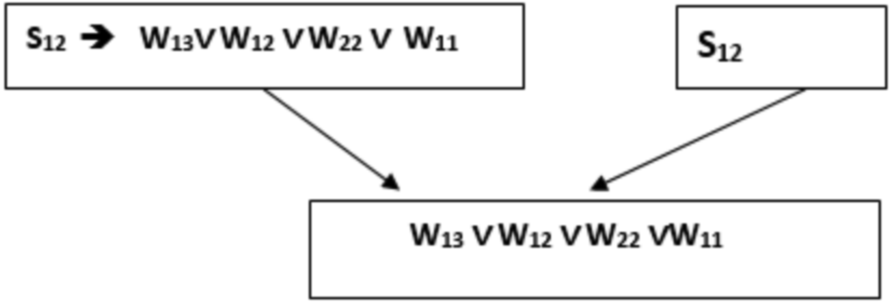
- **Apply And -Elimination rule:**

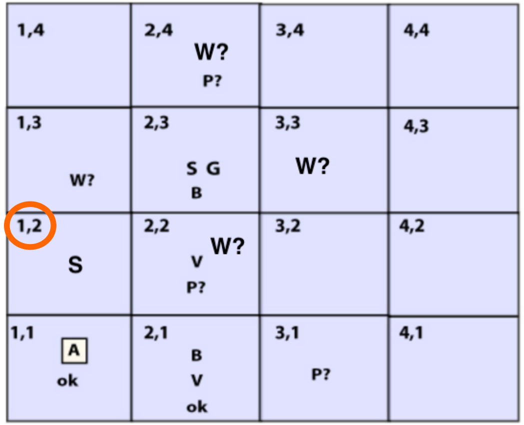
Now again apply And-elimination rule to $\neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$, We will get three statements:

$\neg W_{21}$, $\neg W_{22}$, and $\neg W_{31}$.

- **Apply MP to S_{12} and R4:**

Apply Modus Ponens to S_{12} and R_4 which is $S_{12} \rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$, we will get the output as $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$.



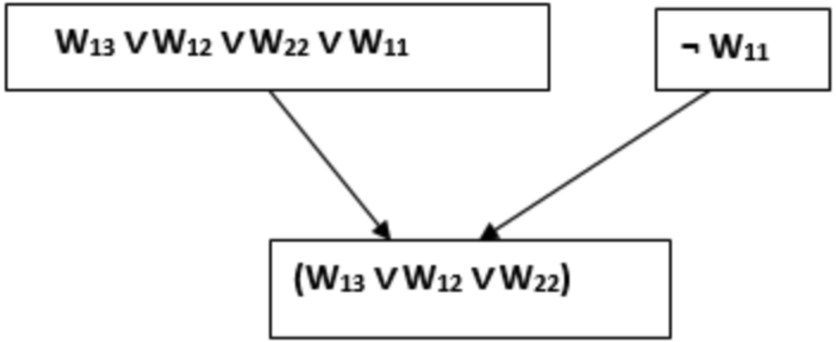


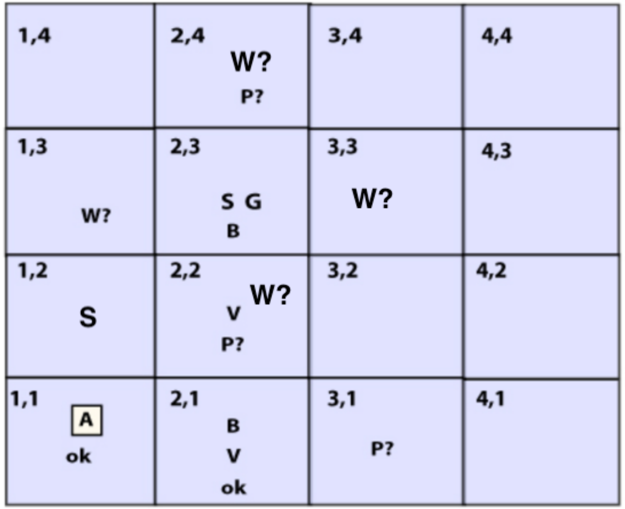
https://www.javatpoint.com/ai-knowledge-base-for-wumpus-world

Wumpus world proof example

- **Apply Unit resolution on $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$ and $\neg W_{11}$:**

After applying Unit resolution formula on $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$ and $\neg W_{11}$ we will get $W_{13} \vee W_{12} \vee W_{22}$.

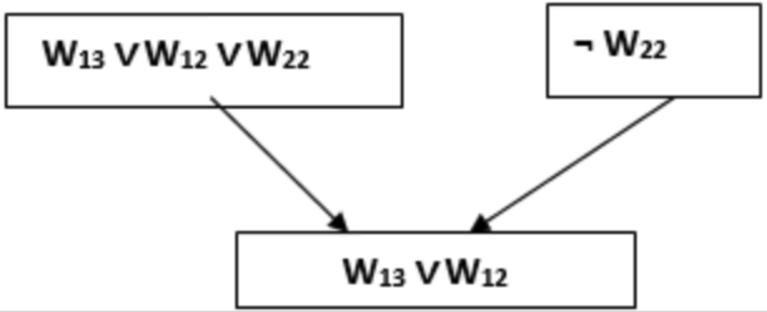


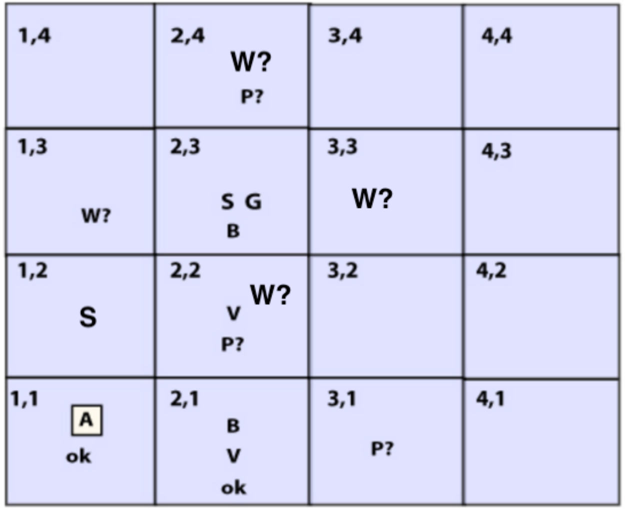


Wumpus world proof example

- **Apply Unit resolution on $W_{13} \vee W_{12} \vee W_{22}$ and $\neg W_{22}$:**

After applying Unit resolution on $W_{13} \vee W_{12} \vee W_{22}$, and $\neg W_{22}$, we will get $W_{13} \vee W_{12}$ as output.

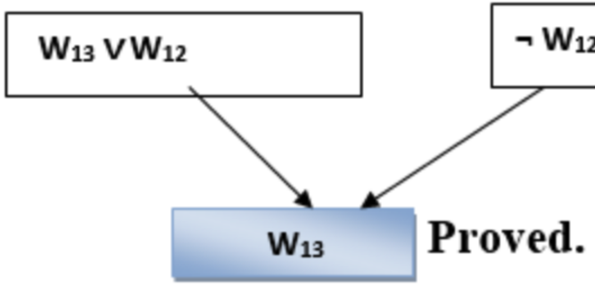




Wumpus world proof example

- **Apply Unit Resolution on $W_{13} \vee W_{12}$ and $\neg W_{12}$:**

After Applying Unit resolution on $W_{13} \vee W_{12}$ and $\neg W_{12}$, we will get W_{13} as an output, hence it is proved that the Wumpus is in the room [1, 3].



Limitations of Propositional Logic

1. It is too weak, i.e., has very limited expressiveness:

- Each rule has to be represented for each situation:
  e.g., "don't go forward if the wumpus is in front of you" takes 64 rules

2. It cannot keep track of changes:

- If one needs to track changes, e.g., where the agent has been before then we need a timed-version of each rule. To track 100 steps we'll then need 6400 rules for the previous example.

Its **hard to write and maintain** such a huge rule-base

Inference becomes intractable

Another example

http://people.cs.pitt.edu/~milos/courses/cs1571-Fall07/Lectures/Class13.pdf

Example. Inference rules approach.

KB: $P \wedge Q \quad P \Rightarrow R \quad (Q \wedge R) \Rightarrow S$ **Theorem:** S

1. $P \wedge Q$
2. $P \Rightarrow R$
3. $(Q \wedge R) \Rightarrow S$

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2. $P \Rightarrow R$
3. $(Q \wedge R) \Rightarrow S$
4. P **From 1 and And-elim**

$$\frac{A_1 \wedge A_2 \wedge \quad A_n}{A_i}$$

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1.	$P \wedge Q$	
2.	$P \Rightarrow R$	
3.	$(Q \wedge R) \Rightarrow S$	
4.	P	**From 1 and And-elim**
5.	R	**From 2,4 and Modus ponens**
6.	Q	**From 1 and And-elim**
7.	$(Q \wedge R)$	**From 5,6 and And-introduction**
8.	S	**From 7,3 and Modus ponens**

$\boxed{\textbf{Proved: } S}$

Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic suffices for some of these tasks

Truth table method is sound and complete for propositional logic